# Inflationary theory and pulsar timing investigations of primordial black holes and gravitational waves

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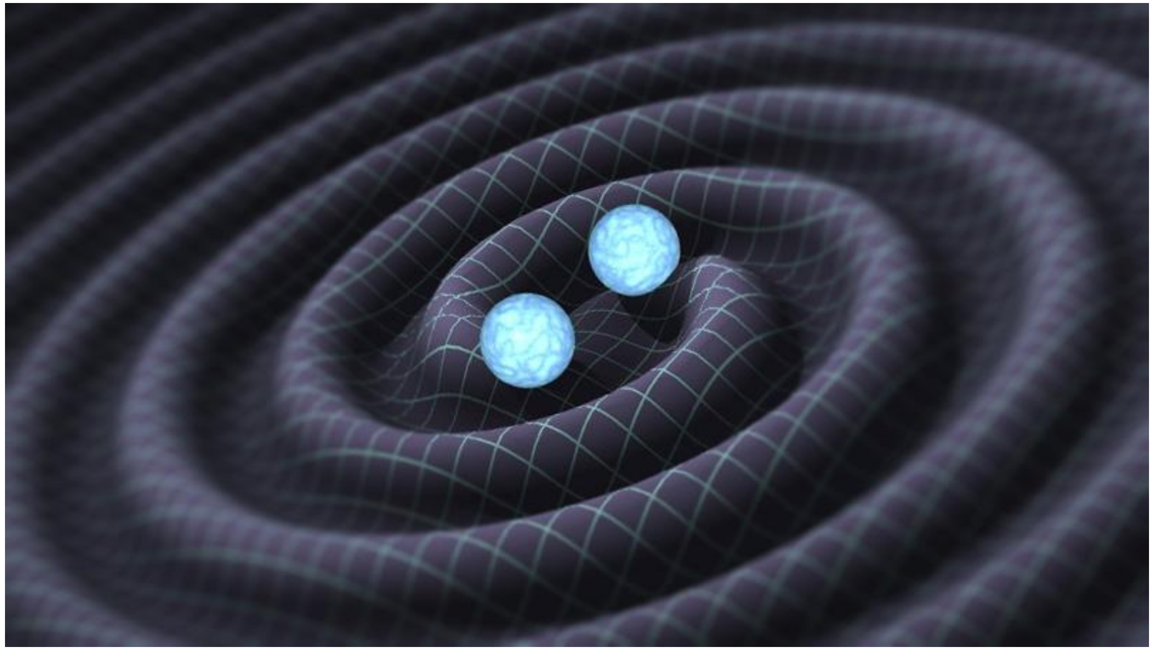
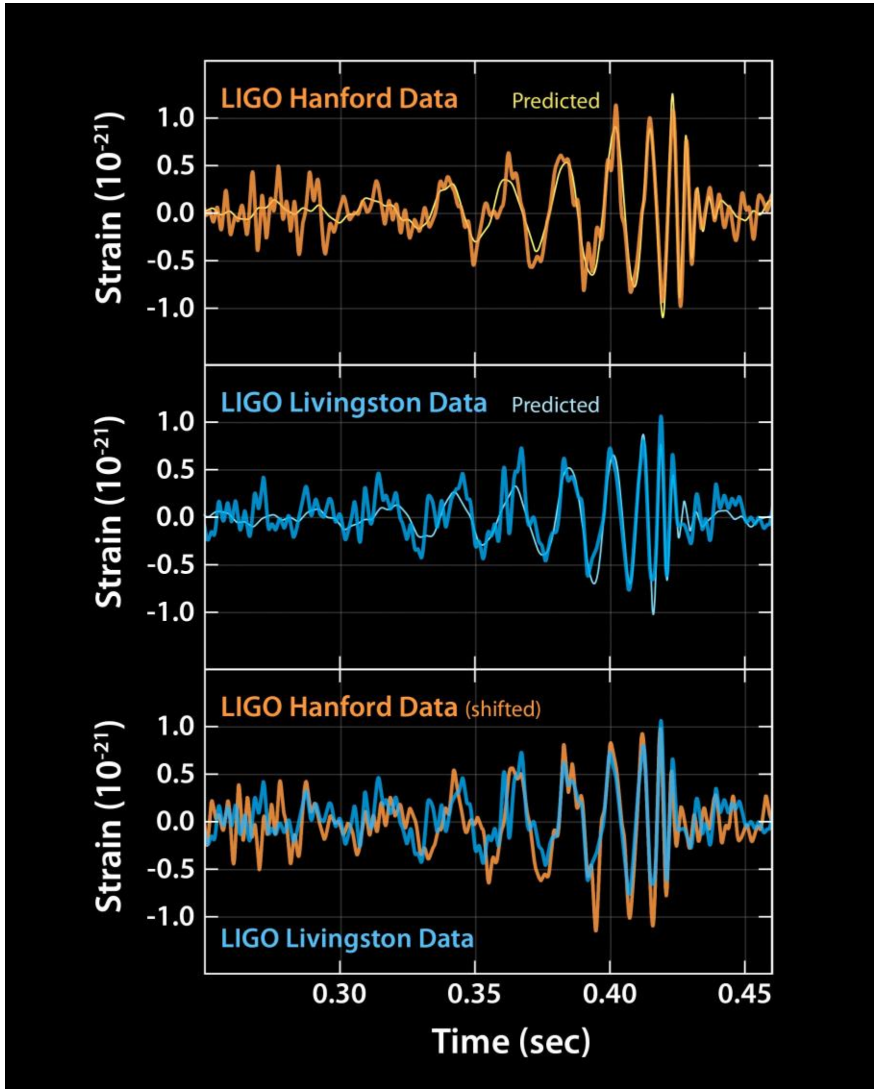





LIGO observes mergers of ~10 to 30 solar mass black holes.

Were these black holes produced primordially?

LIGO

# Abundance?

Bird *et. al.* (2016):
Binaries formed in DM halos could have a merger rate consistent with LIGO if PBHs made up all of the DM abundance.

Sasaki *et. al.* (2016) and Eroshenko (2016):
BH binaries form during the early universe.
The expected merger rate today is higher.
PBHs can only be a small fraction $\sim 10^{-3}$ to $10^{-2}$ of the total DM abundance.

Key point:
Still lots of uncertainty in the PBH DM abundance fraction consistent with the observed LIGO merger rate.

# PBH formation and mass

Overdensities seeded in the early universe.

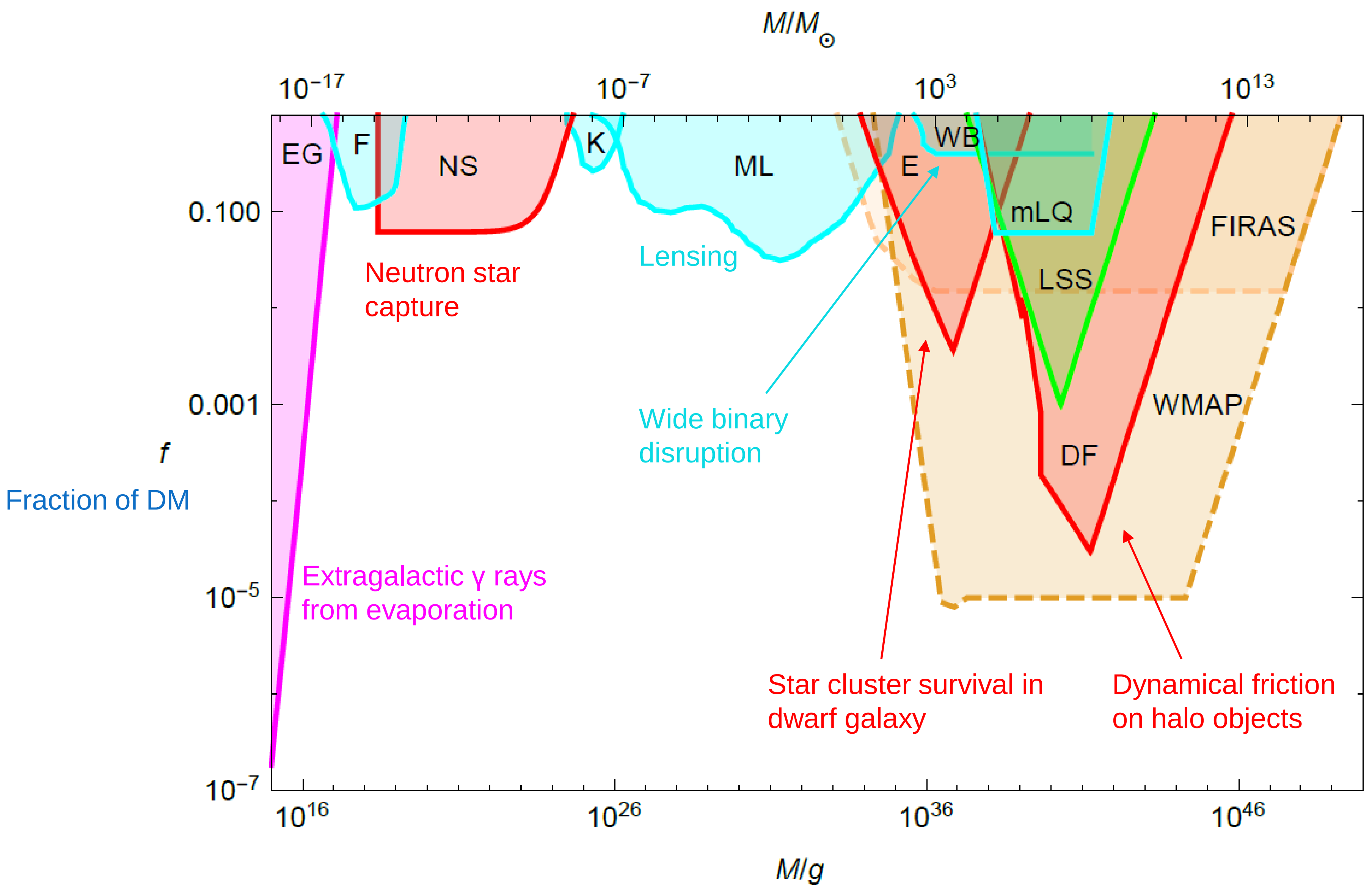
If overdensities are large enough, their gravitational attraction can overcome the rate of Hubble expansion.

Upon reentering the horizon, they immediately collapse to black holes.

Their mass is approximately the mass contained within the horizon:

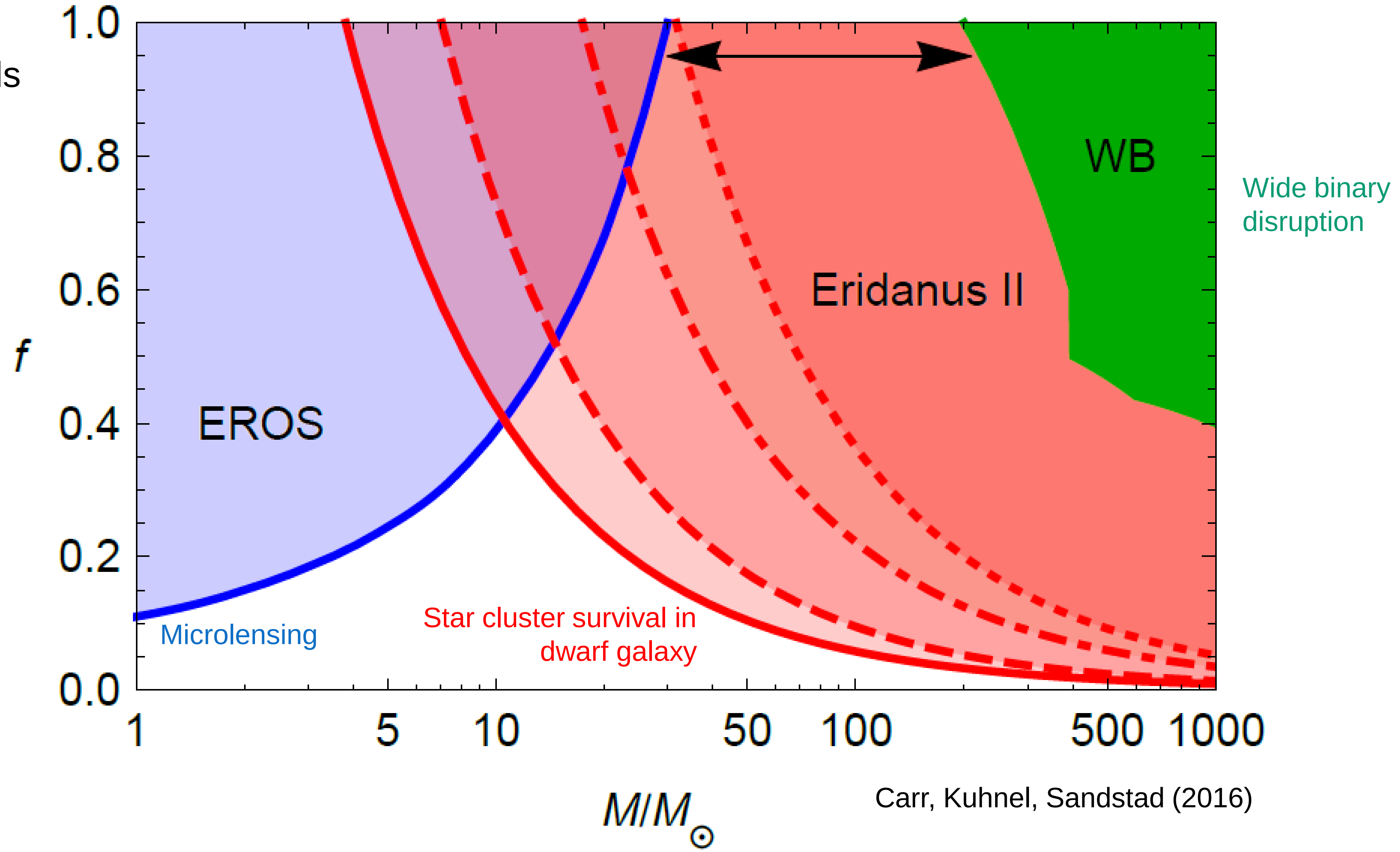
$$M_{\mathrm{PBH}} = \frac{4\pi}{3}\rho_r H^{-3} \simeq 10 M_{\odot} \left(\frac{g_*}{100}\right)^{-1/6} \left(\frac{\mathrm{pc}^{-1}}{k_f}\right)^2$$

(We assume radiation domination (RD) throughout)




Carr, Kuhnel, Sandstad (2016)

Note CMB bounds are not shown




Carr, Kuhnel, Sandstad (2016)

[Green (2016)]: generic examples of extended spectra centered in the LIGO window <u>cannot make up all of DM</u>.

# Secondary gravity waves

How do we know if the BHs are primordial?

The primordial fluctuations must satisfy $\delta\rho/\rho \sim \mathcal{O}(1)$ for matter overdensities to be large enough for PBHs to form.

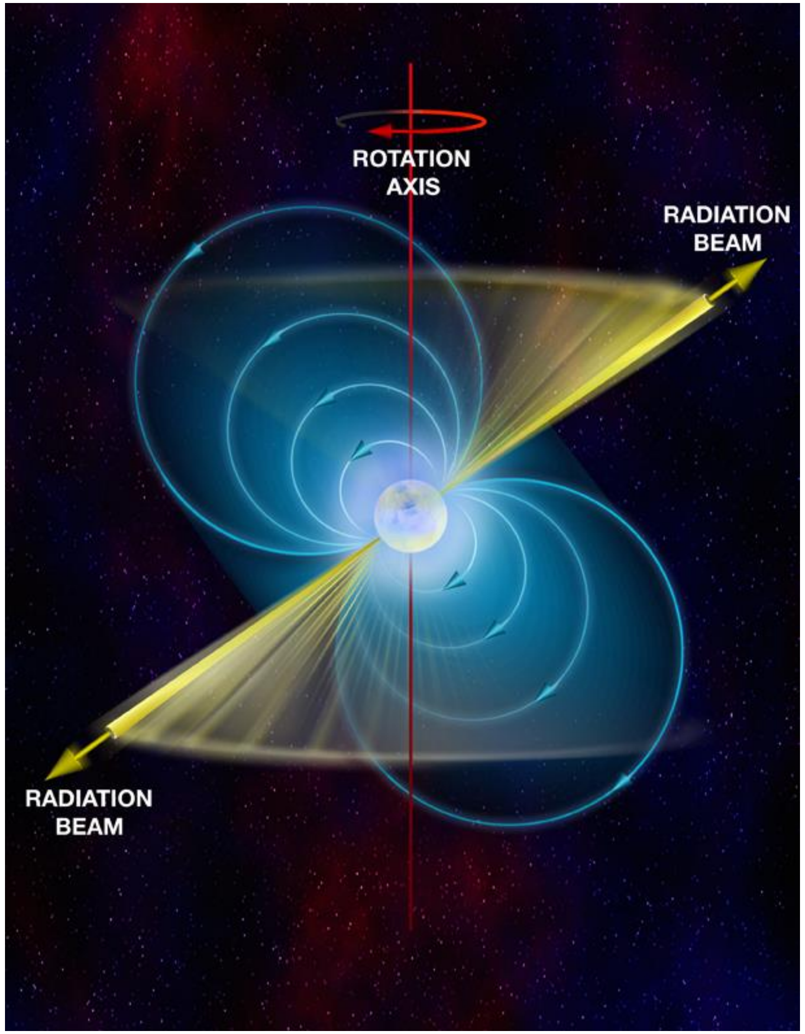
(Compare to CMB temperature fluctuations that are $\mathcal{O}(10^{-5})$.)

These large scalar perturbations lead to tensor perturbations at second order in cosmological perturbation theory.

The tensor perturbations may be observed as gravity waves (SGWs).

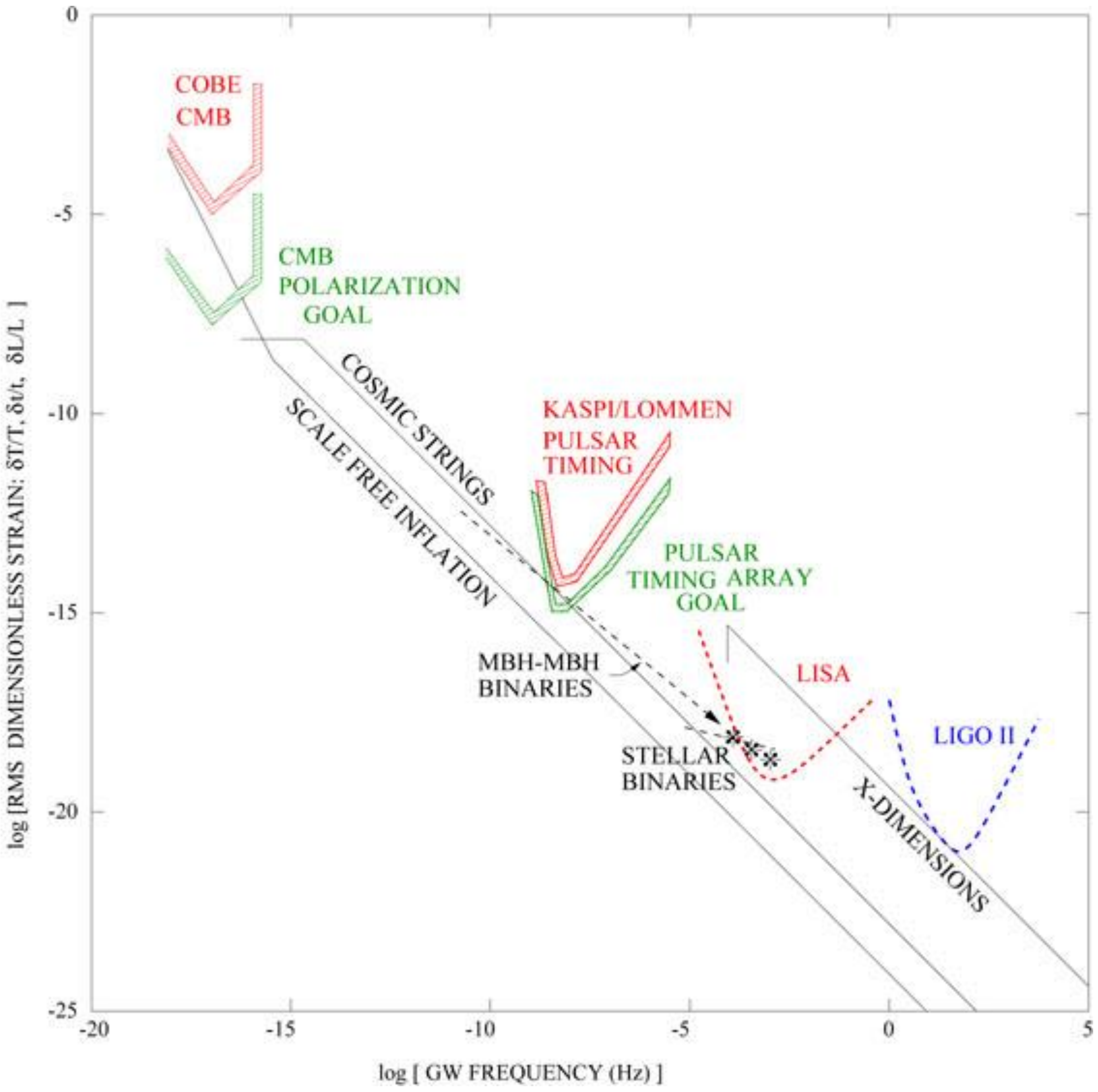
Ananda, Clarkson, Wands (2006)
Baumann, Steinhardt, Takahashi, Ichiki (2007)
Saito, Yokoyama (2008 and 2009)
Bugaev, Klimai (2010)
Alabidi, Kohri, Sasaki, Sendouda (2012)
…

# Pulsar timing arrays




[Bill Saxton, NRAO/AUI/NSF]

# GW detectors comparison




[Backer, Jaffe, Lommen (2003)]

# Dependence on scalar spectrum

PBH abundance: *exponential* dependence (from Press-Schechter formalism)

$$\beta(M) = \frac{d}{d\log M}\frac{\rho_{\rm PBH}(t_f)}{\rho_{\rm tot}(t_f)} = 2\int_{\delta_c}^{\infty} d\delta \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{\delta^2}{2\sigma^2}} = \mathrm{Erfc}\left(\frac{\delta_c}{\sqrt{2}\sigma_{R_M}}\right)$$

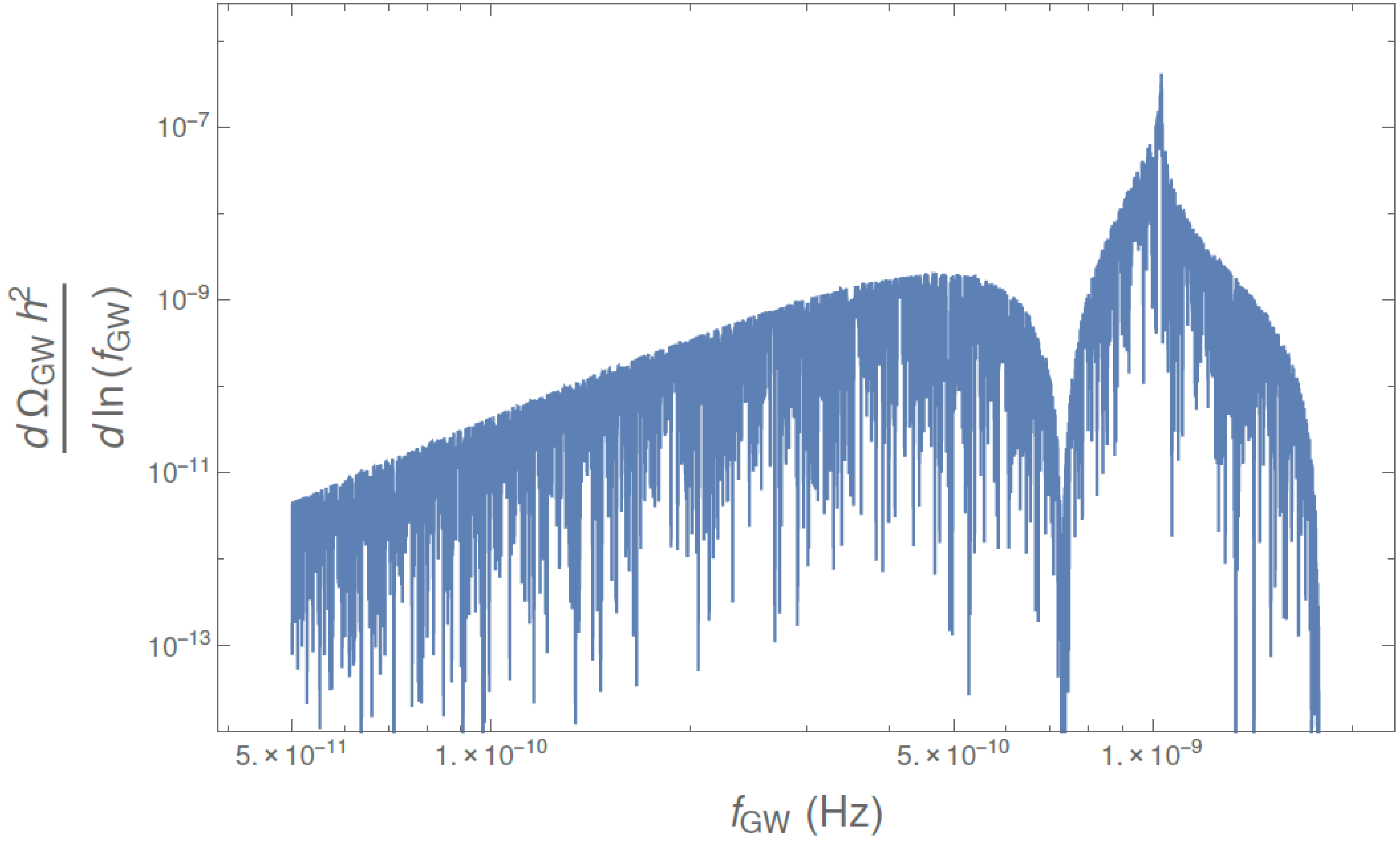
$$\sigma_{R_M}^2 = \int \frac{dk}{k} W(kR_M)^2 \mathcal{P}_\delta(k) = \int \frac{dk}{k} W(kR_M)^2 \left(\frac{4}{9}\right)(kR_M)^4 \mathcal{P}_\Phi(k)$$

GW abundance: *quadratic* dependence

$$\mathcal{P}_h(k,\eta) = \int_0^\infty d\tilde{k} \int_{-1}^{1} d\mu\, \mathcal{P}_\Phi(|\mathbf{k}-\tilde{\mathbf{k}}|)\mathcal{P}_\Phi(\tilde{k})\mathcal{F}(k,\tilde{k},\mu,\eta),$$

Orders of magnitude changes in PBH abundance have only small changes in GW abundance.

# $\delta$-function $P_\Phi$




See also earlier work:
Ananda, Clarkson, Wands (2006)
Saito, Yokoyama (2008 and 2009)

# Extended spectra

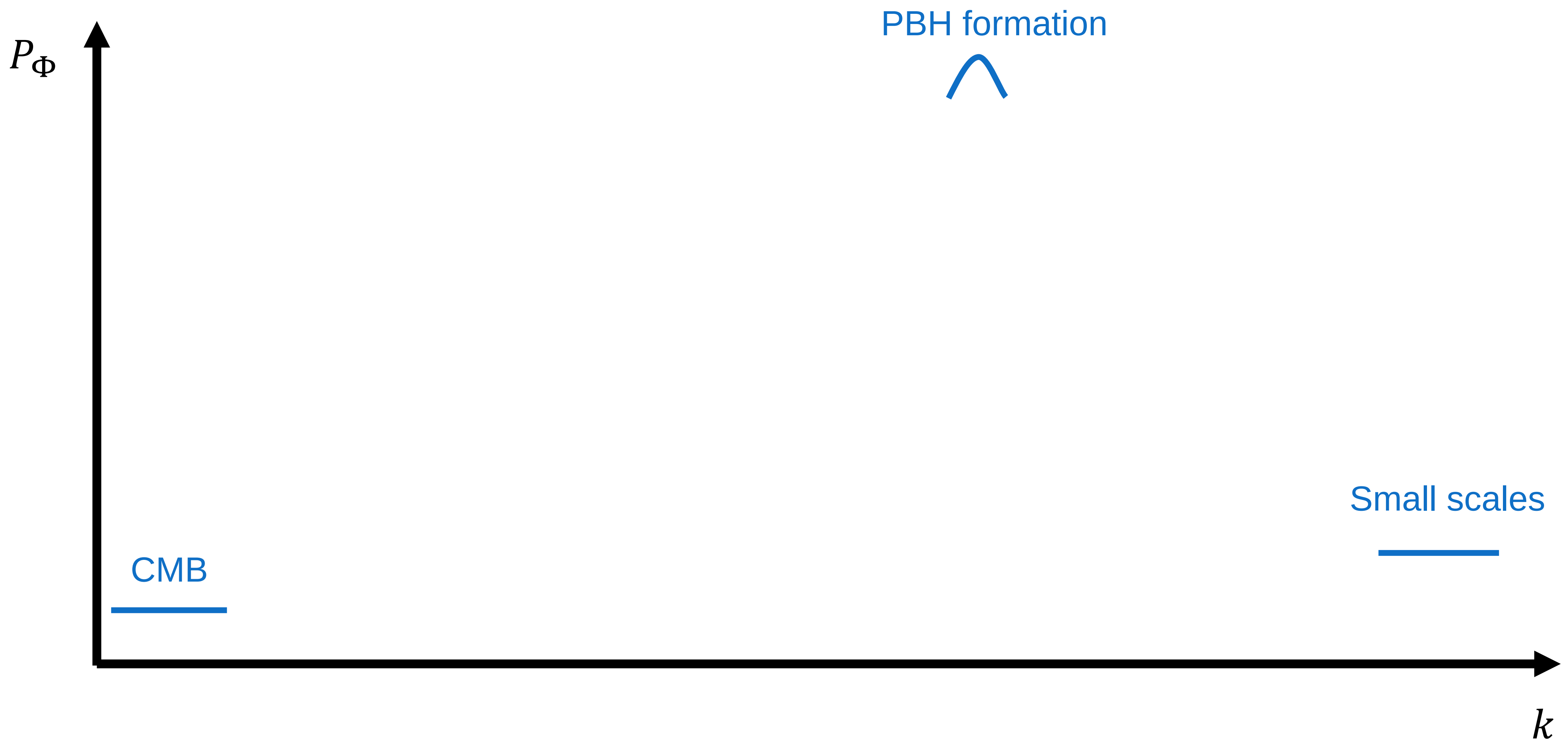
Physical spectra $P_{\Phi}$ are extended.

Our work – consider several working examples for PBH formation:

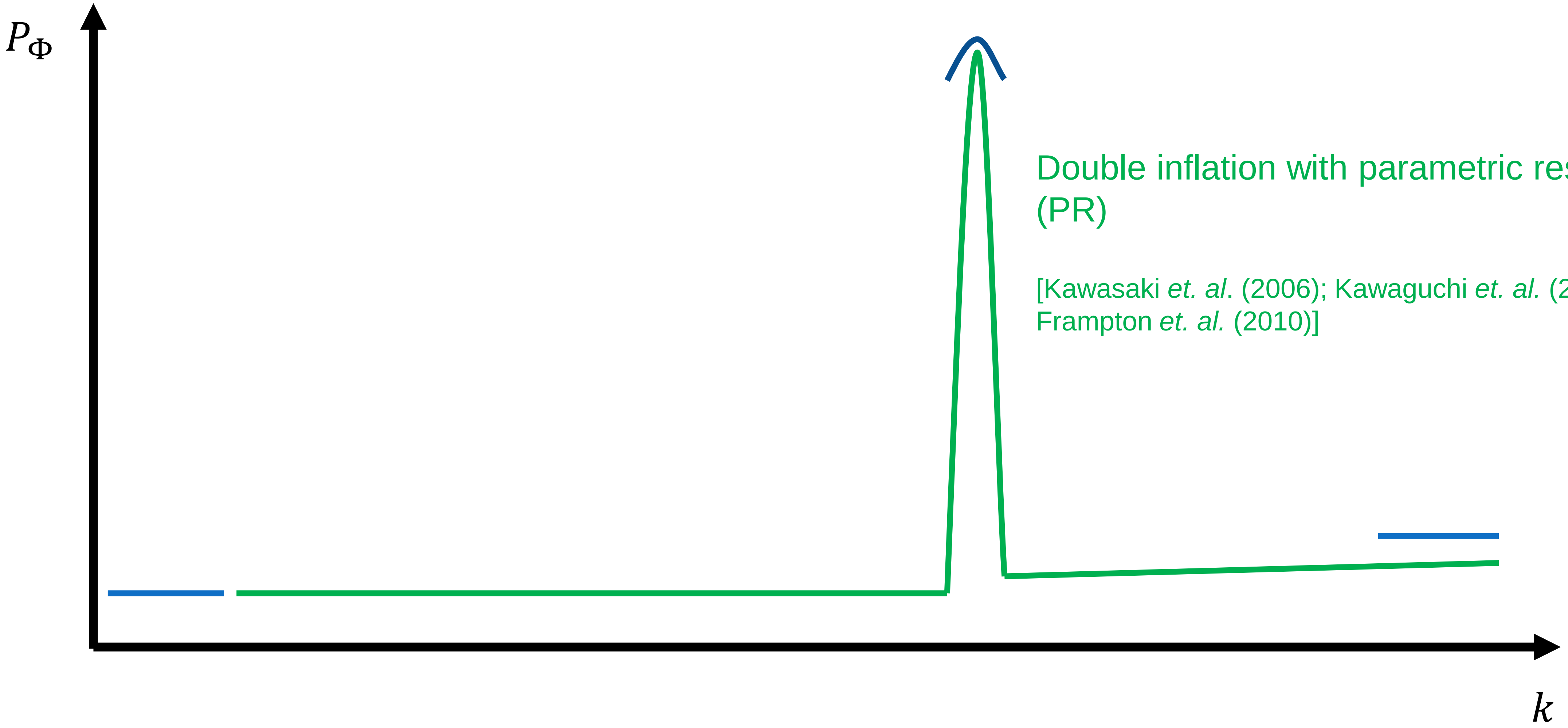
- Narrow spectrum:
  - Double inflation with parametric resonance
- Broad spectra:
  - Double inflation without parametric resonance
  - Running-mass inflation
- Other phenomena:
  - Axion-curvaton

Use these to draw more general conclusions.

# Scalar spectra



# Scalar spectra



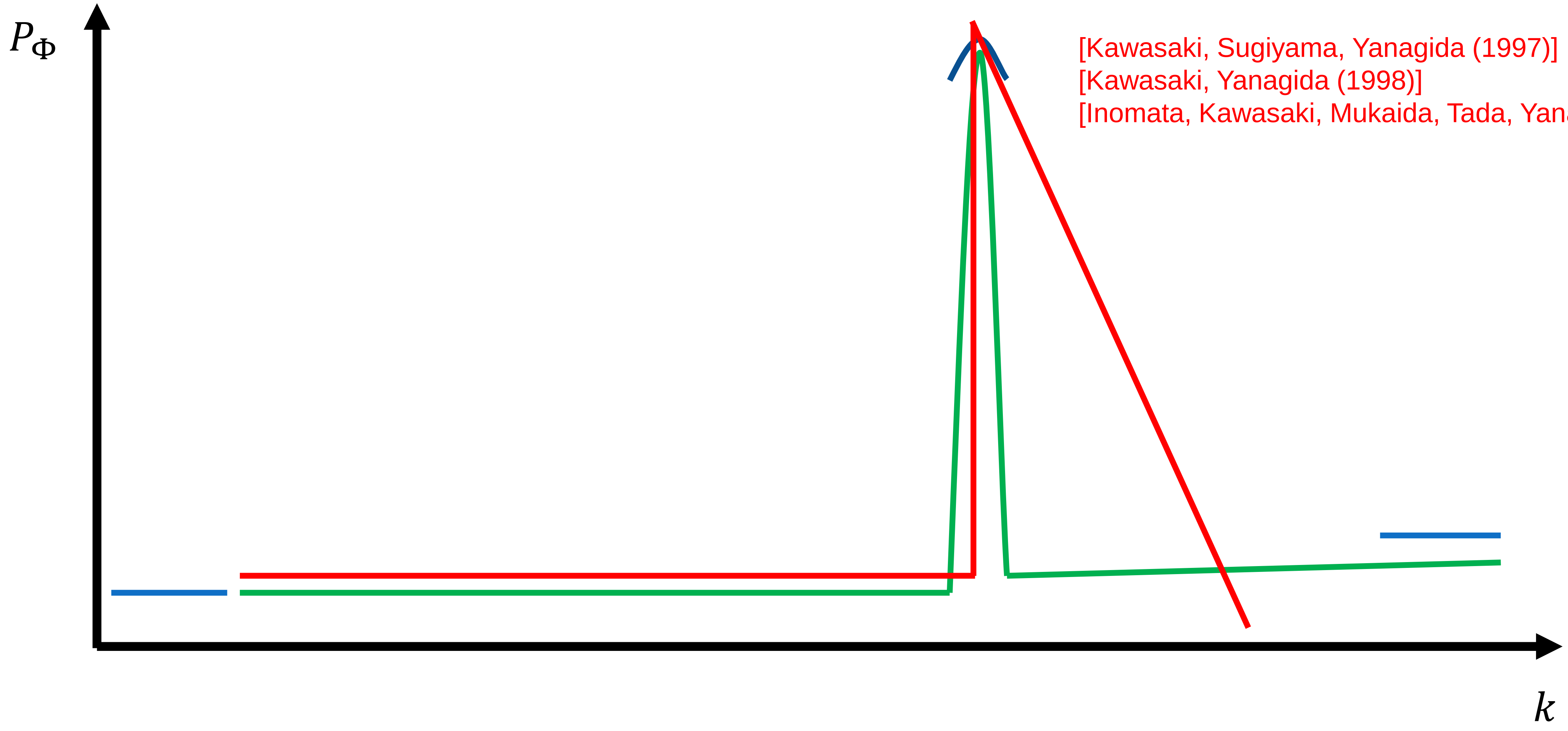

Double inflation with parametric resonance (PR)

[Kawasaki *et. al*. (2006); Kawaguchi *et. al.* (2007); Frampton *et. al.* (2010)]

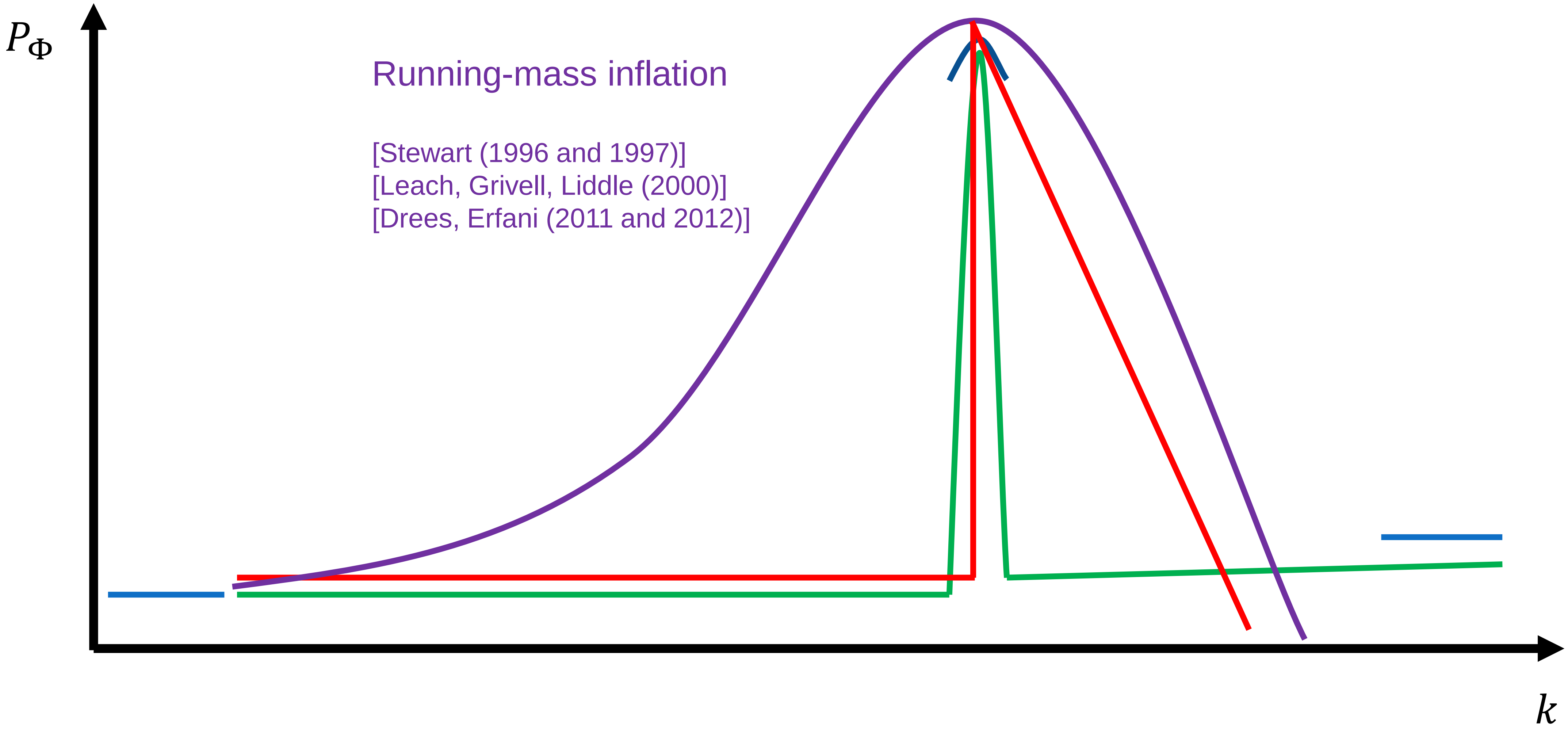
# Scalar spectra

Double inflation (no PR)

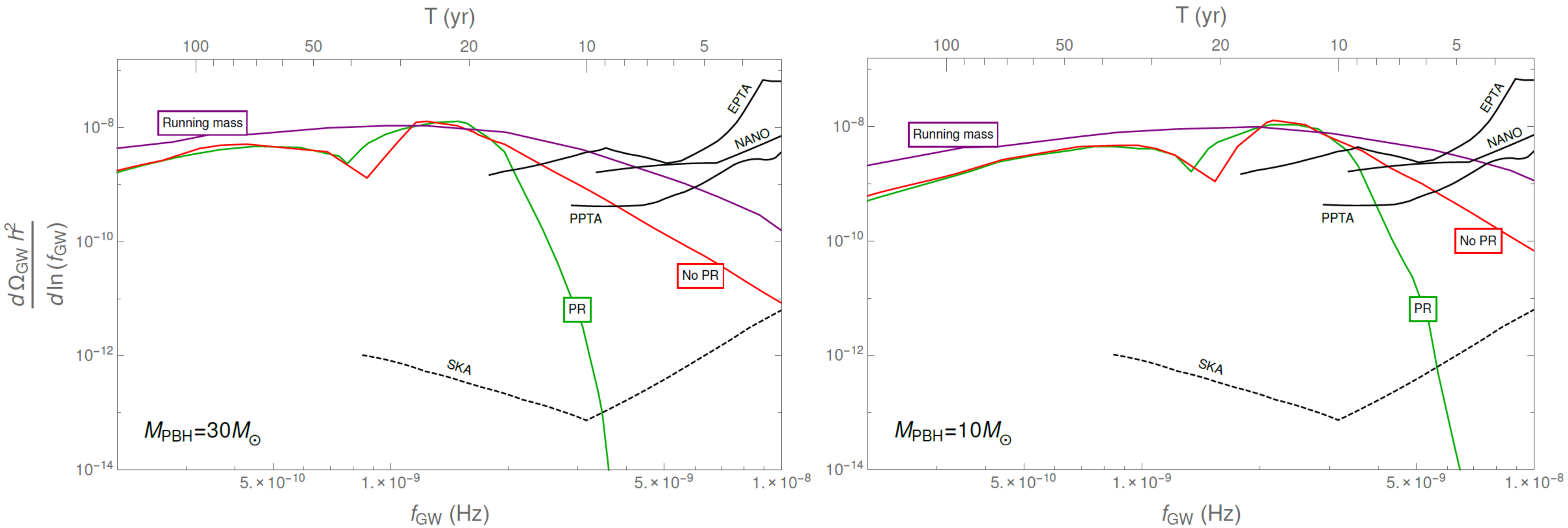
[Kawasaki, Sugiyama, Yanagida (1997)]
[Kawasaki, Yanagida (1998)]
[Inomata, Kawasaki, Mukaida, Tada, Yanagida (2016)]



# Scalar spectra



# Gravity wave spectra




Assume $\Omega_{\mathrm{PBH}} = \Omega_{\mathrm{DM}}$, $\delta_c = 1/3$,
do not account for critical collapse, non-sphericity

Scales on which PBHs do not form may still have large SGW abundance – remember, exponential vs power law.

# Axion-curvaton

[Kasuya, Kawasaki (2009)]
[Kawasaki, Kitajima, Yanagida (2012)]

Radial field reaches its PQ-breaking min. → Density perturbations grow, energy density decreases.

Curvaton decays. → Density perturbations are at their maximum.
PBHs form.
PBH mass is *smaller* than if it had formed immediately because the energy density has decreased.

Thus, $k_*$ must be larger than “expected” to form a PBH at a given mass.

SGWs are formed at larger frequency, <u>cannot detect with PTA</u>!

# Summary

- BH mergers observed by LIGO could be PBHs.
- Probes of GWs by PTAs could be sensitive to the early-universe dynamics responsible for PBH formation.
- GW abundance is largely insensitive to the PBH abundance.
- PTAs can already exclude broader primordial spectra.
  Narrower spectra will soon be probed.
  However, some models can still escape detection by PTAs.

# Backup

# PBH abundance

Use Press-Schechter formalism to calculate the energy fraction of PBHs in the mass range $(M, M + dM)$ at their formation time $t_f$

$$\beta(M) = \frac{d}{d\log M}\frac{\rho_{\rm PBH}(t_f)}{\rho_{\rm tot}(t_f)} = 2\int_{\delta_c}^{\infty} d\delta \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{\delta^2}{2\sigma^2}} = {\rm Erfc}\left(\frac{\delta_c}{\sqrt{2}\sigma_{R_M}}\right)$$

$$\boxed{\delta \equiv \delta\rho/\rho}$$

$$\sigma_{R_M}^2 = \int \frac{dk}{k} W(kR_M)^2 \mathcal{P}_\delta(k) = \int \frac{dk}{k} W(kR_M)^2 \left(\frac{4}{9}\right)(kR_M)^4 \mathcal{P}_\Phi(k)$$

$$W(x) = e^{-x^2/2} \qquad\qquad \delta_c = 1/3$$

Then,

$$\frac{d}{d\log M}\Omega_{\rm PBH}h^2 \simeq 2\times 10^7 \beta(M)\left(\frac{g_{*,i}}{106.75}\right)^{-1/4}\left(\frac{M}{M_\odot}\right)^{-1/2}$$

(assuming Gaussian perturbations, neglecting critical collapse and non-sphericity effects)

# Secondary gravity waves

How to derive:

1. Consider perturbations to the FRW metric to second order

$$\mathrm{d}s^2 = a^2(\eta)\left[-\left(1+2\Phi^{(1)}+2\Phi^{(2)}\right)\mathrm{d}\eta^2 + 2V_i^{(2)}\mathrm{d}\eta\mathrm{d}x^i + \left\{\left(1-2\Psi^{(1)}-2\Psi^{(2)}\right)\delta_{ij} + \frac{1}{2}h_{ij}\right\}\mathrm{d}x^i\mathrm{d}x^j\right]$$

2. Solve the first order EOM for the scalar perturbations (Fourier space)

$$\Phi(\mathbf{k},\eta) = \frac{A(\mathbf{k})}{(\sqrt{w}k\eta)^2}\left(\frac{\sin(\sqrt{w}k\eta)}{\sqrt{w}k\eta} - \cos(\sqrt{w}k\eta)\right)$$

3. Use the Einstein equation to get the EOM for the tensor perturbations

$$h''(\boldsymbol{k},\eta) + \frac{2}{\eta}h'(\boldsymbol{k},\eta) + k^2 h(\boldsymbol{k},\eta) = \mathcal{S}(\boldsymbol{k},\eta),$$

$$\mathcal{S}(\boldsymbol{k},\tilde{\eta}) = \frac{q^{ij}(\boldsymbol{k})}{(2\pi)^{3/2}}\int d^3\tilde{k}\;\tilde{k}_i\tilde{k}_j\left\{12\Phi(\boldsymbol{k}-\tilde{\boldsymbol{k}},\tilde{\eta})\Phi(\tilde{\boldsymbol{k}},\tilde{\eta}) + \left[\tilde{\eta}\Phi(\boldsymbol{k}-\tilde{\boldsymbol{k}},\tilde{\eta}) + \frac{\tilde{\eta}^2}{2}\Phi'(\boldsymbol{k}-\tilde{\boldsymbol{k}},\tilde{\eta})\right]\Phi'(\tilde{\boldsymbol{k}},\tilde{\eta})\right\}$$

$$h(\boldsymbol{k},\eta) = \frac{1}{a(\eta)}\int_{\eta_0}^{\eta} G_k(\eta,\tilde{\eta})a(\tilde{\eta})\mathcal{S}(\boldsymbol{k},\tilde{\eta})d\tilde{\eta},$$

$$G_k(\eta,\tilde{\eta}) = \frac{4}{\pi^2 k}\left[\sin(k\eta)\cos(k\tilde{\eta}) - \cos(k\eta)\sin(k\tilde{\eta})\right].$$

# Secondary gravity waves

Solution for SGW power spectrum:

$$\mathcal{P}_h(k,\eta) = \int_0^\infty d\tilde{k} \int_{-1}^{1} d\mu \; \mathcal{P}_\Phi(|\mathbf{k}-\tilde{\mathbf{k}}|)\mathcal{P}_\Phi(\tilde{k})\mathcal{F}(k,\tilde{k},\mu,\eta),$$

Where:

$$\mathcal{F}(k,\tilde{k},\mu,\eta) = \frac{(1-\mu^2)^2}{a^2(\eta)} \frac{k^3\tilde{k}^3}{|\mathbf{k}-\tilde{\mathbf{k}}|^3} \int_0^\eta d\eta_1 \; a(\eta_1) g_k(\eta,\eta_1) f(\mathbf{k},\tilde{\mathbf{k}},\eta_1)$$
$$\int_0^\eta d\eta_2 \; a(\eta_2) g_k(\eta,\eta_2) \left[ f(\mathbf{k},\tilde{\mathbf{k}},\eta_2) + f(\mathbf{k},\mathbf{k}-\tilde{\mathbf{k}},\eta_2) \right],$$

$$f(\mathbf{k},\tilde{\mathbf{k}},\eta) = 12\Phi(\tilde{k}\eta)\Phi(|\mathbf{k}-\tilde{\mathbf{k}}|\eta) + 8\eta\Phi(\tilde{k}\eta)\Phi'(|\mathbf{k}-\tilde{\mathbf{k}}|\eta) + 4\eta^2\Phi'(\tilde{k}\eta)\Phi'(|\mathbf{k}-\tilde{\mathbf{k}}|\eta)$$

$$g_k(\eta,\eta') = \frac{\sin(k(\eta-\eta'))}{k}$$

SGW abundance today given by:

$$\frac{d}{d\ln k}\Omega_{\rm SGW}(k,\eta_0) \simeq \frac{1}{12}\frac{1}{1+z_{\rm eq}}(k\eta_{\rm eq})^2\mathcal{P}_h(k,\eta_{\rm eq})$$

# Narrow spectrum

Double inflation with parametric resonance

Want to get as close to $\delta$-function spectrum as possible.

After first period of inflation, parametric resonance [Kofman, Linde, Starobinsky (1995, 1998)] excites fluctuations on a particular scale.

This scale is way too small to produce PBHs of the relevant mass. Need a second period of inflation to stretch this scale.

Gives a narrow peak in the power spectrum:

- At any scale---based on length of second inflation
- With any amplitude---based on relationship between efficiency of resonance and decay width of inflaton

[Kawasaki *et. al*. (2006); Kawaguchi *et. al.* (2007); Frampton *et. al.* (2010)]

# Broad spectrum

<u>Double inflation without parametric resonance</u>

First period of inflation gives CMB perturbations,
second period gives larger perturbations relevant for PBH formation.

$$\begin{cases} \mathcal{P}_{\mathcal{R}} = \mathcal{P}_{\mathcal{R}}(k_*)(k/k_*)^{n_s-1} \text{ for } k > k_* \\ \mathcal{P}_{\mathcal{R}} \ll \mathcal{P}_{\mathcal{R}}(k_*) \text{ for } k < k_* \end{cases}$$

Want $n_s$ sufficiently less than 1 so PBHs form predominantly on scale $k_*$.

We take $n_s = -1$, the smallest allowed by slow roll under the model construction given by:

[Kawasaki, Sugiyama, Yanagida (1997)]
[Kawasaki, Yanagida (1998)]

PTA sensitivity also discussed in:

[Inomata, Kawasaki, Mukaida, Tada, Yanagida (2016)]

# Broader spectrum

Running-mass inflation

Single period of inflation with changing power spectrum:

$$\mathcal{P}_\mathcal{R}(k) = \mathcal{P}_\mathcal{R}(k_0)\left(\frac{k}{k_0}\right)^{n(k)-1},$$

$$n(k) = n_s(k_0) + \frac{1}{2!}\alpha_s(k_0)\ln\left(\frac{k}{k_0}\right) + \frac{1}{3!}\beta_s(k_0)\ln^2\left(\frac{k}{k_0}\right) + \frac{1}{4!}\gamma_s(k_0)\ln^3\left(\frac{k}{k_0}\right) + \ldots$$

The power spectrum $P_\mathcal{R}$, spectral tilt $n_s$, and runnings $\alpha_s, \beta_s, \gamma_s$ are constrained on scales relevant to the CMB.

[Stewart (1996 and 1997)]
[Leach, Grivell, Liddle (2000)]
[Drees, Erfani (2011 and 2012)]

# Axion-curvaton

In this model, have complex scalar:

$$\Phi = (\varphi/\sqrt{2})e^{i\sigma/f_\sigma}$$

The axion-curvaton field $\sigma$ has inflationary fluctuations.

However, $\sigma$ is only well-defined after $\varphi \approx f_\sigma$.

Thus, perturbations are only induced on larger scales:

$$\begin{cases} \mathcal{P}_\mathcal{R} = \mathcal{P}_\mathcal{R}(k_*)(k/k_*)^{n_s-1} \text{ for } k < k_* \\ \mathcal{P}_\mathcal{R} \ll \mathcal{P}_\mathcal{R}(k_*) \text{ for } k > k_* \end{cases}$$

Need a blue spectrum ($n_s > 1$) to produce a narrow PBH mass spectrum.

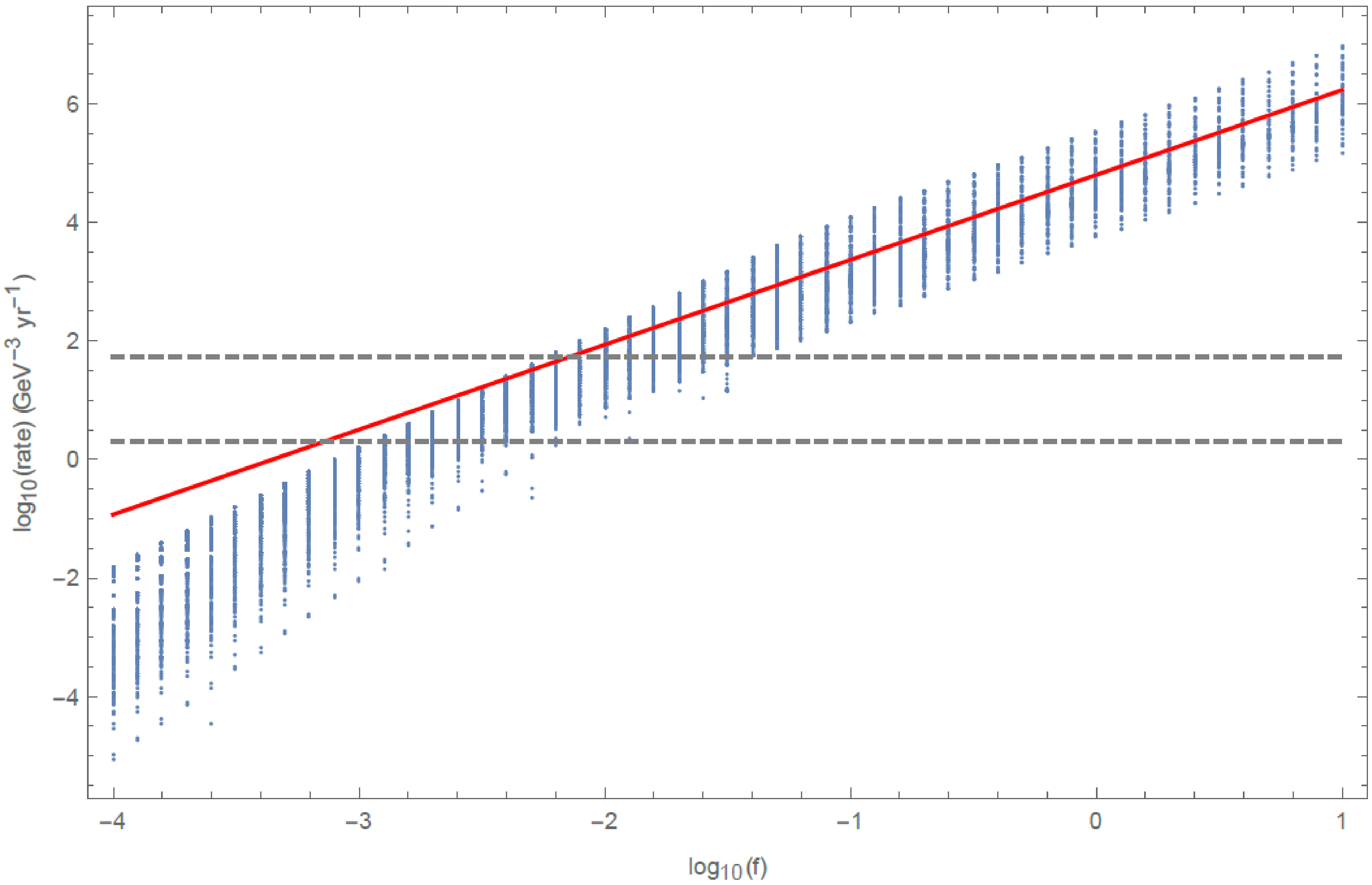
[Kasuya, Kawasaki (2009)]
[Kawasaki, Kitajima, Yanagida (2012)]

# Axion-curvaton

Say that $\sigma$ behaves like matter but *never* dominates the universe's energy density.

Its abundance grows relative to that of radiation, but it decays before it can dominate.

Thus, the perturbations it sources grow until it decays, and PBHs preferentially form at this time.



6
4
2
0
-2
-4
$\log_{10}(\mathrm{rate})\ (\mathrm{GeV}^{-3}\ \mathrm{yr}^{-1})$
-4
-3
-2
-1
0
1
$\log_{10}(f)$