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Decays of neutral pions

Electromagnetic form factors and radiative corrections

Tomáš Husek

Institute of Particle and Nuclear Physics
Charles University, Prague

In collaboration with
K. Kampf, J. Novotný (*Charles University*), S. Leupold (*Uppsala University*)

Malá Skála
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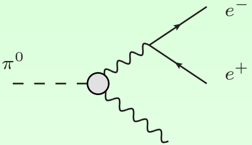
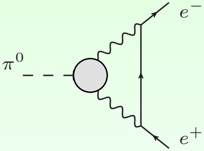
Decay modes of neutral pion:

Process	Branching ratio
$\pi^0 \to \gamma\gamma$	$(98.823 \pm 0.034)\,\%$
$\pi^0 \to e^+e^-\gamma$	$(1.174 \pm 0.035)\,\%$
$\pi^0 \to e^+e^+e^-e^-$	$(3.34 \pm 0.16) \times 10^{-5}$
$\pi^0 \to e^+e^-$	$(6.46 \pm 0.33) \times 10^{-8}$





Rare decay $\pi^0 \to e^+e^-$

- interesting way to study low-energy (long-distance) dynamics in the SM
- systematic theoretical treatment dates back to *Drell*, NC (1959)
- suppressed in comparison to the decay $\pi^0 \to \gamma\gamma$ by a factor of $2(\alpha m_e/M_\pi)^2$
  $\to$ one-loop structure + helicity suppression
  $\to$ may be sensitive to possible effects of new physics

KTeV-E799-II experiment at Fermilab (*Abouzaid et al.*, PRD 75 (2007))
$\to$ precise measurements of branching ratio $\pi^0 \to e^+e^-$ (794 candidates)

$$\frac{\Gamma(\pi^0 \to e^+e^-(\gamma),\, x > 0.95)}{\Gamma(\pi^0 \to e^+e^-\gamma,\, x > 0.232)} = (1.685 \pm 0.064 \pm 0.027) \times 10^{-4}$$

Extrapolate the Dalitz decay branching ratio to full range of x

$$B^{\text{KTeV}}(\pi^0 \to e^+e^-(\gamma),\, x_{\text{D}} > 0.95) = (6.44 \pm 0.25 \pm 0.22) \times 10^{-8}$$

- PDG average value $(6.46 \pm 0.33) \times 10^{-8}$ mainly based on this result
- extrapolate full radiative tail beyond $x > 0.95$ (*Bergström*, Z.Ph.C 20 (1983))
- scale the result back by the overall radiative corrections

$\to$ final result for lowest order (no final state radiation)

$$B^{\text{no-rad}}_{\text{KTeV}}(\pi^0 \to e^+e^-) = (7.48 \pm 0.29 \pm 0.25) \times 10^{-8}$$

Comparison with SM prediction (*Dorokhov and Ivanov*, PRD 75 (2007))

$$B^{\text{no-rad}}_{\text{SM}}(\pi^0 \to e^+e^-) = (6.23 \pm 0.09) \times 10^{-8}$$

$\to$ interpreted as $3.3\,\sigma$ discrepancy between theory and experiment

New physics?

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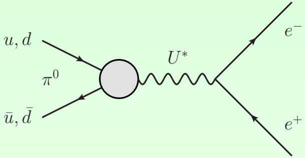


- very fashionable to ascribe eventual discrepancies to effects of new physics

BUT

- first, look for more conventional solution (i.e. within SM)

  $\to$ radiative corrections (usually very important)

  $\to$ form factor modeling

QCD at low energy

- QCD describes the dynamics of quarks and gluons
- at low energy we need to describe interactions of hadrons (e.g. pions) instead
  $\rightarrow$ the relevant degrees of freedom are different
- effective description
  $\rightarrow$ does not need to be necessarily less precise
  $\rightarrow$ in general, the structure at short distances (high energies) is ignored
  $\rightarrow$ many unknown parameters appear (so called LECs)
  $\rightarrow$ set from experiment or more fundamental theory (matching)
- the perturbation series is not based on the coupling
  $\rightarrow$ α_s grows with the distance
  $\rightarrow$ different power counting schemes - derivatives (momenta)
- χPT answers the question how to construct the Lagrangian monomials
  $\rightarrow$ Green functions satisfy the same Ward identities as in QCD

QCD Lagrangian in massless limit

$$\mathcal{L}_{\text{QCD}}^{(m_f=0)} = -\frac{1}{4}G^a_{\mu\nu}G^{a\mu\nu} + i\bar{q}_\text{L}\not{D}q_\text{L} + i\bar{q}_\text{R}\not{D}q_\text{R}$$

where

$$q_{\text{L,R}} = \frac{1}{2}(1 \mp \gamma_5)q\ , \qquad q = \begin{bmatrix} u \\ d \\ s \end{bmatrix}$$

$\rightarrow$ invariant under chiral symmetry group $\text{SU(3)}_\text{L} \times \text{SU(3)}_\text{R}$

$$\Downarrow$$

$$\mathcal{L}_{\chi\text{PT}} = \mathcal{L}_2 + \mathcal{L}_4 + \mathcal{L}_6 + \dots$$

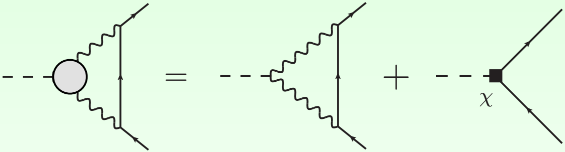
$\mathcal{L}_2 = \frac{1}{4}F_0{}^2\langle D_\mu U(D^\mu U)^\dagger\rangle + \frac{1}{4}F_0{}^2\langle\chi U^\dagger + U\chi^\dagger\rangle\ , \quad \chi = 2B_0(s+ip)$

$\mathcal{L}_4 = \sum_{i=1}^{10} L_i O_4^i$

$\mathcal{L}_6 = \sum_{i=1}^{90} C_i O_6^i$

Leading order

- pions are complicated composite objects
  $\rightarrow$ elementary interactions are not point-like
- electromagnetic pion transition form factor $F_{\pi^0\gamma^*\gamma^*}$ describes this complexity



LO contribution
in QED expansion

its representation
as the LO of χPT

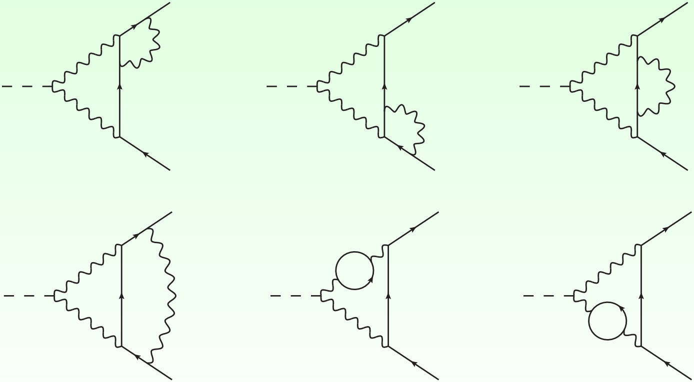
- free parameter $\chi^{(\mathrm{r})}(\mu)$ appears in the finite part of the counter term

$$\chi = [\text{UV-divergent part}] + \chi^{(\mathrm{r})}(\mu)$$

$\rightarrow$ unique for every form factor, e.g. $\chi^{(\mathrm{r})}_{\mathrm{KTeV}}(M_\rho) = 6.0 \pm 1.0$

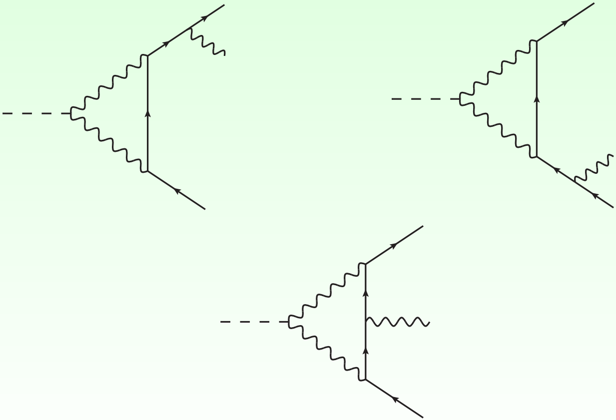
Two-loop virtual radiative corrections

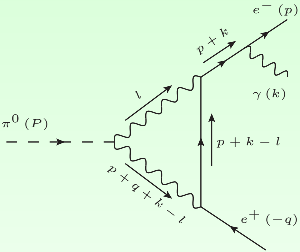
- calculated by *Vaško and Novotný*, JHEP 1110 (2011)

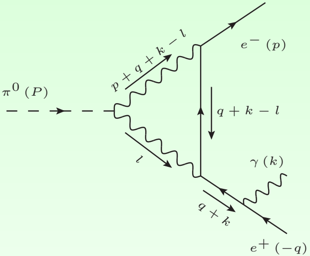


Bremsstrahlung

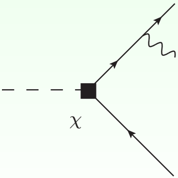
- compensation of infrared divergences in 2-loop contributions
  $\rightarrow$ *TH, Kampf and Novotný*, EPJC 74 (2014)

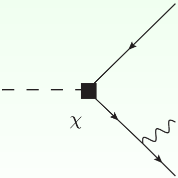






- contain UV subdivergences → counter-term tree diagrams with couplig χ

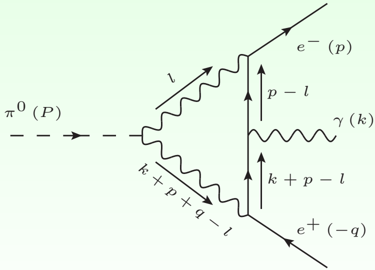




Do not forget the third, box diagram, necessary to satisfy the Ward identities

$$\mathcal{M}_{(\lambda)} = \varepsilon^{*\rho}_{(\lambda)}(k)\mathcal{M}^{\text{BS}}_{\rho} \quad \longrightarrow \quad k^{\rho}\mathcal{M}^{\text{BS}}_{\rho} = 0$$

- finite contribution to bremsstrahlung amplitude



Final matrix element

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$$
\begin{aligned}
i\mathcal{M}_{(\lambda)}(p,q,k) = & \frac{ie^5}{8\pi^2 F}\epsilon^{*\rho}_{(\lambda)}(k) \\
& \times\Big\{ P(x,y)\,[(k\cdot p)q_\rho - (k\cdot q)p_\rho]\,[\bar{u}(p,m)\gamma_5 v(q,m)] \\
& +A(x,y)\Big[\bar{u}(p,m)\,[\gamma_\rho(k\cdot p) - p_\rho \not{k}]\,\gamma_5 v(q,m)\Big] \\
& -A(x,-y)\Big[\bar{u}(p,m)\,[\gamma_\rho(k\cdot q) - q_\rho \not{k}]\,\gamma_5 v(q,m)\Big] \\
& +T(x,y)[\bar{u}(p,m)\gamma_\rho \not{k}\gamma_5 v(q,m)]\Big\}
\end{aligned}
$$

$$
\begin{aligned}
\overline{|\mathcal{M}^{\mathsf{BS}}(x,y)|^2} \equiv & \sum_\lambda \left|\mathcal{M}_{(\lambda)}(p,q,k)\right|^2 = \\
= & \frac{16\pi\alpha^5}{F^2}\frac{M^4(1-x)^2}{8}\Big\{ M^2\left[x(1-y^2)-\nu^2\right]\Big[xM^2\,|P|^2 \\
& + 2\nu M\,\mathrm{Re}\left\{P^*\,[A(x,y)+A(x,-y)]\right\} - 4\,\mathrm{Re}\left\{P^*T\right\}\Big] \\
& + 2M^2(x-\nu^2)(1-y)^2\,|A(x,y)|^2 + (y\to -y) \\
& - 8\nu M y(1-y)\,\mathrm{Re}\left\{A(x,y)T^*\right\} + (y\to -y) \\
& - 4\nu^2 M^2 y^2\,\mathrm{Re}\left\{A(x,y)A(x,-y)^*\right\} + 8(1-y^2)\,|T|^2 \Big\}
\end{aligned}
$$

Final results

Size of the radiative corrections (newly calculated)

$$\delta^{\mathrm{NLO}}(0.95) \equiv \delta^{\mathrm{virt.}} + \delta^{\mathrm{BS}}(0.95) = (-5.5 \pm 0.2)\,\%$$

- can be thought as model-independent
- differs significantly from previous approximate calculations

  Bergström, Z.Ph.C 20 (1983): $\delta(0.95) = -13.8\,\%$

  Dorokhov et al., EPJC 55 (2008): $\delta(0.95) = -13.3\,\%$

- original KTeV vs. SM discrepancy reduced to the $2\,\sigma$ level or less
- contact interaction coupling finite part set to

$$\chi^{(\mathrm{r})}_{\mathrm{LMD}}(M_\rho) = 2.2 \pm 0.9$$

New fit of the coupling $\chi^{(\mathrm{r})}$ value

LMD approximation to the large-N_c spectrum of vector meson resonances
→ theoretically modeled by *Knecht et al.*, PRL 83 (1999)

OR

- numerically fit the coupling to KTeV result using all available corrections
- alternatively, use approximative formula → same result

$$\chi^{(\mathrm{r})}(M_\rho) \simeq \frac{5}{2} + \frac{3}{2}\log\left(\frac{M_\rho^2}{m^2}\right) - \frac{\pi^2}{12} - \frac{1}{4}\log^2\left(\frac{M^2}{m^2}\right)$$

$$+\sqrt{\frac{1}{2}\left(\frac{\pi M}{\alpha m}\right)^2 \frac{B(\pi^0 \to e^+e^-(\gamma),\, x_{\mathrm{D}} > 0.95)}{B(\pi^0 \to \gamma\gamma)\left[1 + \delta^{(\text{2-loop})}(0.95)\right]} - \frac{\pi^2}{4}\log^2\left(\frac{M^2}{m^2}\right)}$$

Final model independent effective value

$$\chi^{(\mathrm{r})}(M_\rho) = 4.5 \pm 1.0$$

One-loop diagrams of order α^2/F^3 for process $\pi^0 \to e^+e^-$

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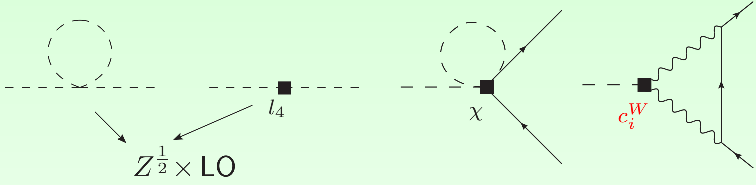

(a) (b) (c) (d)

- the leading log estimation, i.e. taking terms $\sim \log^2 \mu^2$ (up to two loops) $\to$ Weinberg consistency relation
- only the contribution from c_{13}^W diagram survives

The final correction $\to$ stability in the strong sector

$$\Delta^{\mathrm{LL}}\chi^{(\mathrm{r})}(M_\rho) = \frac{1}{36}\left(\frac{M}{4\pi F}\right)^2\left(1 - \frac{10m^2}{M^2}\right)\log^2\left(\frac{M_\rho^2}{m^2}\right) \doteq 0.081$$

End of story?

NLO radiative corrections in the QED sector did not solve the discrepancy

$\rightarrow$ back to LO, but use different model

- hadron physics described by QCD
  → the SU(3) gauge theory of quarks and gluons
- many phenomena like confinement not well understood
- particle spectrum, decay rates or scattering amplitudes
  → difficult to derive from fundamental theory
  → some results from lattice QCD
- exact solution cannot be even imagined
- need for approximation scheme
  → requires expansion parameter
- SU(3) gauge theory with negligible masses has no obvious free parameter

- generalization from 3 to N_c colors, SU(3) $\to$ SU(N_c) gauge group
- possible to solve the theory in large N_c?
  $\to$ $N_c = 3$ theory might be qualitatively/quantitatively close to large N_c limit
- QCD simplifies as N_c becomes large and possesses systematic expansion
  $\to$ resembles the known phenomenology of hadron physics
  $\to$ expansion parameter $1/N_c$

't Hooft, NPB 72 (1974), Witten, NPB 160 (1979)

Is $1/N_c = 1/3$ small enough?

QED

- vertices just $e\gamma^\mu$ with $e = 0.302$
- typical expansion parameter is $\alpha = e^2/4\pi \simeq 1/137$

$\Longrightarrow$ no information about convergence from expansion parameter itself

QCD with N_c colors

- N_c is the only parameter we know about
- phenomenological reasons
- something qualitatively correct $\Longrightarrow$ quantitatively good approximation

In Yang–Mills theory at $N_c = \infty$

1) mesons (and glue states) are free

- meson decay amplitudes of order $1/\sqrt{N_c}$
- meson-meson elastic scattering amplitudes of order $1/N_c$

$\Longrightarrow$ stable and non-interacting

2) infinite number of meson states

3) elastic amplitudes given as sum of tree diagrams

$\rightarrow$ not quarks and gluons, but physical mesons are exchanged

4) Zweig's rule is exact

$\rightarrow$ mesons pure $q\bar{q}$ states

Large N_c limit
Way to derivation

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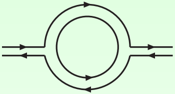
Many colors $\implies$ large combinatoric factors in Feynman diagrams

- gluon fields $N_c \times N_c$ matrices in adjoint representation, $N_c^2(-1)$ components
- quark fields have N_c components

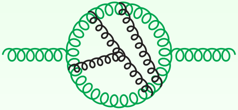
't Hooft's double line notation

$\rightarrow$ gluons have same SU(N_c) QN as $q\bar{q}$



combinatoric factors cancel the vertex factors only in planar diagrams

$\rightarrow$ large N_c limit given therefore given only by sum of planar diagrams



Selection rules

1) in large N_c limit the non-planar diagrams are suppressed by factors of $1/N_c^2$
2) internal quark loops are suppressed by factors of $1/N_c$

Large N_c limit

Important consequence for mesons

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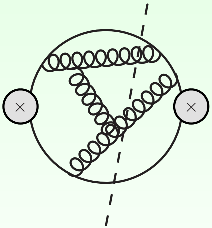
$\Longrightarrow$ leading contributions to matrix elements of quark bilinears are planar graphs with only quarks at the edge

What can be deduced from confining nature and dominance of planar graphs?

Quark-current bilinears are given by

1) infinite sum of
2) one-meson exchange contributions with
3) stable resonances



$$\langle 0|J(q)J(-q)|0\rangle = \sum_n \frac{a_n^2}{q^2 - m_n^2}\,,$$

with e.g. $J(x) := \bar{q}(x)\gamma^\mu q(x)$

ATLAS

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Resonances

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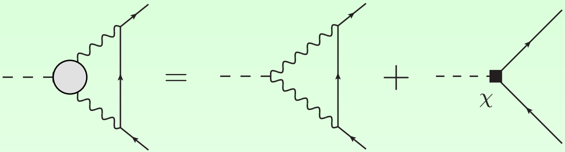
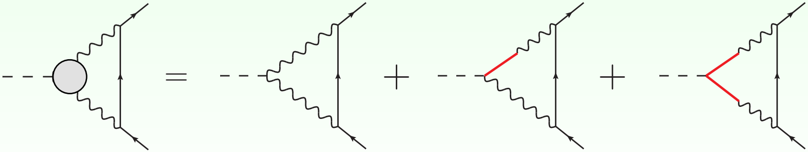


Chiral Perturbation Theory (χPT)

$\Updownarrow$

Resonance Chiral Theory (RχT)



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THS model for PVV correlator

1) Ansatz for Pseudoscalar-Vector-Vector (PVV) correlator

- Two-Hadron-Saturation (THS) - 2 meson multiplets per channel

$$\Pi^{\text{THS}}(r^2;p^2,q^2) \sim \frac{1}{r^2(r^2-M_P^2)}\frac{P(r^2;p^2,q^2)}{(p^2-M_{V_1}^2)(p^2-M_{V_2}^2)(q^2-M_{V_1}^2)(q^2-M_{V_2}^2)}$$

- in numerator stands general polynomial symmetrical in p^2 and q^2
  $\rightarrow$ correlator must drop at large momenta
  $\rightarrow$ 22 free parameters

$$P(r^2;p^2,q^2) = c_0 p^2 q^2 + c_1[(p^2)^3 q^2 + (q^2)^3 p^2] + c_2 (r^2)^2 p^2 q^2 + \ldots$$

2) Use high- and low-energy limits to constrain the parameters

- Operator product expansion (OPE)
- Brodsky–Lepage (B–L) quark counting rules
- chiral anomaly

THS and $\mathcal{F}_{\pi^0\gamma^*\gamma^*}$ form factor

Form factor is in general related to PVV correlator as

$$\mathcal{F}_{\pi^0\gamma^*\gamma^*}(p^2,q^2) \sim \lim_{r^2\to 0} r^2\Pi(r^2;p^2,q^2)$$

→ in our case complicated, but with only one free parameter

$$\mathcal{F}^{\text{THS}}_{\pi^0\gamma^*\gamma^*}(p^2,q^2) = -\frac{N_c}{12\pi^2 F}\left[\frac{M_{V_1}^4 M_{V_2}^4}{(p^2-M_{V_1}^2)(p^2-M_{V_2}^2)(q^2-M_{V_1}^2)(q^2-M_{V_2}^2)}\right] \times\left\{1+\frac{\kappa}{2N_c}\frac{p^2q^2}{(4\pi F)^4}-\frac{4\pi^2F^2(p^2+q^2)}{N_c M_{V_1}^2 M_{V_2}^2}\left[6+\frac{p^2q^2}{M_{V_1}^2 M_{V_2}^2}\right]\right\}$$

κ determined from fit to ω-π transition form factor measurements
→ *NA60*, PLB 677 (2009)

$$\kappa = 21 \pm 3$$

$M_{V_1} \sim \rho, \omega$ vector-meson mass
$M_{V_2} \sim$ between physical masses of first and second vector-meson excitations

$$M_{V_2} \in [1400, 1740]\ \text{MeV}$$

VMD and LMD models

Examples of other approaches

- Vector-Meson Dominance (VMD)

$$\mathcal{F}^{\text{VMD}}_{\pi^0\gamma^*\gamma^*}(p^2,q^2) = -\frac{N_c}{12\pi^2 F}\left[\frac{M_{V_1}^4}{(p^2-M_{V_1}^2)(q^2-M_{V_1}^2)}\right]$$

$\rightarrow$ violates OPE: $\mathcal{F}_{\pi^0\gamma^*\gamma^*}(q^2,q^2) \not\sim \frac{1}{q^2}\,,\ q^2\to-\infty$

- Lowest-Meson Dominance (LMD)

$$\mathcal{F}^{\text{LMD}}_{\pi^0\gamma^*\gamma^*}(p^2,q^2) = \mathcal{F}^{\text{VMD}}_{\pi^0\gamma^*\gamma^*}(p^2,q^2)\left\{1-\frac{4\pi^2F^2(p^2+q^2)}{N_c M_{V_1}^4}\right\}$$

$\rightarrow$ violates BL: $\mathcal{F}_{\pi^0\gamma^*\gamma^*}(0,q^2) \not\sim \frac{1}{q^2}\,,\ q^2\to-\infty$

- none of the models used two meson multiplets in both channels
  $\rightarrow$ vector and pseudoscalar

Results

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Theoretical prediction within THS model

$$B^{\mathsf{THS}}(\pi^0 \to e^+e^-(\gamma),\, x_{\mathrm{D}} > 0.95) = (5.8 \pm 0.2) \times 10^{-8}$$

- recall experimental value: $B^{\mathsf{KTeV}} = (6.44 \pm 0.33) \times 10^{-8}$
  $\to$ disagreement at the level of only 1.8 σ
- matching on LO χPT gives $\chi^{(\mathrm{r})}_{\mathsf{THS}}(M_\rho) = 2.2 \pm 0.7$
- if KTeV result confirmed (e.g. by NA62) $\to$ two scenarios are conceivable:
  a) some aspects of the THS approach not well-suited for $\pi^0 \to e^+e^-$
  b) beyond-Standard Model physics influences the rare pion decay significantly
- under the present circumstances the current discrepancy is inconclusive

Quantity really measured by KTeV

$$\left.\frac{\Gamma(\pi^0 \to e^+e^-(\gamma)\,,\ x > 0.95)}{\Gamma(\pi^0 \to e^+e^-\gamma(\gamma)\,,\ x > 0.2319)}\right|_{\mathsf{KTeV}} = (1.685 \pm 0.064 \pm 0.027) \times 10^{-4}$$

$\to$ Dalitz decay comes into play

All NLO QED radiative corrections for discussed processes are now available
$\rightarrow$ can be taken into account in future experimental analyses

- $\pi^0 \to e^+e^-$
  Vaško and Novotný, JHEP 1110 (2011)
  TH, Kampf and Novotný, EPJC 74 (2014)
- $\pi^0 \to e^+e^-\gamma$
  TH, Kampf and Novotný, PRD 92 (2015)

THS model for $\mathcal{F}_{\pi^0\gamma^*\gamma^*}(p^2, q^2)$

- phenomenologically successful
- satisfies all main theoretical constraints
- *TH and S. Leupold*, EPJC 75 (2015)

Altogether, we get reasonable SM prediction
$\rightarrow$ differs from KTeV by 1.8 σ

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Thank you for listening!