Green functions of chiral currents and their OPE

Tomáš Kadavý

Institute of Particle and Nuclear Physics
Faculty of Mathematics and Physics, Charles University, Prague, Czech Republic

Výjezdní seminář ÚČJF
Malá Skála, April 19 - 23, 2017

April 22, 2017

What is it about?

- What do we do?
  - We study QCD at low energies.
- What do we need?
  - Chiral perturbation theory (χPT) and Resonance chiral theory (RχT).
- What is it?
  - Effective description low-energy QCD.
  - χPT for $E \leq M_\rho$.
    - Spontaneous breaking of the chiral $\mathrm{SU}(3)_L \times \mathrm{SU}(3)_R$ symmetry down to $\mathrm{SU}(3)_V$ in QCD leads to the presence of Goldstone bosons.
    - We identify them with the octet of pseudoscalar mesons (π, K, η) as the lightest hadronic observable states.
  - RχT for $M_\rho \leq E \leq 2\,\mathrm{GeV}$.
    - RχT increases the number of degrees of freedom of χPT by including massive $\mathrm{U}(3)$ multiplets of vector $V(1^{--})$, axial-vector $A(1^{++})$, scalar $S(0^{++})$ and pseudoscalar $P(0^{-+})$ resonances.
- What is it good for?
  - To study important theoretical and phenomenological aspects of QCD.
- What do we use?
  - Green functions of chiral currents.

Green functions of chiral currents

- The amplitudes of physical processes can be computed using LSZ reduction formula from the Green functions, the time ordered products of quantum fields.
- Three-point Green function:

$$\int \mathrm{d}^4 x_1 \mathrm{d}^4 x_2 \, e^{i(p_1 x_1 + p_2 x_2)} \left\langle 0 \middle| \mathrm{T}\left[\mathcal{O}_1(x_1)\mathcal{O}_2(x_2)\mathcal{O}_3(0)\right] \middle| 0 \right\rangle .$$

- $\mathcal{O}_i(x_i)$ stand either for
  - vector and axial-vector currents:

$$V_\mu^a = \overline{q}(x)\gamma_\mu T^a q(x)\,, \qquad A_\mu^a = \overline{q}(x)\gamma_\mu\gamma_5 T^a q(x)\,,$$

  - or scalar and pseudoscalar densities:

$$S^a = \overline{q}(x) T^a q(x)\,, \qquad P^a = i\overline{q}(x)\gamma_5 T^a q(x)\,.$$

- Nontrivial three-point Green functions in QCD:
  - Set I: $\langle SSS\rangle$, $\langle SPP\rangle$, $\langle VVP\rangle$, $\langle AAP\rangle$, $\langle VAS\rangle$, $\langle VVS\rangle$, $\langle AAS\rangle$, $\langle VAP\rangle$.
  - Set II: $\langle VVA\rangle$, $\langle AAA\rangle$, $\langle VVV\rangle$, $\langle ASP\rangle$, $\langle AAV\rangle$, $\langle VSS\rangle$, $\langle VPP\rangle$.

Green functions of chiral currents: Examples

- $\langle VVP \rangle$ Green function.
  - $\left(\Pi_{VVP}(p,q;r)\right)^{abc}_{\mu\nu} = \Pi_{VVP}(p^2,q^2,r^2) d^{abc} \varepsilon_{\mu\nu(p)(q)}$.
  - Direct connection to many phenomenologically important quantities:
    - Transition formfactor $\mathcal{F}^{\mathrm{R}\chi\mathrm{T}}_{\pi^0\to\gamma\gamma}(p^2,q^2,r^2) = \frac{2r^2}{3B_0F}\Pi^{\mathrm{R}\chi\mathrm{T}}_{VVP}(p^2,q^2,r^2)$.
    - Decays $\rho\to\pi\gamma$, $\pi(1300)\to\gamma\gamma$, $\pi(1300)\to\rho\gamma$.
    - Hadronic contribution to light-by-light scattering.
- $\langle VVA \rangle$ Green function.

$$\left(\Pi_{VVA}(p,q;r)\right)^{abc}_{\mu\nu\rho} = d^{abc}\left[w_L \varepsilon_{\mu\nu(p)(q)} r_\rho + w_T^{(1)}\Pi^{(1)}_{\mu\nu\rho} + w_T^{(2)}\Pi^{(2)}_{\mu\nu\rho} + w_T^{(3)}\Pi^{(3)}_{\mu\nu\rho}\right] .$$

  - The tensor part is nontrivial [M. Knecht *et al.* '04]:

$$\Pi^{(1)}_{\mu\nu\rho} = p_\nu \varepsilon_{\mu\rho(p)(q)} - q_\mu \varepsilon_{\nu\rho(p)(q)} - \frac{p^2+q^2-r^2}{r^2}\varepsilon_{\mu\nu(p)(q)} r_\rho + \frac{p^2+q^2-r^2}{2}\varepsilon_{\mu\nu\rho(p-q)} ,$$

$$\Pi^{(2)}_{\mu\nu\rho} = \varepsilon_{\mu\nu(p)(q)}(p-q)_\rho + \frac{p^2-q^2}{r^2}\varepsilon_{\mu\nu(p)(q)} r_\rho ,$$

$$\Pi^{(3)}_{\mu\nu\rho} = p_\nu \varepsilon_{\mu\rho(p)(q)} + q_\mu \varepsilon_{\nu\rho(p)(q)} - \frac{p^2+q^2-r^2}{2}\varepsilon_{\mu\nu\rho(r)} .$$

Operator Product Expansion

- Framework to study behaviour at high energies.
- For $x_1, x_2 \to 0$ (i.e. for large external momenta) Green function can be expanded into a series of nonperturbative parameters with c-number coefficients:

$$\begin{aligned}\langle \mathcal{O}_1(x_1)\mathcal{O}_2(x_2)\mathcal{O}_3(0)\rangle &= C_{\mathbb{1}}\mathbb{1} + C_{\langle \overline{q}q\rangle}\langle 0|\overline{q}q|0\rangle + C_{\langle G^2\rangle}\langle 0|G_{\mu\nu}G^{\mu\nu}|0\rangle \\ &+ C_{\langle \overline{q}Gq\rangle}\langle 0|\overline{q}\sigma_{\mu\nu}G^{\mu\nu}q|0\rangle + C_{\langle 4q\rangle}\langle 0|\overline{q}\,\Gamma_1\,q\overline{q}\,\Gamma_2\,q|0\rangle \\ &+ C_{\langle G^3\rangle}\langle 0|G^a_{\mu\nu}G^b_{\nu\sigma}G^c_{\sigma\mu}|0\rangle f^{abc} + \dots\end{aligned}$$

Operator Product Expansion

- Framework to study behaviour at high energies.
- For $x_1, x_2 \to 0$ (i.e. for large external momenta) Green function can be expanded into a series of nonperturbative parameters with c-number coefficients:

$$\begin{aligned}\langle \mathcal{O}_1(x_1)\mathcal{O}_2(x_2)\mathcal{O}_3(0)\rangle = {} & C_{\mathbb{1}}\mathbb{1} + C_{\langle\overline{q}q\rangle}\langle 0|\overline{q}q|0\rangle + C_{\langle G^2\rangle}\langle 0|G_{\mu\nu}G^{\mu\nu}|0\rangle \\ & + C_{\langle\overline{q}Gq\rangle}\langle 0|\overline{q}\sigma_{\mu\nu}G^{\mu\nu}q|0\rangle + C_{\langle 4q\rangle}\langle 0|\overline{q}\,\Gamma_1\,q\overline{q}\,\Gamma_2\,q|0\rangle \\ & + C_{\langle G^3\rangle}\langle 0|G^a_{\mu\nu}G^b_{\nu\sigma}G^c_{\sigma\mu}|0\rangle f^{abc} + \ldots\end{aligned}$$

- Perturbative contribution (contributes to set II).

Operator Product Expansion

- Framework to study behaviour at high energies.
- For $x_1, x_2 \to 0$ (i.e. for large external momenta) Green function can be expanded into a series of nonperturbative parameters with c-number coefficients:

$$\begin{aligned}\langle \mathcal{O}_1(x_1)\mathcal{O}_2(x_2)\mathcal{O}_3(0)\rangle &= C_{\mathbb{1}}\mathbb{1} + C_{\langle\bar{q}q\rangle}\langle 0|\bar{q}q|0\rangle + C_{\langle G^2\rangle}\langle 0|G_{\mu\nu}G^{\mu\nu}|0\rangle \\ &+ C_{\langle\bar{q}Gq\rangle}\langle 0|\bar{q}\sigma_{\mu\nu}G^{\mu\nu}q|0\rangle + C_{\langle 4q\rangle}\langle 0|\bar{q}\,\Gamma_1\,q\bar{q}\,\Gamma_2\,q|0\rangle \\ &+ C_{\langle G^3\rangle}\langle 0|G^a_{\mu\nu}G^b_{\nu\sigma}G^c_{\sigma\mu}|0\rangle f^{abc} + \ldots\end{aligned}$$

- Perturbative contribution (contributes to set II).
- QCD condensates (with dimension less than 6):
  - Quark condensate (contributes to set I).

Operator Product Expansion

- Framework to study behaviour at high energies.
- For $x_1, x_2 \to 0$ (i.e. for large external momenta) Green function can be expanded into a series of nonperturbative parameters with c-number coefficients:

$$\begin{aligned}\langle \mathcal{O}_1(x_1)\mathcal{O}_2(x_2)\mathcal{O}_3(0)\rangle = {} & C_{\mathbb{1}}\mathbb{1} + C_{\langle \overline{q}q\rangle}\langle 0|\overline{q}q|0\rangle + C_{\langle G^2\rangle}\langle 0|G_{\mu\nu}G^{\mu\nu}|0\rangle \\ & + C_{\langle \overline{q}Gq\rangle}\langle 0|\overline{q}\sigma_{\mu\nu}G^{\mu\nu}q|0\rangle + C_{\langle 4q\rangle}\langle 0|\overline{q}\,\Gamma_1\,q\overline{q}\,\Gamma_2\,q|0\rangle \\ & + C_{\langle G^3\rangle}\langle 0|G^a_{\mu\nu}G^b_{\nu\sigma}G^c_{\sigma\mu}|0\rangle f^{abc} + \dots\end{aligned}$$

- Perturbative contribution (contributes to set II).
- QCD condensates (with dimension less than 6):
  - Quark condensate (contributes to set I).
  - Gluon condensate (contributes to set II).

Operator Product Expansion

- Framework to study behaviour at high energies.
- For $x_1, x_2 \to 0$ (i.e. for large external momenta) Green function can be expanded into a series of nonperturbative parameters with c-number coefficients:

$$\begin{aligned}\langle \mathcal{O}_1(x_1)\mathcal{O}_2(x_2)\mathcal{O}_3(0)\rangle &= C_{\mathbb{1}}\mathbb{1} + C_{\langle \bar{q}q\rangle}\langle 0|\bar{q}q|0\rangle + C_{\langle G^2\rangle}\langle 0|G_{\mu\nu}G^{\mu\nu}|0\rangle \\ &+ C_{\langle \bar{q}Gq\rangle}\langle 0|\bar{q}\sigma_{\mu\nu}G^{\mu\nu}q|0\rangle + C_{\langle 4q\rangle}\langle 0|\bar{q}\,\Gamma_1\,q\bar{q}\,\Gamma_2\,q|0\rangle \\ &+ C_{\langle G^3\rangle}\langle 0|G^a_{\mu\nu}G^b_{\nu\sigma}G^c_{\sigma\mu}|0\rangle f^{abc} + \dots\end{aligned}$$

- Perturbative contribution (contributes to set II).
- QCD condensates (with dimension less than 6):
  - Quark condensate (contributes to set I).
  - Gluon condensate (contributes to set II).
  - Quark-gluon condensate (contributes to set I).

Operator Product Expansion

- Framework to study behaviour at high energies.
- For $x_1, x_2 \to 0$ (i.e. for large external momenta) Green function can be expanded into a series of nonperturbative parameters with c-number coefficients:

$$\begin{aligned}\langle \mathcal{O}_1(x_1)\mathcal{O}_2(x_2)\mathcal{O}_3(0)\rangle &= C_{\mathbb{1}}\mathbb{1} + C_{\langle\overline{q}q\rangle}\langle 0|\overline{q}q|0\rangle + C_{\langle G^2\rangle}\langle 0|G_{\mu\nu}G^{\mu\nu}|0\rangle \\ &+ C_{\langle\overline{q}Gq\rangle}\langle 0|\overline{q}\sigma_{\mu\nu}G^{\mu\nu}q|0\rangle + C_{\langle 4q\rangle}\langle 0|\overline{q}\,\Gamma_1\,q\overline{q}\,\Gamma_2\,q|0\rangle \\ &+ C_{\langle G^3\rangle}\langle 0|G^a_{\mu\nu}G^b_{\nu\sigma}G^c_{\sigma\mu}|0\rangle f^{abc} + \dots\end{aligned}$$

- Perturbative contribution (contributes to set II).
- QCD condensates (with dimension less than 6):
  - Quark condensate (contributes to set I).
  - Gluon condensate (contributes to set II).
  - Quark-gluon condensate (contributes to set I).
  - Four-quark condensate (contributes to set II).

Operator Product Expansion

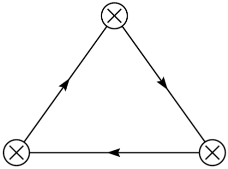
- Framework to study behaviour at high energies.
- For $x_1, x_2 \to 0$ (i.e. for large external momenta) Green function can be expanded into a series of nonperturbative parameters with c-number coefficients:

$$\begin{aligned}\langle \mathcal{O}_1(x_1)\mathcal{O}_2(x_2)\mathcal{O}_3(0)\rangle &= C_{\mathbb{1}}\mathbb{1} + C_{\langle \overline{q}q\rangle}\langle 0|\overline{q}q|0\rangle + C_{\langle G^2\rangle}\langle 0|G_{\mu\nu}G^{\mu\nu}|0\rangle \\ &+ C_{\langle \overline{q}Gq\rangle}\langle 0|\overline{q}\sigma_{\mu\nu}G^{\mu\nu}q|0\rangle + C_{\langle 4q\rangle}\langle 0|\overline{q}\,\Gamma_1\,q\overline{q}\,\Gamma_2\,q|0\rangle \\ &+ C_{\langle G^3\rangle}\langle 0|G^a_{\mu\nu}G^b_{\nu\sigma}G^c_{\sigma\mu}|0\rangle f^{abc} + \dots\end{aligned}$$

- Perturbative contribution (contributes to set II).
- QCD condensates (with dimension less than 6):
  - Quark condensate (contributes to set I).
  - Gluon condensate (contributes to set II).
  - Quark-gluon condensate (contributes to set I).
  - Four-quark condensate (contributes to set II).
  - Three-gluon condensate (vanishes for all quark-bilinear currents).

OPE: Perturbative contribution

- Leading order; example for $\langle VVA \rangle$:

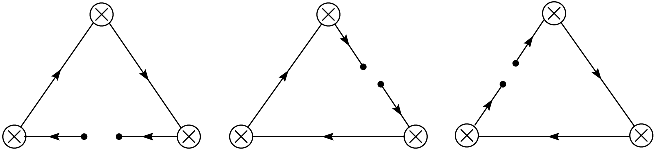


$$= i \int \frac{d^4\ell}{(2\pi)^4} \frac{\mathrm{Tr}[\gamma_\mu \not{\ell} \gamma_\nu (\not{\ell} - \not{q}) \gamma_\rho \gamma_5 (\not{\ell} + \not{p})]}{\ell^2 (\ell - q)^2 (\ell + p)^2} \mathrm{Tr}[T^a T^b T^c] \, .$$

- Anomaly:
  - If vector Ward identities are imposed, the axial Ward identity picks up an extra term.
  - EW for mass fermions: $r^\rho T^{\text{ren.}}_{\mu\nu\rho}(p, q, r; m) = 2m T_{\mu\nu}(p, q, r; m) + \frac{1}{2\pi^2} \varepsilon_{\mu\nu(p)(q)}$.
- Non-renormalization of the $\langle VVA \rangle$ correlator:
  - $\langle VVA \rangle$ is not modified by QCD radiative corrections at two loops.
  - Does it persist at higher orders?
  - Three-loop corrections do not vanish and they are proportional to the QCD β-function [J. Mondejar and K. Melnikov '12].

OPE: Quark condensate contribution $\langle 0|\overline{q}q|0\rangle$

- Leading order:

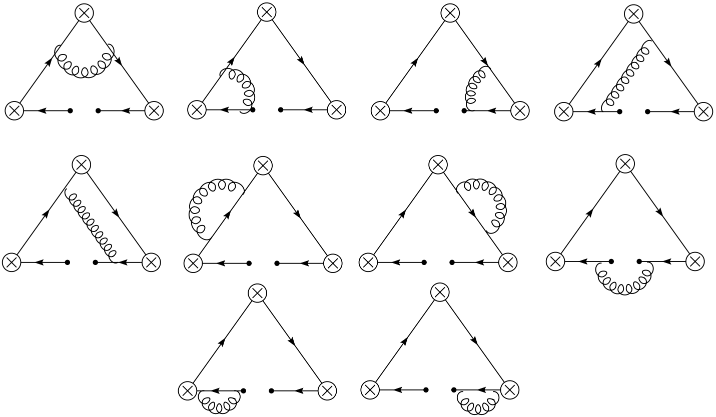


- First nontrivial QCD condensate.
- Responsible for the chiral symmetry breaking.
- Contributes to order parameters of the χSB.
  - Possibility to find out more about it.
- For $\langle VVP\rangle$:

$$C_{VVP}^{\langle\overline{q}q\rangle}(p^2,q^2;r^2) = \frac{p^2+q^2+r^2}{p^2q^2r^2}\,.$$

OPE: Quark condensate contribution $\langle 0|\overline{q}q|0\rangle$

- Next-to-leading order: gluonic corrections at $\mathcal{O}(\alpha_s)$.
- An opportunity to explore the renormalisation dependence of such condensate in full QCD.



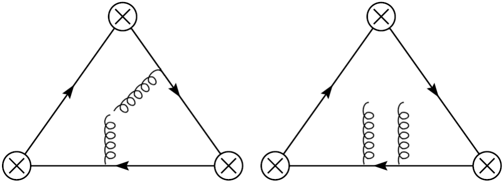
$$C_{\langle\overline{q}q\rangle} = \frac{\alpha_s}{6\pi}\left[L_p \log\left(-\frac{p^2}{\mu^2}\right) + L_q \log\left(-\frac{q^2}{\mu^2}\right) + L_r \log\left(-\frac{r^2}{\mu^2}\right) + L_d C_0 + L_c\right].$$

OPE: Gluon condensate contribution $\langle 0|G_{\mu\nu}G^{\mu\nu}|0\rangle$

- Fock-Schwinger gauge: $x^\mu A_\mu(x) = 0\,.$
- Massless quark propagator in external gluon field:

$$S(x,y) = S_0(x,y) + S_1^{\alpha\beta}(x,y)G_{\alpha\beta}(0) + S_2^{\alpha\beta}(x,y)G_{\alpha\beta}(0) + S_3(x,y)G_{\alpha\beta}(0)G^{\alpha\beta}(0)\,.$$

- Fourier transform needed to convert the result into p-representation.



- Results for $\langle VVA\rangle$ (and $\langle AAA\rangle$) are suprisingly simple [TK, K. Kampf and J. Novotný '17]:

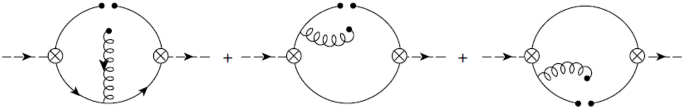
$$w_T^{(1)} = -\frac{g_s^2\langle G^2\rangle}{384\pi^2}\frac{p^2(r^2-4q^2)+p^4+q^2(q^2+r^2)}{p^4q^4r^2}\,,$$

$$w_T^{(2)} = -\frac{g_s^2\langle G^2\rangle}{384\pi^2}\frac{(p^2-q^2)(p^2+q^2+r^2)}{p^4q^4r^2} = -w_T^{(3)}\,.$$

- Cancelation of logarithmic terms!

OPE: Quark-gluon condensate contribution $\langle 0|\overline{q}\sigma_{\mu\nu}G^{\mu\nu}q|0\rangle$

- Fock-Schwinger gauge.



- The last two diagrams are generated by the propagation of the quark condensate condensate in a weak external field:

$$\langle : \overline{q}(x)q(0) : \rangle \ni -\frac{ix^2}{16}\left(\mathbb{1} + \frac{i}{6}m\not{x}\right)\langle : \overline{q}\sigma_{\alpha\beta}G^{\alpha\beta}q : \rangle \, .$$

OPE: Four-quark condensate contribution $\langle 0|\overline{q}\,\Gamma_1\,q\overline{q}\,\Gamma_2\,q|0\rangle$

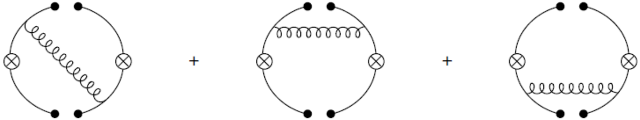
- An *independent* four-quark condensate: $\langle 0|\overline{q}\,\Gamma_1\,q\overline{q}\,\Gamma_2\,q|0\rangle$.
  - Γ_1, Γ_2 stand for any combination of one of the 4×4 matrices $\{\mathbb{1},\gamma_5,\gamma_\mu,\gamma_\mu\gamma_5,\sigma_{\mu\nu}\}$ with one of the 3×3 matrices $\{\mathbb{1},T^a\}$, that preserves the Lorentz invariance.

Notation	Condensate	Notation	Condensate
$\mathcal{Q}_1$	$\langle 0\|\overline{q}q\overline{q}q\|0\rangle$	$\mathcal{Q}_6$	$\langle 0\|\overline{q}\gamma_5 T^a q\overline{q}\gamma_5 T^a q\|0\rangle$
$\mathcal{Q}_2$	$\langle 0\|\overline{q}\gamma_5 q\overline{q}\gamma_5 q\|0\rangle$	$\mathcal{Q}_7$	$\langle 0\|\overline{q}\gamma_\mu\gamma_5 q\overline{q}\gamma^\mu\gamma_5 q\|0\rangle$
$\mathcal{Q}_3$	$\langle 0\|\overline{q}T^a q\overline{q}T^a q\|0\rangle$	$\mathcal{Q}_8$	$\langle 0\|\overline{q}\gamma_\mu\gamma_5 T^a q\overline{q}\gamma^\mu\gamma_5 T^a q\|0\rangle$
$\mathcal{Q}_4$	$\langle 0\|\overline{q}\gamma_\mu q\overline{q}\gamma^\mu q\|0\rangle$	$\mathcal{Q}_9$	$\langle 0\|\overline{q}\sigma_{\mu\nu} q\overline{q}\sigma^{\mu\nu} q\|0\rangle$
$\mathcal{Q}_5$	$\langle 0\|\overline{q}\gamma_\mu T^a q\overline{q}\gamma^\mu T^a q\|0\rangle$	$\mathcal{Q}_{10}$	$\langle 0\|\overline{q}\sigma_{\mu\nu} T^a q\overline{q}\sigma^{\mu\nu} T^a q\|0\rangle$

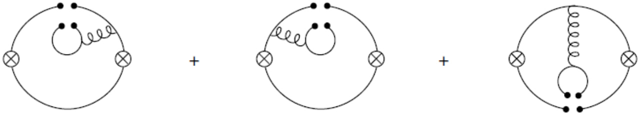
- For $\langle VVA\rangle$, only $\mathcal{Q}_4$, $\mathcal{Q}_5$, $\mathcal{Q}_7$ and $\mathcal{Q}_8$ contribute.

OPE: Four-quark condensate contribution $\langle 0|\bar{q}\,\Gamma_1\, q\bar{q}\,\Gamma_2\, q|0\rangle$

- Diagrams with hard momentum gluon propagator:



- Diagrams with zero momentum gluon propagator:



  - The first two diagrams are generated by the propagation of the quark condensate condensate in a weak external field:

$$\langle :\bar{q}(x)q(0):\rangle \ni \frac{ig_s^2x^2}{288}\not{x}\langle :\bar{q}\gamma_\rho T^a q\sum_f \bar{q}_f\gamma^\rho T^a q_f :\rangle\,.$$

  - The third diagram is generated by the mixed quark-gluon condensate condensate in a weak external field:

$$\langle :\bar{q}(x)G_\rho(y)q(0):\rangle \ni -g_s^2\left(\frac{2i}{3}(y_\mu\gamma_\rho - y_\rho\gamma_\mu) - \frac{1}{2}\not{x}\sigma_{\mu\rho}\right)\langle :\bar{q}\gamma_\mu T^a q\sum_f \bar{q}_f\gamma^\mu T^a q_f :\rangle\,.$$

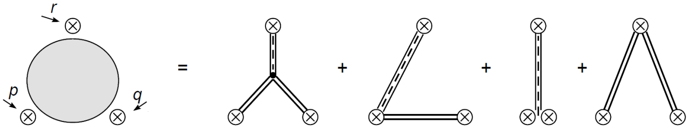
Matching RχT to OPE

- We assume the saturation of dynamics with the lightest resonances.
- At the NLO, relevant Lagrangian in the odd-intrinsic parity sector, was formulated for the first time in [K. Kampf and J. Novotný '11]:

$$\mathcal{L}_R^{(6)} = \sum_X \sum_i \kappa_i^X \widehat{\mathcal{O}}^X_{i\,\mu\nu\alpha\beta} \varepsilon^{\mu\nu\alpha\beta} \,.$$

  - $\mathcal{L}^{(6)}_{\mathrm{R}\chi\mathrm{T}}$: 67 operators and 67 corresponding unknown couplings κ_i^X in total.
  - X stands for the single-resonance fields V, A, S, P, double-resonance fields $VV, AA, SA, SV, VA, PA, PV$ and triple-resonance fields VVP, VAS, AAP.

- Topology of the Feynman diagrams (the crossing is implicitly assumed):



VVA Green function

- Extracted formfactors [TK, K. Kampf and J. Novotný '16]:

$$
\begin{aligned}
w_L &= \frac{N_c}{8\pi^2 r^2}\,, \\
w_T^{(1)} &= -\frac{2\sqrt{2}F_V\left[\kappa_{17}^V(p^2+q^2-2M_V^2)-\sqrt{2}F_V\kappa_3^{VV}\right]}{(p^2-M_V^2)(q^2-M_V^2)}\,, \\
w_T^{(2)} &= -\frac{2\sqrt{2}F_V(p^2-q^2)(2\kappa_{12}^V+\kappa_{16}^V-\kappa_{17}^V)}{(p^2-M_V^2)(q^2-M_V^2)}\,, \\
w_T^{(3)} &= \frac{2\sqrt{2}F_V(p^2-q^2)}{(p^2-M_V^2)(q^2-M_V^2)}\left(2\kappa_{11}^V+2\kappa_{12}^V-\kappa_{17}^V-\frac{\sqrt{2}F_A\kappa_5^{VA}}{r^2-M_A^2}\right).
\end{aligned}
$$

- Impossible to match to the OPE contribution!
  - OPE does not reproduce the perturbative term.
- Generally: the matching RχT to OPE is still unclear.
  - Unclear how to deal with logarithmic terms.
  - Infinite tower of resonances is needed.

VVA Green function: AdS/QCD

- Phenomenologically important formfactor $w_T(Q^2)$:

$$w_T(Q^2) = -16\pi^2 \left[w_T^{(1)}(-Q^2, 0, -Q^2) + w_T^{(3)}(-Q^2, 0, -Q^2)\right].$$

- Expand $w_T(Q^2)$ in terms of Q^2 up to $\mathcal{O}\left(\frac{1}{Q^8}\right)$.
- Why? Soft-wall AdS/QCD and OPE [P. Colangelo *et al.* '12]:

$$w_T(Q^2) = \frac{N_c}{Q^2} + \frac{128\pi^3 \alpha_s \chi \langle \overline{q} q \rangle^2}{9Q^6} + \mathcal{O}\left(\frac{1}{Q^8}\right).$$

- Comparison leads to a system of equations:

$$\frac{N_c}{64\pi^2 F_V} + \sqrt{2}(\kappa_{11}^V + \kappa_{12}^V) = 0\,,$$

$$\frac{F_V \kappa_3^{VV} - F_A \kappa_5^{VA}}{M_V^2} + \sqrt{2}(\kappa_{11}^V + \kappa_{12}^V) = -\frac{N_c}{64\pi^2 F_V}\,,$$

$$\frac{F_V \kappa_3^{VV} - F_A \kappa_5^{VA}}{M_V^2} + \sqrt{2}(\kappa_{11}^V + \kappa_{12}^V) - F_A \kappa_5^{VA} \frac{M_A^2}{M_V^4} = 0\,,$$

$$\frac{F_V \kappa_3^{VV} - F_A \kappa_5^{VA}}{M_V^2} + \sqrt{2}(\kappa_{11}^V + \kappa_{12}^V) - F_A \kappa_5^{VA} \frac{M_A^2}{M_V^4}\left(1 + \frac{M_A^2}{M_V^2}\right) = -\frac{2\pi \alpha_s \chi \langle \overline{q} q \rangle^2}{9 F_V M_V^4}\,.$$

VVA Green function: AdS/QCD

- Phenomenologically important formfactor $w_T(Q^2)$:

$$w_T(Q^2) = -16\pi^2 \left[w_T^{(1)}(-Q^2, 0, -Q^2) + w_T^{(3)}(-Q^2, 0, -Q^2) \right].$$

- Expand $w_T(Q^2)$ in terms of Q^2 up to $\mathcal{O}\left(\frac{1}{Q^8}\right)$.
- Why? Soft-wall AdS/QCD and OPE [P. Colangelo *et al.* '12]:

$$w_T(Q^2) = \frac{N_c}{Q^2} + \frac{128\pi^3 \alpha_s \chi \langle \overline{q} q \rangle^2}{9Q^6} + \mathcal{O}\left(\frac{1}{Q^8}\right).$$

- Comparison leads to a system of equations:

$$\frac{N_c}{64\pi^2 F_V} + \sqrt{2}(\kappa_{11}^V + \kappa_{12}^V) = 0\,,$$

$$\frac{F_V \kappa_3^{VV} - F_A \kappa_5^{VA}}{M_V^2} + \sqrt{2}(\kappa_{11}^V + \kappa_{12}^V) = -\frac{N_c}{64\pi^2 F_V}\,,$$

$$\frac{F_V \kappa_3^{VV} - F_A \kappa_5^{VA}}{M_V^2} + \sqrt{2}(\kappa_{11}^V + \kappa_{12}^V) - F_A \kappa_5^{VA} \frac{M_A^2}{M_V^4} = 0\,,$$

$$\frac{F_V \kappa_3^{VV} - F_A \kappa_5^{VA}}{M_V^2} + \sqrt{2}(\kappa_{11}^V + \kappa_{12}^V) - F_A \kappa_5^{VA} \frac{M_A^2}{M_V^4}\left(1 + \frac{M_A^2}{M_V^2}\right) = -\frac{2\pi \alpha_s \chi \langle \overline{q} q \rangle^2}{9 F_V M_V^4}\,.$$

VVA Green function: Coupling constants constraints

- It is possible to extract the following coupling constants constraints:

$$\kappa_{11}^{V} + \kappa_{12}^{V} = -\frac{N_c}{64\sqrt{2}\pi^2 F_V}\,, \quad \kappa_3^{VV} = -\frac{N_c M_V^4}{64\pi^2 M_A^2 F_V^2}\,, \quad \kappa_5^{VA} = \kappa_3^{VV}\frac{F_V}{F_A}\,.$$

  - Since it is not possible to solve the system of equations completely, the relevance of the constraints should be taken with caution!
- Determination of κ_5^{VA}:
  - Numerically: $\kappa_5^{VA} = -0.086$.
  - From the decay $f_1(1285) \to \rho\gamma$: $\kappa_5^{VA} = -0.062 \pm 0.030$.
- Using the constraints for VVP we can also determine:

$$\kappa_2^{VV} = \frac{1}{64F_V^2}\left(F^2 - \frac{N_c M_V^4}{8\pi^2 M_A^2}\right), \quad \kappa_3^{PV} = -\frac{F^2}{32\sqrt{2}d_m F_V}\left[1 + \frac{N_c M_V^2}{8\pi^2 F^2}\left(\frac{M_V^2}{M_A^2} - 1\right)\right].$$

  - However: BABAR dictates $\kappa_3^{PV} = -\frac{F^2}{32\sqrt{2}d_m F_V}(1+\delta_{\rm BL})$ with the value $\delta_{\rm BL} = -0,055 \pm 0.025$ from VVP.
- However, our prediction from VVA gives $\delta_{\rm BL} = -1.342\,$.

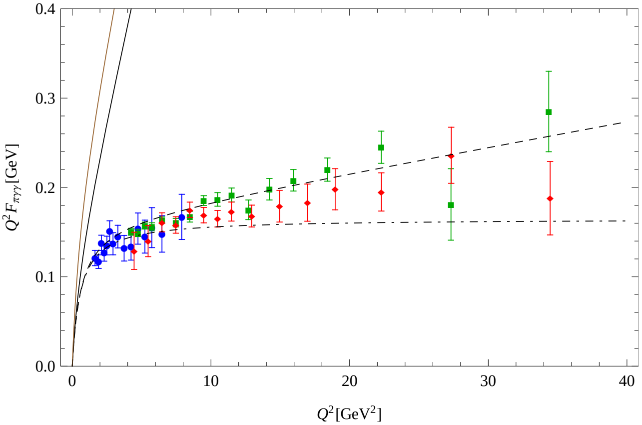


Figure: A plot of BABAR (green), BELLE (red) and CLEO (blue) data fitted with the formfactor $\mathcal{F}^{\mathrm{R}\chi\mathrm{T}}_{\pi^0\gamma\gamma}(0, -Q^2; 0)$ using the modified Brodsky-Lepage condition. The full black line represents our fit with $\delta_{\mathrm{BL}} = -1.342$, and the full brown line is a fit using the LMD formfactor. The dashed line stands for $\delta_{\mathrm{BL}} = -0.055$ and the dot-dashed line for $\delta_{\mathrm{BL}} = 0$.

Conclusion

- Some properties of low-energy QCD and Green functions were summarized.
- We calculated OPE for all nonvanishing three-point Green functions.
- We also study Green functions in the odd-intrinsic parity sector, calculated in the NLO, i.e. up to $\mathcal{O}(p^6)$.
- Examples on two specific correlators were shown:
    - VVP
    - VVA
        - Newest results were presented.
        - OPE with two large momenta is obviously inconsistent with reality, OPE with all three large momenta is needed (and in finishing stages).
- Matching RχT with OPE is still unclear.

Thank you for your attention.