Signatures of Neutrino Mass Hierarchy in Supernova Neutrinos

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Outline

1) **neutrino oscillations in matter & Mikheyev-Smirnov-Wolfenstein effect**
2) **process of supernova explosion**
3) **neutrino detection channels in JUNO – Jiangmen Underground Neutrino Observatory**
4) **search for observables suitable to distinguish between different supernova models and/or neutrino mass hierarchies**

Neutrino oscillations in vacuum

flavor eigenstate basis: $|\nu_f\rangle \quad f = e, \mu, \tau$

$$|\nu_f\rangle = \sum_{i=1}^{3} U_{fi}^{*} |\nu_i\rangle$$

mass eigenstate basis: $|\nu_i\rangle \quad i = 1,2,3$

$$U = \begin{pmatrix} c_{13}c_{12} & c_{13}s_{12} & s_{13}e^{-i\delta} \\ -s_{23}s_{13}c_{12}e^{i\delta} - c_{23}s_{12} & -s_{23}s_{13}s_{12}e^{i\delta} + c_{23}c_{12} & s_{23}c_{13} \\ -c_{23}s_{13}c_{12}e^{i\delta} + s_{23}s_{12} & -c_{23}s_{13}s_{12}e^{i\delta} - s_{23}c_{12} & c_{23}c_{13} \end{pmatrix}$$

$$s_{ij} = \sin(\vartheta_{ij})$$

$$c_{ij} = \cos(\vartheta_{ij})$$

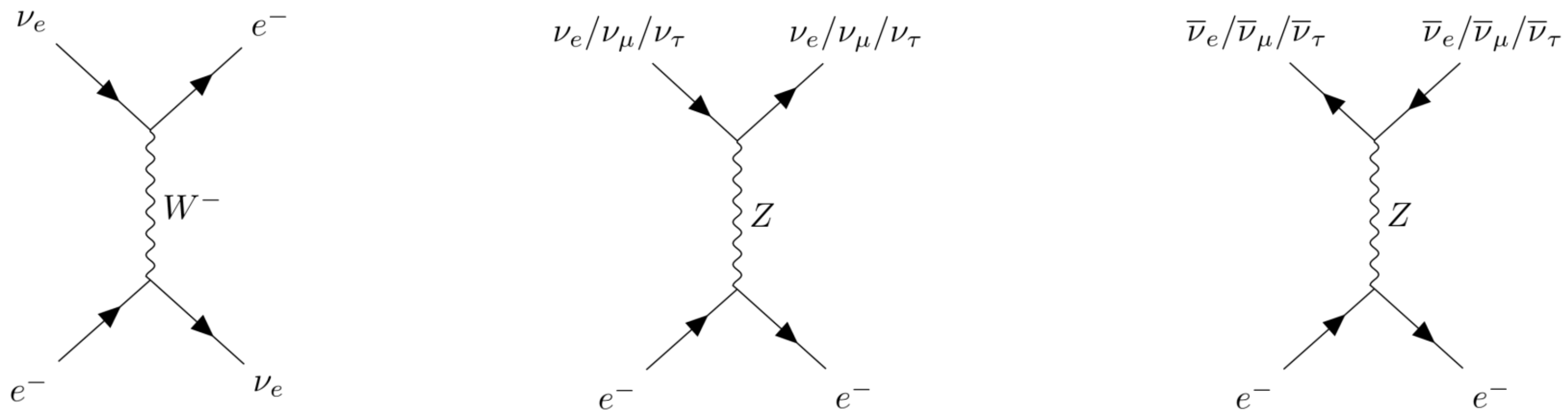
Schrödinger equation in mass eigenstate basis

$$i\hbar c \frac{\partial}{\partial t} \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \end{pmatrix} = H_{\text{mass}} \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \end{pmatrix} \qquad H_{\text{mass}} = \begin{pmatrix} E_1 & 0 & 0 \\ 0 & E_2 & 0 \\ 0 & 0 & E_3 \end{pmatrix}$$

Schrödinger equation in flavor eigenstate basis

$$i\hbar c \frac{\partial}{\partial t} \begin{pmatrix} \psi_e \\ \psi_\mu \\ \psi_\tau \end{pmatrix} = U H_{\text{mass}} U^{\dagger} \begin{pmatrix} \psi_e \\ \psi_\mu \\ \psi_\tau \end{pmatrix} \qquad H_{\text{flavor}} = U H_{\text{mass}} U^{\dagger}$$

Neutrino oscillations in matter



Due to the charged-current interaction between ν_e, $\bar{\nu}_e$ and e^- in ordinary matter, an additional potential V adds to the hamiltonian.

$$V = \begin{pmatrix} \sqrt{2} G_F N_e (\hbar c)^3 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

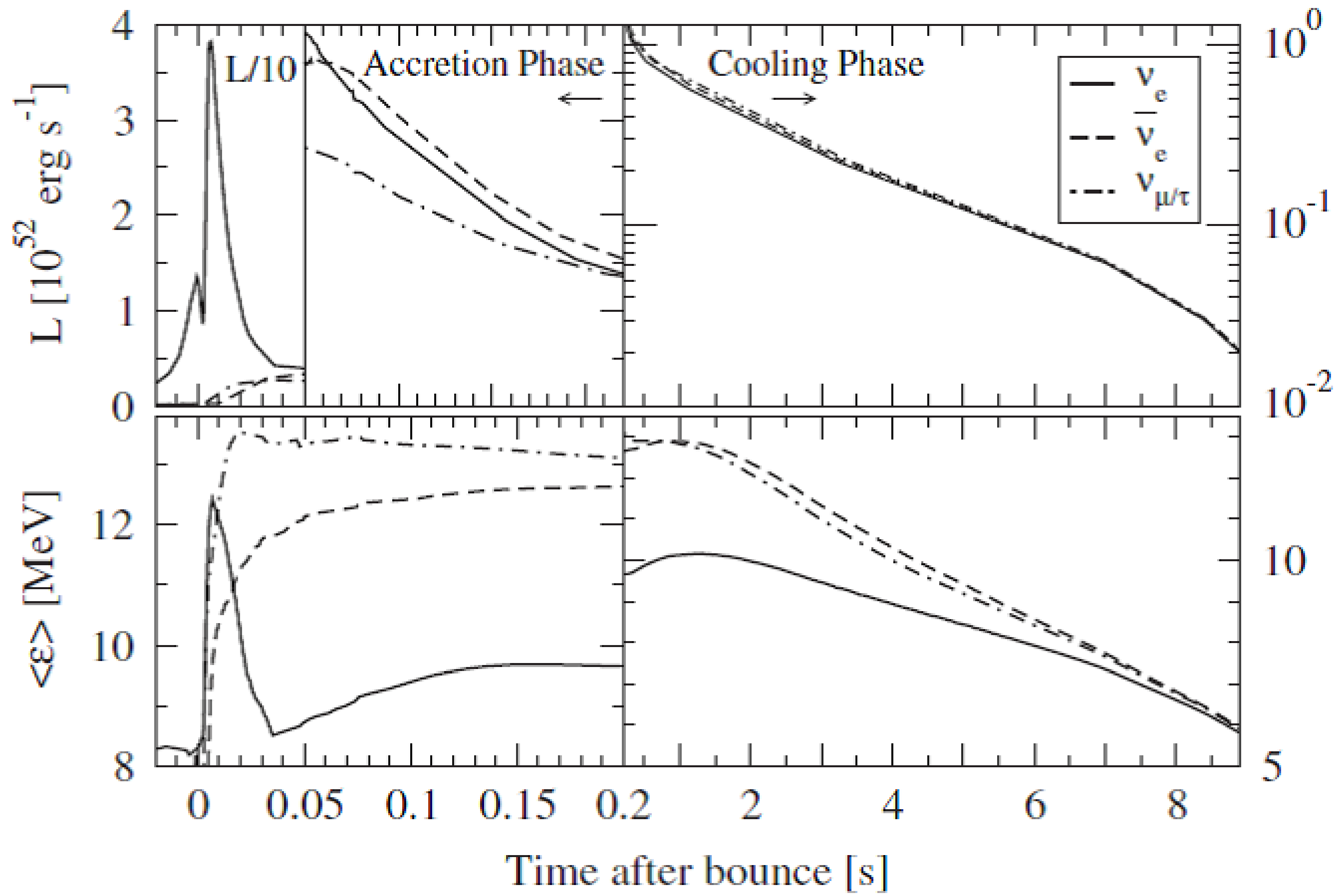
G_F Fermi constant
N_e electron number density

Final hamiltonian in flavor eigenstate basis:

$$H_{\text{neutrino}} = U H_{\text{mass}} U^\dagger + V$$

$$H_{\text{antineutrino}} = U H_{\text{mass}} U^\dagger - V$$

Models of supernova explosion



An 8.8 $M_{\odot}$ supernova simulation of neutrino luminosties and mean energies.

(L. Hudepohl et al., *Neutrino signal of electron-capture supernovae from core collapse to cooling*, Phys. Rev. Lett. 104 (2010) 251101)

Models of supernova explosion

Neutrino mean energies according to three representative SN models.

paper	$\langle E_{\nu_e} \rangle$ [MeV]	$\langle E_{\bar{\nu}_e} \rangle$ [MeV]	$\langle E_{\nu_x} \rangle$ [MeV]
Fogli et al. (2007)	10	15	24
Lai et al (2016)	12	15	18
Hudepohl et al. (2010)	9,4	11,44	11,44

$\nu_x = \nu_\mu, \nu_\tau, \bar{\nu}_\mu$ or $\bar{\nu}_\tau$

typical ordering: $\langle E_{\nu_e} \rangle \leq \langle E_{\bar{\nu}_e} \rangle \leq \langle E_{\nu_x} \rangle$

(G. Fogli et al., *Collective neutrino flavor transitions in supernovae and the role of trajectory averaging*, J. Cosmol. Astropart. Phys. 12 (2007) 010)

(K.C. Lai et al., Probing *neutrino mass hierarchy by comparing the charged-current and neutral-current interaction rates of supernova neutrinos*, J. Cosmol. Astropart. Phys. 7 (2016) 039)

(L. Hudepohl et al., *Neutrino signal of electron-capture supernovae from core collapse to cooling*, Phys. Rev. Lett. 104 (2010) 251101)

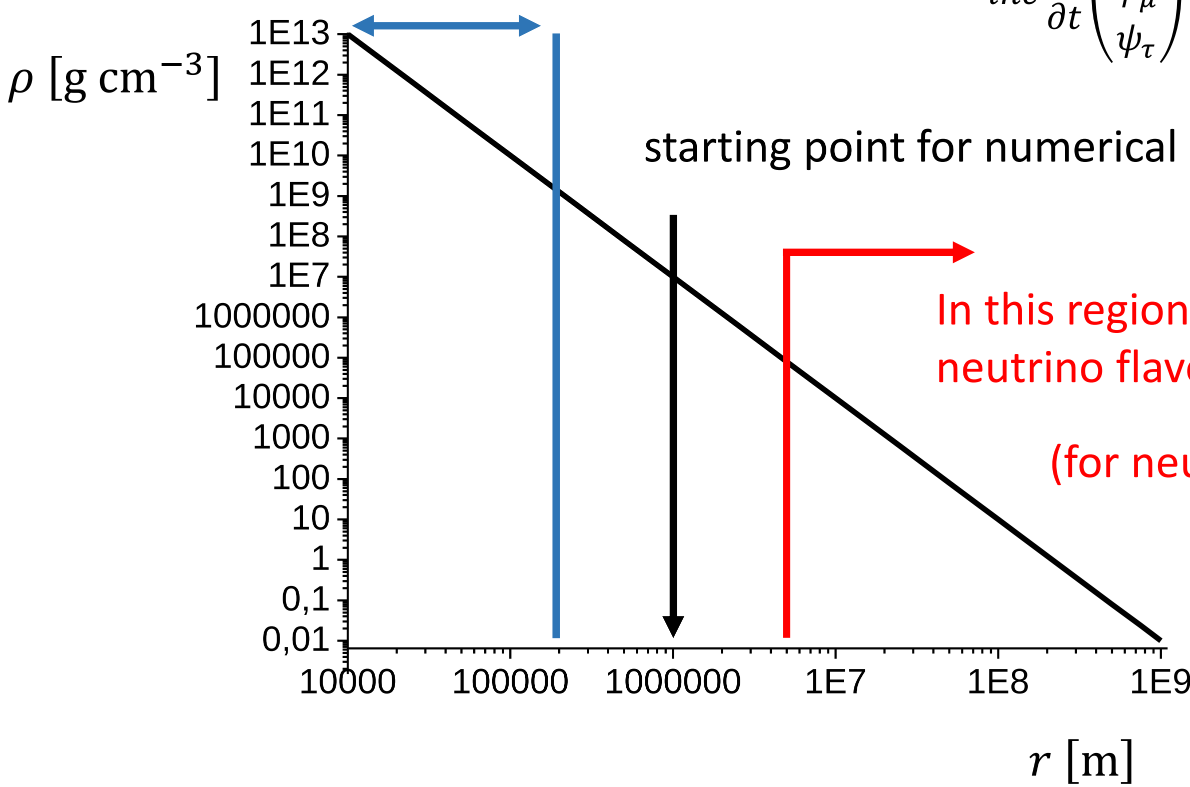
Density profile of supernova & MSW effect

Typical density profile of supernova matter: $\rho(r) = 10^{13}\ \mathrm{g\ cm^{-3}} \left(\frac{10\ km}{r}\right)^3$

region of collective neutrino oscillations

$$i\hbar c \frac{\partial}{\partial t}\begin{pmatrix}\psi_e\\ \psi_\mu\\ \psi_\tau\end{pmatrix} = (U H_{\mathrm{mass}} U^\dagger \pm V)\begin{pmatrix}\psi_e\\ \psi_\mu\\ \psi_\tau\end{pmatrix}$$

starting point for numerical solution of Schrödinger equation.



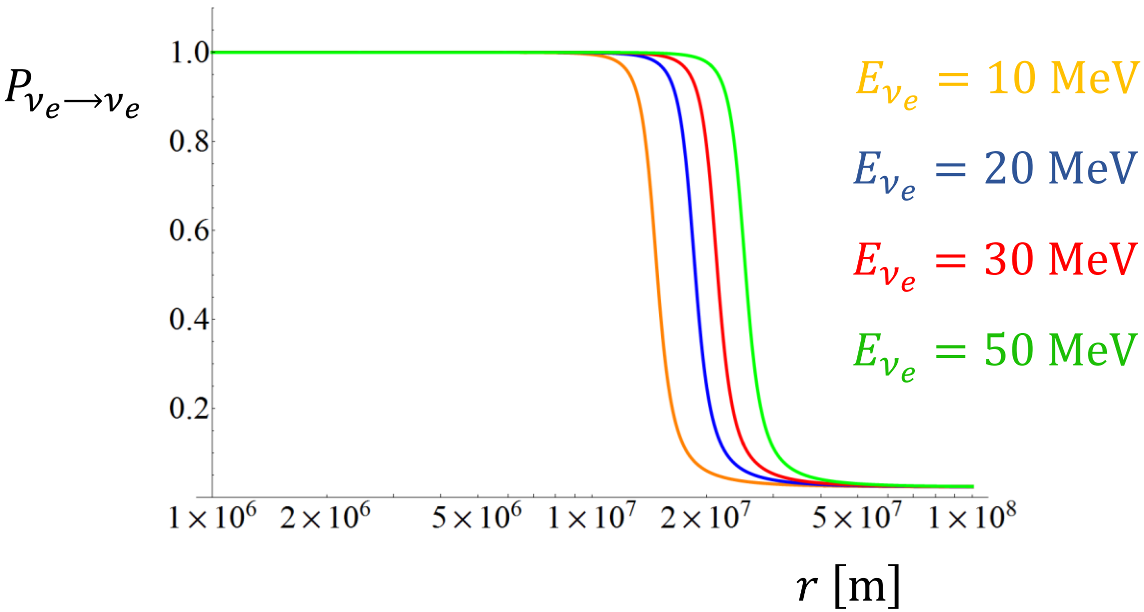
In this region occurs oscillation between neutrino flavors due to MSW effect.

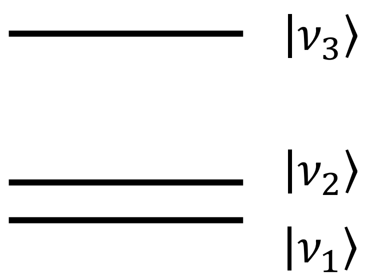
(for neutrino energies higher than 1 MeV)

NEUTRINO OSCILLATIONS DEPEND ON MASS HIERARCHY

MSW effect: neutrinos & normal hierarchy

initial state	final state	probability	check sum	flavor content of the heaviest eigenstate $\|\nu_3\rangle$
ν_e	ν_e	0,023		0,023
ν_e	ν_μ	0,427	1,001	0,427
ν_e	ν_τ	0,551		0,550
ν_μ	ν_e	0,513		
ν_μ	ν_μ	0,284	1,003	
ν_μ	ν_τ	0,206		
ν_τ	ν_e	0,464		
ν_τ	ν_μ	0,293	1,003	
ν_τ	ν_τ	0,246		

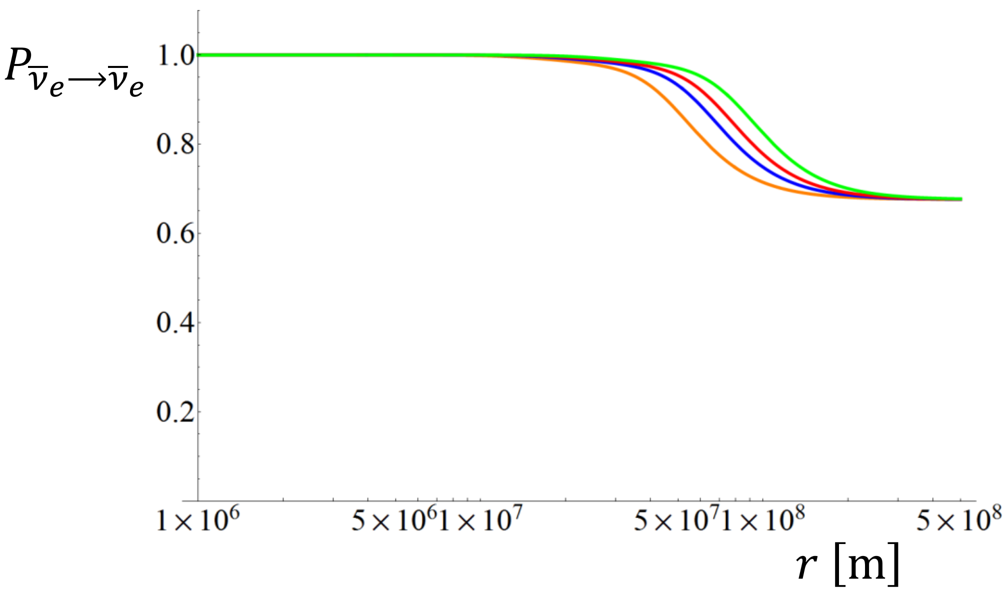




Electron neutrino goes out from supernova as the **heaviest mass eigenstate** $|\nu_3\rangle$.

MSW effect: antineutrinos & normal hierarchy

initial state	final state	probability	check sum	flavor content of the lightest eigenstate $\vert\nu_1\rangle$
$\bar{\nu}_e$	$\bar{\nu}_e$	0,676		0,676
$\bar{\nu}_e$	$\bar{\nu}_\mu$	0,250	0,099	0,251
$\bar{\nu}_e$	$\bar{\nu}_\tau$	0,073		0,074
$\bar{\nu}_\mu$	$\bar{\nu}_e$	0,181		
$\bar{\nu}_\mu$	$\bar{\nu}_\mu$	0,370	1,005	
$\bar{\nu}_\mu$	$\bar{\nu}_\tau$	0,454		
$\bar{\nu}_\tau$	$\bar{\nu}_e$	0,144		
$\bar{\nu}_\tau$	$\bar{\nu}_\mu$	0,383	1,003	
$\bar{\nu}_\tau$	$\bar{\nu}_\tau$	0,476		

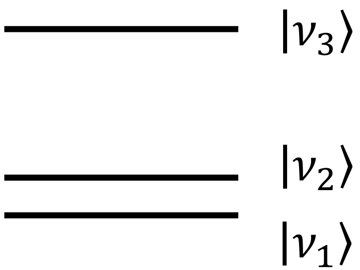


$E_{\bar{\nu}_e} = 10$ MeV

$E_{\bar{\nu}_e} = 20$ MeV

$E_{\bar{\nu}_e} = 30$ MeV

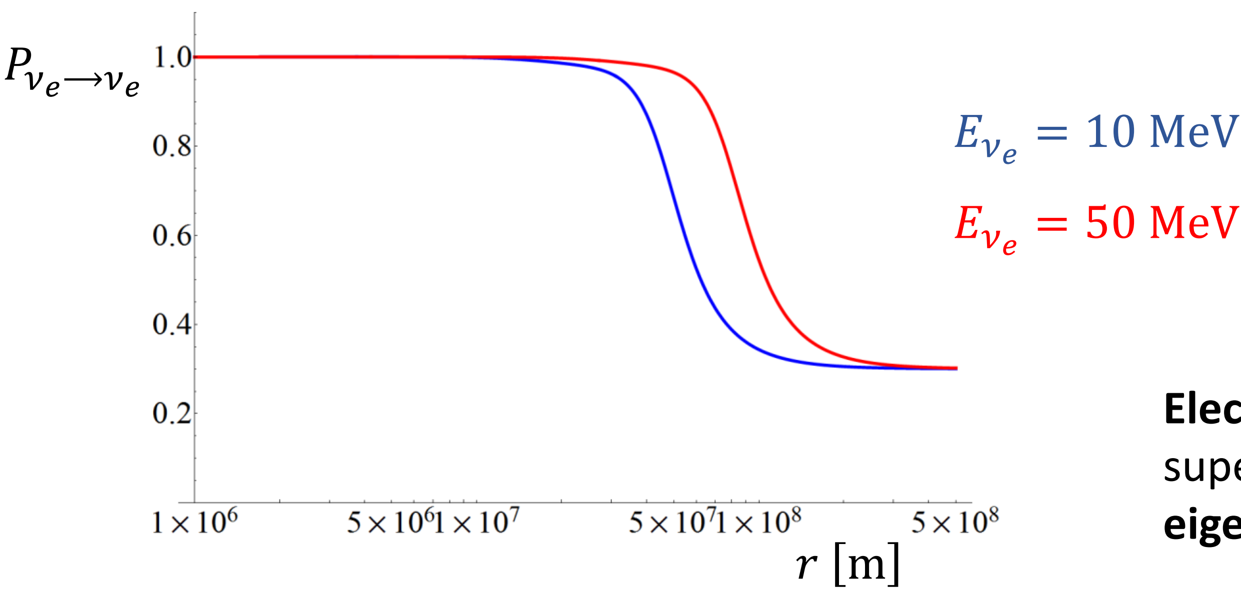
$E_{\bar{\nu}_e} = 50$ MeV

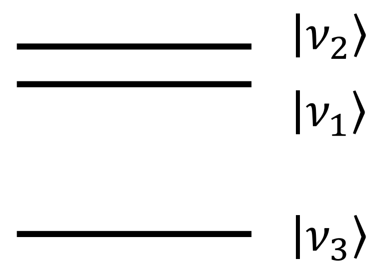


Electron antineutrino goes out from supernova as the **lightest mass eigenstate** $|\nu_1\rangle$.

MSW effect: neutrinos & inverted hierarchy

initial state	final state	probability	check sum	flavor content of the heaviest eigenstate $\|\nu_2\rangle$
ν_e	ν_e	0,301		0,301
ν_e	ν_μ	0,309	1,000	0,309
ν_e	ν_τ	0,390		0,390
ν_μ	ν_e	0,378		
ν_μ	ν_μ	0,339	1,005	
ν_μ	ν_τ	0,288		
ν_τ	ν_e	0,322		
ν_τ	ν_μ	0,356	1,004	
ν_τ	ν_τ	0,326		

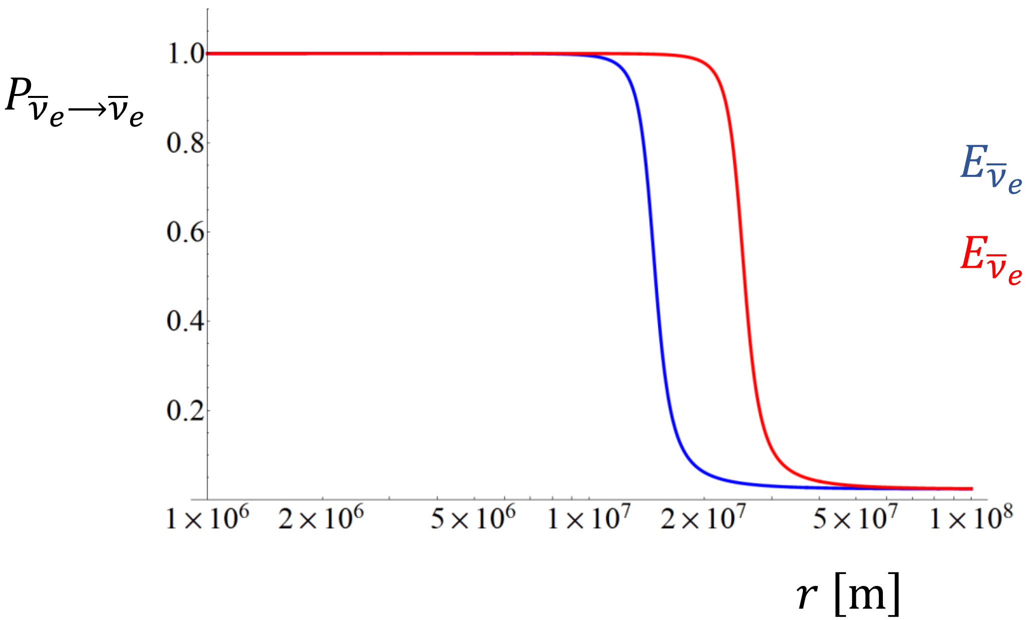




Electron neutrino goes out from supernova as the **heaviest mass eigenstate** $|\nu_2\rangle$.

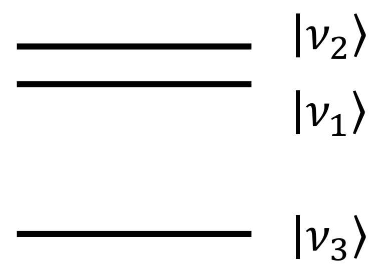
MSW effect: antineutrinos & inverted hierarchy

initial state	final state	probability	check sum	flavor content of the lightest eigenstate $\lvert\nu_3\rangle$
$\bar{\nu}_e$	$\bar{\nu}_e$	0,024		0,024
$\bar{\nu}_e$	$\bar{\nu}_\mu$	0,445	1,002	0,444
$\bar{\nu}_e$	$\bar{\nu}_\tau$	0,533		0,532
$\bar{\nu}_\mu$	$\bar{\nu}_e$	0,472		
$\bar{\nu}_\mu$	$\bar{\nu}_\mu$	0,282	1,002	
$\bar{\nu}_\mu$	$\bar{\nu}_\tau$	0,248		
$\bar{\nu}_\tau$	$\bar{\nu}_e$	0,505		
$\bar{\nu}_\tau$	$\bar{\nu}_\mu$	0,277	1,004	
$\bar{\nu}_\tau$	$\bar{\nu}_\tau$	0,222		



$E_{\bar{\nu}_e} = 10$ MeV

$E_{\bar{\nu}_e} = 50$ MeV

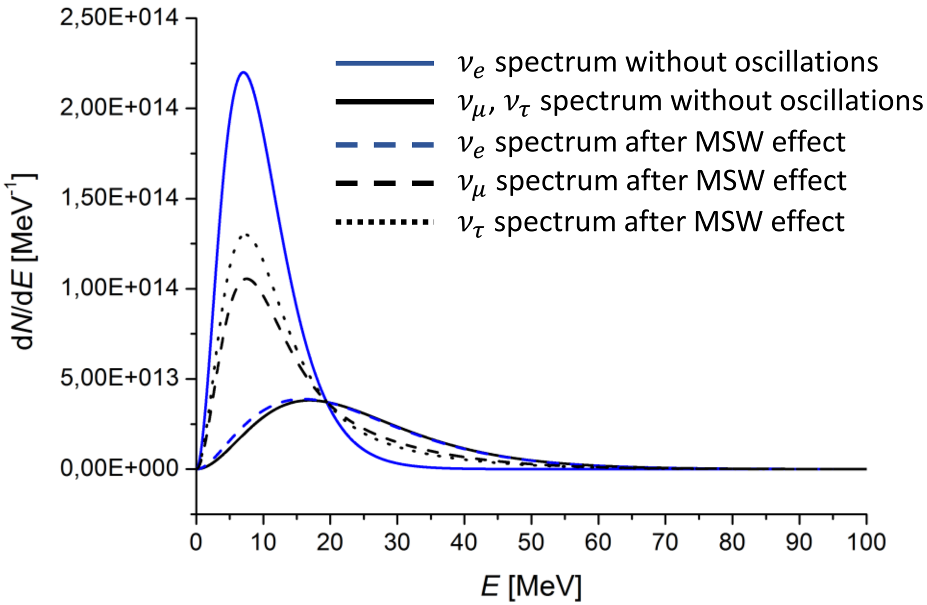


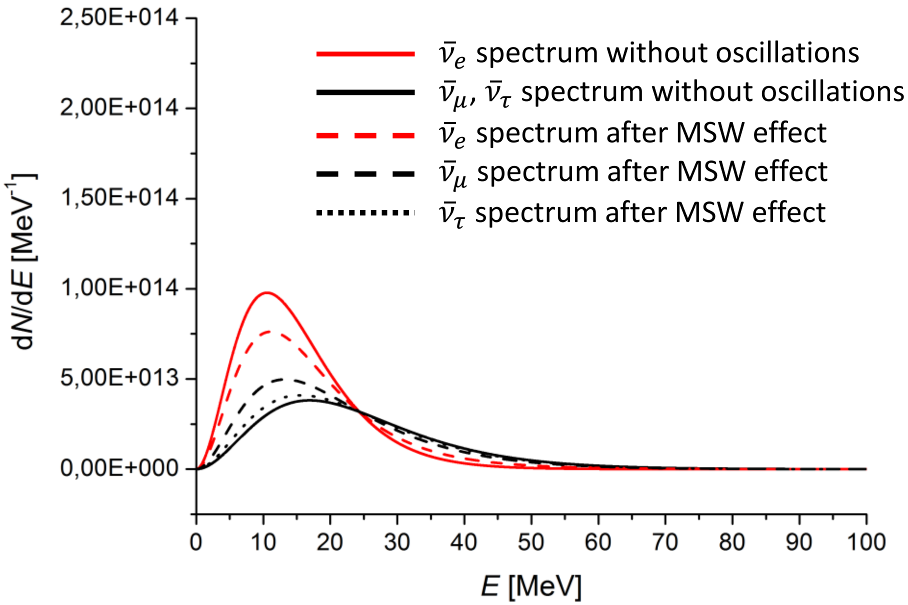
Electron antineutrino goes out from supernova as the **lightest mass eigenstate** $\lvert\nu_3\rangle$.

Time-integrated anti/neutrino spectra before and after MSW effect

Normal hierarchy

$$j_\alpha(E) = \frac{L_\alpha}{4\pi R}\frac{2}{3\zeta(3){T_\alpha}^3}\frac{E^2}{1+\exp\left(\frac{E}{T_\alpha}\right)} \qquad \langle E_\alpha\rangle = \frac{7\pi^4}{180\zeta(3)}T_\alpha$$





Initial mean energies:

$\langle E_{\nu_e}\rangle = 10$ MeV $\qquad \langle E_{\bar{\nu}_e}\rangle = 15$ MeV $\qquad \langle E_{\nu_\mu}\rangle = \langle E_{\nu_\tau}\rangle = \langle E_{\bar{\nu}_\mu}\rangle = \langle E_{\bar{\nu}_\tau}\rangle = 24$ MeV

total released energy: $E_{tot} = 3\cdot 10^{46}$ J $\qquad$ distance: $R = 10$ kpc $\qquad L_\alpha = \frac{E_{tot}}{6\langle E_\alpha\rangle}$

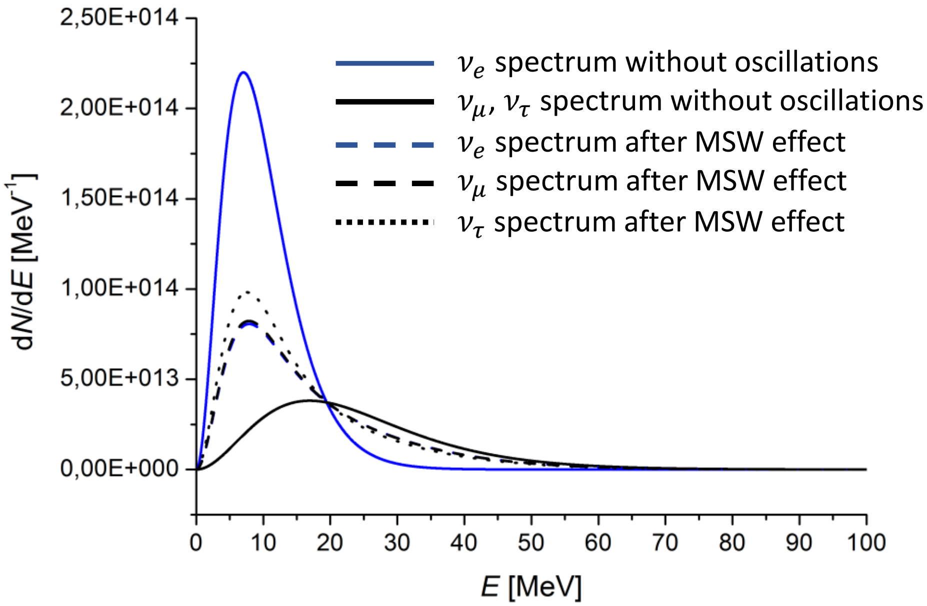
(equally distributed between neutrinos and antineutrinos of all flavors)

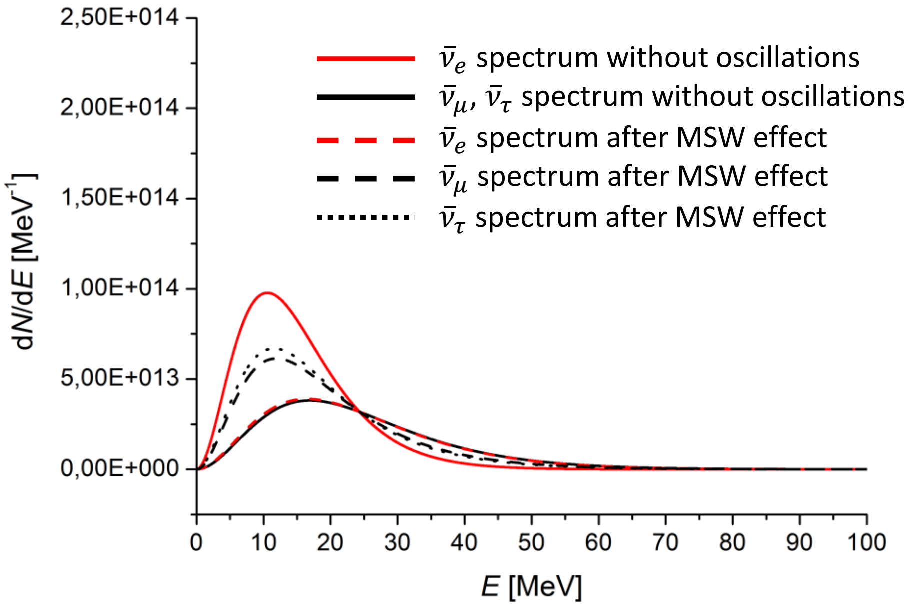
Time-integrated anti/neutrino spectra before and after MSW effect

Inverted hierarchy

$$j_\alpha(E) = \frac{L_\alpha}{4\pi R}\frac{2}{3\zeta(3){T_\alpha}^3}\frac{E^2}{1+\exp\left(\frac{E}{T_\alpha}\right)}$$

$$\langle E_\alpha \rangle = \frac{7\pi^4}{180\zeta(3)}T_\alpha$$





Initial mean energies:

$\langle E_{\nu_e} \rangle = 10$ MeV $\quad \langle E_{\bar{\nu}_e} \rangle = 15$ MeV $\quad \langle E_{\nu_\mu} \rangle = \langle E_{\nu_\tau} \rangle = \langle E_{\bar{\nu}_\mu} \rangle = \langle E_{\bar{\nu}_\tau} \rangle = 24$ MeV

total released energy: $E_{tot} = 3 \cdot 10^{46}$ J

(equally distributed between neutrinos and antineutrinos of all flavors)

distance: $R = 10$ kpc

$$L_\alpha = \frac{E_{tot}}{6\langle E_\alpha \rangle}$$

Detection of SN neutrinos in JUNO

The aim of this study is:

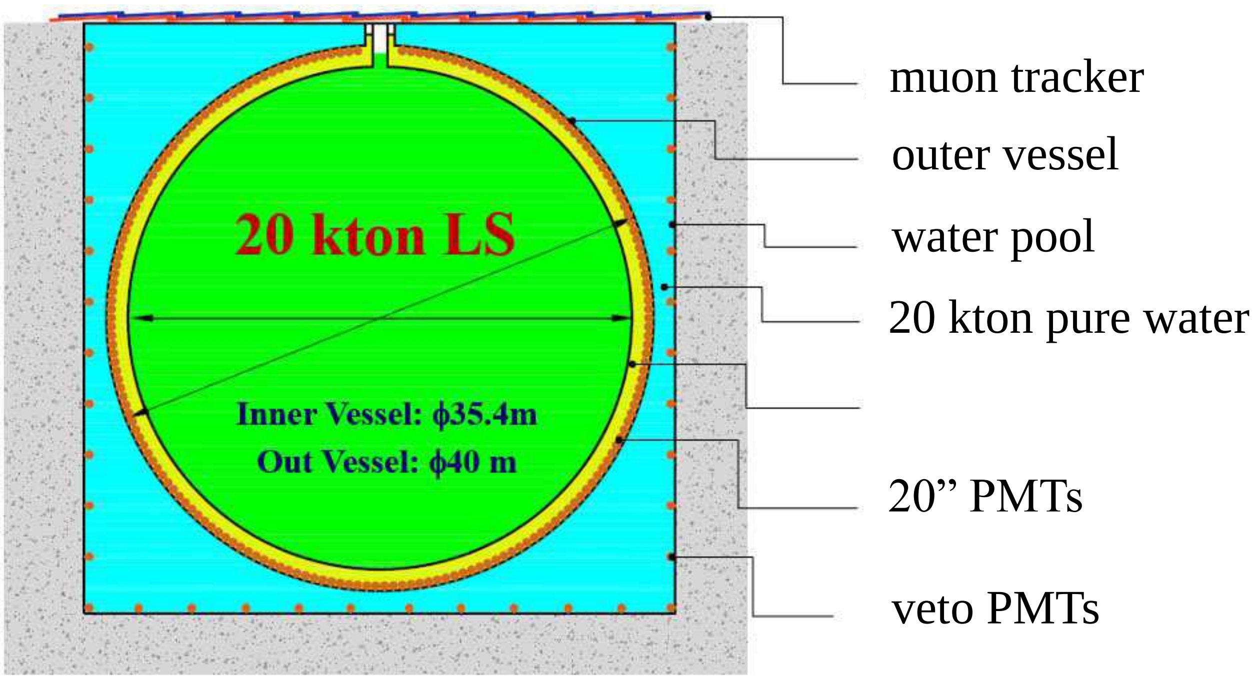
Investigation of the possibilities of JUNO experiment to determine fluxes and spectra of neutrinos coming from a supernova explosion

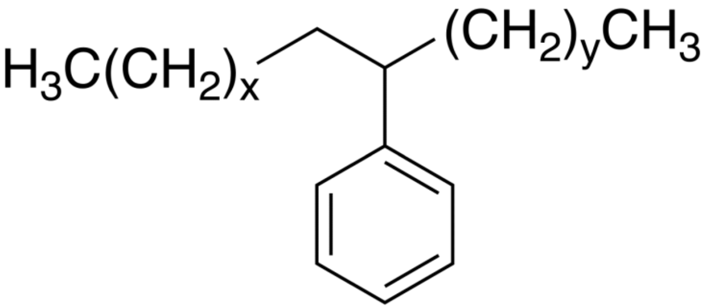
Detection of supernova neutrinos has the potential to supply information about:

1) **neutrino mass hierarchy**
   (neutrino flavor oscillations due to MSW effect are different in the case of normal and inverted hierarchy)

2) **the process of supernova explosion**
   (especially if neutrino mass hierarchy is already known)

JUNO experiment





20 kton liquid scintillator: linear alkylbenzene

$1{,}5 \cdot 10^{33}$ target protons

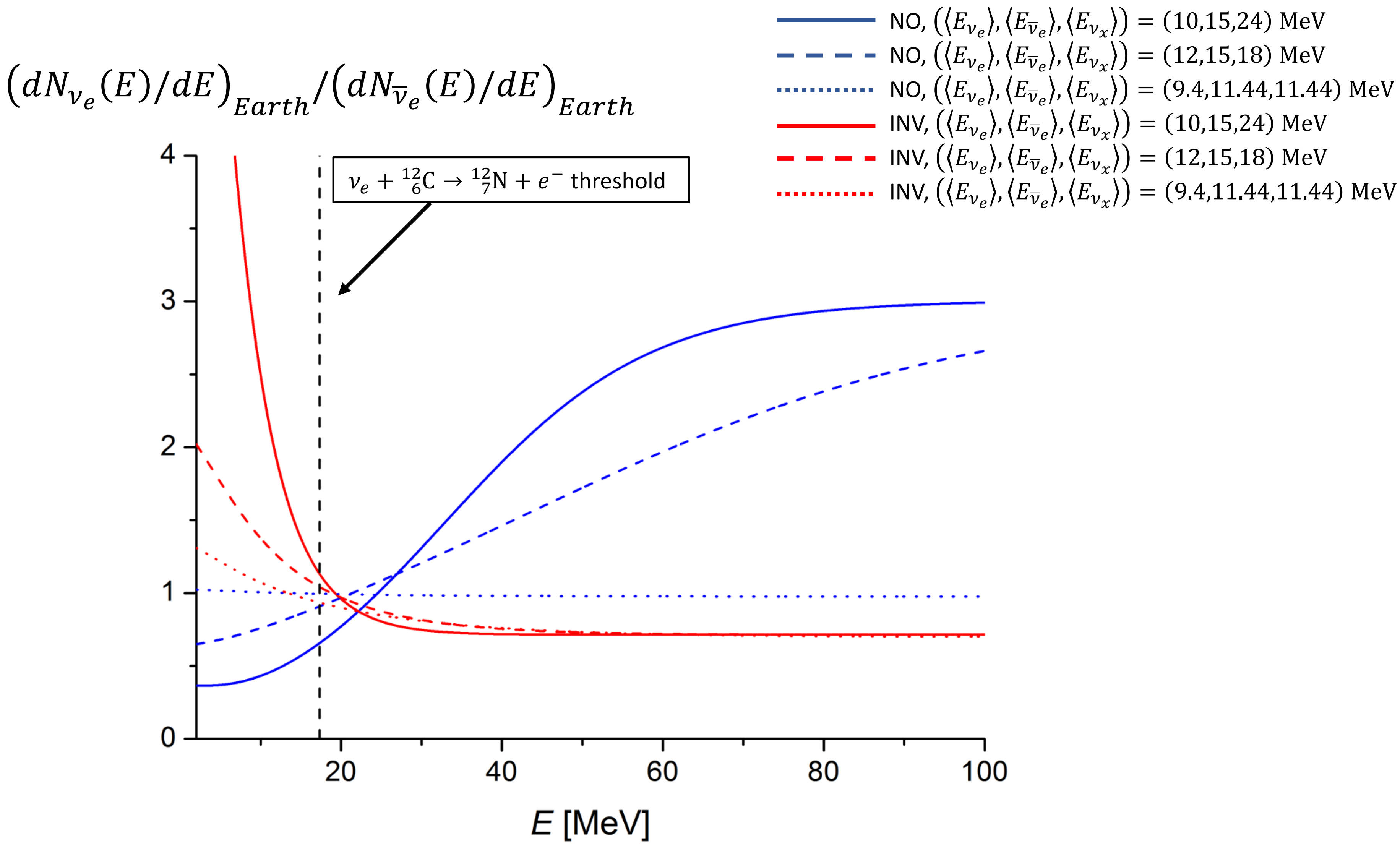
$8{,}8 \cdot 10^{32}$ target ^{12}C nuclei

Detection channels in JUNO

channel	type	threshold [MeV]
$\bar{\nu}_e + p \rightarrow e^+ + n$	CC	1,806
$\nu + p \rightarrow \nu + p$	NC	-
$\nu + e \rightarrow \nu + e$	ES	-
$\nu + ^{12}\mathrm{C} \rightarrow \nu + ^{12}\mathrm{C}^*$	NC	15,11
$\nu_e + ^{12}\mathrm{C} \rightarrow e^- + ^{12}\mathrm{N}$	CC	17,35
$\bar{\nu}_e + ^{12}\mathrm{C} \rightarrow e^+ + ^{12}\mathrm{B}$	CC	14,40

In this study we will focus on charged-current detection channels.

Ratio of ν_e and $\bar{\nu}_e$ spectra coming to Earth



Relations between neutrino fluxes: NORMAL hierarchy

$$\left(\frac{dN_{\nu_e}(E)}{dE}\right)_{Earth} \cong \frac{1}{2}\left(\frac{dN_{\nu_\mu}(E)}{dE}\right)_{initial} + \frac{1}{2}\left(\frac{dN_{\nu_\tau}(E)}{dE}\right)_{initial} = \left(\frac{dN_{\nu_x}(E)}{dE}\right)_{initial}$$

$$\left(\frac{dN_{\overline{\nu}_e}(E)}{dE}\right)_{Earth} \cong \frac{2}{3}\left(\frac{dN_{\overline{\nu}_e}(E)}{dE}\right)_{initial} + \frac{1}{6}\left(\frac{dN_{\overline{\nu}_\mu}(E)}{dE}\right)_{initial} + \frac{1}{6}\left(\frac{dN_{\overline{\nu}_\tau}(E)}{dE}\right)_{initial} =$$

$$= \frac{2}{3}\left(\frac{dN_{\overline{\nu}_e}(E)}{dE}\right)_{initial} + \frac{1}{3}\left(\frac{dN_{\overline{\nu}_x}(E)}{dE}\right)_{initial}$$

$$\frac{\left(\frac{dN_{\nu_e}(E)}{dE}\right)_{Earth}}{\left(\frac{dN_{\overline{\nu}_e}(E)}{dE}\right)_{Earth}} \cong \frac{\left(\frac{dN_{\nu_x}(E)}{dE}\right)_{initial}}{\frac{2}{3}\left(\frac{dN_{\overline{\nu}_e}(E)}{dE}\right)_{initial} + \frac{1}{3}\left(\frac{dN_{\overline{\nu}_x}(E)}{dE}\right)_{initial}}$$

Relations between neutrino fluxes: INVERTED hierarchy

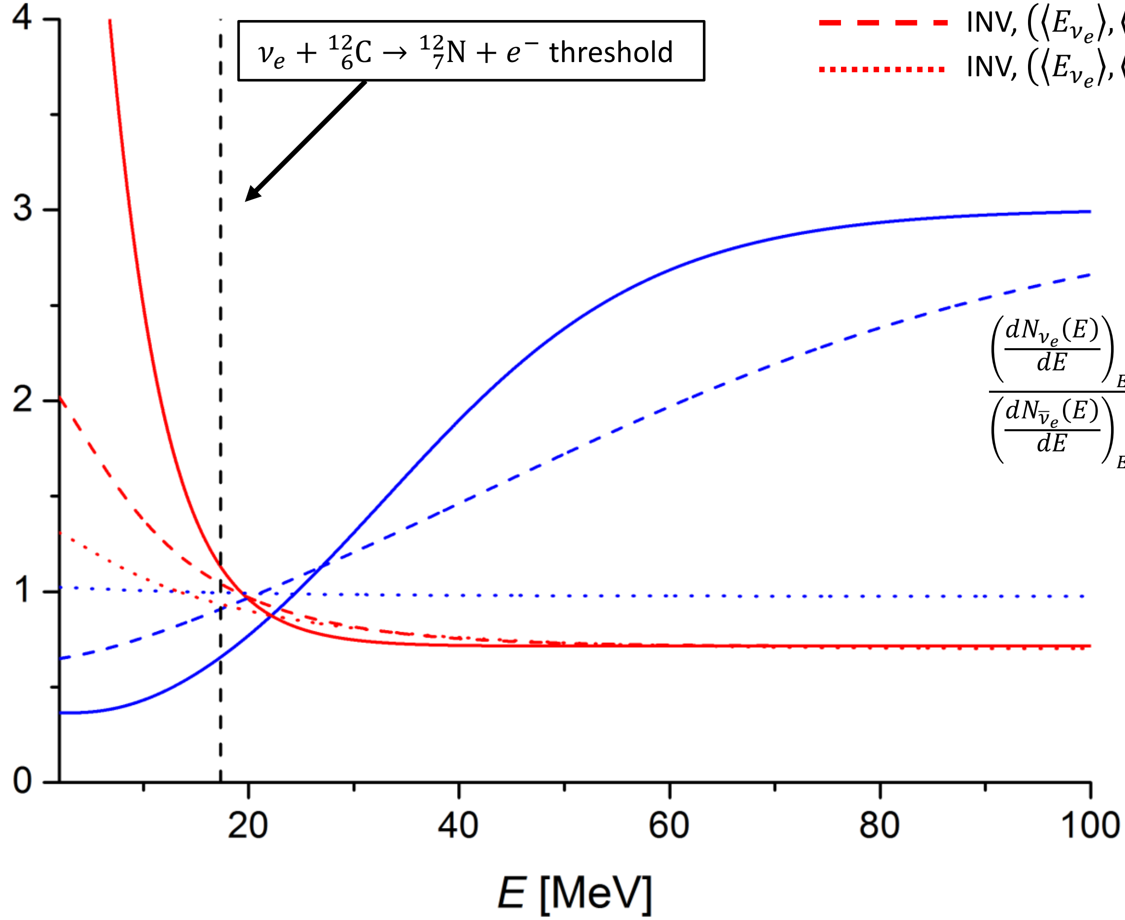
$$\left(\frac{dN_{\nu_e}(E)}{dE}\right)_{Earth} \cong \frac{1}{3}\left(\frac{dN_{\nu_e}(E)}{dE}\right)_{initial} + \frac{1}{3}\left(\frac{dN_{\nu_\mu}(E)}{dE}\right)_{initial} + \frac{1}{3}\left(\frac{dN_{\nu_\tau}(E)}{dE}\right)_{initial} =$$

$$= \frac{1}{3}\left(\frac{dN_{\nu_e}(E)}{dE}\right)_{initial} + \frac{2}{3}\left(\frac{dN_{\nu_x}(E)}{dE}\right)_{initial}$$

$$\left(\frac{dN_{\overline{\nu}_e}(E)}{dE}\right)_{Earth} \cong \frac{1}{2}\left(\frac{dN_{\overline{\nu}_\mu}(E)}{dE}\right)_{initial} + \frac{1}{2}\left(\frac{dN_{\overline{\nu}_\tau}(E)}{dE}\right)_{initial} = \left(\frac{dN_{\overline{\nu}_x}(E)}{dE}\right)_{initial}$$

$$\frac{\left(\frac{dN_{\nu_e}(E)}{dE}\right)_{Earth}}{\left(\frac{dN_{\overline{\nu}_e}(E)}{dE}\right)_{Earth}} \cong \frac{2}{3} + \frac{1}{3}\frac{\left(\frac{dN_{\nu_e}(E)}{dE}\right)_{initial}}{\left(\frac{dN_{\overline{\nu}_x}(E)}{dE}\right)_{initial}}$$

Ratio of ν_e and $\bar{\nu}_e$ spectra coming to Earth



NORMAL hierarchy:

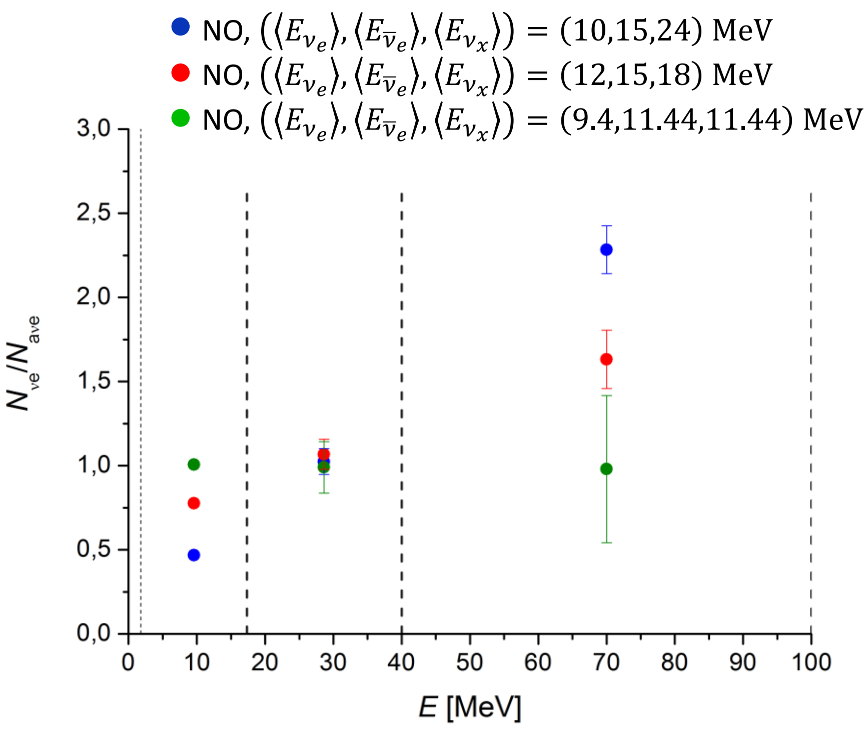
$$\frac{\left(\frac{dN_{\nu_e}(E)}{dE}\right)_{Earth}}{\left(\frac{dN_{\bar{\nu}_e}(E)}{dE}\right)_{Earth}} \cong \frac{\left(\frac{dN_{\nu_x}(E)}{dE}\right)_{initial}}{\frac{2}{3}\left(\frac{dN_{\bar{\nu}_e}(E)}{dE}\right)_{initial} + \frac{1}{3}\left(\frac{dN_{\bar{\nu}_x}(E)}{dE}\right)_{initial}}$$

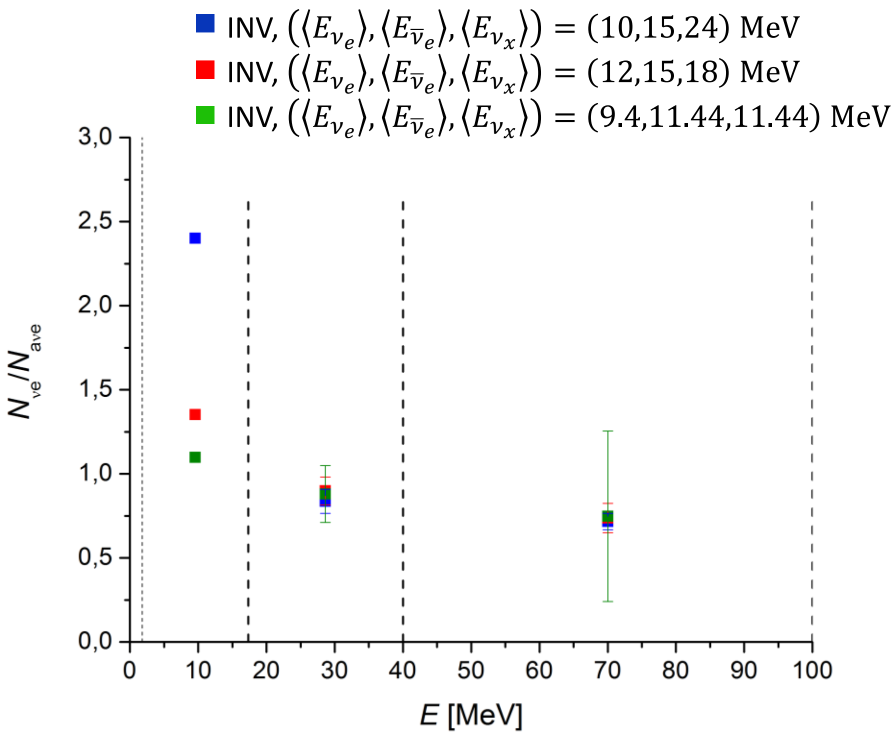
INVERTED hierarchy:

$$\frac{\left(\frac{dN_{\nu_e}(E)}{dE}\right)_{Earth}}{\left(\frac{dN_{\bar{\nu}_e}(E)}{dE}\right)_{Earth}} \cong \frac{2}{3} + \frac{1}{3}\frac{\left(\frac{dN_{\nu_e}(E)}{dE}\right)_{initial}}{\left(\frac{dN_{\bar{\nu}_x}(E)}{dE}\right)_{initial}}$$

Ratio of total numbers of ν_e and $\bar{\nu}_e$

Ratio of numbers of ν_e and $\bar{\nu}_e$ coming to Earth and having energy in one of the energy intervals marked by vertical dashed lines: 1,8 – 17,34 MeV, 17,34 – 40 MeV and 40 – 100 MeV





In the energy interval 17,34 – 40 MeV all SN models predict almost the same value $N_{\nu_e}/N_{\bar{\nu}_e}$ ratio regardless of neutrino mass hierarchy.

A value of $N_{\nu_e}/N_{\bar{\nu}_e}$ far from 1 in the energy interval 17,34 – 40 MeV would suggest that a physical process occurs during the SN collapse that is not included in current SN models.

Charged-current reactions on ^{12}C

$\nu_e + {}^{12}_{6}\mathrm{C} \rightarrow {}^{12}_{7}\mathrm{N} + e^-$ followed by ${}^{12}_{7}\mathrm{N} \rightarrow {}^{12}_{6}\mathrm{C} + e^+ + \nu_e$

$$E_{\nu_e}^{\mathrm{thr}} = \frac{(m_\mathrm{N} + m_e)^2 - m_\mathrm{C}^2}{2m_\mathrm{C}} = 17{,}35\ \mathrm{MeV}$$

$\bar{\nu}_e + {}^{12}_{6}\mathrm{C} \rightarrow {}^{12}_{5}\mathrm{B} + e^+$ followed by ${}^{12}_{5}\mathrm{B} \rightarrow {}^{12}_{6}\mathrm{C} + e^- + \bar{\nu}_e$

$$E_{\bar{\nu}_e}^{\mathrm{thr}} = \frac{(m_\mathrm{B} + m_e)^2 - m_\mathrm{C}^2}{2m_\mathrm{C}} = 14{,}40\ \mathrm{MeV}$$

Nuclear masses:

$$m_\mathrm{B} = m_\mathrm{B}^{atom} - 5m_e = 11188{,}74\ \mathrm{MeV}$$

$$m_\mathrm{C} = m_\mathrm{C}^{atom} - 6m_e = 11174{,}86\ \mathrm{MeV}$$

$$m_\mathrm{N} = m_\mathrm{N}^{atom} - 7m_e = 11191{,}69\ \mathrm{MeV}$$

Atomic masses:

$$m_\mathrm{B}^{atom} = 12{,}014352658\ \mathrm{u}$$

$$m_\mathrm{C}^{atom} = 12{,}000000000\ \mathrm{u}$$

$$m_\mathrm{N}^{atom} = 12{,}018613187\ \mathrm{u}$$

$$1\mathrm{u} = 931{,}494\ \mathrm{MeV}$$

Delayed beta decays

$${}^{12}_{7}\mathrm{N} \rightarrow {}^{12}_{6}\mathrm{C} + e^{+} + \nu_e \qquad \tau_{1/2}({}^{12}_{7}\mathrm{N}) = 11{,}00\ \mathrm{ms}$$

$$Q_{\beta^+} = m_\mathrm{N} - m_\mathrm{C} - m_e = 16{,}32\ \mathrm{MeV}$$

$$T_e^{\max} = \frac{m_\mathrm{N}^2 + m_e^2 - m_\mathrm{C}^2}{2m_\mathrm{N}} - m_e = 16{,}30\ \mathrm{MeV}$$

positron annihilation

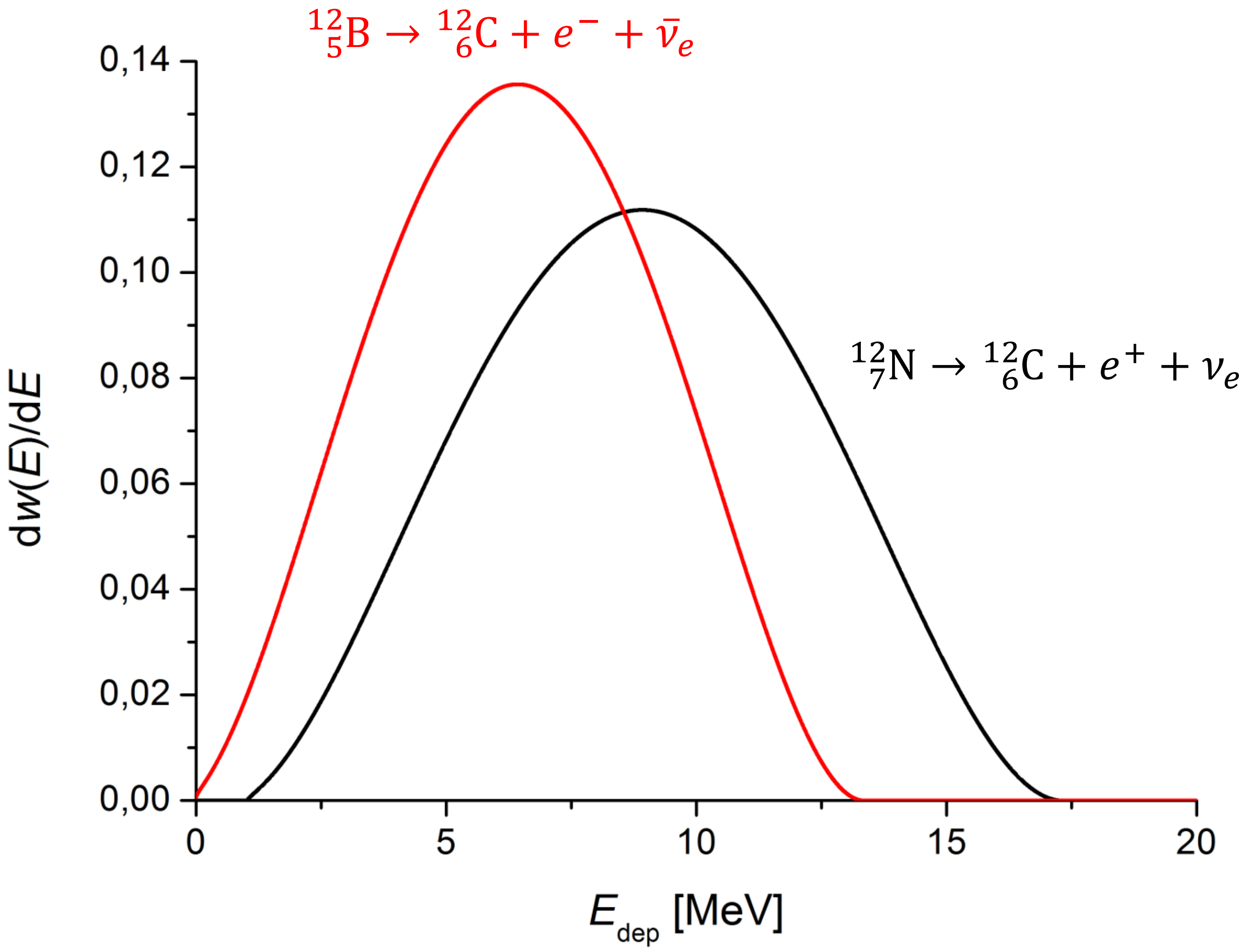
$$\Delta E = 1{,}022\ \mathrm{MeV}$$

$${}^{12}_{5}\mathrm{B} \rightarrow {}^{12}_{6}\mathrm{C} + e^{-} + \bar{\nu}_e \qquad \tau_{1/2}({}^{12}_{5}\mathrm{B}) = 20{,}20\ \mathrm{ms}$$

$$Q_{\beta^-} = m_\mathrm{B} - m_\mathrm{C} - m_e = 13{,}37\ \mathrm{MeV}$$

$$T_e^{\max} = \frac{m_\mathrm{B}^2 + m_e^2 - m_\mathrm{C}^2}{2m_\mathrm{B}} - m_e = 13{,}36\ \mathrm{MeV}$$

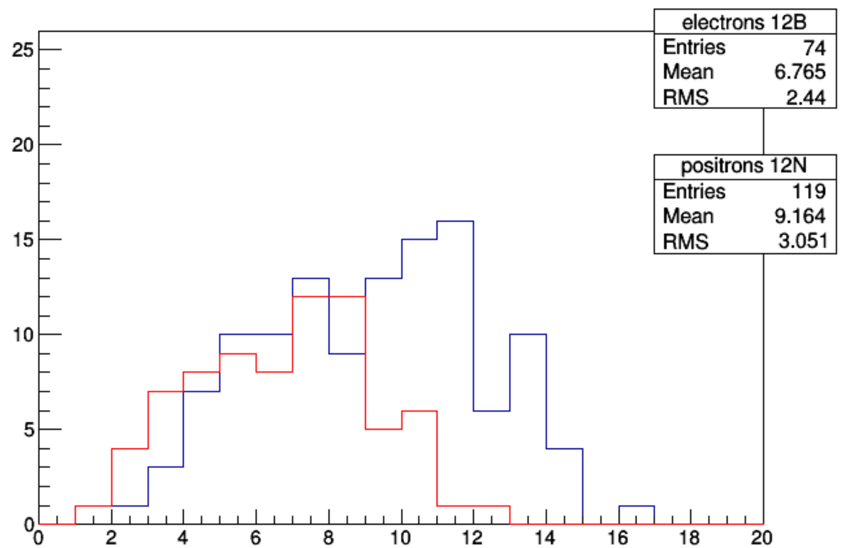
Distribution of deposited energy – delayed beta decays



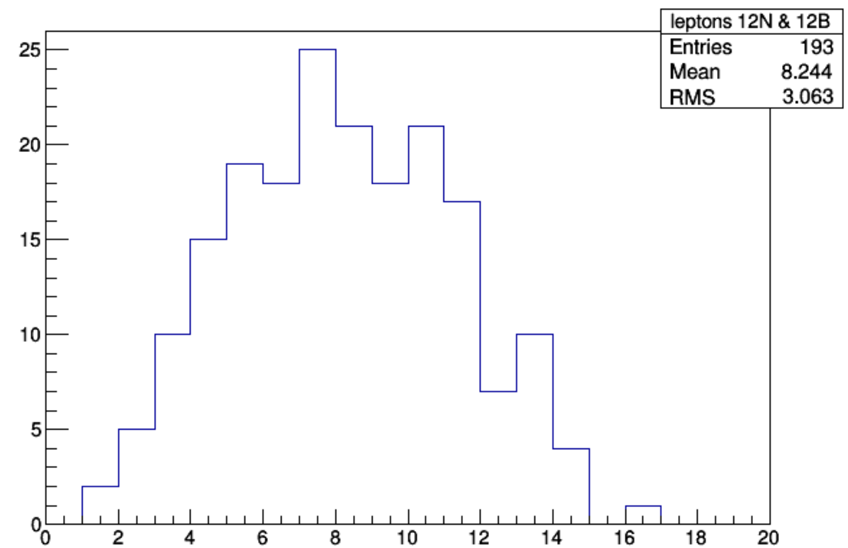
Distribution of deposited energy – delayed beta decays

$^{12}_{5}\mathrm{B} \rightarrow {}^{12}_{6}\mathrm{C} + e^- + \bar{\nu}_e$

$^{12}_{7}\mathrm{N} \rightarrow {}^{12}_{6}\mathrm{C} + e^+ + \nu_e$



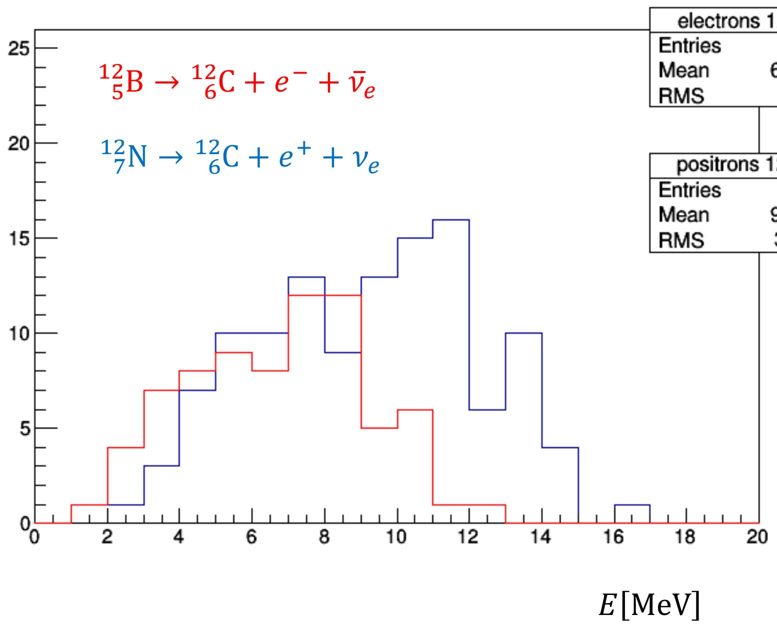
$E\,[\mathrm{MeV}]$

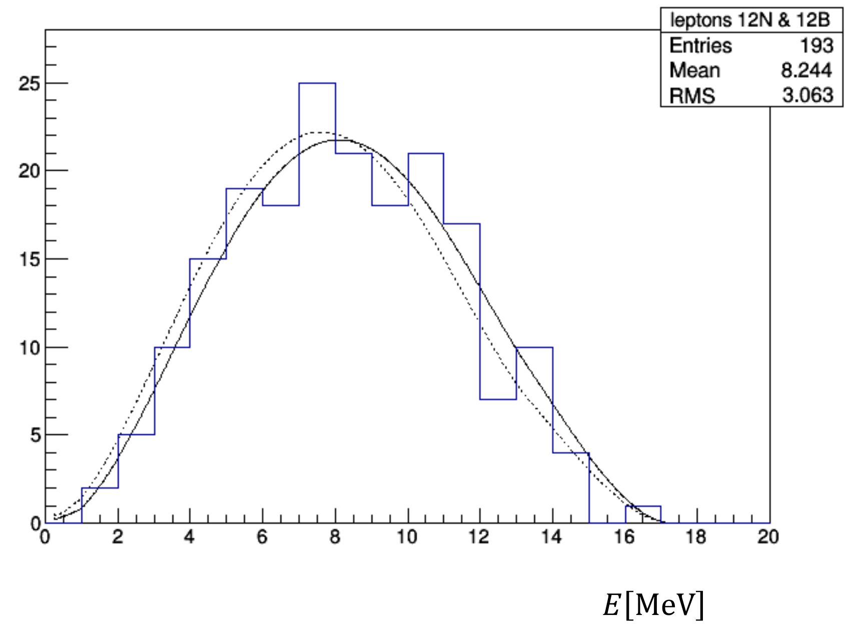


$E\,[\mathrm{MeV}]$

theoretical spectrum: $\beta(E) = \theta\beta_{^{12}N}(E) + (1-\theta)\beta_{^{12}B}(E)$

Distribution of deposited energy – delayed beta decays





theoretical spectrum: $\beta(E) = \theta\beta_{^{12}\mathrm{N}}(E) + (1-\theta)\beta_{^{12}\mathrm{B}}(E)$

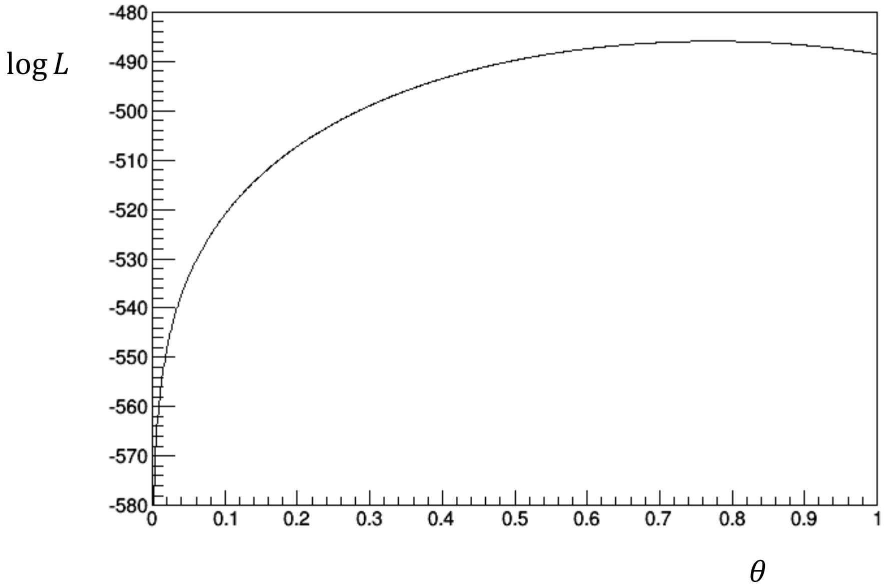
fixed total number of events $N_{tot} = 193$

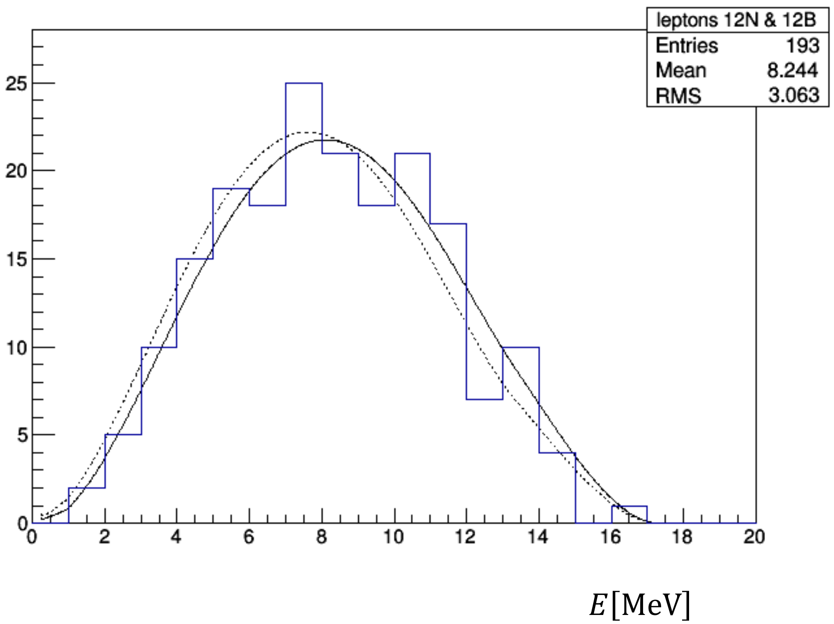
ML: $\hat{\theta} = 0{,}77$ $\hat{\sigma}_{\hat{\theta}} = 0{,}09$

$\theta = 0{,}62$

decay	MC	ML estimate
$^{12}\mathrm{N}$	119	150 ± 20
$^{12}\mathrm{B}$	74	44 ± 20

Distribution of deposited energy – delayed beta decays





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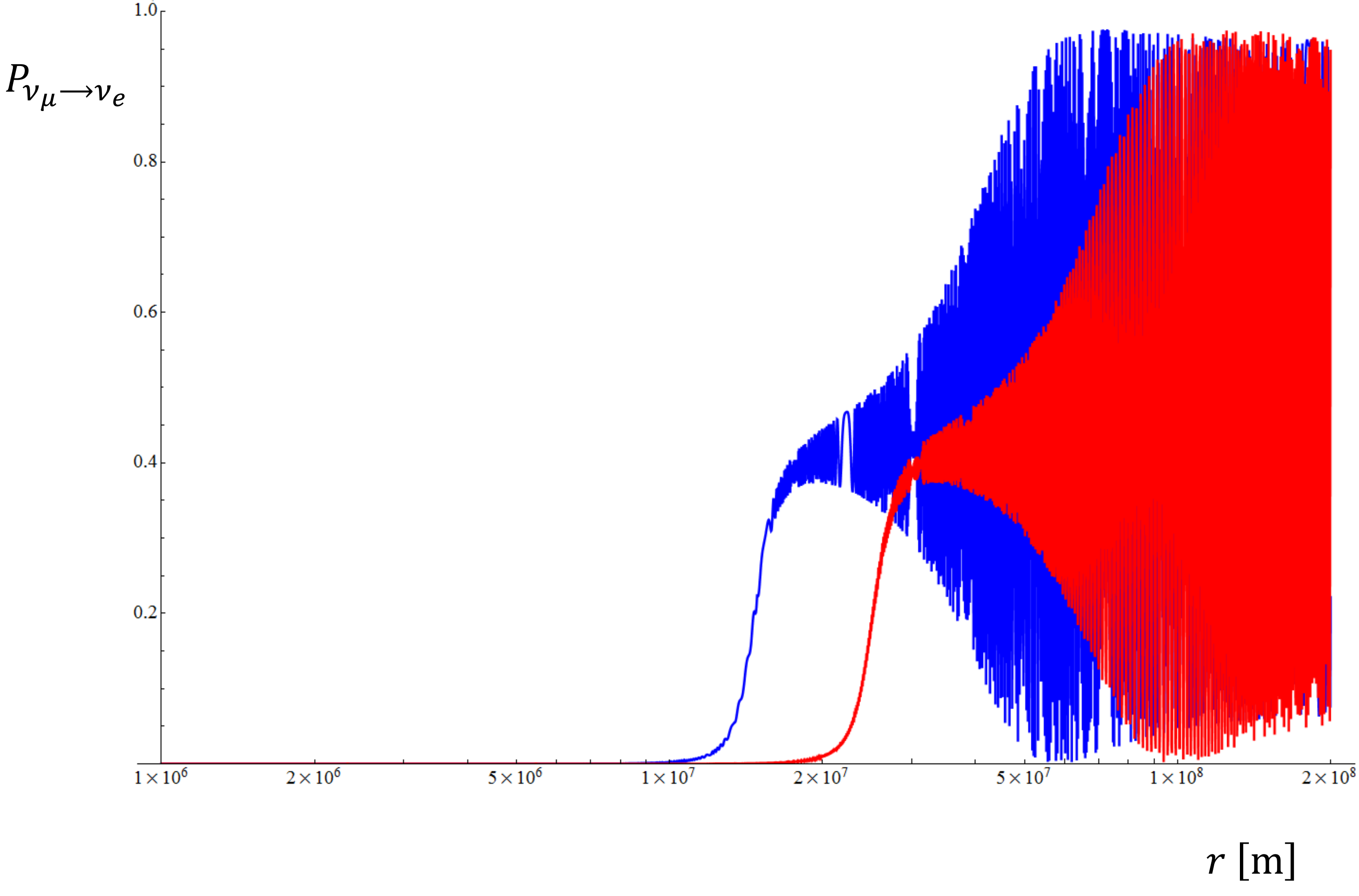
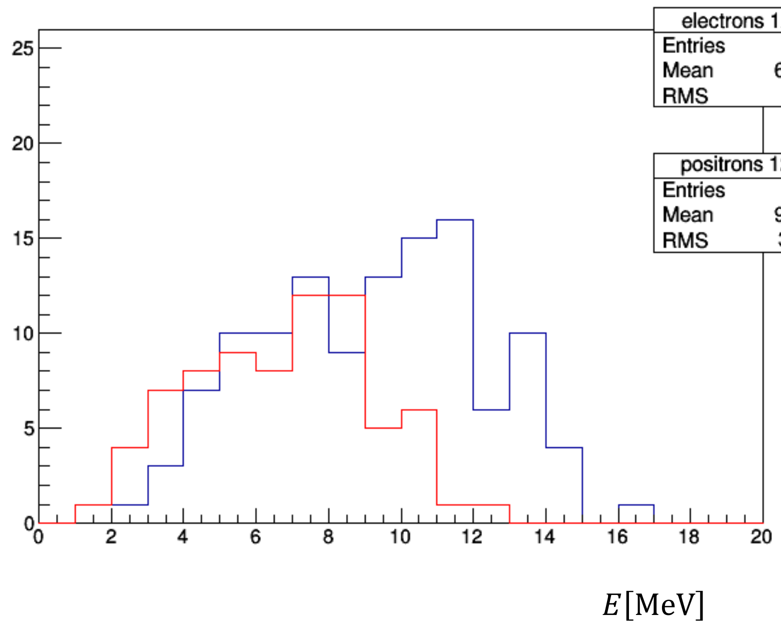
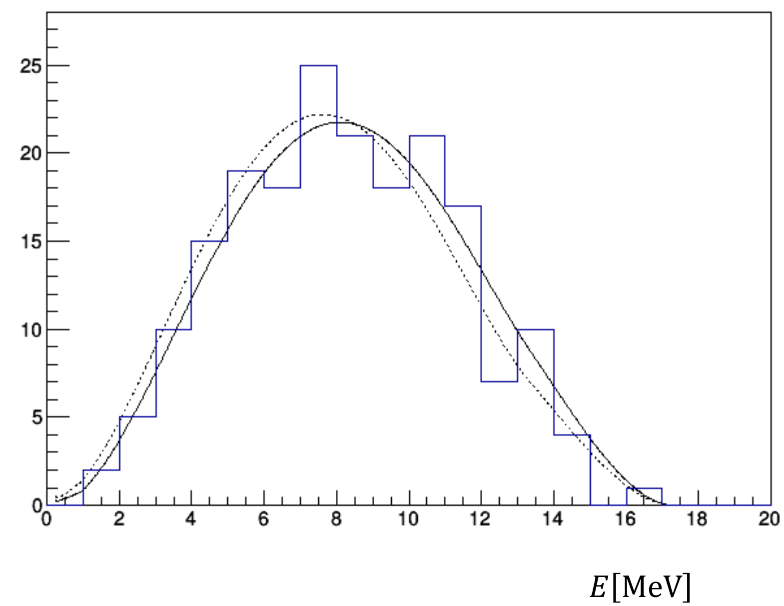
Thank you for your attention

Normal hierarchy $P_{\nu_\mu \to \nu_e}$



Distribution of deposited energy – delayed beta decays





theoretical spectrum: $\beta(E) = N_{^{12}\mathrm{N}}\, \beta_{^{12}\mathrm{N}}(E) + N_{^{12}\mathrm{B}}\, \beta_{^{12}\mathrm{B}}(E)$

total number of events $N_{tot} = N_{^{12}\mathrm{N}} + N_{^{12}\mathrm{B}}$ is a free parameter

decay	MC	ML estimate
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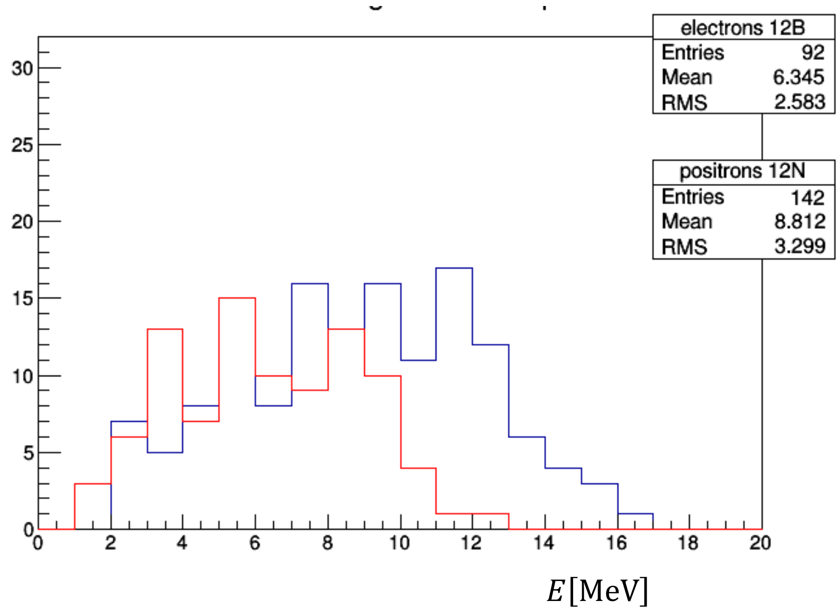
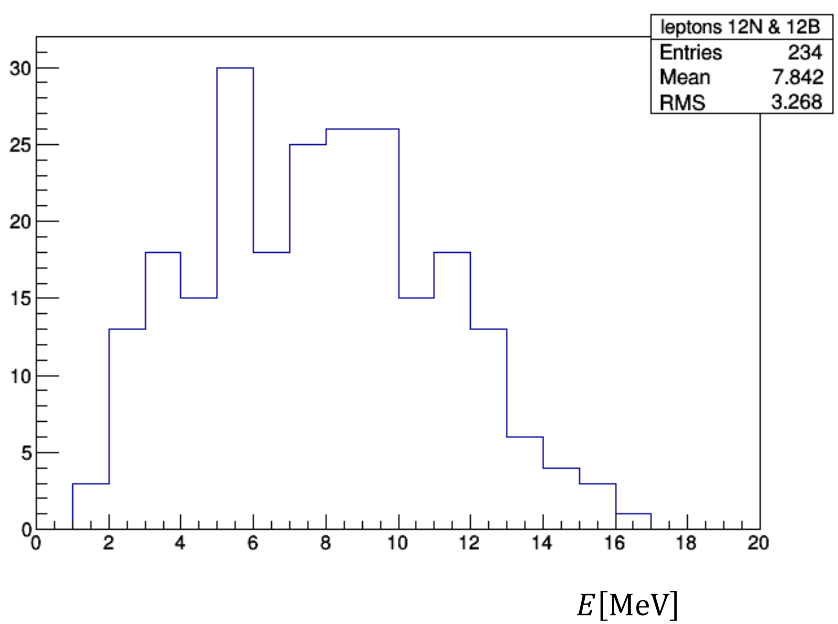
Distribution of deposited energy – delayed beta decays





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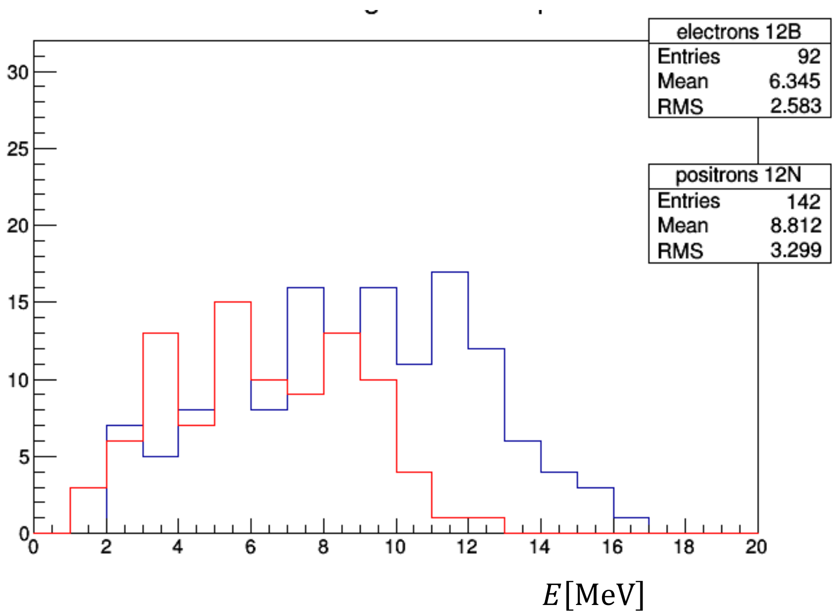
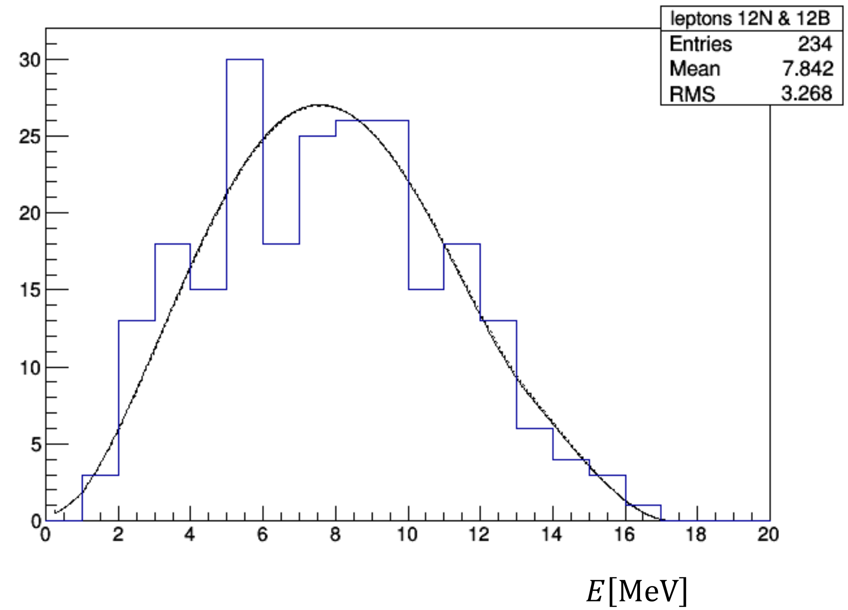
Distribution of deposited energy – delayed beta decays





theoretical spectrum: $\beta(E) = \theta\beta_{^{12}\mathrm{N}}(E) + (1-\theta)\beta_{^{12}\mathrm{B}}(E)$

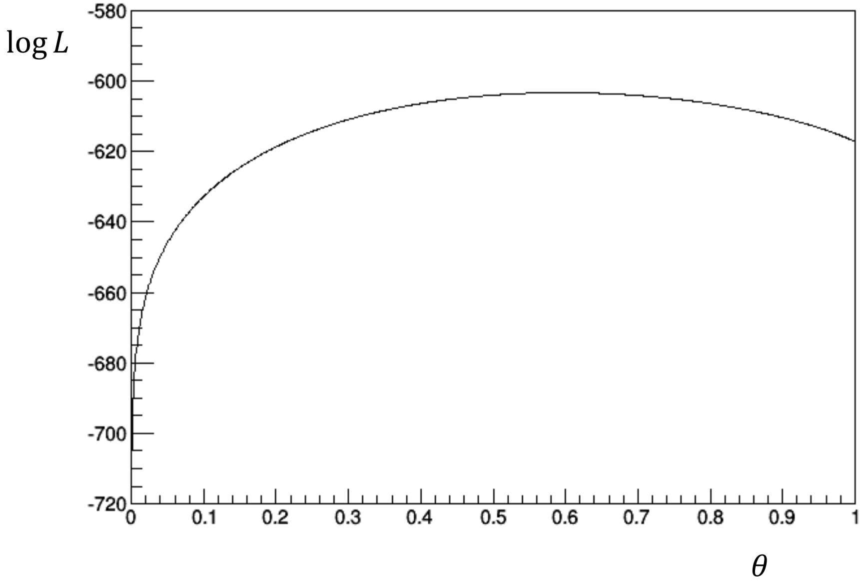
fixed total number of events $N_{tot} = 234$

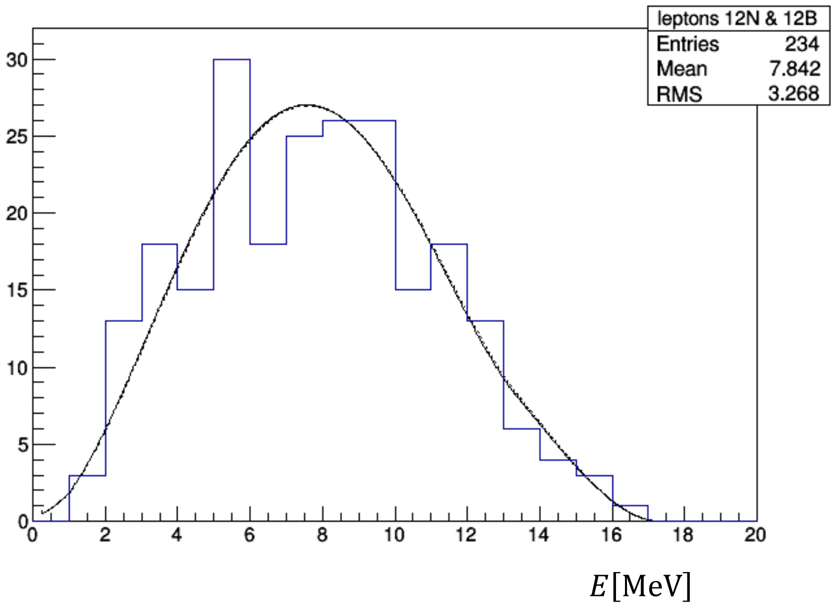
$\hat{\theta} = 0{,}59$ $\hat{\sigma}_{\hat{\theta}} = 0{,}08$

$\theta = 0{,}61$

decay	detected	estimated
$^{12}\mathrm{N}$	142	140 ± 20
$^{12}\mathrm{B}$	92	95 ± 20

Distribution of deposited energy – delayed beta decays





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