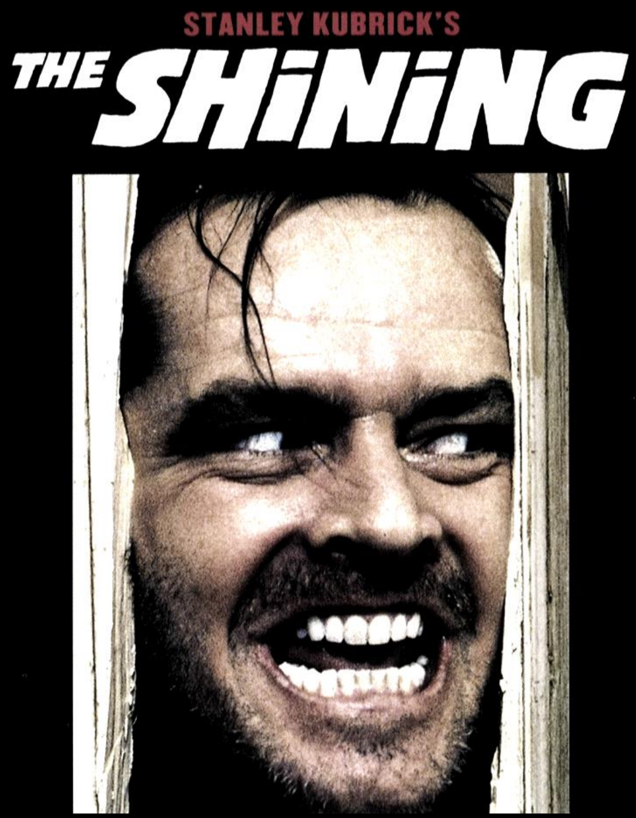
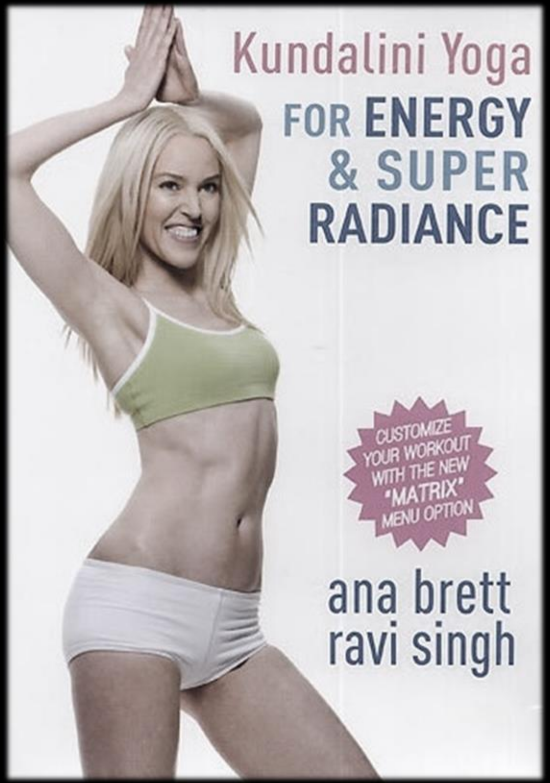
Pavel Cejnar
IPNP MFF UK

Superradiance





Legacy of Robert Dicke

Robert Henry Dicke (1916-97)

1946 – **Dicke radiometer**
(development of radar)

1953 – **Dicke narrowing**
(analogous to Mössbauer effect, 1958)

1954 – Dicke superradiance

1956 – **infrared laser**
(US patent in 1958, same as Townes & Schawlow in US, and Prokhorov in CCCP)

1965 – prediction (with Peebles, Roll & Wilkinson)
of **cosmic microwave background**
(1965 discovered by Penzias & Wilson using Dicke radiometer)

1957-70 – renaissance of **gravitation and cosmology**
(alternative theory to general relativity, anthropic principle…)

Dicke Superradiance

PHYSICAL REVIEW VOLUME 93, NUMBER 1 JANUARY 1, 1954

Coherence in Spontaneous Radiation Processes

R. H. DICKE
Palmer Physical Laboratory, Princeton University, Princeton, New Jersey
(Received August 25, 1953)

By considering a radiating gas as a single quantum-mechanical system, energy levels corresponding to certain correlations between individual molecules are described. Spontaneous emission of radiation in a transition between two such levels leads to the emission of coherent radiation. The discussion is limited first to a gas of dimension small compared with a wavelength. Spontaneous radiation rates and natural line breadths are calculated. For a gas of large extent the effect of photon recoil momentum on coherence is calculated. The effect of a radiation pulse in exciting "super-radiant" states is discussed. The angular correlation between successive photons spontaneously emitted by a gas initially in thermal equilibrium is calculated.

IN the usual treatment of spontaneous radiation by a gas, the radiation process is calculated as though the separate molecules radiate independently of each other. To justify this assumption it might be argued that, as a result of the large distance between molecules and subsequent weak interactions, the probability of a given molecule emitting a photon should be independent of the states of other molecules. It is clear that this model is incapable of describing a coherent spontaneous radiation process since the radiation rate is proportional

triplet and singlet states of the particles. The triplet state is capable of radiating to the ground state (triplet) but the singlet state will not couple ... system. Consequent...

..., no photon ... neutrons are in a singlet state and ... impossible to predict which neutron is the excited one.

WoS April 2017: ~4000 citations (+2)

Cited in: Quantum optics, Condensed matter + Solid state physics (incl. graphene), Astrophysics, Nuclear physics, Biophysics + Biology (incl. neurosciences), Engineering …

Dicke Superradiance

A collection of phenomena named *“superradiance”*

1) Superradiant flash

enhanced non-exponential irradiation of coherent sources

2) Superradiant phase transitions

thermal/quantum phase transitions from normal to superradiant phase
in interacting matter–field systems

3) Non-hermitian superradiance

separation of long- and short-lived states in systems coupled to continuum

4) Zel’dovich-Misner-Unruh superradiance

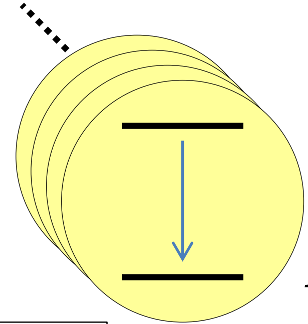
amplification of radiation by rotating black holes

....

....

Superradiant flash

A sample of N two-level atoms



n^*

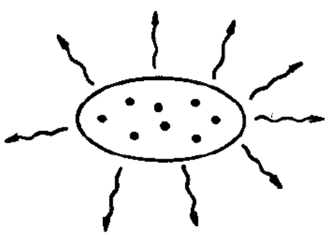
$N - n^*$

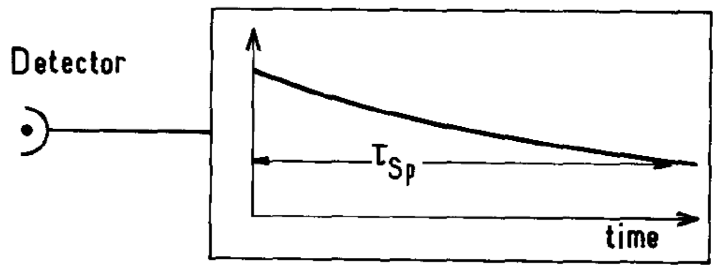
M. Gross, S. Haroche, *Phys. Rep.* 93, 301 (1982)

1) Independent radiators

$$n^*(t) = N e^{-t/\tau}$$

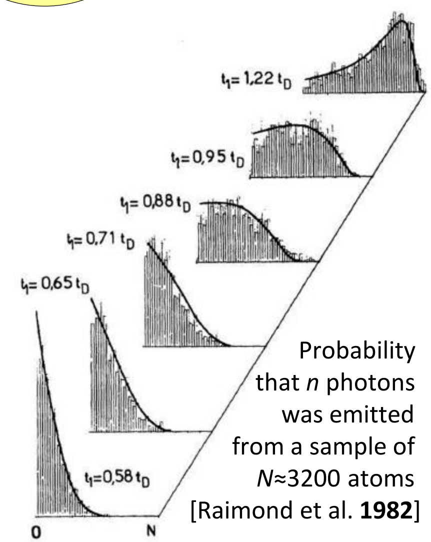
$$I(t) = -\frac{d}{dt} n^*(t) = \frac{N}{\tau} e^{-t/\tau}$$





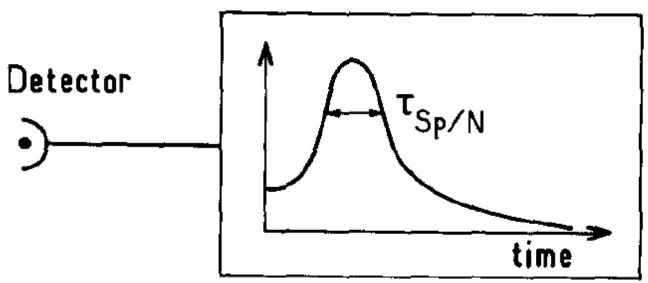
$$\Delta t \approx \tau$$

$$I \propto N$$



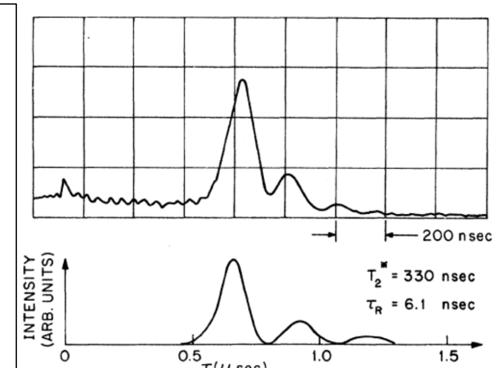
Probability that n photons was emitted from a sample of N≈3200 atoms [Raimond et al. **1982**]

2) Coherent (collective) radiators



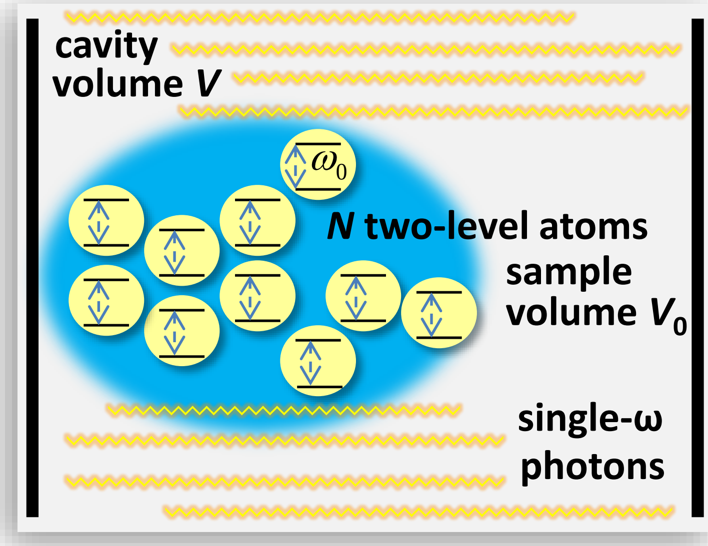
$$\Delta t \approx \frac{\tau}{N}$$

$$I \propto N^2$$



Superradiant pulse in optically pumped HF gas [Skribanowitz et al. **1973**]

Superradiant flash



Dicke model

A schematic model for cavity QED: interaction of single-mode radiation with two-level atoms

$$H = \omega_0 \sum_{i=1}^{N} \tfrac{1}{2}\underbrace{\sigma_z^i}_{\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}} + \omega \overbrace{b^+ b}^{n} + \sum_{i=1}^{N} \frac{\lambda_i}{\sqrt{N}} \underbrace{\sigma_x^i}_{D \propto \begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix} = \frac{1}{2}\sigma_+^i + \frac{1}{2}\sigma_-^i} \overbrace{(b^+ + b)}^{\propto \mathcal{E}}$$

$$\lambda_i = \frac{\omega_0 d}{\omega \mathcal{E}_0} \sqrt{\frac{V_0}{V} \underbrace{\frac{N}{V_0}}_{\rho}} \cos\varphi_i \quad \text{(phase)}$$

- dipole approximation
  D, d ... dipole operator, matrix element
  $\mathcal{E}$ ... electric field intensity
- neglect of the A^2 term

$$V_0 \ll V \;\Rightarrow\; \varphi_i \approx \varphi \;\Rightarrow\; \lambda_i \approx \lambda$$

$$H = \omega_0 \underbrace{\sum_{i=1}^{N} \tfrac{1}{2}\sigma_z^i}_{J_0} + \omega b^+ b + \frac{\lambda}{\sqrt{N}} \underbrace{\sum_{i=1}^{N} \left(\tfrac{1}{2}\sigma_+^i + \tfrac{1}{2}\sigma_-^i\right)}_{(J_+ + J_-)} (b^+ + b)$$

$$\mathbf{J} = \sum_{i=1}^{N} \tfrac{1}{2}\boldsymbol{\sigma}^i$$

total quasi-spin operators

$$H = \omega_0 J_0 + \omega b^+ b + \frac{\lambda}{\sqrt{N}} (J_+ + J_-)(b^+ + b)$$

Conserved quantity

$$\mathbf{J}^2 \to j(j+1)$$

Superradiant flash

Dicke model

Hilbert space $\mathcal{H} = \mathcal{H}_{\mathrm{A}} \otimes \mathcal{H}_{\mathrm{F}}$

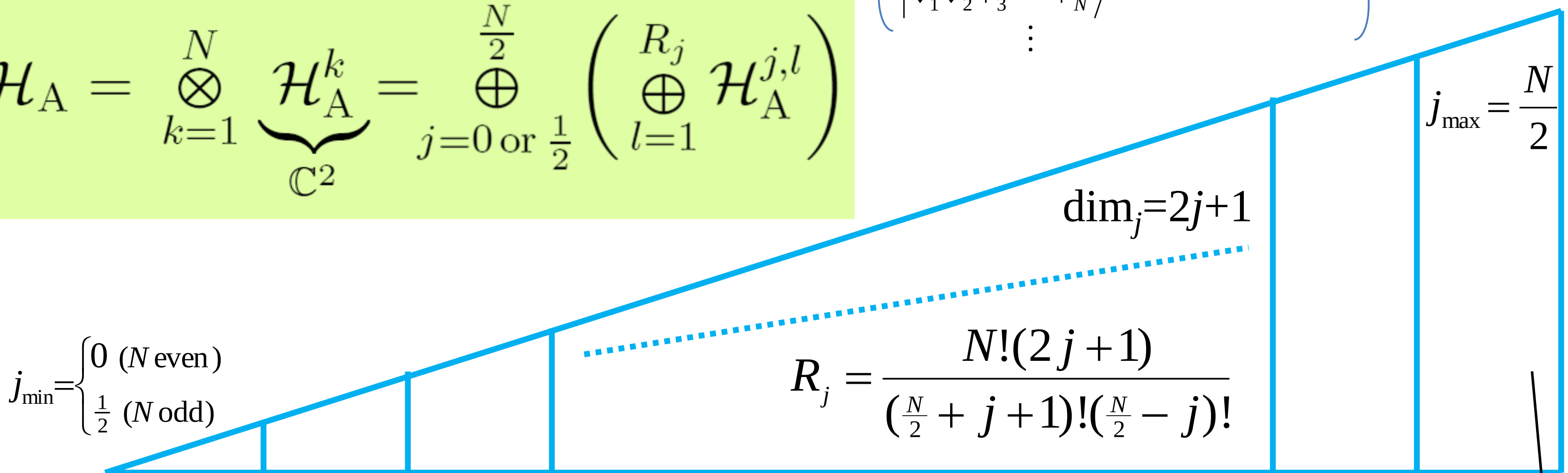
$\dim_{\mathrm{A}} = 2^N$

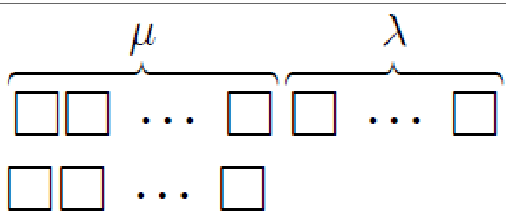
$$\left\{ \begin{array}{l} |\uparrow_1\uparrow_2\uparrow_3\cdots\uparrow_N\rangle \\ |\downarrow_1\uparrow_2\uparrow_3\cdots\uparrow_N\rangle \\ |\downarrow_1\downarrow_2\uparrow_3\cdots\uparrow_N\rangle \\ \vdots \end{array} \right.$$

bases $|n\rangle \equiv |0\rangle, |1\rangle, |2\rangle \cdots$

$\dim_{\mathrm{F}} = \infty$

$$\mathcal{H}_{\mathrm{A}} = \underbrace{\bigotimes_{k=1}^{N} \mathcal{H}_{\mathrm{A}}^{k}}_{\mathbb{C}^2} = \bigoplus_{j=0 \text{ or } \frac{1}{2}}^{\frac{N}{2}} \left(\bigoplus_{l=1}^{R_j} \mathcal{H}_{\mathrm{A}}^{j,l} \right)$$

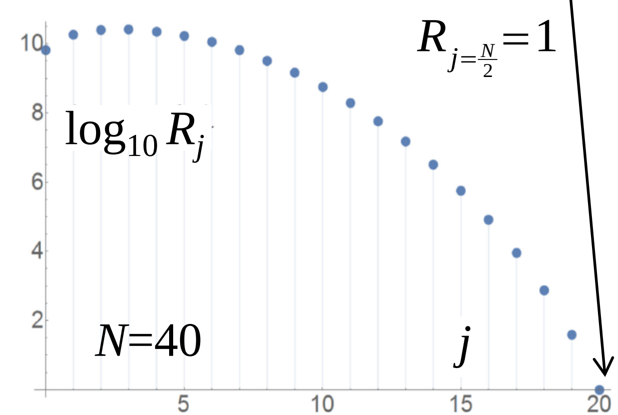




$\lambda = 2j$

$\mu = \frac{N}{2} - j$

$n^* = j + m \in [0, 2j]$ =**number of exc.atoms**



Superradiant flash

Dicke model

$$H = \omega_0 J_0 + \omega b^+ b + \underbrace{\frac{\lambda}{\sqrt{N}} \overbrace{(J_+ + J_-)(b^+ + b)}^{\approx J_+ b + J_- b^+ \text{ decay}}}_{H_{\text{int}}}$$

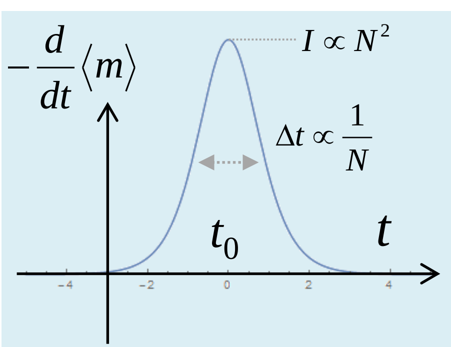
$$\langle j(m-1)|J_-|jm\rangle = \sqrt{j(j+1) - m(m-1)} = \sqrt{(j+m)(j-m+1)}$$

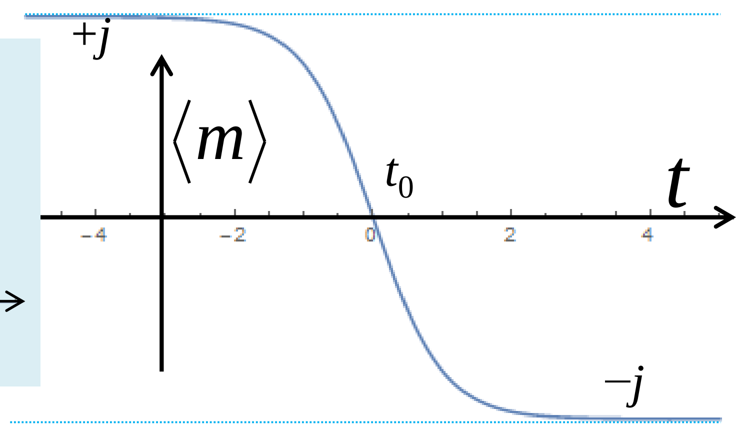
Approximate solution of the decay dynamics:

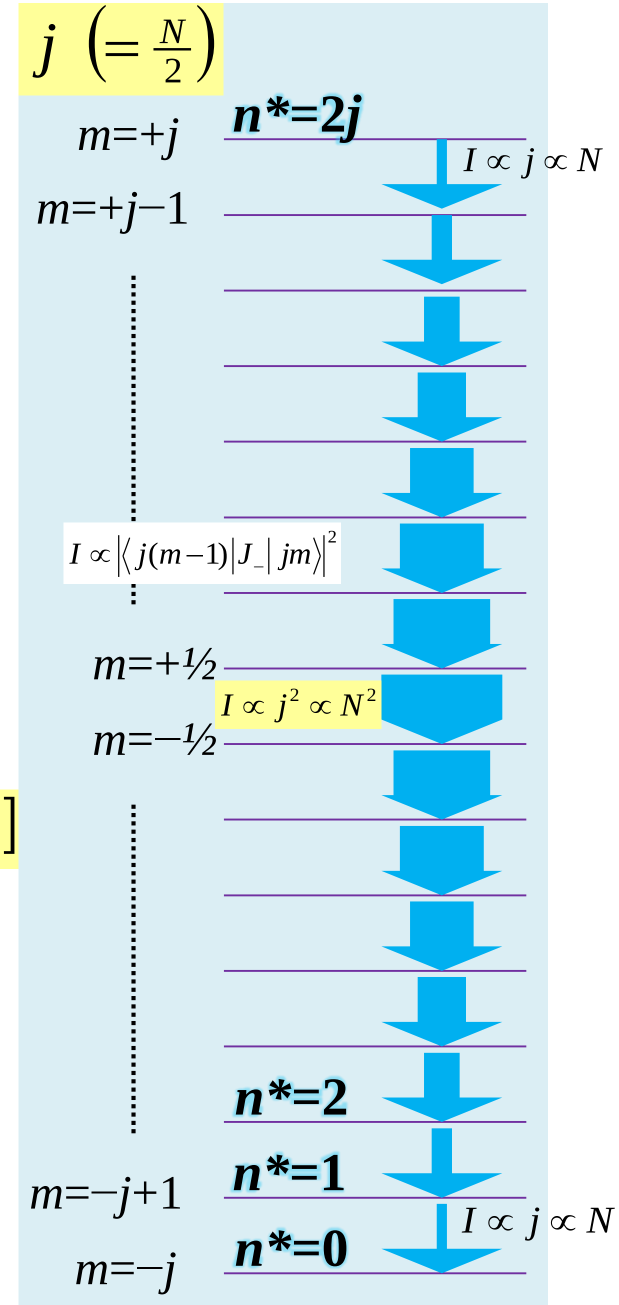
$$\frac{d}{dt} P_{jm} \approx -\underbrace{\left|\langle F, j(m-1)|H_{\text{int}}|F, jm\rangle\right|^2}_{\Gamma[j(j+1)-m(m-1)]} P_{jm} + \underbrace{\left|\langle F, jm|H_{\text{int}}|F, j(m+1)\rangle\right|^2}_{\Gamma[j(j+1)-(m+1)m]} P_{j(m+1)}$$

$$\frac{d}{dt}\langle m\rangle \approx \sum_m m \frac{d}{dt} P_{jm} = -\Gamma \underbrace{(j+m)(j-m-1)}_{\approx j^2 - m^2}$$

$$\langle m\rangle \approx -j \tanh[j\Gamma(t-t_0)]$$







Superradiant phase transitions

John
Daniel Edward
"Jack" Torrance
(? – ?)

Dicke model

$$H=\underbrace{\omega_0 J_0}_{m}+\underbrace{\omega\, b^+ b}_{n}+\underbrace{\frac{\lambda}{\sqrt{N}}\overbrace{\left(J_+ + J_-\right)\left(b^+ + b\right)}^{\approx\, J_+ b + J_- b^+}}_{H_{\rm int}}$$

In this approximation, the model conserves the quantity $M=n+\overbrace{m+j}^{n^*}$ $\Rightarrow$

$M=0: \quad |n,n^*\rangle=|0,0\rangle$

$M=1: \quad |n,n^*\rangle=|0,1\rangle \quad |1,0\rangle$

$M=2: \quad |n,n^*\rangle=|0,2\rangle \quad |1,1\rangle \quad |2,0\rangle$

$\vdots \qquad \ddots$

Assume the **resonant case** $\omega_0=\omega$

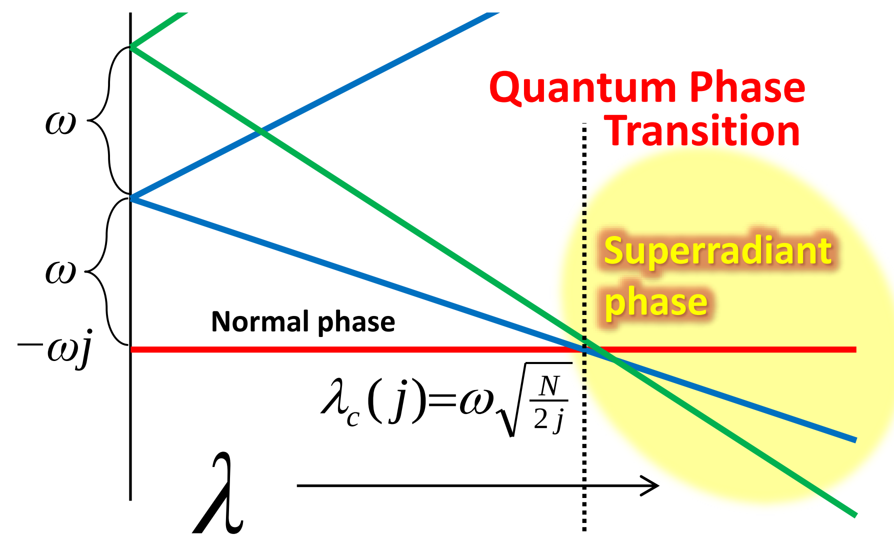
$$\mathcal{H}=\mathcal{H}_{\rm A}\otimes\mathcal{H}_{\rm F}\supset\mathcal{H}_{\rm A}^{j,l}\otimes\mathcal{H}_{\rm F}=\bigoplus_{M=0}^{\infty}\mathcal{H}_M^{j,l}$$

M=0: $H_0=-\omega j+0$

M=1:
$$H_1=-\omega(j-1)+\frac{\lambda}{\sqrt{N}}\begin{pmatrix}0 & \sqrt{2j}\\ \sqrt{2j} & 0\end{pmatrix}$$

M=2:
$$H_2=-\omega(j-2)+\frac{\lambda}{\sqrt{N}}\begin{pmatrix}0 & \sqrt{2(2j-1)} & 0\\ \sqrt{2(2j-1)} & 0 & \sqrt{4j}\\ 0 & \sqrt{4j} & 0\end{pmatrix}$$

$\vdots$

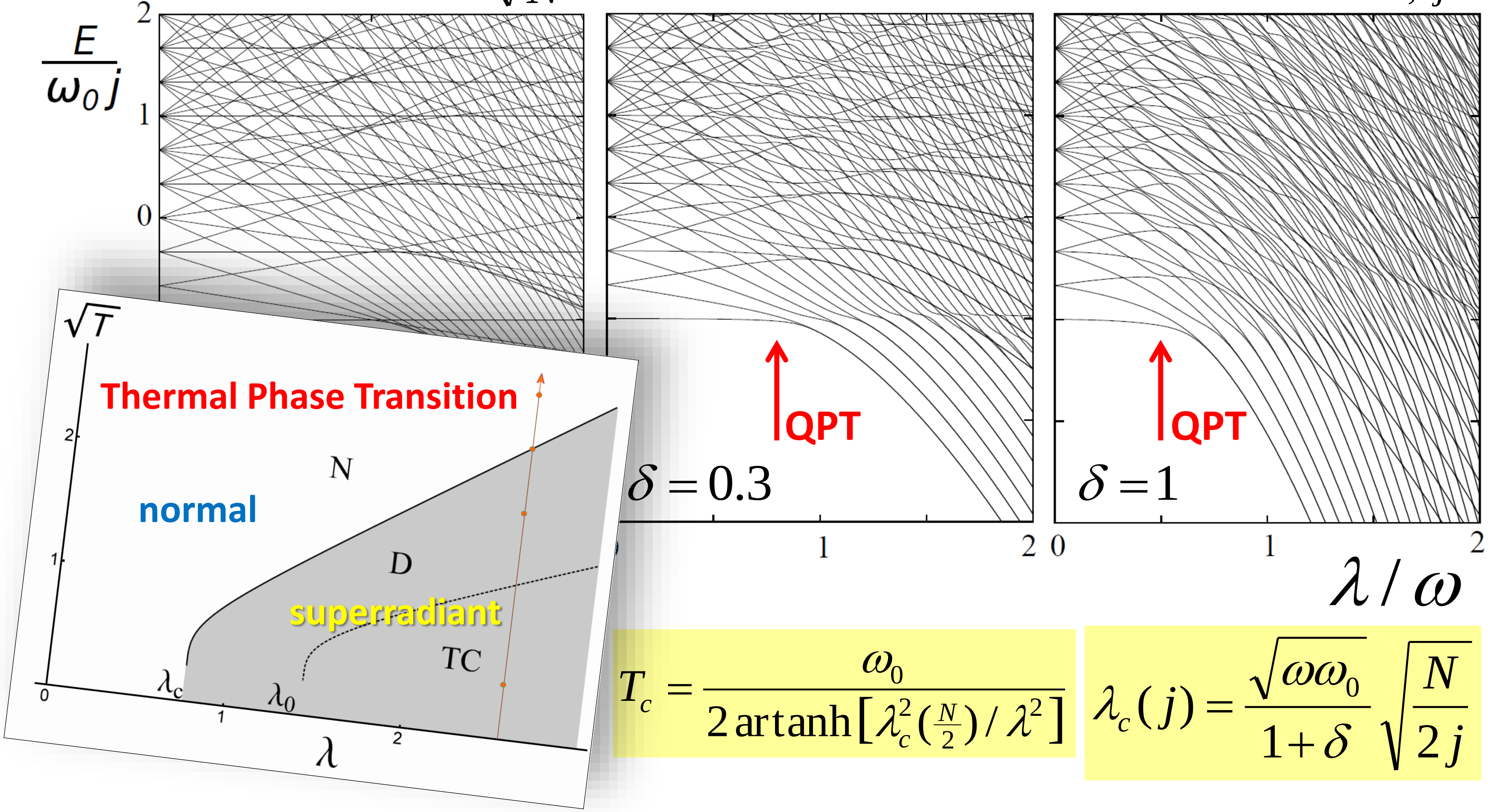


Superradiant phase transitions

Dicke model → **extended** $\delta \in [0,1]$

$$H = \omega_0 J_0 + \omega b^{+} b + \frac{\lambda}{\sqrt{N}}\left(\hat{b}^{+} J_{-} + b J_{+} + \delta [\, b^{+} J_{+} + b J_{-}]\right)$$

$N = 6, \; j = \frac{N}{2}$



$$T_c = \frac{\omega_0}{2\,\mathrm{artanh}\left[\lambda_c^2(\tfrac{N}{2}) / \lambda^2\right]}$$

$$\lambda_c(j) = \frac{\sqrt{\omega\omega_0}}{1+\delta}\sqrt{\frac{N}{2j}}$$

Superradiant phase transitions

Theory of superradiant phase transitions:

Thermal: Y.K. Wang, F.T. Hioe, *Phys. Rev.* A 7, 831 (**1973**)
K. Hepp, E.H. Lieb, *Phys. Rev.* A 8, 2517 (**1973**)

Quantum: C. Emary, T. Brandes, *Phys. Rev. Lett.* 90, 044101 (**2003**)

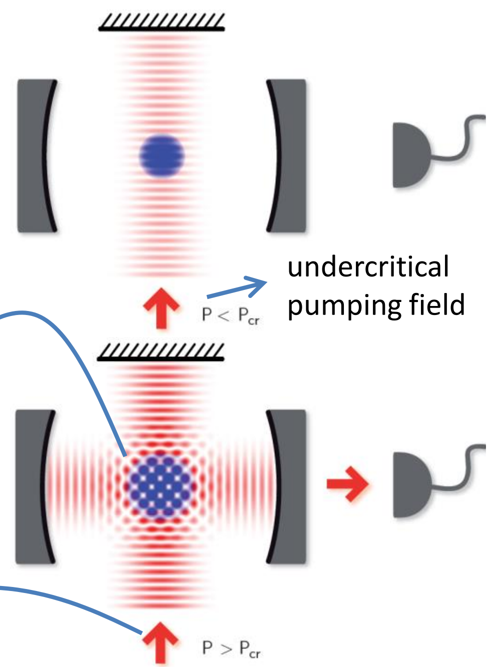
Experiment

F. Dimer et al., Phys. Rev. A 75, 013804 (**2007**) … **proposal**

K. Baumann et al., *Nature* 464, 1301 **(2010)**, Phys. Rev. Lett. 107, 140402 (2011)

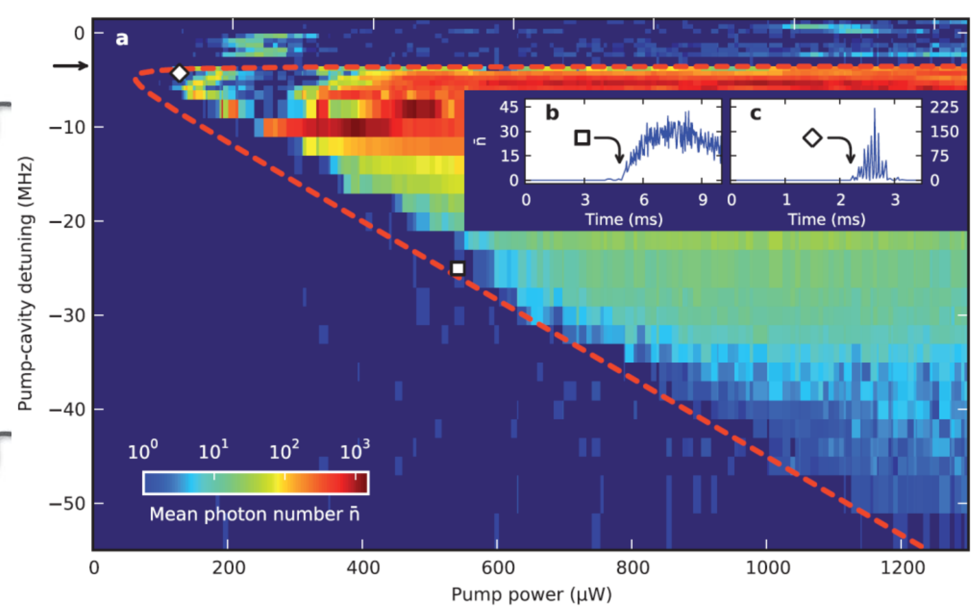
Realization by means of BEC at $T \approx 50\,\mathrm{nK}$ in optical cavity

Spatial redistribution of BEC leads to constructive interference of reflected waves



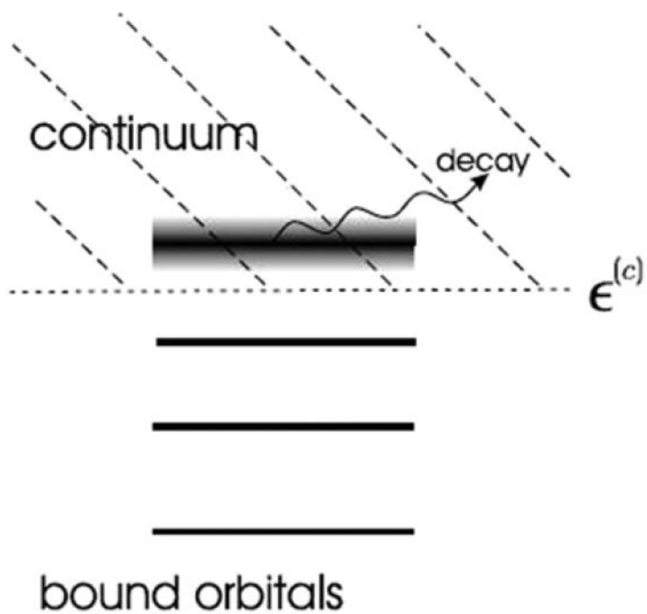
undercritical pumping field

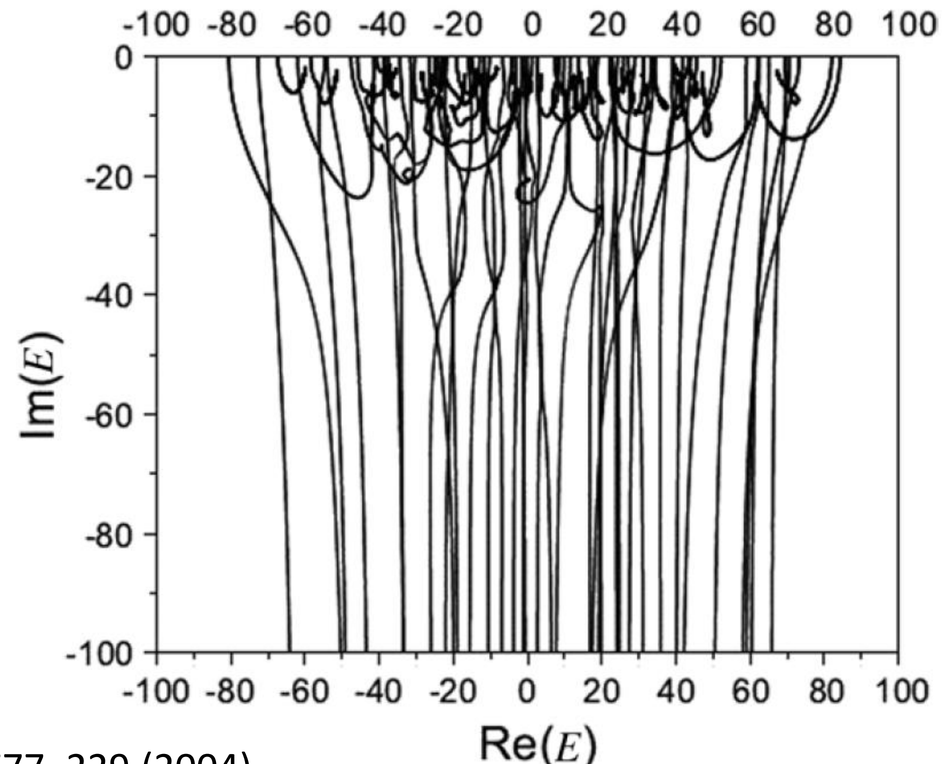
pumping field of overcritical power



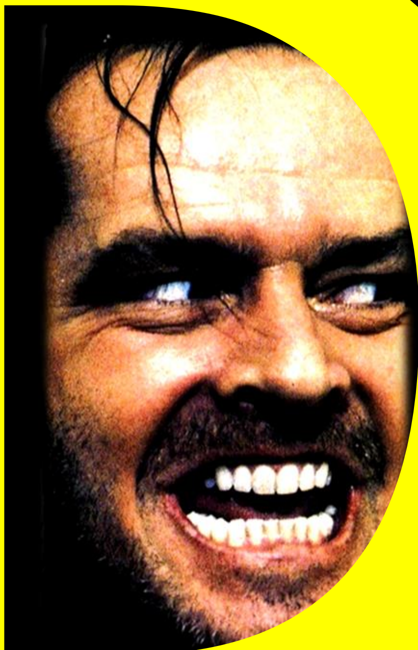
Non-hermitian superradiance

Quasi-bound quantum system with a common set of decay channels. Increasing coupling to continuum leads to seggregation of long-lived (compound) and short-lived (superradiant) resonance states. Applications in nuclear physics (e.g. giant & pygmy resonances…), particle physics (baryon resonances) biophysics (photosynthesis), graphene… **Recent reviews**: N. Auerbach, V. Zelevinsky, *Rep. Prog. Phys.* 74, 106301 (2011), I. Rotter, J.P. Bird, *Rep. Prog. Phys.* 78, 114001 (2015)





A. Volya, V. Zelevinsky, AIP Conf.Proc. 777, 229 (2004)



Děkuji !

Thank you !