pEDM - general plan
Madx, lattice design
Madx and Leapfrog orbit tracking,
Spink spin tracking

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The magic condition

For proton edm measurement in a storage ring[1] we will use a beam of spin-polarized protons where the spin dynamics is governed by the covariant Thomas-Bargman-Michel-Telegdi T-BMT equation[2]

$$\frac{d\mathbf{s}}{dt} = -\frac{q}{m\gamma}\mathbf{f} \times \mathbf{s}. \tag{1}$$

Here $\mathbf{s}$ is the real 3-dimensional spin vector of a 1/2-spin particle, and $\mathbf{f}$ is a function of the position and the momentum of the proton and of the electro-magnetic field encountered by it along its trajectory. We can say that spin is a passenger on the orbit.

In a pure electrostatic ring, *e.g.* with no magnets or RF cavities, **f** reduces to

$$\mathbf{f} = \left(a\gamma - \frac{\gamma}{\gamma^2 - 1}\right)\frac{\mathbf{E} \times \mathbf{v}}{c^2}, \tag{2}$$

where a is the spin anomaly, ratio between proton momentum and spin.

At the magic momentum value $pc = mc^2/\sqrt{a}$, it is $\mathbf{f} = \mathbf{0}$, and the spin remains frozen in its longitudinal direction imposed at injection.

Measurement of the proton electric dipole moment 'pEDM'

In the pure electric ring the spin of these 0.7 GeV polarized protons should remain frozen in the longitudinal direction for billion of turns.

However, if the protons possesses an electrical dipole moment that would be parallel to its spin, this electric moment will be relativistically seen in the Laboratory frame as a very small vertical spin component that could be measured. The result of this measurement is the purpose of the experiment.

Preliminary stage of the project

This experiment requires the design and construction of a large electrostatic synchrotron with a polarized proton source to produce and store a beam of 0.5 Gev (the magic energy) polarized protons. Preliminary stages of the project to address and discuss in detail are

1. design of the synchrotron

2. preliminary tracking of the proton beam at full energy, when the synchrotron is a storage ring

Synchrotron lattice

The ring has a FODO structure with electric bends, quadrupoles, and straight sections for beam injection, extraction and devices like, say, a RF cavity and a RF solenoid, sextupoles (to preserve spin coherence as it will be discussed),

For this we used the CERN package MAD [3], modified to accept electric bends and other devices. We run MAD through a UNIX scrip[4] to create tables of horizontal and vertical tune and max and min of beta function, varying the quadrupole strengths. Such a table shows the islands of stability of the ring. The values of β_{max}, then of the size of the beam, are growing at the edges of the islands where the betatron tunes decrease.

madX input file = y2.madin

K1F	K1D	Q1	Q2	BETXMAX	BETYMAX	GAMMATR
0.04	-0.04	0.7527416133	0.7344357921	70.35019034	65.88048455	1.51914812
0.04	-0.0399	0.7677519866	0.7132495545	68.92797464	67.77007236	1.51914812
0.04	-0.0398	0.7824842064	0.6914395641	67.58426395	69.83859752	1.51914812
0.04	-0.0397	0.7969537274	0.6689447034	66.31201144	72.11593235	1.51914812
0.04	-0.0396	0.8111746362	0.6456932835	65.10502622	74.63942772	1.51914812
0.04	-0.0395	0.8251598151	0.6216002936	63.95784412	77.45650557	1.51914812
0.04	-0.0394	0.8389210811	0.5965636573	62.86562152	80.62844147	1.51914812
0.04	-0.0393	0.8524693057	0.5704590161	61.82404748	84.23604223	1.51914812
0.04	-0.0392	0.8658145174	0.543132265	60.82927074	88.38845322	1.51914812
0.04	-0.0391	0.8789659902	0.5143885299	59.87783851	93.23736217	1.51914812
0.04	-0.039	0.8919323209	0.483975263	58.96664494	99.00100005	1.51914812
0.04	-0.0389	0.9047214963	0.4515550945	58.09288752	106.0070769	1.51914812
0.04	-0.0388	0.9173409523	0.4166596512	57.25403002	114.7752848	1.51914812
0.04	-0.0387	0.9297976253	0.3786049411	56.44777079	126.1912142	1.51914812
0.04	-0.0386	0.9420979984	0.3363200488	55.67201557	141.9219457	1.51914812
0.04	-0.0385	0.9542481414	0.28794713	54.92485402	165.6066661	1.51914812
0.04	-0.0384	0.966253747	0.2296708277	54.20453952	207.4313141	1.51914812
0.04	-0.0383	0.9781201626	0.1503876941	53.50947145	316.489104	1.51914812

Preliminary orbit in the pEDM lattice

Once the design of the lattice was done, we performed a preliminary orbit tracking with first order MAD matrices.It is a ring of 800 m length (taylored on the BNL-AGS tunnel, easily scalable) with 72 bends of 9 m length. 80 FODO quadrupoles of 2×0.5 m length, 4 drifts of 2×9 m. Values of basic parameters are

```
runname_________________________=     z2
Lbend [m]                       =   9.00000000
Ldrift [m]                      =   9.00000000
Lhalfquad [m]                   =   0.50000000
nbend                           =  72
pcs [GeV]  = mc2/dsqrt(anom)    =   0.70074061
x,xp,y,yp                       =   0.00000000  0.00010000  0.00000000  0.00010000
dp/p                            =   0.01000000
pc [GeV]                        =   0.70074061
Ut [GeV] = dsqrt(mc2^2+pc^2)    =   1.17106461
gam   = Ut/mc2                  =   1.24810740
beta  = dsqrt(1-1/(gam*gam))    =   0.59837912
brho  = 1.d9*pcs/cl             =   2.33741907
theta  [rad]                    =   0.08726646
rho [m] = nbend*Lbend/twopie    = 103.13240308
Bo [V-s/m^2] = brho/rho         =   2.26642549E-02
Eo [V/m] = Bo*beta*cl           =   4.06573039E+06
k1F,k1D,k2F,k2D                 =   0.0410 -0.0370  0.0410 -0.0370
beta tunes (oscill count)       =   3.3000  1.5500
xmax,ymax                       =     4.825242E-03    8.060333E-03
```

The ring, with 4×18 bends an 8 straights

[5]

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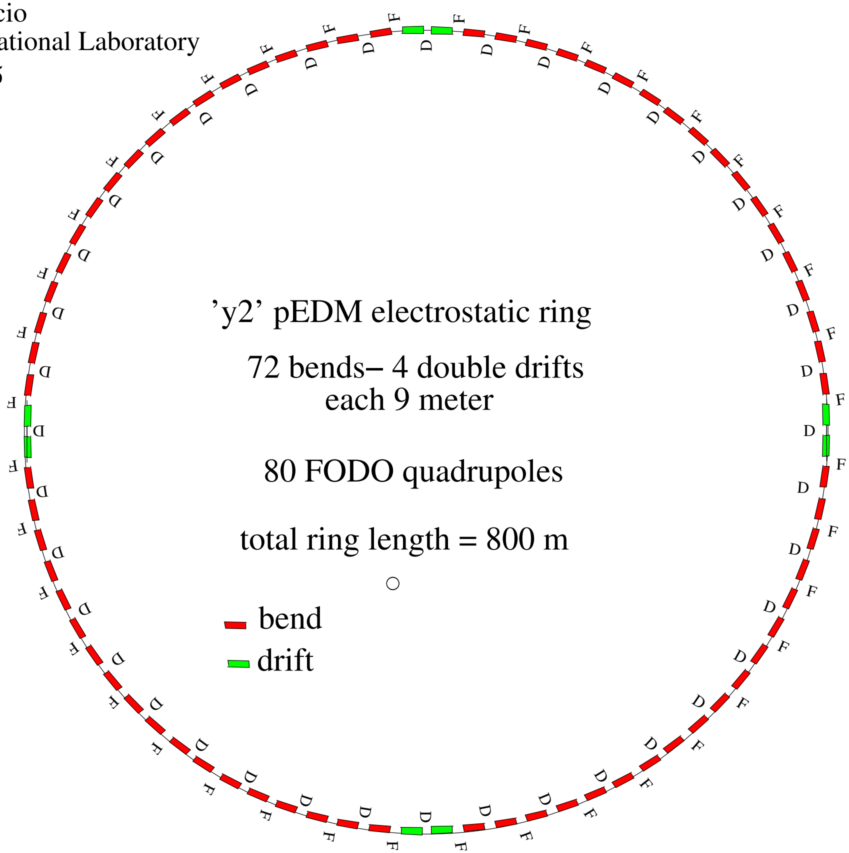
February 2015



Electrostatic bends and quads

1. El.static bend matrices to be used in MAD are[7]

   $$\alpha = \sqrt{2-\beta^2}, a1 = \cos(\alpha\theta),\ a2 = (\rho/\alpha)\sin(\alpha\theta),\ a3 = -(\alpha/\rho)\sin(\alpha\theta),$$

   $$M = \begin{pmatrix} a1 & a2 & 0 & 0 \\ a3 & a1 & 0 & 0 \\ 0 & 0 & 1 & L_b \\ 0, & 0 & 0 & 1 \end{pmatrix}, L_b = \text{length of bend}$$

   other matrices are Madx′s

2. El.static quadrupoles

   $$gap = 2a = 10cm,$$
   $$\text{El.gradient}: G_E = \frac{2V_0}{a^2}, k = \frac{e}{mc^2}\frac{G_E}{\beta^2\gamma}, V_0 = \frac{a^2}{2}\frac{mc^2}{e}(\beta^2\gamma)k$$

3. For Leapfrog tracking we used a field expansion [9].

Value (MKSA) and dimension of all ring parameters are

$$
\begin{array}{lcl}
\text{magic proton of} & & \\
\beta & = & 0.59837912., \\
\gamma & = & 1.24810740, \\
\beta^2\gamma & = & 0.44689430, \\
k & = & 0.043, \\
G_E & = & [kg \cdot A^{-1} \cdot s^{-3}] \\
V_0 & = & 1.42245166.10^5 V, \\
E_q \text{in the quads} & = & 2.844903.10^6 V/m, \\
E_b \text{in the bends} & = & 2.54972867.10^6 V/m
\end{array}
$$

Compared with the Stability Table, the above shows that the working γ of this particle is less that γ_T (transition) as is desirable for the pEDM search..

Note that for the optics the quantity of importance is

$$\sqrt{k} L_q$$

with L_q the length of the quadrupole. The field in the quadrupole is proportional to k. Therefore increasing the length of the quadrupole, but at the same time decreasing k and keeping $\sqrt{k} L_q$ constant, can effectively reduce the field.

Issues for electric accelerator lattices

An electrostatic lattice behaves differently than a classical magnetic lattice. In an electric ring device, such as a bend or a quadrupole, the kinetic energy of a particle is modulated, while in a magnetic device it is not, since in the Lorentz equation of motion

$$\frac{\mathbf{dp}}{dt} = e\mathbf{E} + e\mathbf{v} \times \mathbf{B} \tag{3}$$

only the scalar driving term eE appears. In a magnetic lattice it is the vector term to act, where the force is perpendicular to the velocity.

In the present design we adopted simple cylindrical electrodes for the bends, that produce only a radial field far from edges. Note that while magnetic bends do not focus the beam, electric cylindric bends produce a small horizontal focusing, so that to produce a FODO (focus-defocus) lattice, that we favor, the focusing and defocusing quadrupoles are slightly different.

An optional geometry for the bends, other that cylindrical, is with also a vertical curvature. For the moment we are not considering this since such geometry is more hard and expensive to construct with the desired accuracy.

Examine and compare algorithms and codes for production orbit tracking

Simulation of an electrostatic storage ring for the pEDM should be bste done by several competitive algorithms for mutual comparison and benchmark.

All codes should be symplectic for stability in the long range, In particular the continuously calculated Hamiltonian should remain constant. Tracking should also be fast, because the number ring turns to measure the pEDM will number in the billions.

It is important to distinguish wether a lattice is dynamic i.e. does contain elements or devices, like a RF cavity or a RF solenoid that run on their own cycle, or, otherwise, static because, In the dynamic case the lattice is changing during orbit tracking.

The Madx tracking described before was based on first order matrices, It is not symplectic, and was done only for check the lattice in a first pass.

Basic types of codes for orbit production tracking

Call a lattice static if does not contain a variable element, such as a RF cavity or a RF solenoid, with quantities that vary on their own cycle. Otherwise call the lattice dynamic. Among existing orbit tracking codes some visit in order the elements of the lattice. They work for static and dynamic lattices. No.4 below lumps the entire lattice in a precalculated manifold, then is hard for use for a dynamic lattice. Some existing codes (no pretense to be complete) are:

1. track using matrices (Madx2)[3]

2. Runge-Kutta integration of the differential Lorentz equation (slow) [7]

$$d\mathbf{p}/dt = e\mathbf{E} + e\mathbf{v} \times \mathbf{B} \tag{4}$$

3. track by kick integration of the same [Teapot-Spink] [8,9]

4. track using a precalculated symplectic manifold for the entire lattice (fast). [Cosy-Infinity] [10]

5. In our work we first used matrices for preliminary tracking and, after, [LeapFrog] kick integration.

Our tracking

Value (MKSA) and dimension of all ring parameters are

magic proton of

$$
\begin{aligned}
\beta &= 0.59837912., \\
\gamma &= 1.24810740, \\
\beta^2\gamma &= 0.44689430, \\
k &= 0.043, \\
G_E &= [kg \cdot A^{-1} \cdot s^{-3}] \\
V_0 &= 1.42245166.10^5 V, \\
E_q \text{in the quads} &= 2.844903.10^6 V/m, \\
E_b \text{in the bends} &= 2.54972867.10^6 V/m
\end{aligned}
$$

Compared with the Stability Table, the above shows that the working γ of this particle is less that γ_T (transition) as is desirable for the pEDM search..

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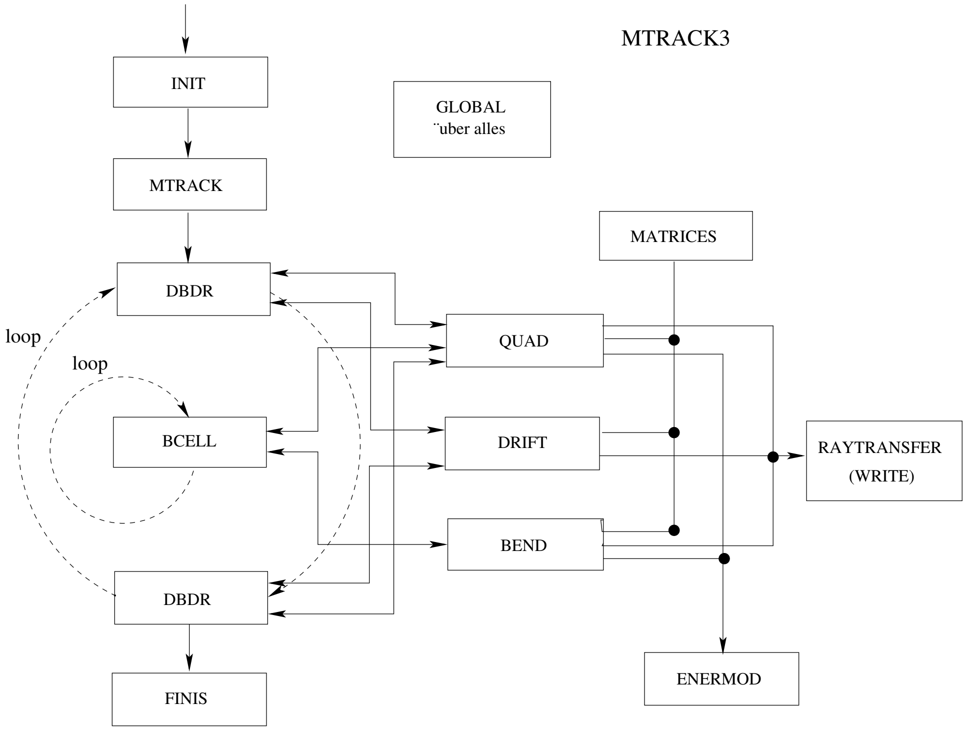
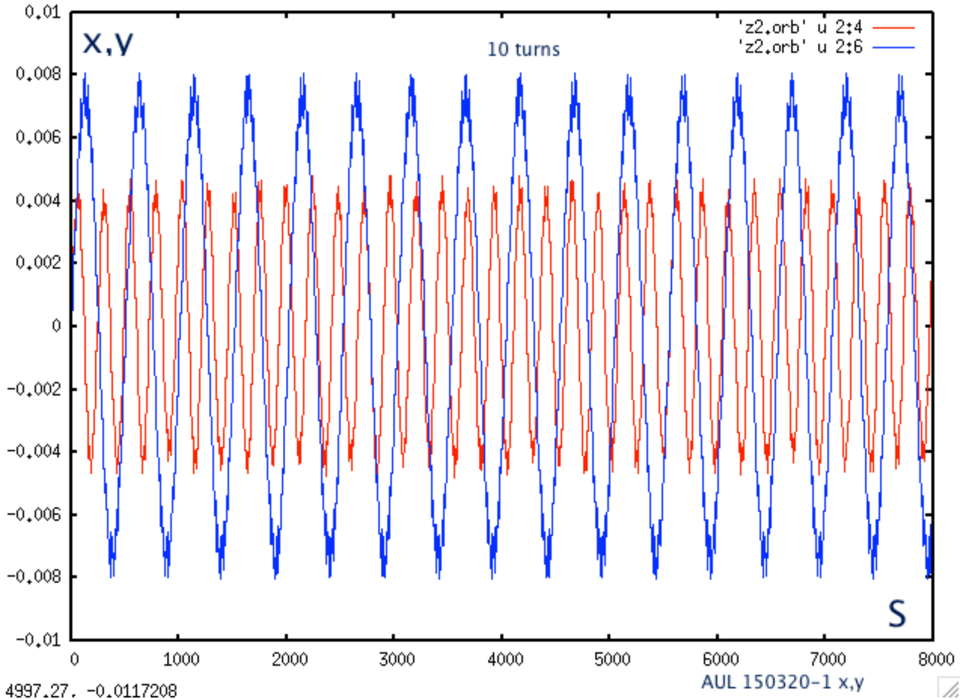
with L_q the length of the quadrupole. The field in the quadrupole is proportional to k. Therefore increasing the length of the quadrupole, but at the same time decreasing k and keeping $\sqrt{k} L_q$ constant, can effectively reduce the field.

The matrix driven tracking program



flowchart

betatron oscillations- by matrix tracking



Our production orbit tracking by Leapfrog

For a production tracking going beyond matrix orbit tracking, for simplecticity and speed, we considered canonical integration of the Lorentz diff. equation of motion

$$\frac{d\mathbf{p}}{dt} = e\mathbf{E} + e\mathbf{V} \times \mathbf{v} \tag{5}$$

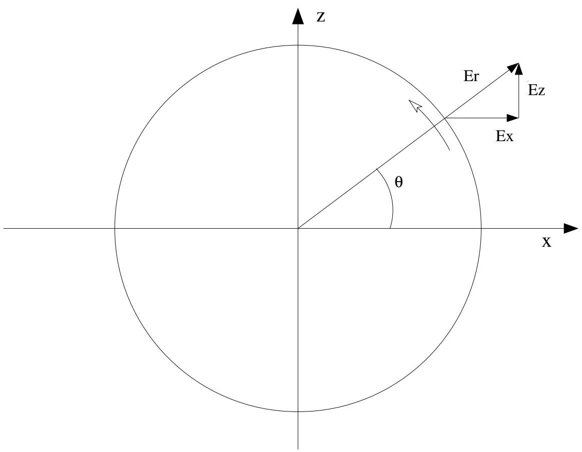
by Leapfrog or Verlet kicks[11], method invented by the astronomer Delambre[12] in 1791, and revisiteded for accelerators by Ronald Ruth[13]. It is a kick integration method that interleaves drifts, where only space coordinates are advanced, with symplectic kick bends where the momentum components are advanced. Leapfrog is an algorithm accurate to 2.nd order in time step. TEAPOT, by R.Talman and L.Schachinger [14] and [15] is similarly constructed.

Integration algorithms, like Runge-Kutta, used by Semertzidis and Haciameroglu[16] are accurate to 4.th order in time, are written with mathematical accuracy in mind, but are comparatively slow. The 2.nd order Leapfrog is exactly symplectic, fast and and written with physical accuracy in mind.

See also comments in [18] and [19].

Orbit coordinates

In Leapfrog we use Cartesian "laboratory" coordinates (x, z, y),with $\hat{x}$ and $\hat{z}$ axes in the horizontal plane, and time as the independent variable. Electric field components are calculated by a power expansion in the "horizontal" x, z plane of the ring



The circular ring lattice shown is obtained by tracking a "reference particle" *i.e* at nominal energy injected tangentially.

Orbit Leapfrog formalism basics

Ménagerie of quantities for the game are:

$r_o[m]$	$=$	radius of curvature
a	$=$	magnetic anomaly
$U_o[GeV]$	$=$	mc^2, mass – energy
$\wp \equiv pc[GeV]$	$=$	$U_o/\sqrt{a}$ moment
$U_T[GeV]$	$=$	$\sqrt{\wp^2 + U_o^2}$, total energy
γ	$=$	$U_T/mc^2, \quad \beta = \sqrt{1 - 1/\gamma^2}$
$B\rho[V \cdot s/m]$	$=$	$10^9 \wp/c$, rigidity
$eE[eV/m]$	$=$	$(= \wp/r_0)\beta c$ Electric bend field

Leapfrog tracking conserves the value of the Hamiltonian, that is being continuously recalculated during runs.

$$\mathcal{H} = \sqrt{(\wp - eA)^2 + (mc^2)^2} + e\phi. \tag{6}$$

To show how Leapfrog works, we propose 4 examples (1) circular ring, (2) race-track structure, (3) 8-super-period structure with 8 bends, 8 drifts and 8 electrostatic quadrupoles, (4) simple magnetic structure (helix)

A basic leapfrog cell is a sequence

$$\boxed{\text{drift + momentum kick + drift}}$$

Momentum kicks are done by kick integration of the Lorentz equation, for an electric or magnetic bend, respectively

$$\frac{d\mathbf{p}}{dt} = e\mathbf{E}, \text{ or : } = e\mathbf{B}\times\mathbf{v} \text{ with } \mathbf{E} = -\nabla\phi, \quad \mathbf{B} = \nabla\times\mathbf{A}. \tag{7}$$

The potential, needed for the Hamiltonian, should obey the Laplace equation

$$\nabla^2\phi = 0. \tag{8}$$

(Explicit expressions for ϕ and **A** are found by power expansion across the plane.)

The reference particle, around which the whole beam dances, is the magic one whose spin would remain frozen in position during the propagation.

Leapfrog cell - Electric bend

This is how a reference particle moves on the horizontal plane

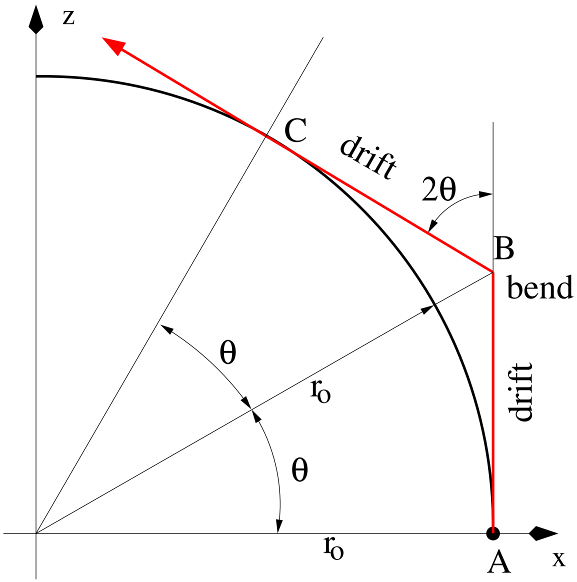


Fig.1

A →B →C,
drift, kick-bend, drift

drift A-B

Start in **A** with Initial coordinates

$$(\mathbf{A})\quad x = r_o,\ z = 0,\ \wp_x = 0,\ \wp_z = \wp.$$

Eq's for the drift, with a time step dt for the drift **A**$\rightarrow$**B**:

$$\frac{dx}{dt} = \frac{\wp_x}{U_o\gamma}c, \qquad \frac{dz}{dt} = \frac{\wp_z}{U_o\gamma}c, \ \text{or}$$
$$x := x + \wp_x/(U_o\gamma)c\,dt, \qquad z := z + \wp_z/(U_o\gamma)c\,dt \tag{9}$$

using the identity $\wp = U_o\beta\gamma$, we obtain at the kick bend **B** the new position

$$(\mathbf{B})\quad x = r_o,\ z = \beta c\,dt,\ \wp_x = 0,\ \wp_z = \wp.$$

kick in B

In **B** a kick is imparted to the momentum **p**c, using the Lorentz Equation, with a time step δt, different from the dt of the drift.

$$\wp_x := \wp_x - eE_x c\,\delta t, \qquad \wp_z := \wp_z - eE_z c\,\delta t \tag{10}$$

For cylindrical bend the field E is purely radial, with components

$$eE_x = -eE\,(r_o/r)\cos\theta \qquad eE_z = eE\,(r_o/r)\sin\theta. \tag{11}$$

Now | find the relation between dt and δt for leapfrog | *i.e*:

1. Through the bend the value of the total momentum pc must be conserved,

2. The trajectory in **C** should return tangent to the circle, as in the figure. Namely:

$$\arccos\left[(\mathbf{p}(A)\cdot\mathbf{p}(C))/p^2\right] = 2\theta \tag{12}$$

If both conditions hold, the basic trajectory will be a polygon circumscribed to the circle. Other particles in the beam will dance around it in betatron oscillations.

For condition (1): moment conservation, combining the preceding equations

$$\wp_x = -\wp/r \cos\theta\, \beta c\, \delta t, \;\; \wp_z = \wp\, (1 - (1/r) \sin\theta\, \beta c\, \delta t) \tag{13}$$

then after kick (**C**):

$$\wp_x^2 + \wp_z^2 = (pc)^2 \left[1 + ((\beta c/r)\delta t)^2 - (2/r) \sin\theta\, \beta c\, \delta t\right]. \tag{14}$$

Since: $\cos\theta = z/r$, $\sin\theta = x/r$, taking the value of x from Eq.(9), the term in [ ] in Eq.(14) above reduces to 1 when

$$\boxed{\delta t = 2\, dt}$$

For condition (2): new angle, it is calculated from the scalar product of the momentum before and after the kick

- (**A**) before kick: $\wp_x = 0$, $\wp_z = \wp$
- (**C**) after kick: $\wp_x = -(\wp/r) \cos\theta\, \beta c\, \delta t$, $\wp_z = \wp \left(1 - 2 \sin^2\theta\right)$

$$\text{angle} = \arccos \frac{\wp(A) \cdot \wp(B)}{(pc)^2} = \arccos\left(1 - 2\sin^2\theta\right) = \boxed{2\theta} \;\; q.e.d.$$

Reference Trajectory

Let us produce a reference trajectory on the horizontal plane by Leapfrog tracking along a polygonal pattern tangent to a structure made of straights (drifts) and circular arcs (bends). So, The leap-frog orbit is slightly longer than the reference orbit. The more kicks we put in a bend the lesser this difference is.

In an example of a structure with 8 bends and 8 drifts of circa 270 m of total length, using 32 kicks in each bend of 36 m of radius, the difference in effective radius between the geometrical base line and the polygon is about 1 mm.

The step is much larger than the required step of a solution by integration for similar accuracy, with a very large gain in computing speed.

Tracking a reference particle will create a reference trajectory. An example is shown in the following picture.

Reference Trajectory by tracking
Four bends and four straights

Note that using Leapfrog the reference particle is actually // designing the lattice !

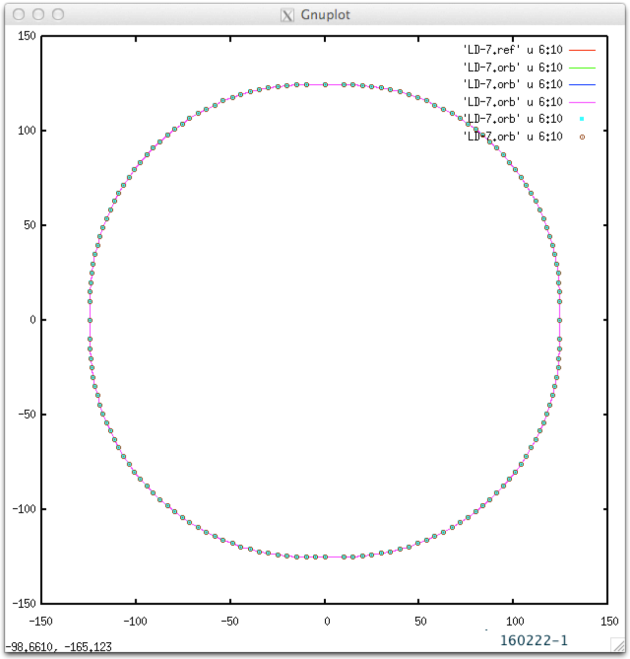


Fig.2

72 bends
bend length=28.276 m
drift length 4×20 m
curvature radius $\rho = 114.59m$
entire ring length = 800 m
$E_{cyl} = 3.65915710^7$ V/m

Evaluation of the electric field

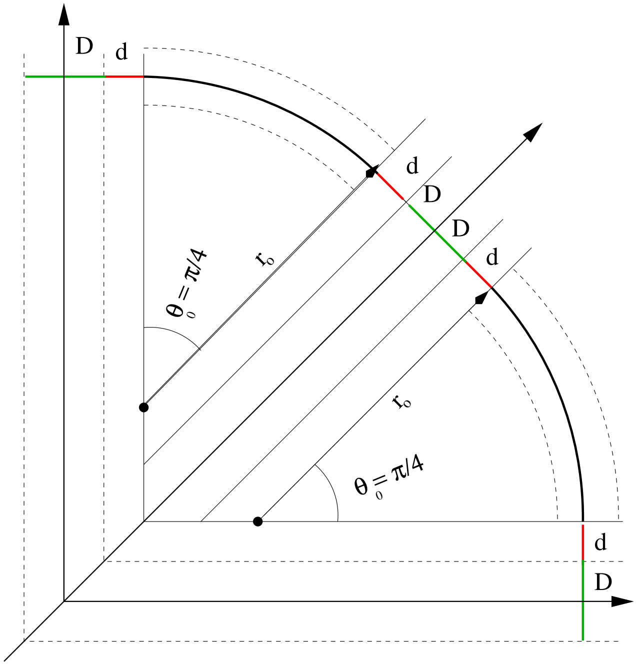


Fig.3

In a general lattice the center of curvature for the calculation of the electric field continuously changes and has to be re-evaluated every time

The sketch (for the preceding lattice) suggests how

'D' is any added drift space
'd' is a leapfrog inner-bend drift

General tracking

The Leapfrog formalism extends to 3 dimensions and applies unchanged to particles that don't have a magic energy or are injected in the lattice on a finite transverse emittance.

Eqs.(9) and .(10) in 3 dimensions are

$$\left\{ \begin{array}{l} x := x + \wp_x/(U_o\gamma)c\,dt \\ y := y + \wp_y/(U_o\gamma)c\,dt \\ z := z + \wp_z/(U_o\gamma)c\,dt \end{array} \right. , \quad \begin{array}{l} \wp_x := \wp_x - eE_x 2c\,dt \\ \wp_y := \wp_y - eE_y 2c\,dt \\ \wp_z := \wp_z - eE_z 2c\,dt \end{array} . \tag{15}$$

However, In a general case the leapfrog conditions (1) for momentum and angle are not fully satisfied in a bend because, due to transverse oscillations, the particle sees a tangential component of the electric field that modulates the energy.

During tracking the Hamiltonian is continuously calculated. It conserves its initial vaiue.

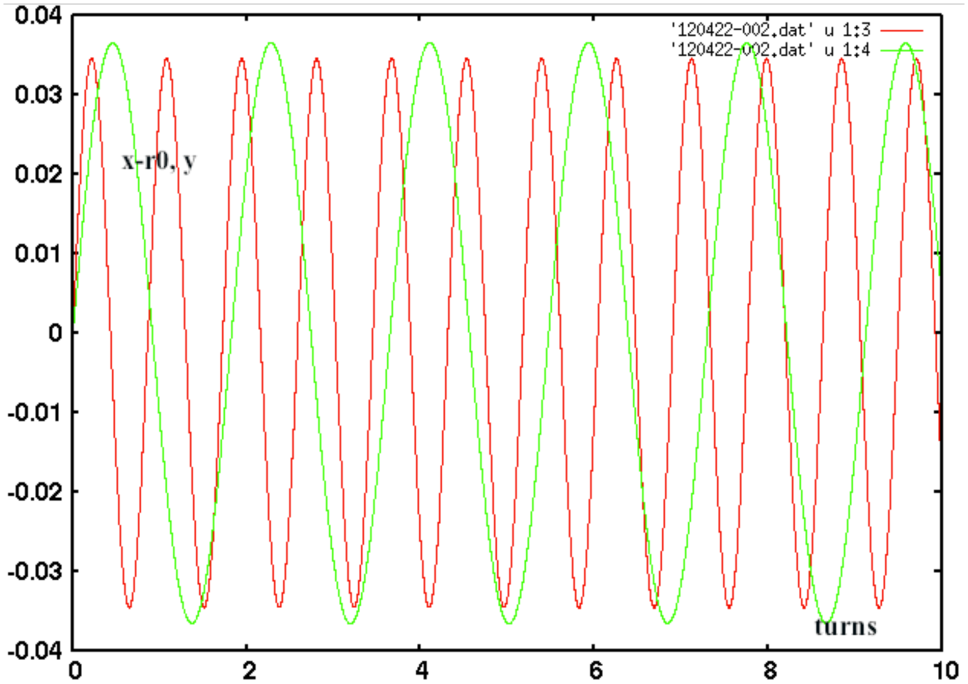


Fig.4 x,y betatron oscillations vs. turn #

Add a RF - Example of RF bucket

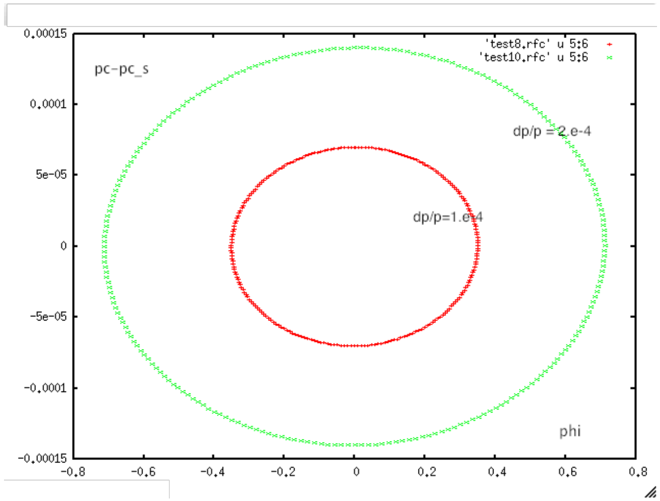


Fig.5 - Phase space of $\Delta \times pc$ for two particles, with $dp/p = 1.e^{-4}$ and $2.e^{-4}$, with $V_{RF} = 1000V$ and $h = 24$. Number of turns for a complete oscillations is 335, corresponding synchrotron frequency $\nu_s = 0.002985$ oscillations per turn

Briefly on Spin Dynamics: EDM

Spin kicks, applied at each Bend and Quad, follow the leapfrog pattern of the orbit.

At the magic energy it is $\mathbf{F} = \mathbf{0}$ and the spin remains frozen If the proton has an EDM, the spin is Not completely frozen: in the rest frame of the particle, the electric field appears as a magnetic field **B'** $\perp$ to **E** and another small term is added to **f** in Eq.(2)

$$\mathrm{B}' = -\gamma\vec{\beta} \times \mathbf{E}. \quad \mathbf{f} := \mathbf{f} + \eta \mathrm{B}' \times \mathbf{v}. \tag{16}$$

The spin will make a precession around this magnetic field and a spin vertical component will appear, that can be measured. For a magic proton this is the only non vanishing additional spin component.

Spin dynamics of a frozen spin

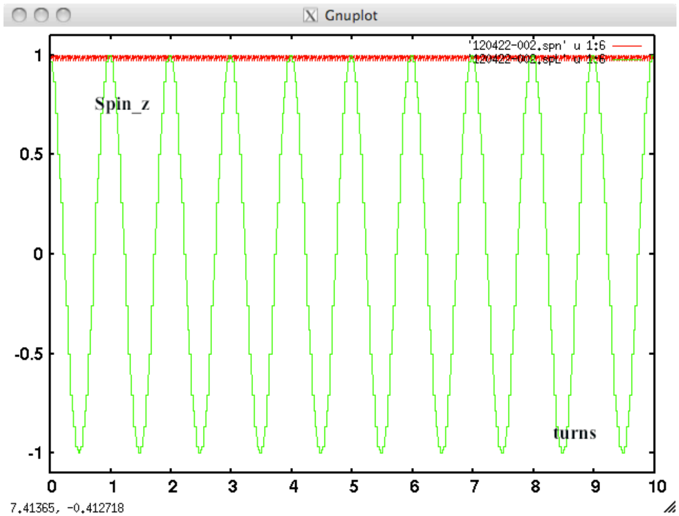


Fig.8 - Longit. component of the frozen spin: red line in accelerator coordinates, green line, in laboratory coordinates. The red line shows little wiggles because the responsible proton is on purpose not perfectly magic and there are betatron oscillations.

At the very beginning of tracking, at **B**, after the drift, it is

$$v_x = 0, \quad v_z = v, \tag{17}$$

so, according to (18) only p_x would receive a kick, which is paradoxical -but a toll one pays using kick formalism- because a magnetic force cannot change the total momentum. After the kick we may therefore write

$$\wp_x = \wp\frac{v}{r_0}\delta t, \quad \wp_z = \wp\sqrt{1-\frac{\wp_x^2}{\wp^2}} \tag{18}$$

For a magnetic kick the **1.st leapfrog condition** should be satisfied for free by default. The **2.nd leapfrog condition** for the angle still calls for a relation between dt, the time step for a drift, and δt, the time step for a bend.

We see from Fig.**??**, that at the beginning, the polygonal condition for the kick bend angle requires:

$$\frac{\wp_x}{\wp_z} = \tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}. \tag{19}$$

(1)Tracking a full beam
(2)Spin Coherence

The pEDM game is only complete when we address two very important aspects:

1. Tracking a full beam of many particles, with a certain emittance and energy spread

2. Address the issue of spin coherence.

Both Task(s1) can be addressed by parallel computing on a multi-processor mainframe. One should first produce a population of particles that represents the beam and then run in parallel for each particle. Typically, we worked on samples of, say 240 particle and processes using a parallel library like MPI (Message Passing Interface)[19].

For Task(2) we should calculate the spin tune from the eigenvalues of the spin tune matrix of each particle. Spin tunes for the beam form a spin tune line. The spin coherence of the beam is inversely proportional to the spin tune linewidth.

In our study of the spin coherence during simulation on the spin polarization measurements done on the polarized proton beam of COSY[12], it was confirmed that use of sextupoles in the ring would increase the spin coherence of the beam.

level-AGS Proposal, BNL (2008)
2. J.D.Jackson - Classical Electrodynamics Wiley, 2.nd edition, p.558
3. J.Ch.Iselin, F.Schmidt and others - MAD-X: Methodical Accelerator Design Library, URL: cdsweb.cern,com/indexw665527g65184413.pdf
4. by Nicholas D'Imperio - Brookhaven National Laboratory
5. M.Conte - Electrostatic Storage Ring
University of Genova, **PACS 29.20.db** *Oct.24, 2012*
6. by Y.Semertzidis
7. by S.Hachameroglu and Y,Semertzidis = Brookhaven
8. TEAPOT - by R.Talman, L.Schachinger
9. A.Luccio et Al. UAL simulation, Bad Honnef presentation, July 6,2011
10. Cosy-Infinity by Martin Berz
11. L.Verlet Computer experiments in classical fluids
Phys.Rev. **159-98-103**
12 J.B.Delambre Tables du soleil *1792*
13. R.D.Ruth A Canonical Integration Technique
IEEE Trans. on Nuclear Sciences **NS-3**, *No.4, August 1983 14 L.Schachinger and R.Talman - TEAPOT: A Thin-element Accelerator Program*
Particle Accelerators, **22** *1987*
15. A.U.Luccio et al . TEAPOT-SPINK simulation of orbit and spin in COSY. Bad Honnef Symposium, July 31,2013
16 Y.Semertzidis and S.Hachameroglu. A Runge-Kutta tracking
17. J.M.Sanz-Serna. Symplectic integrators for Hamiltonian problems
Acta Numerica (1991), 243-286 QA 297,A21 1992
18. W.H.Press et Al. Numerical Recipes *Cambridge Uni.Press,1992//*
19. W.Gropp, E.Lusk and A.Skjellum, MPI, MIT Press 1996