Induced global $U(1)$ symmetries of models containing $SU(3)$ or $SU(4)$ gauge symmetry

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Outline

Induced global $U(1)$ symmetries of models containing $SU(3)$ or $SU(4)$ gauge symmetry

- Induction $SU(N) \quad \rightarrow \quad U(N) = SU(N) \otimes U(1)$
- Standard model
  - Short review
  - Relationship between $SU(3)_c$ and Baryon number
- Simple extensions of the SM
  - SM $\oplus\ (3, 2, +7/6)_s$
  - SM $\oplus\ (3, 1, +2/3)_s$
  - SM $\oplus\ (3, 1, -4/3)_s$
- Pati–Salam $SU(4)_c$ models
  - Introduction
  - Global Matter symmetry of the model
  - Baryon number conservation in minimal P–S model

Revision: symmetries

Question: Why are symmetries of action important?

Anwser (Noether):

They imply the existence of conserved quantities.

Example: $U(1)$ symmetry of the action

$$S = \int \mathrm{d}^4x \; \mathcal{L}(\phi_A, \partial\phi_A)$$

$$\phi_A \to e^{iQ_A\alpha}\phi_A$$

implies conservation of charge Q carried by the fields with

$$Q(\phi_A) = Q_A$$

U(1) induction

Suppose we want to form invariants from field multiplets $\phi, \chi, \ldots$ which transform under various IRs of $SU(N)$, e.g.,

$$\phi \xrightarrow{\mathcal{U}} D^{\phi}(\mathcal{U})\,\phi, \qquad \chi \xrightarrow{\mathcal{U}} D^{\chi}(\mathcal{U})\,\chi, \qquad \mathcal{U} \in SU(N).$$

Theorem: Any IR of $SU(N)$ can be realized by a tensor with a certain number of lower and upper fundamental indices, though with extra restrictions like (anti)symmetry or tracelessness. Then the action of the group may look like

$$\phi^{ij} \xrightarrow{\mathcal{U}} \mathcal{U}^{i}{}_{k}\,\mathcal{U}^{j}{}_{l}\,\phi^{kl}, \qquad \chi_m \xrightarrow{\mathcal{U}} \chi_k\,\mathcal{U}^{*}_{m}{}^{k}, \qquad \mathcal{U} \in SU(N)$$

The invariants are then easily obtained by contracting the indices, e.g.,

$$\phi^{ij}\phi^{\dagger}_{ij}, \qquad \chi_m\,\chi^{\dagger m}, \qquad \phi^{ij}\,\chi_i\,\chi_j,$$

or, if $N = 2$, $\chi_i\phi^{ij}\varepsilon_{jk}\chi^{\dagger k}.$

U(1) induction

$$\phi^{ij}\phi^{\dagger}_{ij} \qquad \chi_m \chi^{\dagger m} \qquad \phi^{ij}\chi_i\chi_j$$

Key idea: The $SU(N)$ invariants above are also invariant w.r. to an overall $U(1)$ transformation

$$\phi^{ij} \xrightarrow{\alpha} e^{i\alpha Q_\phi}\phi^{ij} \qquad \chi_m \xrightarrow{\alpha} e^{i\alpha Q_\chi}\chi_m$$

where generally

$$\boxed{Q = (\#\,\text{upper indices}) - (\#\,\text{lower indices})}$$

i.e., $Q_\phi = +2$ $Q_\chi = -1$

This idea fails for the terms with Levi-Civita, e.g.

$$\chi_i\phi^{ij}\varepsilon_{jk}\chi^{\dagger k}$$

Standard model

Postulated gauge symmetry:

$$SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$$

Accidental global symmetries:

$$U(1)_B \otimes U(1)_e \otimes U(1)_\mu \otimes U(1)_\tau$$

. . . *massless neutrinos*

$$U(1)_B \otimes U(1)_L$$

. . . *massive oscilating neutrinos*

Standard model

$$
\begin{aligned}
\mathcal{L}_{SM} = &-\frac{1}{4}B_{\mu\nu}B^{\mu\nu} - \frac{1}{2}A^{i}_{j\mu\nu}A^{j\mu\nu}_{i} - \frac{1}{2}G^{\alpha}_{\mu\nu\beta}G^{\beta\mu\nu}_{\alpha} \\
&+ \overline{L}_i i \not{D}^{i}_{j} L^j + \overline{e} i \not{D} e + \overline{Q}_{i\alpha} i \not{D}^{i\alpha}_{j\beta} Q^{j\beta} + \overline{u}_{\alpha} i \not{D}^{\alpha}_{\beta} u^{\beta} + \overline{d}_{\alpha} i \not{D}^{\alpha}_{\beta} d^{\beta} \\
&+ (D_\mu H)^{\dagger}_{k} D^{k\mu}_{j} H^j + \mu^2 H^{\dagger}_{i} H^i - \lambda \left(H^{\dagger}_{i} H^i\right)^2 \\
&+ \left(\overline{Q}_{i\alpha} y_{(d)} d^{\alpha} \varepsilon^{ij} H^{\dagger}_{j} + \overline{Q}_{i\alpha} y_{(u)} u^{\alpha} H^i + \overline{L}_i y_{(e)} e H^i\right) + \mathrm{h.c.}
\end{aligned}
$$

Indices:

$i, j \in \{1, 2\}$	$SU(2)_L$ fundamental
$\alpha, \beta \in \{1, 2, 3\} = \{r, g, b\}$	$SU(3)_C$ fundamental
$\mu, \nu \in \{0, 1, 2, 3\}$	Lorentz $SO(1, 3)$ fundamental
implicit	Lorentz spinor indices

Standard model

$$\mathcal{L}_{SM} = -\frac{1}{4}B_{\mu\nu}B^{\mu\nu} - \frac{1}{2}A^{i}_{j\mu\nu}A^{j\mu\nu}_{i} - \frac{1}{2}G^{\alpha}_{\mu\nu\beta}G^{\beta\mu\nu}_{\alpha}$$
$$+ \overline{L}_i i\not{D}^i_j L^j + \overline{e} i\not{D} e + \overline{Q}_{i\alpha} i\not{D}^{i\alpha}_{j\beta} Q^{j\beta} + \overline{u}_\alpha i\not{D}^\alpha_\beta u^\beta + \overline{d}_\alpha i\not{D}^\alpha_\beta d^\beta$$
$$+ (D_\mu H)^\dagger_k D^{k\mu}_j H^j + \mu^2 H^\dagger_i H^i - \lambda \left(H^\dagger_i H^i\right)^2$$
$$+ \left(\overline{Q}_{i\alpha} y_{(d)} d^\alpha \varepsilon^{ij} H^\dagger_j + \overline{Q}_{i\alpha} y_{(u)} u^\alpha H^i + \overline{L}_i y_{(e)} e\, H^i\right) + \mathrm{h.c.}$$

Two-component fermion fields and thieir $SU(2)_L$ structure:

$$e \equiv e_R, \qquad u \equiv u_R, \qquad d \equiv d_R$$

$$L \equiv \begin{pmatrix} L^1 \\ L^2 \end{pmatrix} \equiv \begin{pmatrix} \nu_L \\ e_R \end{pmatrix}, \qquad Q^\alpha \equiv \begin{pmatrix} Q^{1\alpha} \\ Q^{2\alpha} \end{pmatrix} \equiv \begin{pmatrix} u^\alpha_L \\ d^\alpha_L \end{pmatrix}$$

Standard model

$$\mathcal{L}_{SM} = -\frac{1}{4}B_{\mu\nu}B^{\mu\nu} - \frac{1}{2}A^i_{j\mu\nu}A^{j\mu\nu}_i - \frac{1}{2}G^\alpha_{\mu\nu\beta}G^{\beta\mu\nu}_\alpha$$
$$+ \overline{L}_i i\not{D}^i_j L^j + \overline{e} i\not{D} e + \overline{Q}_{i\alpha} i\not{D}^{i\alpha}_{j\beta} Q^{j\beta} + \overline{u}_\alpha i\not{D}^\alpha_\beta u^\beta + \overline{d}_\alpha i\not{D}^\alpha_\beta d^\beta$$
$$+ (D_\mu H)^\dagger_k D^{k\mu}_j H^j + \mu^2 H^\dagger_i H^i - \lambda \left(H^\dagger_i H^i\right)^2$$
$$+ \left(\overline{Q}_{i\alpha} y_{(d)} d^\alpha \varepsilon^{ij} H^\dagger_j + \overline{Q}_{i\alpha} y_{(u)} u^\alpha H^i + \overline{L}_i y_{(e)} e H^i\right) + \text{h.c.}$$

Non-Abelian gauge fields (fundamental vs. adjoint indices):

$$A^i_{j\mu} = \frac{1}{2}(\sigma^a)^i{}_j A^a_\mu, \quad a \in \{1,2,3\}$$

$$G^\alpha_{\beta\mu} = \frac{1}{2}(\lambda^c)^\alpha_\beta G^c_\mu \quad c \in \{1,2,\ldots,8\}$$

Standard model

$$
\begin{aligned}
\mathcal{L}_{SM} = & -\frac{1}{4}B_{\mu\nu}B^{\mu\nu} - \frac{1}{2}A^{i}_{j\mu\nu}A^{j\mu\nu}_{i} - \frac{1}{2}G^{\alpha}_{\mu\nu\beta}G^{\beta\mu\nu}_{\alpha} \\
& + \overline{L}_i i \not{D}^{i}_{j} L^{j} + \overline{e} i \not{D} e + \overline{Q}_{i\alpha} i \not{D}^{i\alpha}_{j\beta} Q^{j\beta} + \overline{u}_{\alpha} i \not{D}^{\alpha}_{\beta} u^{\beta} + \overline{d}_{\alpha} i \not{D}^{\alpha}_{\beta} d^{\beta} \\
& + (D_{\mu}H)^{\dagger}_{k}\, D^{k\mu}_{j} H^{j} + \mu^{2} H^{\dagger}_{i} H^{i} - \lambda \left(H^{\dagger}_{i} H^{i}\right)^{2} \\
& + \left(\overline{Q}_{i\alpha} y_{(d)} d^{\alpha} \varepsilon^{ij} H^{\dagger}_{j} + \overline{Q}_{i\alpha} y_{(u)} u^{\alpha} H^{i} + \overline{L}_{i} y_{(e)} e\, H^{i}\right) + \mathrm{h.c.}
\end{aligned}
$$

Covariant derivative – with explicit indices, e.g.

$$
D^{i\alpha}_{j\beta\mu} = \delta^{i}_{j}\delta^{\alpha}_{\beta}\partial_{\mu} - ig'\delta^{i}_{j}\delta^{\alpha}_{\beta} Y B_{\mu} - ig_{3}\delta^{i}_{j} G^{\alpha}_{\beta\mu} - ig\,\delta^{\alpha}_{\beta} A^{i}_{j\mu}
$$

or implicitly, e.g., in the field strength definition:

$$
A^{i}_{j\mu\nu} \equiv \left(D_{[\mu}A_{\nu]}\right)^{i}_{\ j} = \partial_{[\mu}A^{i}_{\nu]j} - igA^{i}_{k[\mu}A^{k}_{\nu]j}.
$$

Standard model

$$\mathcal{L}_{SM} = -\frac{1}{4}B_{\mu\nu}B^{\mu\nu} - \frac{1}{2}A^{i}_{j\mu\nu}A^{j\mu\nu}_{i} - \frac{1}{2}G^{\alpha}_{\mu\nu\beta}G^{\beta\mu\nu}_{\alpha}$$
$$+ \overline{L}_i i \not{D}^i_j L^j + \overline{e} i \not{D} e + \overline{Q}_{i\alpha} i \not{D}^{i\alpha}_{j\beta} Q^{j\beta} + \overline{u}_\alpha i \not{D}^\alpha_\beta u^\beta + \overline{d}_\alpha i \not{D}^\alpha_\beta d^\beta$$
$$+ (D_\mu H)^\dagger_k D^{k\mu}_j H^j + \mu^2 H^\dagger_i H^i - \lambda \left(H^\dagger_i H^i\right)^2$$
$$+ \left(\overline{Q}_{i\alpha} y_{(d)} d^\alpha \varepsilon^{ij} H^\dagger_j + \overline{Q}_{i\alpha} y_{(u)} u^\alpha H^i + \overline{L}_i y_{(e)} e H^i\right) + \mathrm{h.c.}$$

3 generations:

$$e \to \begin{pmatrix} e \\ \mu \\ \tau \end{pmatrix}, \quad L^i \to \begin{pmatrix} L^i_{(e)} \\ L^i_{(\mu)} \\ L^i_{(\tau)} \end{pmatrix}, \quad u^\alpha \to \begin{pmatrix} u^\alpha \\ c^\alpha \\ t^\alpha \end{pmatrix}, \quad d^\alpha \to \begin{pmatrix} d^\alpha \\ s^\alpha \\ b^\alpha \end{pmatrix},$$

$$Q^{i\alpha} \to \begin{pmatrix} Q^{i\alpha}_{(1)} \\ Q^{i\alpha}_{(2)} \\ \cdots \end{pmatrix}, \qquad y_{(e)}, y_{(d)}, y_{(u)} \in \mathbb{C}^{3\times 3}$$

Baryon number

An extra global symmetry $U(1)_B$ related to $SU(3)_c$.
The "charges" are ascribed as follows:

- $+1/3$ for each upper $SU(3)$ index α
- $-1/3$ for each lower $SU(3)$ index α

Therefore:

field	B
$u^\alpha, d^\alpha, Q^{i\alpha}$	$+1/3$
$\bar{u}_\alpha, \bar{d}_\alpha, \bar{Q}_\alpha$	$-1/3$
$G^\alpha_{\ \beta\mu}$	0
$L^i, e, H^i, B_\mu, A^i_{\ j\mu}$	0

Nothing similar concerning SU(2)

The same procedure does not work for $SU(2)_L$, since

- there is an antisymmetric ε in the Lagrangian:

$$\overline{Q}_{i\alpha} y_{(d)} d^{\alpha} \varepsilon^{ij} H_j^{\dagger} \quad + \quad \text{h.c.}$$

- $SU(2)_L$ is spontaneously broken

Conservation of colors

- Combine gauge and global symmetry:

$$SU(3)_C \otimes U(1)_B = U(3)_{BC}$$

- Action of this global symmetry group on all SM fields realized via the multiplication by the same matrix $\mathcal{U} \in U(3)_{BC}$.
- Cartan subalgebra of $U(3)_{BC}$ spanned by

$$\lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \lambda_8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}, \quad B = \frac{1}{3} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

- Change the basis:

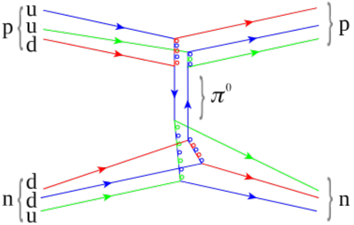
$$r = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad g = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad b = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix},$$

Conservation of colors

- The Cartan subgroup

$$U(1)_r \otimes U(1)_g \otimes U(1)_b \;\subset\; U(3)_{BC}$$

hence relates to the conservation of individual colors r,g,b

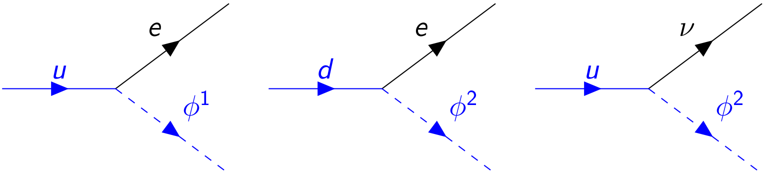


- This is not the case for the BSM theories with BNV

Scalar extensions of the SM

Example 1: Scalar leptoquark $\Phi^{\alpha i} \sim (3, 2, +7/6)$

$$\begin{aligned}\mathcal{L} = \mathcal{L}_{\rm SM} &+ (D_\mu \Phi)^\dagger_{i\alpha} (D^\mu \Phi)^{i\alpha} \\ &- \left(\overline{Q}_{i\alpha} y_{(1)} e\, \Phi^{i\alpha} + \text{h.c.}\right) - \left(\overline{u}_\alpha y_{(2)} L^i \varepsilon_{ij}\, \Phi^{j\alpha} + \text{h.c.}\right) \\ &- m_\phi^2 \left(\Phi^\dagger_{i\alpha} \Phi^{i\alpha}\right) - \kappa \left(\Phi^\dagger_{i\alpha} \Phi^{i\alpha}\right)^2 - \rho \left(\Phi^\dagger_{i\alpha} \Phi^{i\alpha}\right) \left(H^\dagger_j H^j\right)\end{aligned}$$



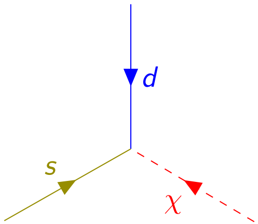
According to the general prescription,

$$B_\phi = +1/3$$

Scalar extensions of the SM

Example 2: Scalar diquark $\chi^\alpha \sim (3, 1, +2/3)$

$$\mathcal{L} = \mathcal{L}_{\mathrm{SM}} + (D_\mu \chi)^\dagger_\alpha \, (D^\mu \chi)^\alpha - \left(d^{\alpha\,\mathsf{T}} \mathcal{C} \, y \, d^\beta \chi^\gamma \varepsilon_{\alpha\beta\gamma} + \mathrm{h.c.} \right)$$
$$- m_\chi^2 \left(\chi^\dagger_\alpha \chi^\alpha \right) - \kappa \left(\chi^\dagger_\alpha \chi^\alpha \right)^2 - \rho \left(\chi^\dagger_\alpha \chi^\alpha \right) \left(H^\dagger_j H^j \right) .$$



Scalar extensions of the SM

Example 2: Scalar diquark $\chi^\alpha \sim (3, 1, +2/3)$

$$\mathcal{L} = \mathcal{L}_{\rm SM} + (D_\mu\chi)^\dagger_\alpha (D^\mu\chi)^\alpha - \left(d^{\alpha\mathsf{T}}\mathcal{C}\, y\, d^\beta \chi^\gamma \varepsilon_{\alpha\beta\gamma} + \text{h.c.}\right)$$
$$- m_\chi^2 \left(\chi^\dagger_\alpha \chi^\alpha\right) - \kappa \left(\chi^\dagger_\alpha \chi^\alpha\right)^2 - \rho \left(\chi^\dagger_\alpha \chi^\alpha\right)\left(H^\dagger_j H^j\right).$$

Redefinition:

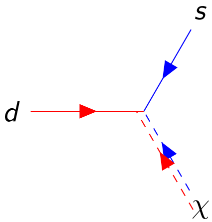
$$\chi_{\alpha\beta} = \chi^\gamma \varepsilon_{\alpha\beta\gamma} \qquad \Leftrightarrow \qquad \chi^\gamma = \frac{1}{2}\chi_{\alpha\beta}\, \varepsilon^{\alpha\beta\gamma}$$

the whole Lagrangian takes the form

$$\mathcal{L} = \mathcal{L}_{\rm SM} + \frac{1}{2}(D_\mu\chi)^{\dagger\alpha\beta} (D^\mu\chi)_{\alpha\beta} - \left(d^{\alpha\mathsf{T}}\mathcal{C}\, y\, d^\beta \chi_{\alpha\beta} + \text{h.c.}\right)$$
$$- \frac{1}{2}m_\chi^2 \left(\chi^{\dagger\alpha\beta} \chi_{\alpha\beta}\right) - \frac{1}{2}\kappa \left(\chi^{\dagger\alpha\beta} \chi_{\alpha\beta}\right)^2 - \frac{1}{2}\rho \left(\chi^{\dagger\alpha\beta} \chi_{\alpha\beta}\right)\left(H^\dagger_j H^j\right).$$

Example 2: Scalar diquark $\chi_{\alpha\beta} \sim (3, 1, +2/3)$

$$B\chi = -2/3$$



$$\mathcal{L} = \mathcal{L}_{\rm SM} + \frac{1}{2}(D_\mu\chi)^{\dagger\alpha\beta}(D^\mu\chi)_{\alpha\beta} - \left(d^{\alpha\,\mathsf{T}}\mathcal{C}\,y\,d^\beta\chi_{\alpha\beta} + \mathrm{h.c.}\right)$$
$$- \frac{1}{2}m_\chi^2\left(\chi^{\dagger\alpha\beta}\chi_{\alpha\beta}\right) - \frac{1}{2}\kappa\left(\chi^{\dagger\alpha\beta}\chi_{\alpha\beta}\right)^2 - \frac{1}{2}\rho\left(\chi^{\dagger\alpha\beta}\chi_{\alpha\beta}\right)\left(H_j^\dagger H^j\right).$$

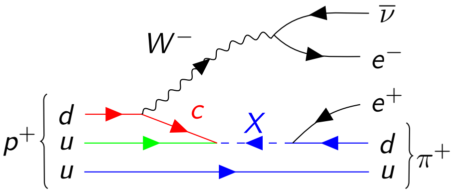
Scalar extensions of the SM

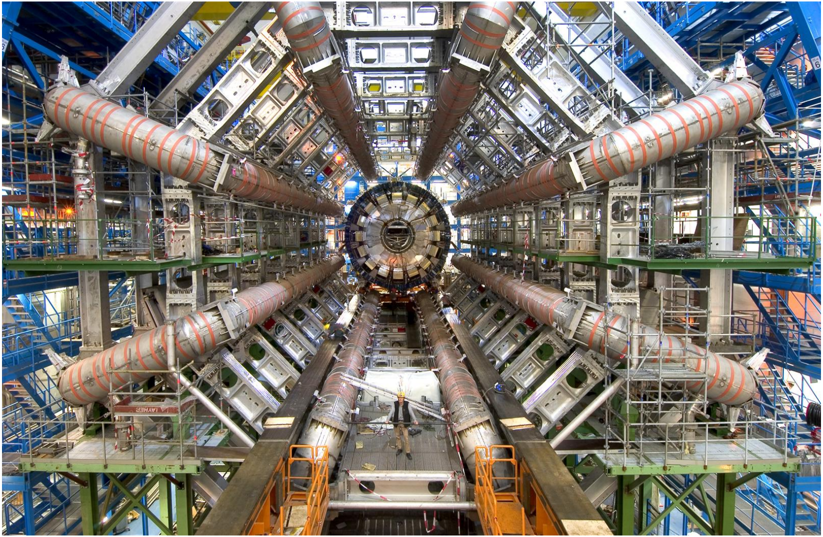
Example 3: BNV scalar $X^\alpha \sim (3, 1, -4/3)$

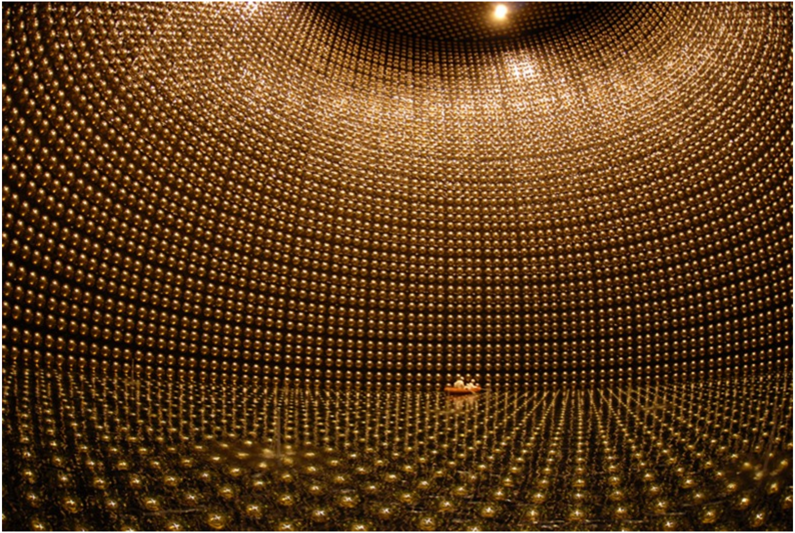
$$\mathcal{L}_{X\psi\psi} = u^{\alpha\,\mathsf{T}}\mathcal{C}\, y_{(1)}\, u^\beta X^\gamma \varepsilon_{\alpha\beta\gamma} + d^{\alpha\,\mathsf{T}}\mathcal{C}\, y_{(2)}\, e\, X^\dagger_\alpha \quad + \quad \text{h.c.}$$

Redefinition $X_{\alpha\beta} = X^\gamma\, \varepsilon_{\alpha\beta\gamma}$ would not help.

Proton decay:







Pati – Salam models

Introduction, main ideas:

- Lepton number as the fourth color,
  Phys.Rev. D10 (1974) 275-289
- $SU(3)_c$ replaced by $SU(4)_c$,
  quarks and leptons placed in IRs of this group together
- The observed difference between quarks and leptons achieved by spontaneous symmetry breaking

$$SU(4)_c \to SU(3)_c$$

Pati – Salam models

Particular minimal model [Perez, Wise]

- Gauge group and its SSB:

$$
\begin{aligned}
& SU(4)_C \otimes SU(2)_L \otimes U(1)_R \\
& \quad \supset \quad SU(3)_C \otimes U(1)_{B-L} \otimes SU(2)_L \otimes U(1)_R \\
& \quad \rightarrow \quad SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \equiv G_{\mathrm{SM}} \\
& \quad \rightarrow \quad SU(3)_C \otimes U(1)_Q .
\end{aligned}
$$

$$
\begin{aligned}
Y &= R + \frac{1}{2\sqrt{3}} T_{4C} = R + \frac{[B-L]}{2} \\
Q &= T_{3L} + Y
\end{aligned}
$$

Pati – Salam models

Particular minimal model [Perez, Wise]

- Gauge group and its SSB:

$$SU(4)_C \otimes SU(2)_L \otimes U(1)_R \to SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$$

- Fermions:

$$\begin{aligned}
\mathrm{F}^{ia} &= \left(Q^{i\alpha}, \quad L^i\right)_L && \sim \quad (4, 2, 0) \\
\mathrm{u}^a &= \left(u^{\alpha}, \quad \nu\right)_R && \sim \quad (4, 1, +1/2) \\
\mathrm{d}^a &= \left(d^{\alpha}, \quad e\right)_R && \sim \quad (4, 1, -1/2) \\
\psi & && \sim \quad (1, 1, 0)
\end{aligned}$$

Indices: $a \in \{1, 2, 3, 4\}, \qquad \alpha \in \{1, 2, 3\}$

Pati – Salam models

Particular minimal model [Perez, Wise]

- Gauge group and its SSB:

$$SU(4)_C \otimes SU(2)_L \otimes U(1)_R \to SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$$

- Scalars:

$$\begin{aligned} &H^i && \sim (1, 2, +1/2) \\ &\Phi^{ia}_b = \begin{pmatrix} \Phi^{i\alpha}_\beta & \phi^i_\beta \\ \varphi^{i\alpha} & 0 \end{pmatrix} + \frac{1}{2\sqrt{6}} \begin{pmatrix} 1_{3x3} & 0 \\ 0 & -3 \end{pmatrix} H^i_2 && \sim (15, 2, +1/2) \\ &\chi^a = (\chi^\alpha, \chi^L) && \sim (4, 1, +1/2) \end{aligned}$$

Indices: $a \in \{1, 2, 3, 4\}$, $\alpha \in \{1, 2, 3\}$

P–S: Lagrangian

$$\begin{aligned}\mathcal{L}_{PS} = &\bar{u}_a y_{(1)} F^{ia} \varepsilon_{ij} H^j + \bar{u}_a y_{(1)} F^{ib} \varepsilon_{ij} \Phi^{aj}_b \\ &+ \bar{d}_a y_{(3)} F^{ia} H^\dagger_i + \bar{d}_a y_{(4)} F^{ia} \Phi^{\dagger a}_{ib} \\ &+ \bar{u}_\alpha y_{(5)} \psi \chi^a + \frac{1}{2} \mu \psi \mathcal{C} \psi \quad \text{h.c.} \\ &+ \text{boring stuff}\end{aligned}$$

Pati – Salam models

No invariant term containing ε_{abcd} with mass–dimension ≤ 4 can be written

$\Rightarrow$ Extra global $U(1)_M$ symmetry – matter number M

field	F	u	d	ψ	χ	H	Φ
M	$+1$	$+1$	$+1$	0	$+1$	0	0

$$U(1)_M \otimes SU(4)_C = U(4)_{MC}$$

Pati – Salam models

$$\tilde{\lambda}_3 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \qquad \tilde{\lambda}_8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -2 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix},$$

$$[B-L] = \frac{1}{3} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -3 \end{pmatrix} \qquad M = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Alternative basis:

$$r = \mathrm{diag}(1,0,0,0)$$
$$g = \mathrm{diag}(0,1,0,0)$$
$$b = \mathrm{diag}(0,0,1,0)$$
$$L = \mathrm{diag}(0,0,0,1)$$

There is also

$$B = \frac{1}{3} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$B = (r+g+b)/3$$

Pati – Salam models

Spontaneous symmetry breaking

$$SU(4)_c \to SU(3)_C$$

SSB in practice:

- shift the relevant components of the scalar fields
- split the incides $a \to \alpha, L$
  For example: $\bar{F}_{ia}\, y_F\, u^a H^i = \bar{Q}_{i\alpha}\, y_F\, u^\alpha H^i + \bar{L}_i\, y_F\, \nu H^i$

The global $U(1)_M$ rises and falls together with $SU(4)_c$.
The only way how $\varepsilon_{\alpha\beta\gamma}$ could occur in the broken–phase $\mathcal{L}$ arises from $\varepsilon_{abcd} \to$ in the symmetrical phase.
But there is no such a term in the theory.
$\Rightarrow$ Baryon number conservation!

Conclusion

- $SU(N)$ invariants are often invariant also w.r.t. to $U(1)$
- $SU(3)_C$ in SM supports the conservation of B
- Main idea of Pati–Salam model
- Minimal P–S model does not decay proton