EDM kick-off meeting

Analytical, E&B-field related studies

Alexander J. Silenko

BLTP, Joint Institute for Nuclear Research, Dubna, Russia

Research Institute for Nuclear Problems, BSU, Minsk, Belarus

March 13-14, 2017, CERN, Geneva

OUTLINE

- **Analytical description of spin dynamics**
  - **noncontinuous and nonaligned fields**
  - **oscillating transverse fields**
- **Resonance EDM experiments with a rf electric-field flipper and a rf Wien filter**
- **Systematical errors in an experiment with the rf Wien filter**
- **Summary**

Analytical description of spin dynamics

noncontinuous and nonaligned fields

Is it possible to describe a spin motion without a calculation of beam dynamics?

Probably not, but some steps ahead can be made

First of all, a standard description of spin motion relative to detectors can be used, i.e., an equation of spin motion in the cylindrical or Frenet-Serret coordinates can be applied.

Equation of spin motion in the Cartesian coordinates:

$$\frac{d\zeta}{dt} = (\Omega_{T-BMT} + \Omega_{EDM}) \times \zeta,$$

$$\Omega_{T-BMT} = \frac{e}{m}\left[\left(G + \frac{1}{\gamma+1}\right)\beta \times \boldsymbol{E} - \left(G + \frac{1}{\gamma}\right)\mathrm{B} + \frac{G\gamma}{\gamma+1}(\beta \cdot \boldsymbol{B})\beta\right]$$

$$\Omega_{EDM} = -\frac{e\eta}{2m}\left[\boldsymbol{E} - \frac{\gamma}{\gamma+1}(\beta \cdot \boldsymbol{E})\beta + \beta \times \boldsymbol{B}\right], \quad \beta = \frac{\mathbf{v}}{c},$$

$G = (g-2)/2$, $\eta = 2mcd/(es)$, and d is the EDM.

Equation of spin motion in the cylindrical coordinates:

$$\boldsymbol{\Omega}^{(cyl)} = -\frac{e}{m}\left\{G\boldsymbol{B} - \frac{G\gamma}{\gamma+1}\boldsymbol{\beta}(\boldsymbol{\beta}\cdot\boldsymbol{B}) + \left(\frac{1}{\gamma^2-1} - G\right)(\boldsymbol{\beta}\times\boldsymbol{E}) + \frac{1}{\gamma}\left[\boldsymbol{B}_{\|} - \frac{1}{\beta^2}(\boldsymbol{\beta}\times\boldsymbol{E})_{\|}\right] + \frac{\eta}{2}\left(\boldsymbol{E} - \frac{\gamma}{\gamma+1}\boldsymbol{\beta}(\boldsymbol{\beta}\cdot\boldsymbol{E}) + \boldsymbol{\beta}\times\boldsymbol{B}\right)\right\}.$$

Equation of spin motion in the Frenet-Serret coordinates:

$$\boldsymbol{\Omega}^{(FS)} = -\frac{e}{m}\left[G\boldsymbol{B} - \frac{G\gamma}{\gamma+1}\boldsymbol{\beta}(\boldsymbol{\beta}\cdot\boldsymbol{B}) + \left(\frac{1}{\gamma^2-1} - G\right)(\boldsymbol{\beta}\times\boldsymbol{E}) + \frac{\eta}{2}\left(\boldsymbol{E} - \frac{\gamma}{\gamma+1}\boldsymbol{\beta}(\boldsymbol{\beta}\cdot\boldsymbol{E}) + \boldsymbol{\beta}\times\boldsymbol{B}\right)\right].$$

PHYSICAL REVIEW SPECIAL TOPICS - ACCELERATORS AND BEAMS
9, 034003 (2006)

Equation of spin motion in storage rings in the cylindrical coordinate system

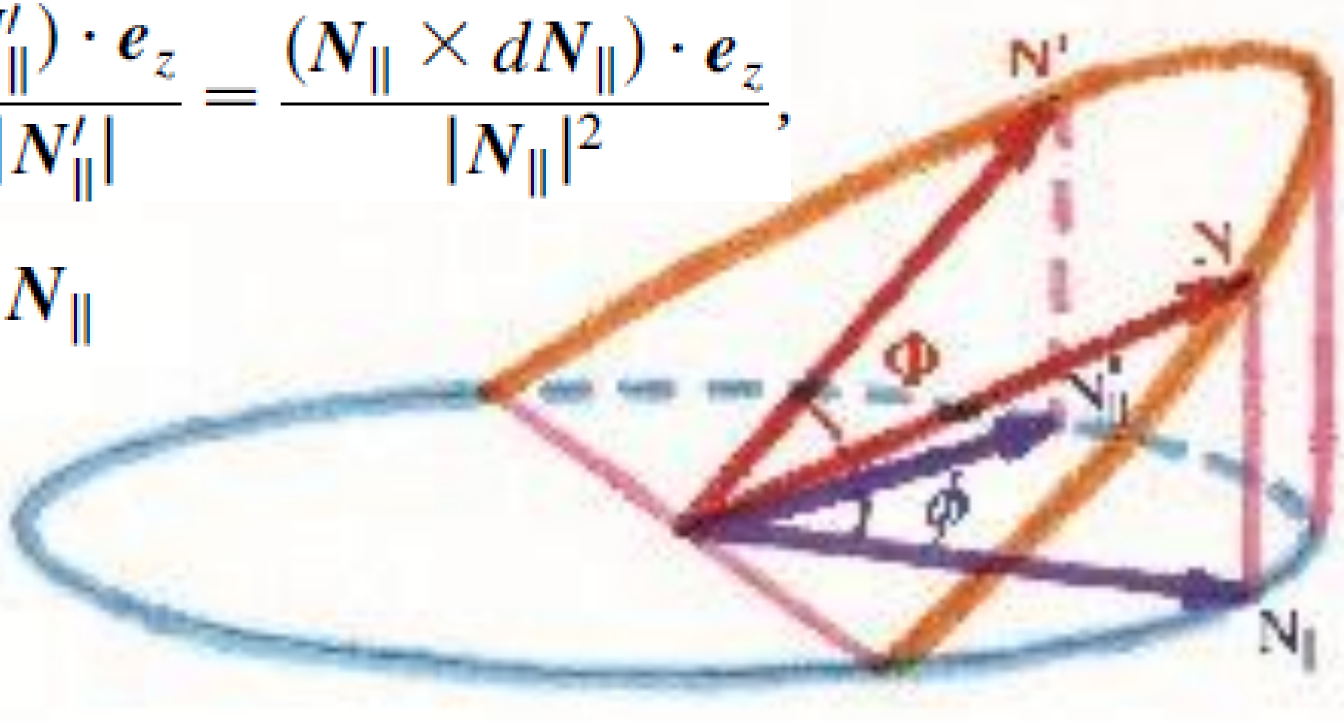
Alexander J. Silenko
Institute of Nuclear Problems, Belarusian State University, 220080 Minsk, Belarus
(Received 23 January 2004; revised manuscript received 14 December 2004; published 29 March 2006)

We can derive the equation of spin motion in storage rings in the cylindrical coordinate system with allowance for field misalignments and imperfections and also for fields which are orthogonal to main ones. Besides this, we can consider spin motion in periodical external fields and can solve the spin motion equations.

The (pseudo)-vector of angular velocity (i.e., stable spin axis) becomes tilted. The instantaneous plane of particle motion does not coincide with the horizontal plane, and the instantaneous plane of rotation of vector $\boldsymbol{N}=\boldsymbol{p}/p$ *is not* horizontal. The angle Φ between two positions of the rotating vector $\boldsymbol{N}$ in the tilted plane is not equal to the angle ϕ between two corresponding horizontal projections. Therefore, the instantaneous angular velocity of particle motion is changed. The infinitesimal angle of particle rotation in the xy *plane,* $d\phi$, is given by

$$d\phi = \frac{(\boldsymbol{N}_{\|} \times \boldsymbol{N}'_{\|}) \cdot \boldsymbol{e}_z}{|\boldsymbol{N}_{\|}| \cdot |\boldsymbol{N}'_{\|}|} = \frac{(\boldsymbol{N}_{\|} \times d\boldsymbol{N}_{\|}) \cdot \boldsymbol{e}_z}{|\boldsymbol{N}_{\|}|^2},$$

$$d\boldsymbol{N}_{\|} = \boldsymbol{N}'_{\|} - \boldsymbol{N}_{\|}$$



$$\dot{\phi} \equiv \frac{d\phi}{dt} = \frac{(\boldsymbol{N}_{\|} \times \dot{\boldsymbol{N}}_{\|}) \cdot \boldsymbol{e}_z}{|\boldsymbol{N}_{\|}|^2} = \omega_z - o,$$

$$o = \frac{(\omega_x N_x + \omega_y N_y) N_z}{1 - N_z^2} = \frac{(\omega_\rho N_\rho + \omega_\phi N_\phi) N_z}{1 - N_z^2}.$$

As a rule, *o* **is a rather small correction. These equations are exact.**

$$\dot{\phi} = \omega_z = -\frac{e}{\gamma m}\left(B_z - \frac{(\boldsymbol{N} \times \boldsymbol{E})_z}{\beta}\right).$$

The instantaneous angular velocity of spin rotation in the horizontal plane, $\dot{\psi}$**, is characterized by the change of angle** ψ **determining the spin orientation in this plane.**

$$\dot{\psi} \equiv \frac{d\psi}{dt} = \frac{(\xi_{\|} \times \dot{\xi}_{\|}) \cdot e_z}{|\xi_{\|}|^2} = (\omega_a)_z - O, \qquad \xi = s/s$$

$$O = \frac{[(\omega_a)_x \xi_x + (\omega_a)_y \xi_y]\xi_z}{1 - \xi_z^2} = \frac{[(\omega_a)_\rho \xi_\rho + (\omega_a)_\phi \xi_\phi]\xi_z}{1 - \xi_z^2}$$

As a rule, O **is a rather small correction. These equations are exact.**

Analytical description of spin dynamics

oscillating transverse fields

General classical and quantum-mechanical description of magnetic resonance: An application to electric-dipole-moment experiments

Alexander J. Silenko

Research Institute for Nuclear Problems,

Belarusian State University, Minsk 220030, Belarus and

Bogoliubov Laboratory of Theoretical Physics,

Joint Institute for Nuclear Research, Dubna 141980, Russia

(Dated: December 30, 2016)

Abstract

A general theoretical description of a magnetic resonance is presented. This description is necessary for a detailed analysis of spin dynamics in electric-dipole-moment experiments in storage rings. General formulas describing a behavior of all components of the polarization vector at the magnetic resonance are obtained for an arbitrary initial polarization. These formulas are exact on condition that the nonresonance rotating field is neglected. The spin dynamics is also calculated at frequencies far from resonance with allowance for both rotating fields. A general quantum-mechanical analysis of the spin evolution at the magnetic resonance is fulfilled and the full agreement between the classical and quantum-mechanical approaches is shown. Quasimagnetic resonances for particles and nuclei moving in noncontinuous perturbing fields of accelerators and

Constant vertical magnetic field and oscillating horizontal magnetic field

$$\mathbf{B}_0 = B_0 \mathbf{e}_z, \qquad \mathbf{B}_{||} = \boldsymbol{\mathcal{B}} \cos(\omega t + \chi).$$

Spin-dependent part of the classical Hamiltonian is given by

$$H = \omega_0 \cdot \zeta + 2\mathfrak{C} \cdot \zeta \cos(\omega t + \chi), \quad \omega_0 = -\frac{g_N \mu_N}{\hbar} B_0,$$

$$\mathfrak{C} = -\frac{g_N \mu_N}{2\hbar} \boldsymbol{\mathcal{B}},$$

For particles, $g_N \mu_N$ should be replaced with $eg\hbar/(2m)$,

where $g = 2mc\mu/(es)$.

The amplitudes of the rotating magnetic fields are equal to $\boldsymbol{\mathcal{B}}/2$.

Let us direct $\mathfrak{E}$ along the x axis: $\mathfrak{E} = \mathfrak{E}\boldsymbol{e}_x$.

This direction is not important in the considered case.

In the rotating frame, the total angular velocity of the spin rotation is equal to

$$\Omega = \omega_0 - \omega + \mathfrak{E}, \quad \Omega = \sqrt{(\omega_0 - \omega)^2 + \mathfrak{E}^2}.$$

We can derive exact formulas for spin dynamics in the rotating frame and then pass to the lab one.

When the initial spin direction is defined by the spherical angles θ **and** ψ**,**

$$P_x(0) = \sin\theta\cos\psi, \quad P_y(0) = \sin\theta\sin\psi, \quad P_z(0) = \cos\theta,$$

the final result is given by

$$
\begin{aligned}
P_x(t) = {} & \cos \Omega t \sin \theta \cos (\omega t + \psi) + \frac{\mathfrak{E}^2}{\Omega^2}(1 - \cos \Omega t) \sin \theta \cos (\psi - \chi) \cos (\omega t + \chi) \\
& - \frac{\omega_0 - \omega}{\Omega} \sin \Omega t \sin \theta \sin (\omega t + \psi) \\
& + \frac{\mathfrak{E}}{\Omega} \left[\frac{\omega_0 - \omega}{\Omega} (1 - \cos \Omega t) \cos (\omega t + \chi) + \sin \Omega t \sin (\omega t + \chi) \right] \cos \theta, \\
P_y(t) = {} & \frac{\omega_0 - \omega}{\Omega} \sin \Omega t \sin \theta \cos (\omega t + \psi) + \cos \Omega t \sin \theta \sin (\omega t + \psi) \\
& + \frac{\mathfrak{E}^2}{\Omega^2} (1 - \cos \Omega t) \sin \theta \cos (\psi - \chi) \sin (\omega t + \chi) \\
& + \frac{\mathfrak{E}}{\Omega} \left[\frac{\omega_0 - \omega}{\Omega} (1 - \cos \Omega t) \sin (\omega t + \chi) - \sin \Omega t \cos (\omega t + \chi) \right] \cos \theta, \\
P_z(t) = {} & \frac{(\omega_0 - \omega)\mathfrak{E}}{\Omega^2} (1 - \cos \Omega t) \sin \theta \cos (\psi - \chi) + \frac{\mathfrak{E}}{\Omega} \sin \Omega t \sin \theta \sin (\psi - \chi) \\
& + \left[1 - \frac{\mathfrak{E}^2}{\Omega^2} (1 - \cos \Omega t) \right] \cos \theta.
\end{aligned}
$$

Both the vertical and horizontal initial polarizations are useful

At frequencies far from resonance, effects of two magnetic fields rotating in opposite directions are comparable

$$P_x(t) = \sin\theta\cos(\omega_0 t + \psi) + \left\{ \frac{\mathfrak{E}}{\omega_0 + \omega}\left[\cos(\omega t + \chi) - \cos(\omega_0 t - \chi)\right] \right.$$

$$\left. + \frac{\mathfrak{E}}{\omega_0 - \omega}\left[\cos(\omega t + \chi) - \cos(\omega_0 t + \chi)\right] \right\} \cos\theta,$$

$$P_y(t) = \sin\theta\sin(\omega_0 t + \psi) + \left\{ -\frac{\mathfrak{E}}{\omega_0 + \omega}\left[\sin(\omega t + \chi) + \sin(\omega_0 t - \chi)\right] \right.$$

$$\left. + \frac{\mathfrak{E}}{\omega_0 - \omega}\left[\sin(\omega t + \chi) - \sin(\omega_0 t + \chi)\right] \right\} \cos\theta,$$

$$P_z(t) = \cos\theta + \left(\frac{\mathfrak{E}}{\omega_0 + \omega}\left\{\cos(\psi + \chi) - \cos\left[(\omega_0 + \omega)t + \psi + \chi\right]\right\} \right.$$

$$\left. + \frac{\mathfrak{E}}{\omega_0 - \omega}\left\{\cos(\psi - \chi) - \cos\left[(\omega_0 - \omega)t + \psi - \chi\right]\right\} \right) \sin\theta.$$

We have also given the quantum-mechanical description of the magnetic resonance

The transformation (22) brings the equation for the matrix Hamiltonian to the form

$$i\frac{dC(t)}{dt} = \frac{1}{2}\begin{pmatrix} \omega_0 - \omega & \mathfrak{G}\exp\left[i(\omega t + \chi)\right] \\ \mathfrak{G}\exp\left[-i(\omega t + \chi)\right] & -\omega_0 + \omega \end{pmatrix} C(t). \tag{24}$$

The terms in Eq. (24) oscillating with the angular frequency 2ω can be neglected and this equation takes the form

$$i\frac{dC(t)}{dt} = \frac{1}{2}\begin{pmatrix} \omega_0 - \omega & \mathfrak{E} \\ \mathfrak{E} & -\omega_0 + \omega \end{pmatrix} C(t). \tag{25}$$

The solution of Eq. (25) is given by

$$\begin{aligned} C_+(t) &= \left(\cos\frac{\Omega t}{2} - i\frac{\omega_0 - \omega}{\Omega}\sin\frac{\Omega t}{2}\right)C_+(0) - i\frac{\mathfrak{E}}{\Omega}\sin\frac{\Omega t}{2}C_-(0), \\ C_-(t) &= -i\frac{\mathfrak{E}}{\Omega}\sin\frac{\Omega t}{2}C_+(0) + \left(\cos\frac{\Omega t}{2} + i\frac{\omega_0 - \omega}{\Omega}\sin\frac{\Omega t}{2}\right)C_-(0), \end{aligned} \tag{26}$$

where Ω is defined by Eq. (3).

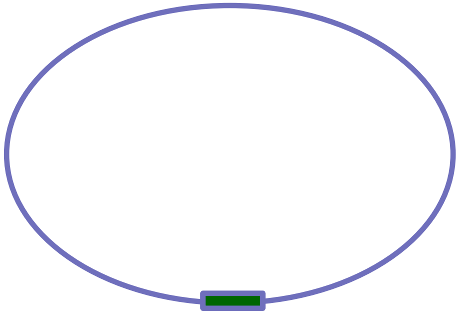
If we use Eq. (26) for a derivation of $P'_i(t)$ in terms of $P'_i(0)$ $(i = x, y, z)$, we come to Eq. (7). This fact clearly demonstrates the full agreement of results obtained by the classical

Resonance EDM experiments with a rf electric-field flipper and a rf Wien filter

A distinguishing feature of storage ring EDM experiments is a discontinuity of perturbing fields

$l \ll C$

l **is the rf flipper/filter length,**
C **is the circumference**



rf flipper/filter

Since lengths of the flipper and the filter are small as compared with the ring circumference, an approximation of the noncontinuous perturbing field by the delta function is permissible. An expansion of the delta function into the Fourier series is defined by

$$\sum_{n=-\infty}^{\infty} \delta(\Phi - 2\pi n) = \frac{1}{2\pi} + \frac{1}{\pi}\sum_{n=1}^{\infty} \cos(n\Phi) = \frac{1}{2\pi}\sum_{n=-\infty}^{\infty} \cos(n\Phi),$$

where $\Phi = \omega_c t$ is the phase.

$$\sin(\omega' t + \chi)\sum_{n=-\infty}^{\infty} \delta(\Phi - 2\pi n) = \frac{1}{2\pi}\sum_{n=-\infty}^{\infty} \sin[(\omega' + n\omega_c)t + \chi] = \frac{1}{2\pi}\sum_{n=-\infty}^{\infty} \sin[(n+\nu)\Phi + \chi],$$

$$\cos(\omega' t + \chi)\sum_{n=-\infty}^{\infty} \delta(\Phi - 2\pi n) = \frac{1}{2\pi}\sum_{n=-\infty}^{\infty} \cos[(\omega' + n\omega_c)t + \chi] = \frac{1}{2\pi}\sum_{n=-\infty}^{\infty} \cos[(n+\nu)\Phi + \chi],$$

where $\nu = \omega'/\omega_c$ is the modulation tune.

More adequately, the electric fields of the flipper and the filter can be defined as oscillating fields on the finite length *l*:

$$\mathcal{E}(\Phi) = \begin{cases} \boldsymbol{E}_0 & \text{if} \quad \Phi \in \left[-\dfrac{\pi l}{C} + 2\pi n, \dfrac{\pi l}{C} + 2\pi n\right] \\ 0 & \text{if} \quad \Phi \notin \left[-\dfrac{\pi l}{C} + 2\pi n, \dfrac{\pi l}{C} + 2\pi n\right] \end{cases}$$

$$n = 0, \pm 1, \pm 2, \ldots$$

In this case, an expansion into the Fourier series has the form

$$\mathcal{E}(\Phi) = \boldsymbol{E}_0 \sum_{n=-\infty}^{\infty} a_n \cos(n\Phi), \quad a_0 = \frac{l}{C}, \quad a_n = \frac{1}{\pi n} \sin \frac{\pi n l}{C}.$$

Angular velocity of the spin rotation is given by

$$\Omega_{\|} = -\frac{e\eta}{2m} \boldsymbol{E}_0 \sum_{n=-\infty}^{\infty} a_n \cos\left[(\omega' + n\omega_c)t + \chi\right] = -\frac{e\eta}{2m} \boldsymbol{E}_0 \sum_{n=-\infty}^{\infty} a_n \cos\left[(n + \nu)\Phi + \chi\right].$$

Important distinguishing features of storage ring EDM experiments are also a simultaneous influence of external fields on the electric and magnetic dipole moments and the existence of a resonance effect even when the stimulating torque acting *on the EDM* is equal to zero.

For a more precise Fourier expansion, one can use real parameters of the resonator fields.

1. ``Resonance" stimulated by the oscillating vertical magnetic field in the storage ring with the main magnetic field ***B₀***

$$\mathcal{B}(\Phi)\cos\left(\omega' t+\chi\right)=B_0^{(osc)}\sum_{n=-\infty}^{\infty}a_n\cos\left[(\omega'+n\omega_c)t+\chi\right]$$

The angular velocity of the spin rotation in the cylindrical coordinates is given by **($\boldsymbol{\beta}=\beta\boldsymbol{e}_\phi$)**

$$\boldsymbol{\Omega}^{(cyl)}=\omega_0\left[1+b_z\cos\left(\omega t+\chi\right)\right]\boldsymbol{e}_z-\frac{e\eta}{2m}\beta B_0\left[1+b_r\cos\left(\omega t+\chi\right)\right]\boldsymbol{e}_r,$$

$$\omega_0=-\frac{eG}{m}B_0.$$

In the considered case,

$$b_r=b_r^{(m)}=b_z=b_z^{(m)}=\frac{a_nB_0^{(osc)}}{B_0}.$$

Evidently, the constant and oscillating parts of the vector $\mathbf{\Omega}^{(cyl)}$ are collinear.
This vector forms the small angle with the z axis

$$\vartheta = \sin\vartheta = -\frac{e\eta}{2m\omega_0}\beta B_0 = \frac{\eta\beta}{2G}$$

Any resonance effect does not exist:

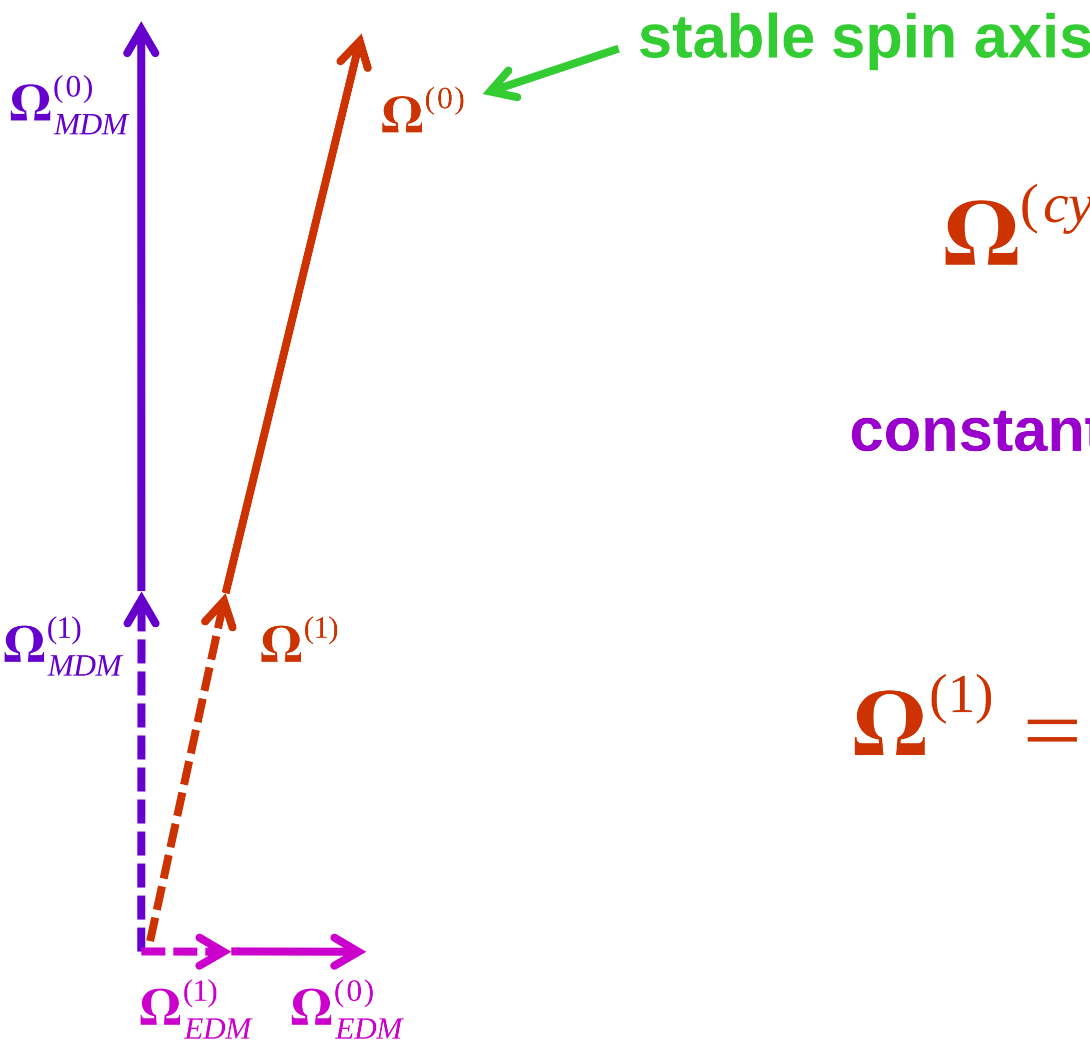
$$\mathbf{\Omega}^{(cyl)} = \omega_0\left[1 + b_z\cos\left(\omega t + \chi\right)\right]\boldsymbol{e}_\vartheta, \qquad \boldsymbol{e}_\vartheta = \boldsymbol{e}_z + \vartheta\boldsymbol{e}_r.$$

$$\mathbf{\Omega}^{(cyl)} = \mathbf{\Omega}^{(0)} + \mathbf{\Omega}^{(1)}$$

constant part **oscillating part**

$$\mathbf{\Omega}^{(0)} = \omega_0\mathbf{e}_\vartheta, \qquad \mathbf{\Omega}^{(1)} = \omega_0 b_z\cos\left(\omega t + \chi\right)\mathbf{e}_\vartheta.$$

Figure 1. Magnetic-field ``flipper”.



$$\mathbf{\Omega}^{(cyl)} = \mathbf{\Omega}^{(0)} + \mathbf{\Omega}^{(1)}$$

constant part **oscillating part**

$$\mathbf{\Omega}^{(0)} = \omega_0 \mathbf{e}_{\vartheta},$$

$$\mathbf{\Omega}^{(1)} = \omega_0 b_z \cos\left(\omega t + \chi\right) \mathbf{e}_{\vartheta}.$$

2. Resonance stimulated by the oscillating radial electric field in the storage ring with the main magnetic field B_0

$$\boldsymbol{E}_0 = E_0 \boldsymbol{e}_r, \qquad b_r = b_r^{(e)} = \frac{a_n E_0}{\beta B_0}, \qquad b_z = b_z^{(e)} = -\frac{\beta a_n E_0}{G B_0}\left(\frac{1}{\gamma^2 - 1} - G\right)$$

It is very convenient to switch to the new axes, $\boldsymbol{e}_\zeta = \boldsymbol{e}_r - \vartheta \boldsymbol{e}_z$, $\boldsymbol{e}_\phi$ and $\boldsymbol{e}_\vartheta$

$$\boldsymbol{\Omega}^{(cyl)} = \omega_0 \left[1 + b_z \cos(\omega t + \chi)\right] \boldsymbol{e}_\vartheta + \omega_0 \vartheta \left(b_r - b_z\right) \cos(\omega t + \chi) \boldsymbol{e}_\zeta.$$

$$b_r^{(e)} - b_z^{(e)} = \frac{a_n E_0}{\beta B_0} \cdot \frac{G+1}{G\gamma^2} = -\frac{e g a_n E_0}{2m\beta\gamma^2\omega_0}.$$

The horizontal spin polarization at the initial vertical spin direction is given by

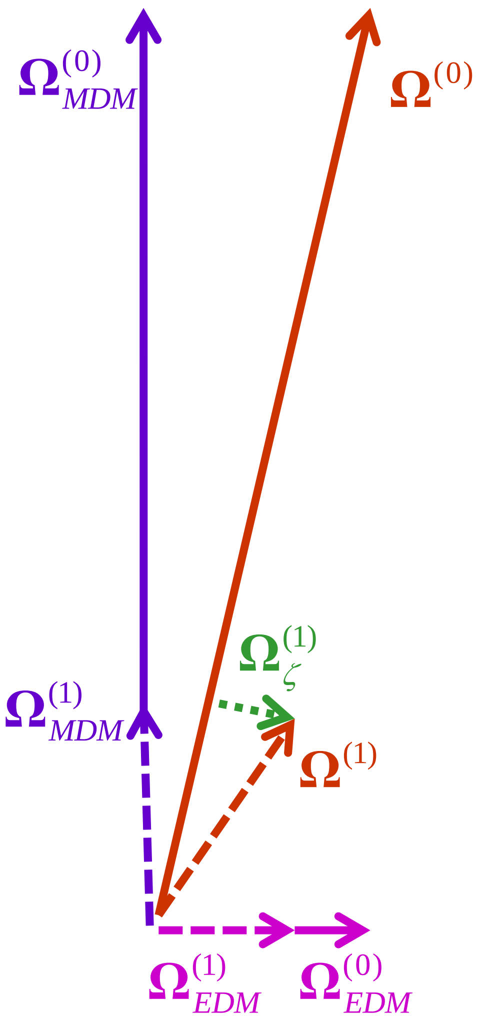
$$P_x(t) = \mathfrak{E} t \sin(\omega t + \chi),$$

$$P_y(t) = -\mathfrak{E} t \cos(\omega t + \chi)$$

$$\mathfrak{E} = \frac{1}{2}\omega_0 \vartheta \left(b_r^{(e)} - b_z^{(e)}\right) = -\frac{e\eta}{4m} \cdot \frac{G+1}{G\gamma^2} a_n E_0.$$

Figure 2. Electric-field flipper.

stable spin axis



For the E-field flipper,

$$\mathbf{\Omega}^{(cyl)} = \mathbf{\Omega}^{(0)} + \mathbf{\Omega}^{(1)},$$

$$\mathbf{\Omega}^{(0)} = \omega_0 \mathbf{e}_{\vartheta},$$

$$\mathbf{\Omega}^{(1)} = \omega_0 b_z^{(e)} \cos\left(\omega t + \chi\right) \mathbf{e}_{\vartheta}$$

$$+ \underbrace{\omega_0 \vartheta \left(b_r^{(e)} - b_z^{(e)}\right) \cos\left(\omega t + \chi\right) \mathbf{e}_{\zeta}}_{\parallel \atop \mathbf{\Omega}_{\zeta}^{(1)}}.$$

3. Resonance stimulated by the rf Wien filter in the storage ring with the main magnetic field ***B₀***

There is not any oscillating force acting on a particle:

$$B_0^{(osc)} + \beta E_0 = 0.$$

Oscillating fields should be synchronized:

$$\mathbf{E} = \mathbf{E}_0 \cos(\omega t + \chi), \qquad \mathbf{B}^{(osc)} = \mathbf{B}_0^{(osc)} \cos(\omega t + \chi).$$

The angular velocity of the spin rotation takes the form

$$\Omega^{(cyl)} = \omega_0 \left[1 + \left(b_z^{(e)} + b_z^{(m)}\right) \cos(\omega t + \chi)\right] e_z + \underline{\omega_0 \vartheta e_r},$$

There is not a resonance field acting on the EDM!

However, $\boldsymbol{e}_\vartheta$ and $\boldsymbol{e}_z$ are not collinear

To determine the resonance effect, it is convenient to pass to the axes $\mathbf{e}_\vartheta$ and $\mathbf{e}_\zeta$:

$$\mathbf{e}_\vartheta = \mathbf{e}_z + \vartheta \mathbf{e}_r, \qquad \mathbf{e}_\zeta = \mathbf{e}_r - \vartheta \mathbf{e}_z.$$

In this case,

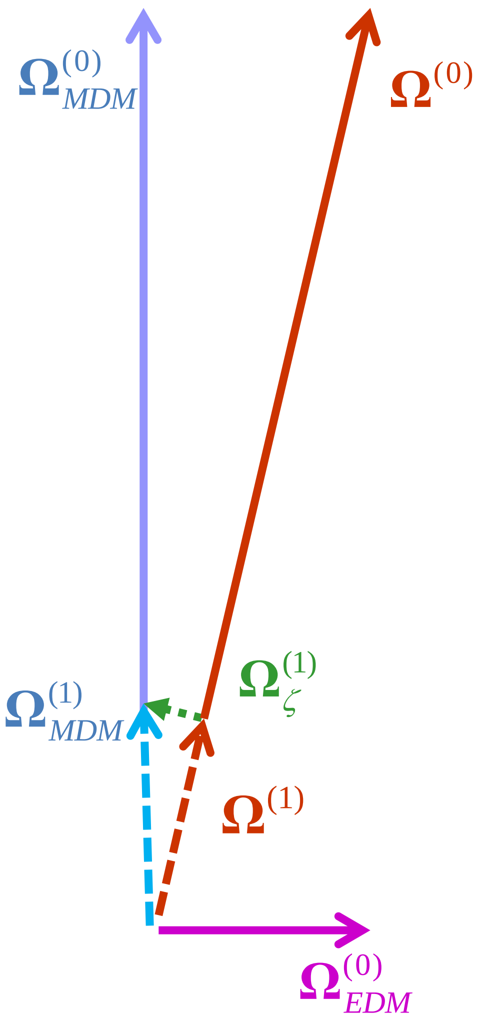
$$\Omega^{(cyl)} = \omega_0 \left[1 + \delta \cos\left(\omega t + \chi\right)\right] e_\vartheta - \omega_0 \vartheta \delta \cos\left(\omega t + \chi\right) e_\zeta,$$

where

$$\delta = b_z^{(e)} + b_z^{(m)} = -b_r^{(e)} + b_z^{(e)} = -\frac{a_n E_0}{\beta B_0} \cdot \frac{G+1}{G\gamma^2}.$$

The resonance EDM effect is provided by the oscillating torque acting on the MDM

Figure 3. rf Wien filter.



stable spin axis

For the rf Wien filter,

$$\mathbf{\Omega}^{(cyl)} = \mathbf{\Omega}^{(0)} + \mathbf{\Omega}^{(1)},$$

$$\mathbf{\Omega}^{(0)} = \omega_0 \mathbf{e}_{\vartheta},$$

$$\mathbf{\Omega}^{(1)} = -\omega_0 \left(b_r^{(e)} - b_z^{(e)} \right) \cos\left(\omega t + \chi \right) \mathbf{e}_{\vartheta}$$

$$\underbrace{+\omega_0 \vartheta \left(b_r^{(e)} - b_z^{(e)} \right) \cos\left(\omega t + \chi \right) \mathbf{e}_{\zeta}}_{= \mathbf{\Omega}_{\zeta}^{(1)}}.$$

It can be checked that

$$b_r^{(e)} - b_z^{(e)} = \frac{a_n E_0}{\beta B_0} \cdot \frac{G+1}{G\gamma^2} = -\frac{ega_n E_0}{2m\beta\gamma^2\omega_0}.$$

If b_z<<1, the general equations obtained for the magnetic resonance can be used. In this case,

$$\mathfrak{E} = \frac{1}{2}\omega_0\vartheta\left(b_r^{(e)} - b_z^{(e)}\right) = -\frac{e\eta}{4m} \cdot \frac{G+1}{G\gamma^2} a_n E_0.$$

When the initial spin direction is horizontal,

$$P_z(t) = \mathfrak{E}t\sin\left(\psi - \chi\right) = -\frac{e\eta}{4m} \cdot \frac{G+1}{G\gamma^2} a_n E_0 t \sin\left(\psi - \chi\right).$$

This equation agrees with previous results.

$\sin(\psi-\chi)=1$ in the experiment planned

The more precise derivation of the beam polarization along the $\mathbf{e}_\vartheta$ axis during one spin revolution:

$$\Delta P_\vartheta = -\omega_0 \vartheta \delta \int_{-T/2}^{T/2} \cos\left(\omega_0 t + \delta \sin \omega_0 t\right) \cos \omega_0 t \, dt,$$

We can apply properties of the Bessel functions (*n* is integer):

$$J_n(z) = \frac{1}{\pi} \int_0^\pi \cos\left(nx - z \sin x\right) dx, \qquad J_{-n}(z) = (-1)^n J_n(z).$$

We obtain the following *exact* formula:

$$\Delta P_\vartheta = \pi \vartheta [J_0(|\delta|) + J_2(|\delta|)] \delta.$$

When $\delta \ll 1$, $J_0(\delta)+J_2(\delta) \approx 1 - \delta^2/8$.

As a result, the average build-up of the vertical spin polarization is given by

$$P_z(t) = -\frac{\omega_0 \vartheta \delta}{2} [J_0(|\delta|) + J_2(|\delta|)] t = -\frac{e\eta}{4m} \cdot \frac{G+1}{G\gamma^2} a_n E_0 [J_0(|\delta|) + J_2(|\delta|)] t.$$

Systematical errors in an experiment with the rf Wien filter

The vertical electric field and the radial and longitudinal magnetic fields may create a resonance effect imitating the presence of the EDM. This effect can take place due to misalignments and imperfections of the oscillating fields in the rf Wien filter. Similarly directed constant imperfection fields can also exist in the storage ring. However, they do not create any resonance effect.

PHYSICAL REVIEW SPECIAL TOPICS - ACCELERATORS AND BEAMS **16**, 114001 (2013)

rf Wien filter in an electric dipole moment storage ring: The "partially frozen spin" effect

William M. Morse,[1] Yuri F. Orlov,[2] and Yannis K. Semertzidis[1,*]

[1]*Brookhaven National Laboratory, Upton, New York 11973-5000, USA*

[2]*Cornell University, Ithaca, New York 14853, USA*

(Received 15 April 2013; published 27 November 2013)

An rf Wien filter (WF) can be used in a storage ring to measure a particle's electric dipole moment (EDM). If the WF frequency equals the spin precession frequency without WF, and the oscillating WF fields are chosen so that the corresponding transverse Lorentz force equals zero, then a large source of systematic errors is canceled but the EDM signal is not. This effect, discovered by simulation, can be called the "partially frozen spin" effect.

To decrease systematic errors, it is necessary to avoid any dependence of the particle motion on the fields of the rf Wien filter. This means canceling the Lorenz force in both radial and vertical directions. We obtain the formula

$$\left(\mathbf{E}_0 + \boldsymbol{\beta} \times \mathbf{B}_0^{(osc)}\right)\Big|_z = 0.$$

In general,

$$\mathbf{B}_0^{(osc)} = \mathfrak{B}^{(r)} + \mathfrak{B}^{(\phi)} + \mathfrak{B}^{(z)} = \mathfrak{B}_0^{(r)}\mathbf{e}_r + \mathfrak{B}_0^{(\phi)}\mathbf{e}_\phi + B_0^{(osc)}\mathbf{e}_z.$$

For the vertical electric and the radial magnetic fields, we obtain the formula

$$\Omega_{\parallel} = -\frac{e}{m} \cdot \frac{G+1}{\gamma^2} a_n \mathfrak{B}^{(r)}, \qquad \mathfrak{E} = -\frac{e}{2m} \cdot \frac{G+1}{\gamma^2} a_n \mathfrak{B}_0^{(r)}.$$

This defines the corresponding systematic error.

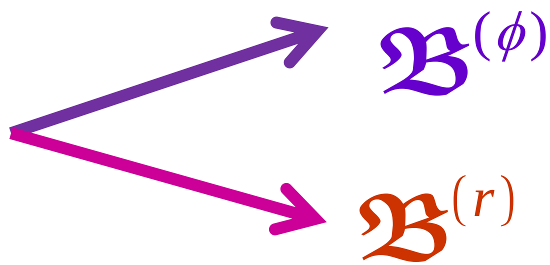
In agreement with Morse, Orlov, and Semertzidis work

The longitudinal magnetic field causes the resonance effect defined by

$$\Omega_{\|} = -\frac{e}{m} \cdot \frac{G+1}{\gamma} a_n \mathfrak{B}^{(\phi)}, \qquad \mathfrak{E} = -\frac{e}{2m} \cdot \frac{G+1}{\gamma} a_n \mathfrak{B}_0^{(\phi)}.$$

What is a difference between the two systematical errors?

The longitudinal magnetic field turns the spin around the longitudinal direction while the EDM effect consists in the spin rotation around the radial direction. As a result, phases of rotating horizontal spin components appearing due to the longitudinal magnetic field and the EDM effect differ on π/2.



Summary

- General description of spin dynamics at the magnetic resonance has been carried out. Formulas have been obtained for all polarization components, an arbitrary initial beam polarization, an arbitrary phase of a perturbing field, and also for the spin evolution at frequencies far from resonance
- The results obtained can be effectively used for a quasimagnetic resonance in storage ring EDM experiments with allowance for their distinguishing features
- Spin dynamics in resonance EDM experiments with a rf electric-field flipper and a rf Wien filter has been theoretically described in detail
- Systematical errors in resonance EDM experiments with the rf Wien filter have been calculated


Thank you for your attention