

# Quantum aspects of theories with extended gauge symmetries

Helena Kolešová

Under supervision of Michal Malinský

Faculty of Nuclear Sciences and Physical Engineering  
Czech Technical University in Prague

DISSERTATION THESIS

Quantum aspects of beyond-Standard-model theories  
with extended gauge symmetries

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Helena Kolečová

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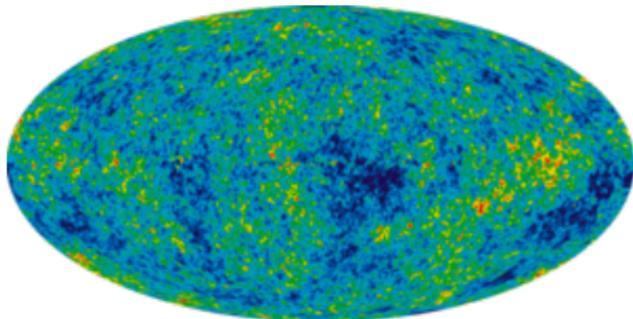
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## Non-zero neutrino masses?



*“The Nobel Prize in Physics 2015 was awarded jointly to Takaaki Kajita and Arthur B. McDonald for the discovery of neutrino oscillations, which shows that neutrinos have mass”*

$$\Delta m_{sol}^2 = 7.4 \times 10^{-5} \text{ eV}^2, \quad \Delta m_{atm}^2 = 2.5 \times 10^{-3} \text{ eV}^2 \quad \Rightarrow \quad m_{\nu_{heaviest}} \gtrsim 0.05 \text{ eV}$$



[Planck collaboration (2016)]

$$\sum_{a=1}^3 m_{\nu_a} \leq 0.23 \text{ eV}$$

# Unbearable lightness of neutrinos



## Dirac mass as for other SM fermions?

- Addition of right-handed neutrinos necessary!

$$-\mathcal{L}_Y \ni Y_\nu \bar{L} i \sigma_2 H^* \nu_R + h.c.$$

- $Y_\nu \sim 10^{-12}$  in order to achieve  $m_\nu \sim 0.1 \text{ eV}$



## Majorana mass?

$$-\mathcal{L}_Y \ni \frac{1}{2} M_\nu \nu_L^T C \nu_L + h.c.$$

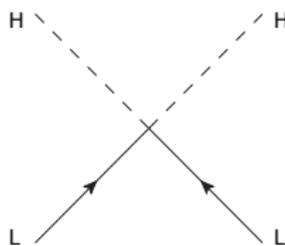
- can not be written in  $SU(2)_L$  invariant and in the same time renormalizable way (if SM scalar sector assumed)

# The Weinberg operator

- But what about non-renormalizable operators?

$$\mathcal{L} = \mathcal{L}^{d=4} + \frac{1}{\Lambda} \sum_k C_k^{d=5} \mathcal{O}_k^{d=5} + \frac{1}{\Lambda^2} \sum_k C_k^{d=6} \mathcal{O}_k^{d=6} + \dots$$

- The only  $d = 5$  operator breaks the lepton number and generates Majorana neutrino masses! [Weinberg (1979)]



$$\frac{C_\nu}{\Lambda} (L^T i\sigma_2 H) C (H^T i\sigma_2 L) + h.c.$$

$$m_\nu \sim \frac{C_\nu}{\Lambda} v^2 \sim 0.1 \text{ eV} \Rightarrow \text{new physics @ } \Lambda \sim 10^{14} \text{ GeV?}$$

# “Opening” of the non-ren. operators: SM example

Fermi interaction

(modern form):

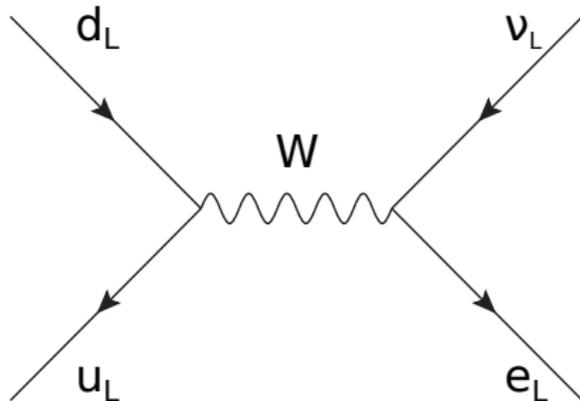
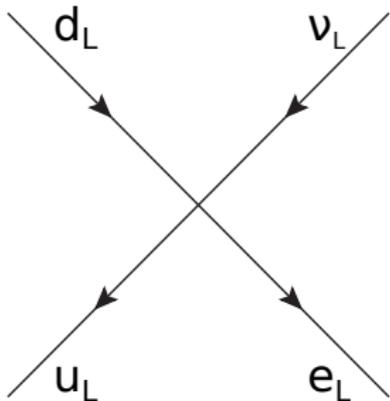
$$G (\bar{u}_L \gamma_\mu d_L) (\bar{e}_L \gamma^\mu \nu_L), [G] = M^{-2}$$

→

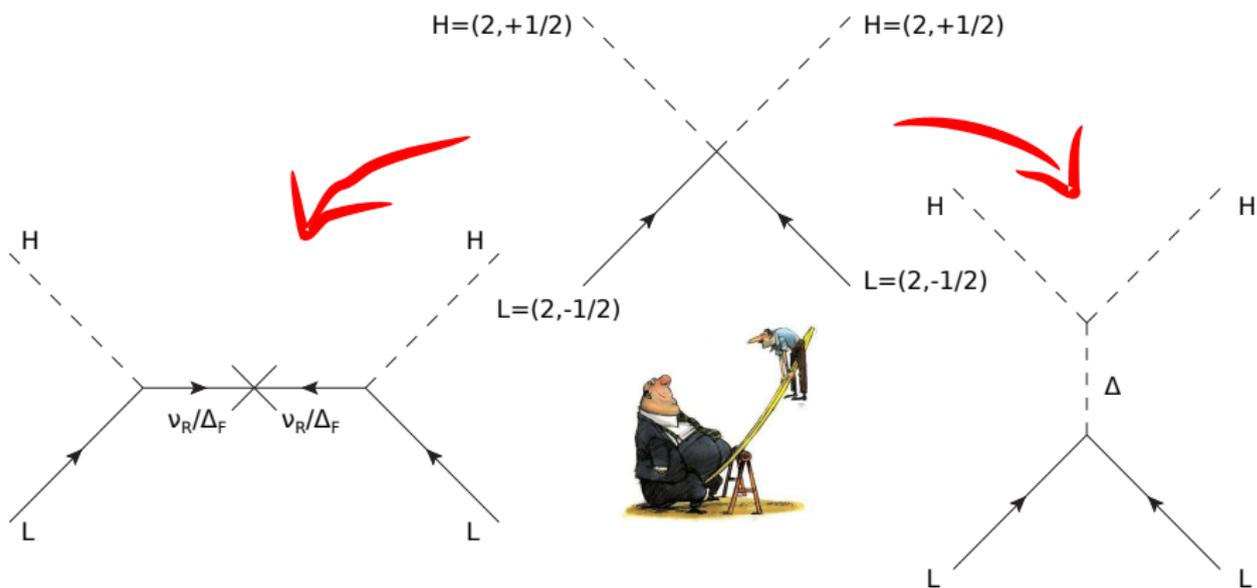
Weak charged

current interactions:

$$G \propto \frac{1}{m_W^2}$$



# “Opening” of the Weinberg operator at the tree level



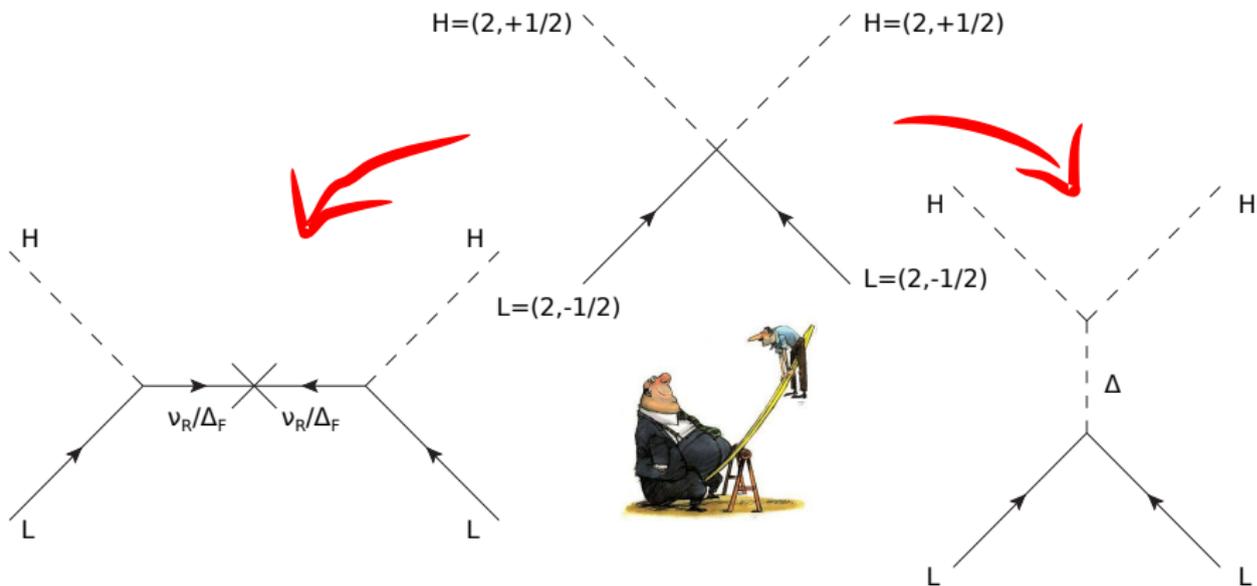
**Fermionic mediator:**  
 $SU(2)_L$  singlet/triplet,  $Y = 0$   
 Type I/Type III seesaw

[Minkowski(1977); Yanagida(1979);  
 Mohapatra, Senjanovic(1980)]/[Foot, Lew, He, Joshi(1989)]

**Scalar mediator:**  
 $SU(2)_L$  triplet,  $Y = 1$   
 Type II seesaw

[Schechter, Valle (1980); Magg, Wetterich (1980); Mohapatra, Senjanovic (1981)]

# “Opening” of the Weinberg operator at the tree level



## Type I seesaw:

$\nu_R$ : Dirac mass  $m_D$  & large Majorana mass  $m_R$

$$m_\nu = \frac{m_D^2}{m_R}$$

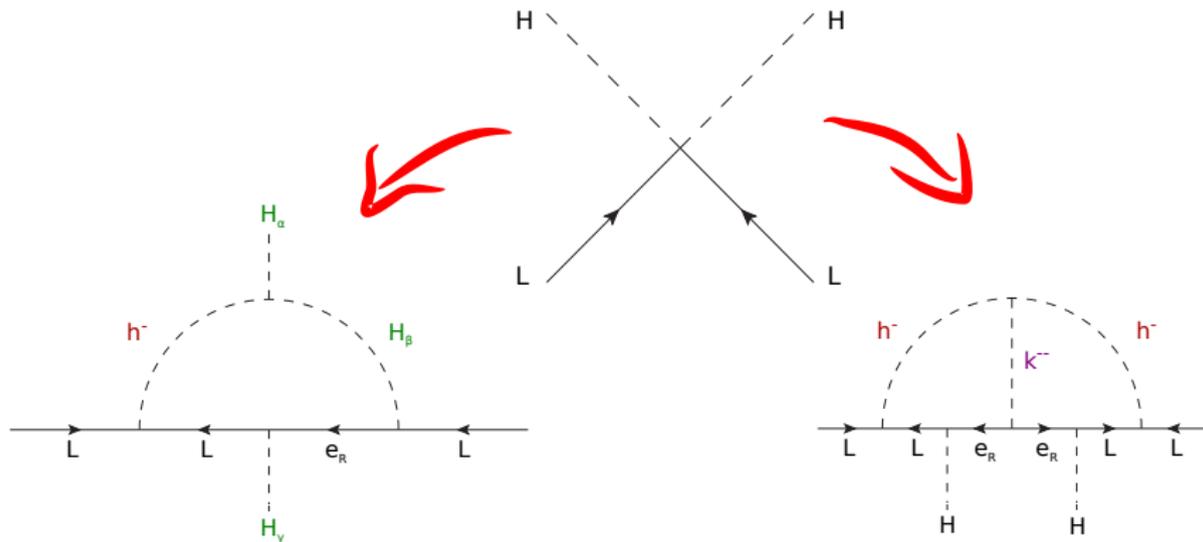
$$m_D \sim v \Rightarrow m_R \sim 10^{14} \text{ GeV}$$

## Scalar mediator:

$SU(2)_L$  triplet,  $Y = 1$   
Type II seesaw

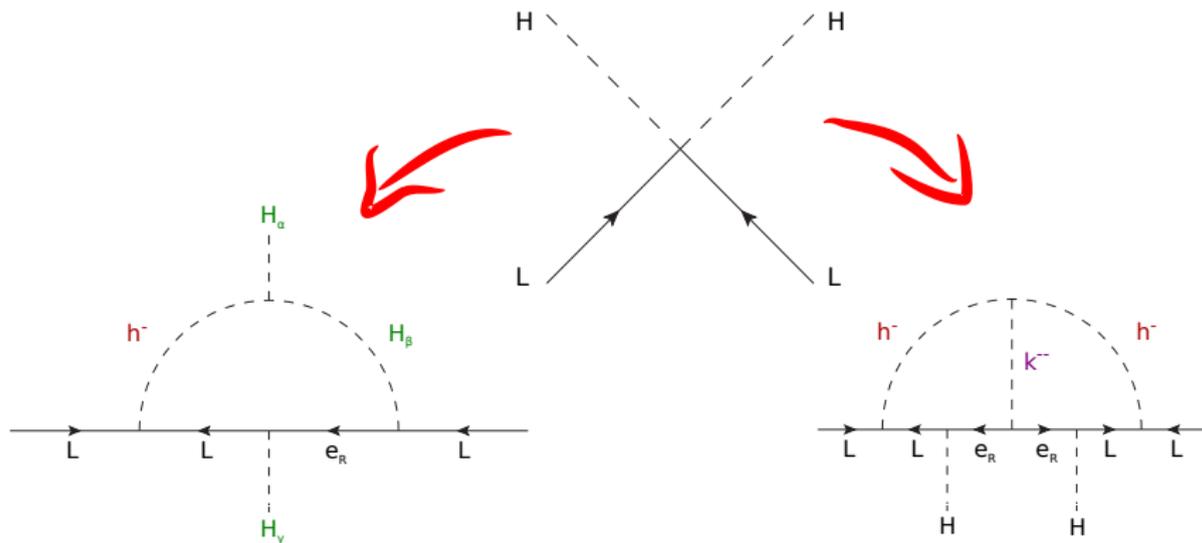
[Schechter, Valle (1980); Magg, Wetterich (1980); Mohapatra, Senjanovic (1981)]

# Loop realization of the Weinberg operator



**Smallness of neutrino masses also due to loop suppression!**

# Loop realization of the Weinberg operator



**Smallness of neutrino masses also due to loop suppression!**

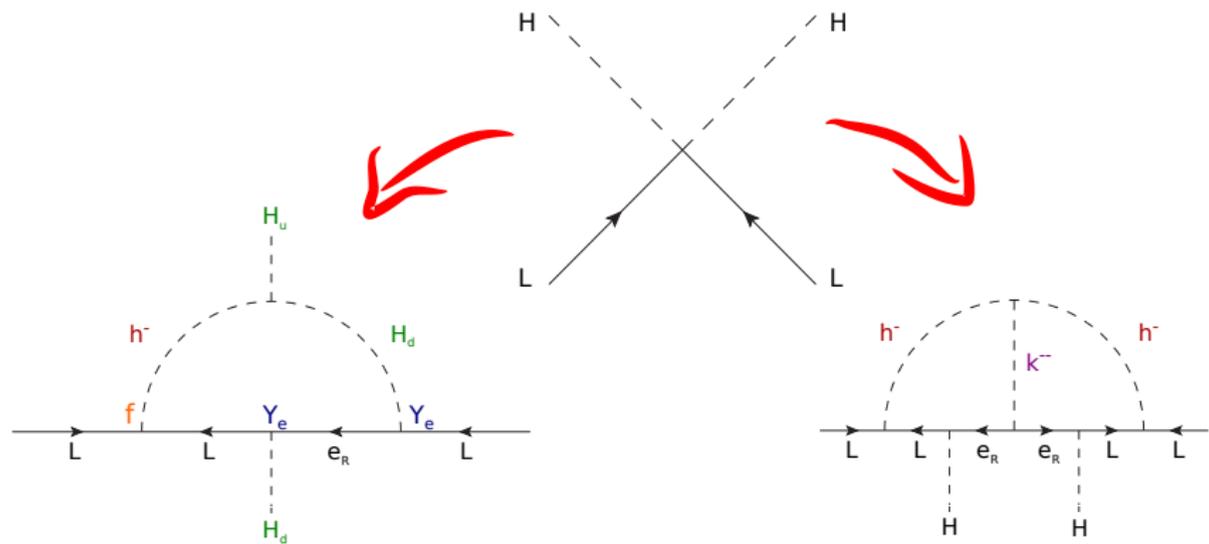
**Zee model:** [Zee (1980)]

$h^-$ :  $SU(2)_L$  singlet,  $Y=Q=-1$

$V \ni \mu_{\alpha\beta} H_\alpha^T i\sigma_2 H_\beta h^-$

$\mu_{\alpha\beta} = -\mu_{\beta\alpha} \Rightarrow$  2nd Higgs doublet

# Loop realization of the Weinberg operator



**Smallness of neutrino masses also due to loop suppression!**

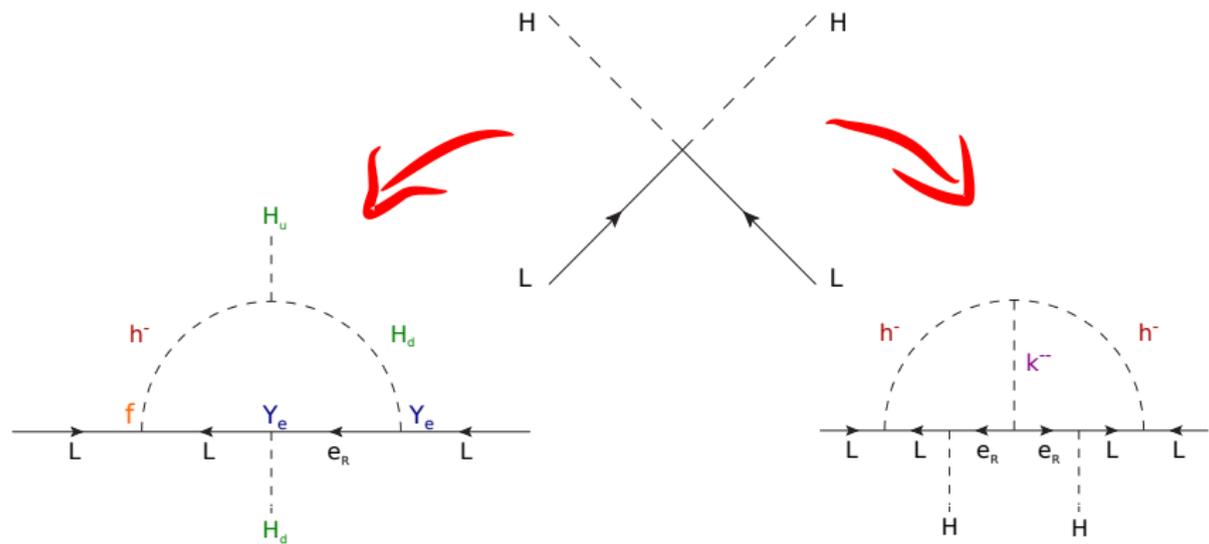
**Zee model:** [Zee (1980)]

$$\mathcal{L}_Y \ni f_{jk} L_j^T C i \sigma_2 L_k h^+, \quad f_{jk} = -f_{kj}$$

$$(m_\nu)_{ij} \propto f_{ij} (m_{e_j}^2 - m_{e_i}^2) \Rightarrow \text{excluded!}$$

Realistic  $\nu$  mixing: e.g. [Babu, Julio (2014)]

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**Zee-Babu model:** [Zee(1986), Babu(1988)]

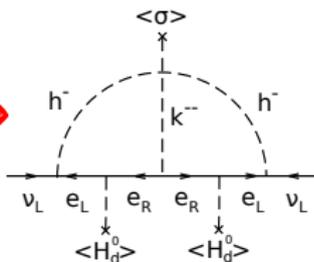
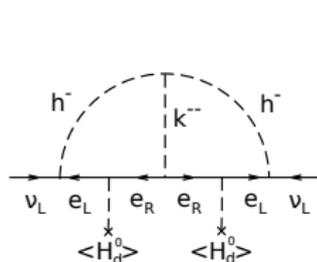
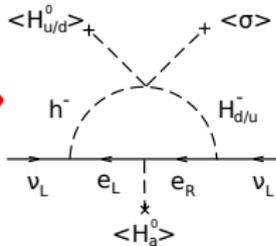
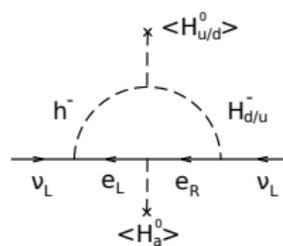
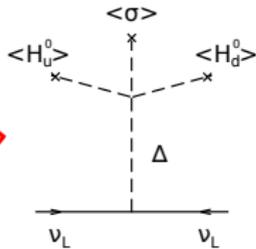
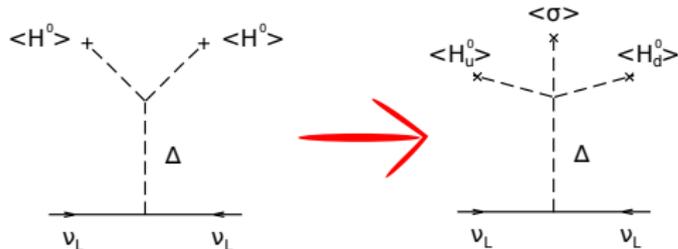
$h^-$ :  $SU(2)_L$  singlet,  $Y=Q=-1$

$k^{--}$ :  $SU(2)_L$  singlet,  $Y=Q=-2$

TeV scale scalars observable @ LHC!

[Herrero-Garcia, Nebot, Rius, Santamaria(2014)]

# Scalar extensions of the SM with $\nu$ -axion interconnection



[Bertolini, DiLuzio, Kolečová, Malinsky (2015);

Bertolini, DiLuzio, Kolečová, Malinsky, Vasquez (2016)]

- Dimensionful coupling  $\rightarrow$  interaction with a scalar  $\sigma$

$\sigma$ :  $SU(2)_L$  singlet,  $Y = 0$

- $\langle \sigma \rangle \sim 10^9$  GeV
- Peccei-Quinn symmetry introduced [P., Q. (1977)]

$\Rightarrow$  DFSZ axion [Dine, Fischler, Srednicki (1981); Zhitnitsky (1980)]

$\Rightarrow$  Dark matter candidate!

# Conclusions

- Majorana neutrino masses can be generated by a  $d = 5$  operator
- tree level “opening” of such an operator suggests new scalars/fermions @  $\sim 10^{14}$  GeV
- loop realizations include scalars with lower masses, even observable at the LHC
- scalar extensions of the SM mentioned above supplemented by another singlet scalar  $\Rightarrow$  models with an interplay between the neutrino sector and the axionic dark matter!

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**Thank you for attention!**