OSFT Techniques in Condensed Matter

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in collaboration with

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Outline

1. Preliminary notions
   - Conformal Field Theory
   - Bosonic string, BCFT and OSFT
2. OSFT in condensed matter
   - OSFT/BCFT relation
   - QSHI: a case study

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Massless free boson in 2d: classical

- euclidean action for $\phi(z,\bar{z}) : \mathbb{C} \to \mathbb{R}$

$$S = \frac{1}{2\pi} \int d^2 z \, \partial\phi \, \bar{\partial}\phi \quad \overset{\delta S = 0}{\Longrightarrow} \quad \partial\bar{\partial}\phi(z,\bar{z}) = 0.$$

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- 2d conformal symmetry: $(z,\bar{z}) \to (w,\bar{w}) = (f(z), \bar{f}(\bar{z}))$, generated by

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- conformal Noether currents $z^{m+1}T(z), \bar{z}^{n+1}\bar{T}(\bar{z})$, charges (Laurent)

$$L_m = \frac{1}{2\pi i}\oint_{\mathcal{C}(0)} dz\, z^{n+1}\,T(z)\,, \qquad \bar{L}_m = \frac{1}{2\pi i}\oint_{\mathcal{C}(0)} d\bar{z}\,\bar{z}^{n+1}\,\bar{T}(\bar{z})$$

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- Hilbert space $|m_1, \ldots \bar{n}_1, \ldots\rangle = \alpha_{-m_1}\ldots\bar{\alpha}_{-m_1}\ldots|0\rangle\otimes\overline{|0\rangle}$, $m_i, n_i > 0$,

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- quantum conformal algebra

$$[L_m,L_n] = (m-n)L_{m+n} + \frac{c}{12}(m^3-m)\delta_{m+n,0}\,, \qquad \text{[Virasoro]}$$

with $c=1$, same for $\bar L_m$s with $[L_m,\bar L_n]=0$

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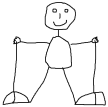

akin to lifting oneself by pulling straps attached to one's boots

CFTs: beacons in theory space

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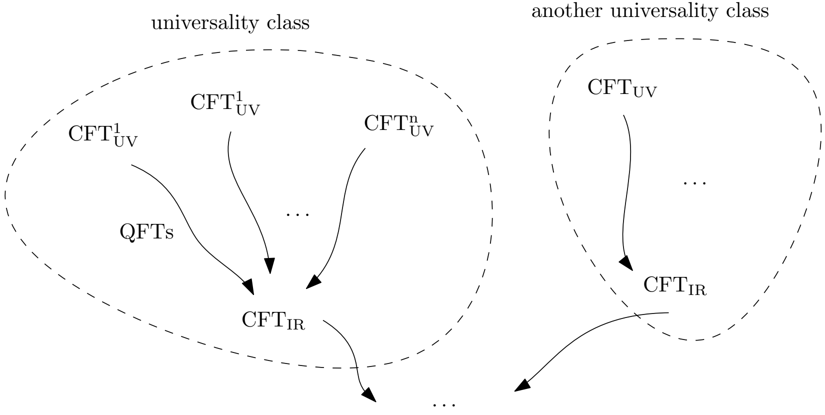
$\rightarrow$ CFTs as signposts in the QFT landscape



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Closed bosonic string in flat background

- extremization of worldsheet area gives $(\mu, \nu = 1, \ldots, D,\ a, b = 1, 2)$

$$S_{\mathrm{Poly}} = \frac{1}{4\pi} \int d^2\sigma \sqrt{-h} h^{ab} \eta_{\mu\nu} \partial_a X^\mu(\sigma) \partial_b X^\nu(\sigma)$$

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with $Q_{\mathrm{B}}=-c_0+\sum_n c_n L^{\mathrm{m}}_{-n}+\sum_{m,n}\frac{m-n}{2}:c_m c_n b_{-m-n}:$ and

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- $\mathcal{H}_{\rm phys} = \mathrm{Ker}\,Q_{\rm B}/\mathrm{Im}\,Q_{\rm B}$ at $\#_{\rm gh} = 1$ with level matching

$$(\text{creation ops.})|\Omega,k\rangle\,,\quad |\Omega,k\rangle = e^{ik\cdot X(0,0)}c_1|0\rangle\otimes\bar c_1\overline{|0\rangle}$$

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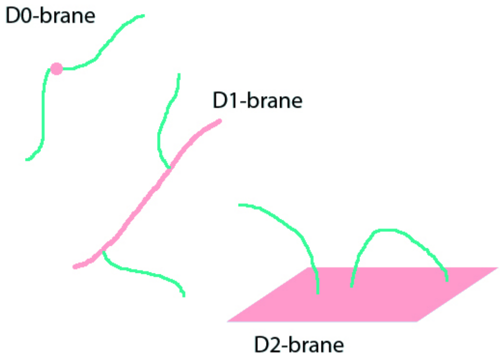
- solution: Dirichlet/Neumann BCs ($\mathrm{D}p$-branes)



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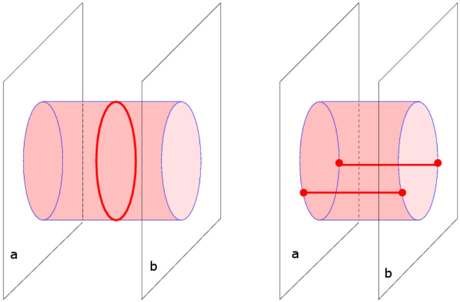

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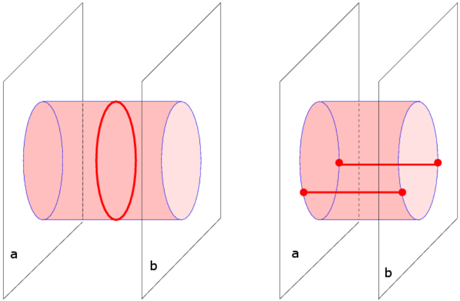

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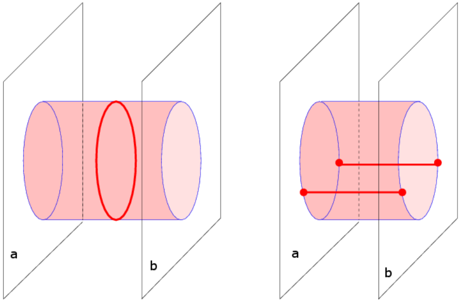

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$\rightarrow$ boundary state $\| B \rangle\rangle$: formal limit of bulk states

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- symmetry algebra $\mathcal{W} \supset \mathrm{Vir}$: allow gluing automorphism Ω s.t.

$$(W_n - \Omega \bar{W}_{-n})\|B\rangle\rangle = 0\,, \qquad \Omega L_n = L_n\,, \quad \Omega \bar{L}_n = \bar{L}_n$$

for $\mathcal{W} = \widehat{\mathrm{u}}(1)$: $W_n = \alpha_n$, D/N BCs $\subset$ free boson BCs ($\hat{\mathrm{u}}(1)$-breaking)

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BCFT: conformal boundaries II

- condition for conformal symmetry

$$(L_n - \bar{L}_{-n})\|B\rangle\rangle = 0$$

  - 1-1 correspondence with spinless bulk primaries $\phi_{h,h}$ [Ishibashi]

$$\|B\rangle\rangle = \sum_{h\in\mathrm{Spec}} B_h|\phi_{h,h}\rangle\rangle\,, \qquad g_B := B_0$$

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- BCFT deformations $S \to S + \int_{-\infty}^{\infty} dx\, V[\phi(x)]$ generally induce RG flow to a new BCFT$'$ with $g' < g$ [Affleck, Friedan, Konechny]

Cubic OSFT: stringy background

- free ends from now on (WLOG): most general state (string field)

$$|\Psi\rangle = \int d^{26}k \, \left(\tilde{\phi}(k) + \tilde{A}_\mu(k)\alpha^\mu_{-1} + \tilde{B}_{\mu\nu}(k)\alpha^\mu_{-1}\alpha^\nu_{-1} + \dots\right) e^{ik\cdot X(0)} c_1|0\rangle$$

$\phi, A_\mu, \dots$ have action (free string)

$$S_2 = \frac{1}{2}\langle\Psi|Q_{\mathrm{B}}|\Psi\rangle = \frac{1}{2}\langle\mathcal{I}\circ\Psi(0)Q_{\mathrm{B}}\Psi(0)\rangle \overset{\delta S_2=0}{\Longrightarrow} Q_{\mathrm{B}}\Psi = 0\,,$$

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$$S_{\mathrm{OSFT}} = S_2 + S_3 \quad \overset{\delta S_{\mathrm{OSFT}}=0}{\Longrightarrow} \quad Q_{\mathrm{B}}\Psi + \Psi*\Psi = 0$$

  with gauge freedom $\delta\Psi = Q_{\mathrm{B}}\chi + \Psi*\chi - \chi*\Psi$

Outline

1. Preliminary notions
   - Conformal Field Theory
   - Bosonic string, BCFT and OSFT
2. OSFT in condensed matter
   - OSFT/BCFT relation
   - QSHI: a case study

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- similar picture to BCFT deformation: $\mathrm{BCFT}_c \xrightarrow{\text{OSFT}} \mathrm{BCFT}'_c$, but OSFT can go against RG flow [Kudrna, Rapčák & Schnabl]

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Helical spinful TLL at criticality

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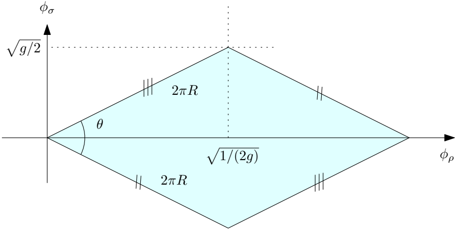
- bosonised fields: ϕ_ρ, ϕ_σ compactified on twisted torus with

$$R_1 = R_2 = R = \sqrt{(1+g^2)/(2g)}\,, \quad \tan(\theta/2) = (1/g)$$



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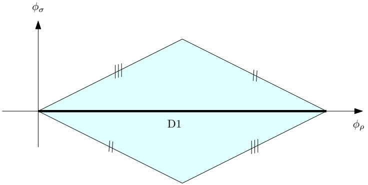
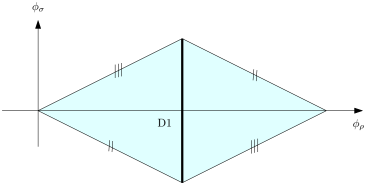


(a) CI phase



(b) IC phase

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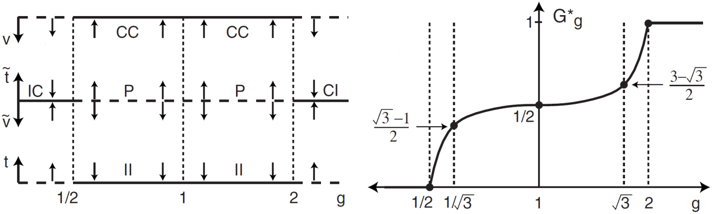

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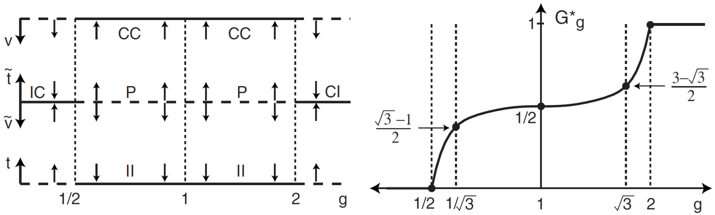

- analytical predictions for critical conductivity

$$G_\rho^* = \begin{cases} 1/2 + \mathcal{O}(\varepsilon^3)\,, & g = 1 - \varepsilon \\ (\sqrt{3} - 1)/2\,, & g = 1/\sqrt{3} \\ \pi^2 \varepsilon\,, & g = 1/2 + \varepsilon \end{cases}$$

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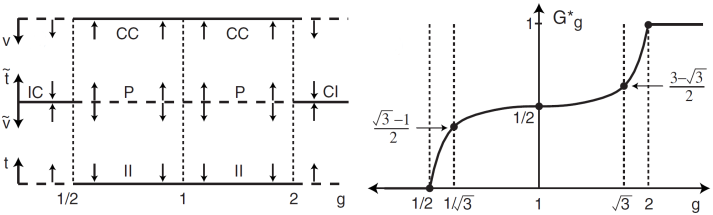

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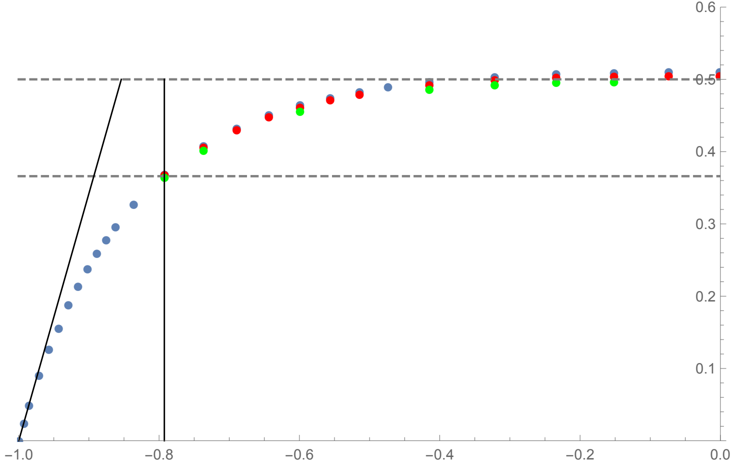
- can determine critical conductivity from BCFT as $G_\rho^* = f(B_{\partial X})$

G_ρ^*: level truncation results

- OSFT level truncation $G^*(g)$ data (levels 11, 12, 13)



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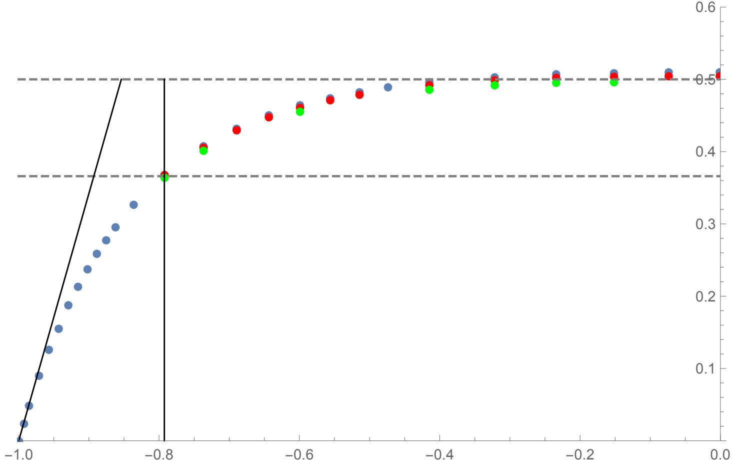

- P is a $\mathcal{W} = \widehat{\mathrm{su}}(2)_1$ WZW boundary state at $g = 1$!

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- other applications: junctions of quantum wires, impurities, ...

References and further reading

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work in progress.