Natural Low-Scale Inflation and the Relaxion

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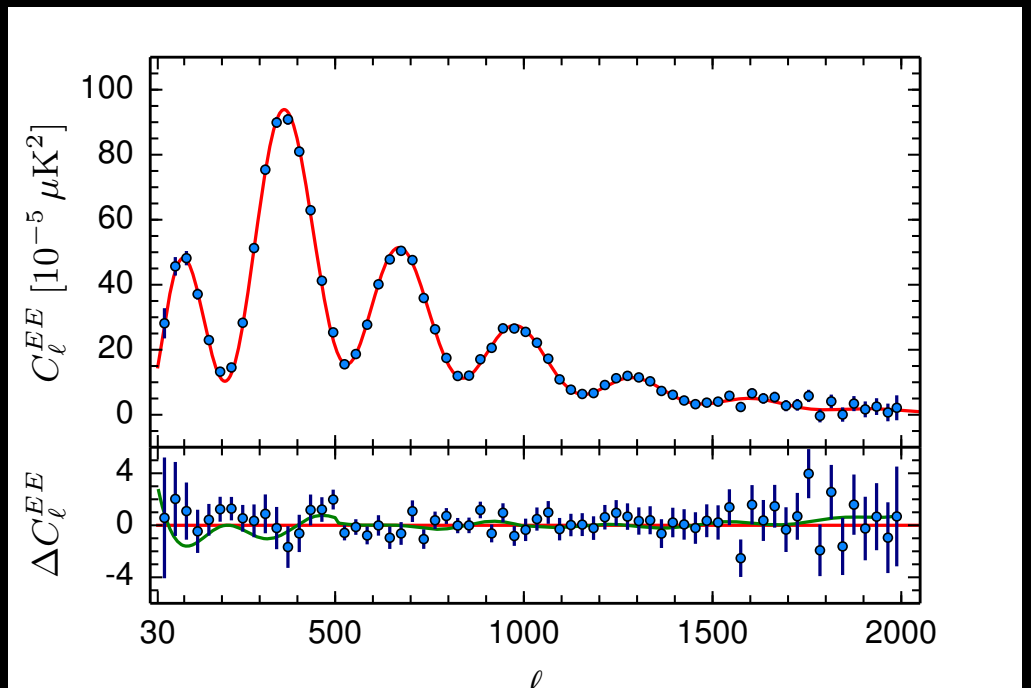
JLE, Gherghetta, Nagata, Peloso arXiv:1704.03695

Seeds of the CMB Anisotropy

- Quantum fluctuations give $\Delta T/T \sim 10^{-5}$ in early universe
- Power spectrum determines size of fluctuations

$$\langle \mathcal{R}\mathcal{R} \rangle = \int_0^\infty A_\mathcal{R}\, d\ln k \qquad A_\mathcal{R} = \left(\frac{H}{\dot{\phi}}\right)^2 \frac{H^2}{(2\pi)^2}$$



- CMB determines $A_\mathcal{R}$
- $C_\ell^{EE} = \pi \int \frac{dk}{k} A_\mathcal{R}(k) \Delta^2_{E\ell}(k)$

Inflation Parameters

- Normalization of C_ℓ^{XY} determine power spectrum

$$(A_{\mathcal{R}})^{1/2} \simeq 5 \times 10^{-5} \left(\frac{H}{10^{13}\ \mathrm{GeV}}\right) \left(\frac{2.6 \times 10^{-3}}{\epsilon}\right)^{1/2}$$

- ϵ related to slope of potential

$$\epsilon = \frac{M_P^2}{2}\left(\frac{V_\phi}{V}\right)^2 \qquad \eta = M_P^2 \frac{V_{\phi\phi}}{V}.$$

- Experimental constraints on ϵ becoming marginal
  - ϵ already borderline for explaining n_s

$$r \lesssim 0.09\ (95\%) \qquad \rightarrow \qquad \epsilon \lesssim 5 \times 10^{-3}$$

- Spectral tilt requires largish slopes for either ϵ or η

$$n_s - 1 = 2\eta - 6\epsilon \simeq -0.03$$

Relaxions and the Hubble Scale

$$-\mathcal{L} \supset \left(-M^2 + g\phi\right)|H|^2 + \tfrac{1}{2}(g\phi)^2 + \Lambda^4(H,T)\cos\left(\tfrac{\phi}{f}\right) \quad \left(+m_T^2|T|^2\right)$$

- Three important parts: m_H^2, m_ϕ^2, and barrier
- Classical rolling restricts size of H

$$H_I < (gM^2)^{1/3} \simeq \left(\frac{m_\pi^2 f_\pi^2}{f}\right)^{1/3} = 6\times 10^{-5}\left(\frac{10^9\ \mathrm{GeV}}{f}\right)^{1/3}$$

- Supersymmetric two-field model a little better

$$H_I < (m_S m_{SUSY} f_\phi)^{1/3} \simeq 4.6\ \mathrm{GeV}\ \left(\frac{m_S}{10^{-7}\ \mathrm{GeV}}\right)^{1/3}\left(\frac{f_\phi}{10^5\ \mathrm{GeV}}\right)^{1/3}\left(\frac{m_{SUSY}}{10^5\ \mathrm{GeV}}\right)^{1/3}$$

Axions and the Hubble Scale

- Breaking of $U(1)_{PQ}$ lead to strings and domain walls
- If $U(1)_{PQ}$ breaks before inflation strings inflate away
- During inflation, quantum fluctuations arise in axion
  - If axions contribute to CDM, get isocurvature perturbations
  - Low-scale inflation suppresses isocurvature perturbations

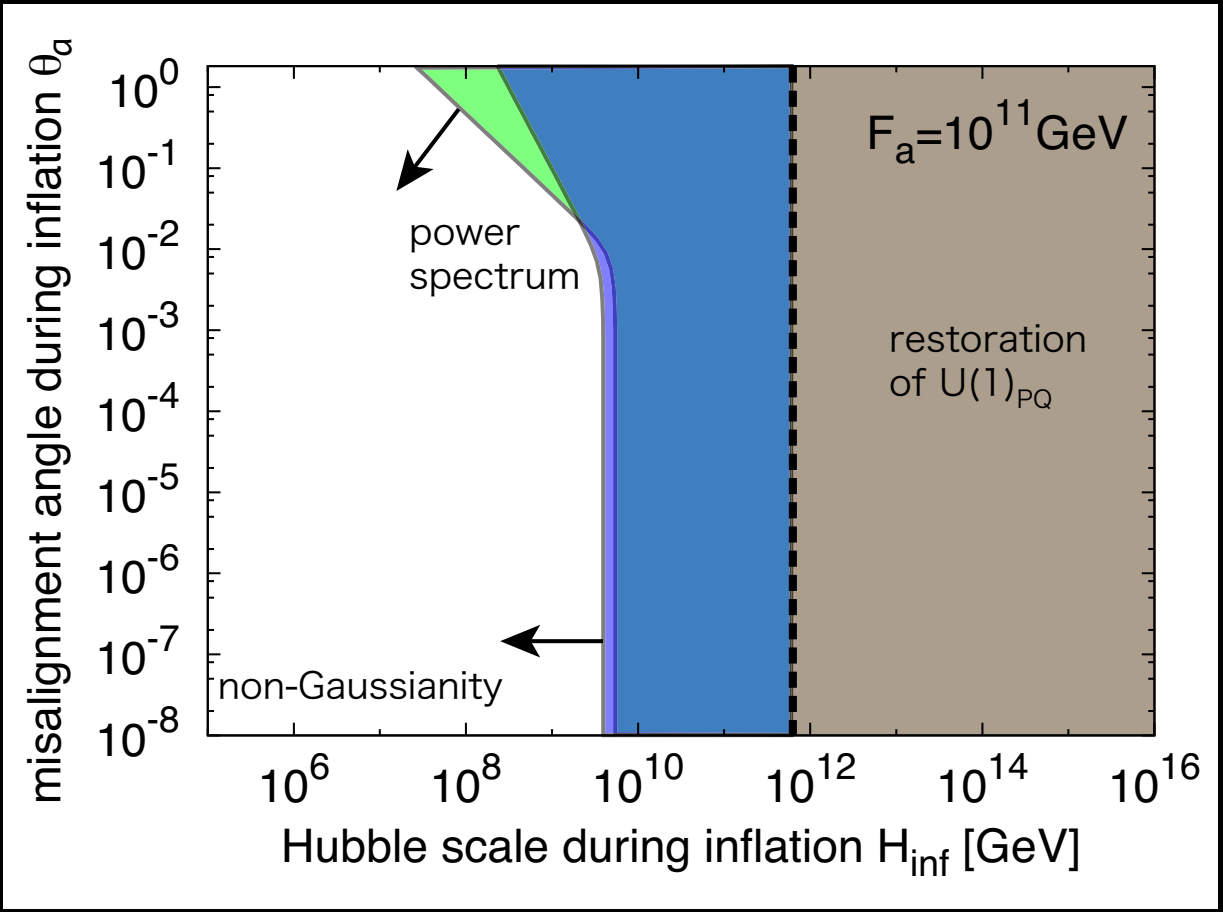
$$\left[\theta_a^2 + \left(\frac{H_I}{2\pi F_a}\right)^2\right]\left(\frac{H_I}{2\pi F_a}\right)^2\left(\frac{F_a}{10^{12}\ GeV}\right)^{2.38} < 3.6 \times 10^{-11}$$

- If No isocurvature perturbations can be dark matter

$$\Omega_a h^2 = 0.18\left[\theta_a^2 + \left(\frac{H_I}{2\pi F_a}\right)^2\right]\left(\frac{F_a}{10^{12}\ GeV}\right)^{1.19}$$

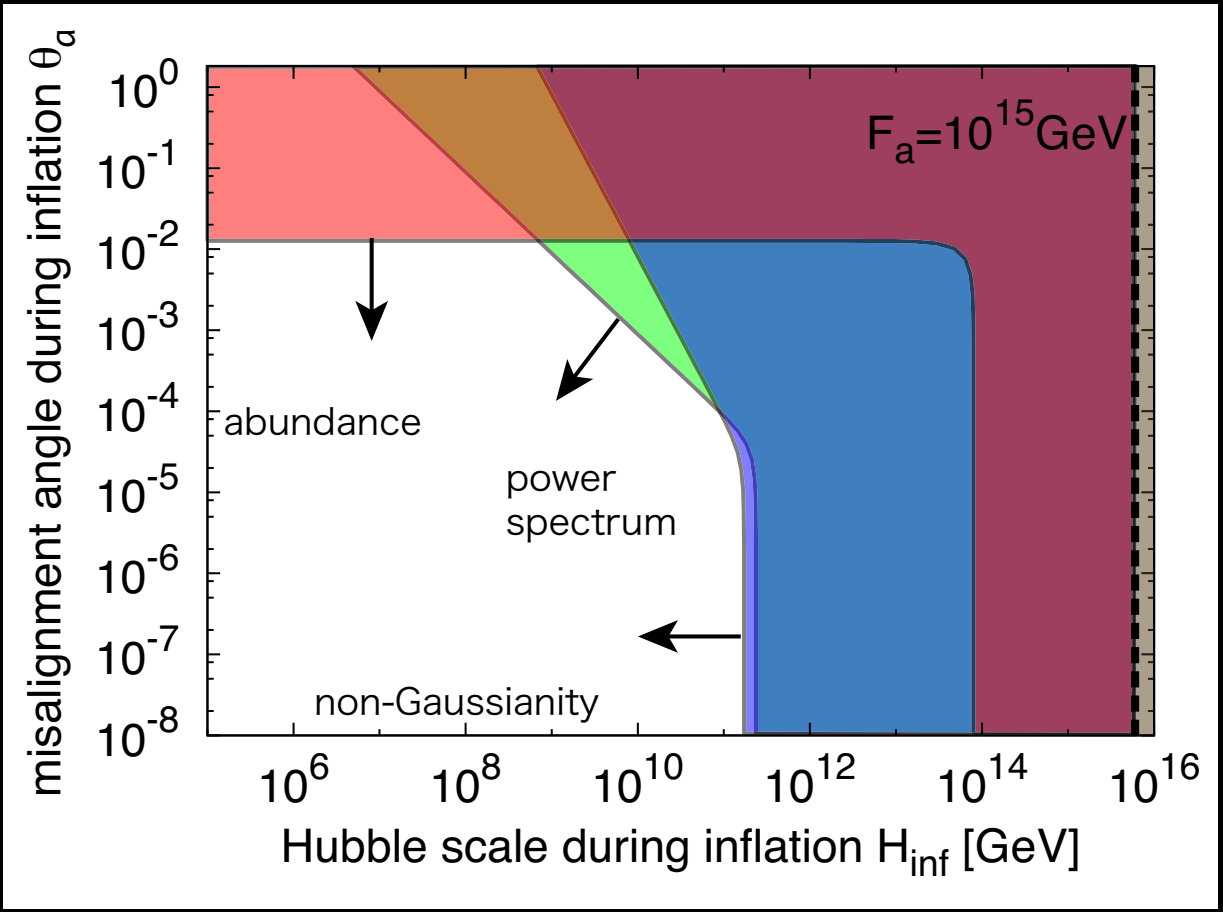
Axion Parameter Space

- PQ breaking before inflation with $F_a = 10^{11}$ GeV



Axion Parameter Space

- PQ breaking before inflation prefers with $F_a = 10^{15}$ GeV



Low-Scale Inflation

$$\epsilon = \frac{M_P^2}{2}\left(\frac{V_\phi}{V}\right)^2 \qquad \eta = M_P^2 \frac{V_{\phi\phi}}{V}.$$

- Difficulties of low scale inflation

$$\epsilon = 8.6 \times 10^{-21}\left(\frac{H}{10^5\ \mathrm{GeV}}\right)^2$$

- For low-scale inflation we know η

$$n_s - 1 = 2\eta - 6\epsilon \simeq 2\eta \simeq -0.03 \qquad \eta \simeq -1.5 \times 10^{-2}$$

- Need potential very flat with large 2nd derivative
  - For relaxion, needs to be technically natural

D-term Inflation

- D-term with FI term

$$V_D = \frac{g^2}{2}\left[|\phi_+| - |\phi_-| - \xi\right]^2$$

- Inflaton couples directly to $U(1)$ charged particles

$$W = \kappa T \phi_+ \phi_-$$

- $\langle \phi_\pm \rangle = 0$ during inflation

$$|\kappa T| > \sqrt{g^2 \xi}$$

- Potential perfectly flat at tree level

$$V_D = \frac{g^2}{2}\xi^2$$

Coleman-Weinberg Potential

- During inflation $\phi_\pm$ massive
  - integrating $\phi_\pm$ generates potential for T
- Potential to one-loop for $|\kappa T| > \sqrt{g^2\xi}$

$$V = \frac{g^2\xi^2}{2}\left(1 + \frac{g^2}{8\pi^2}\ln\left[\frac{|\kappa T|^2}{Q^2}\right]\right)$$

- Slow roll parameters

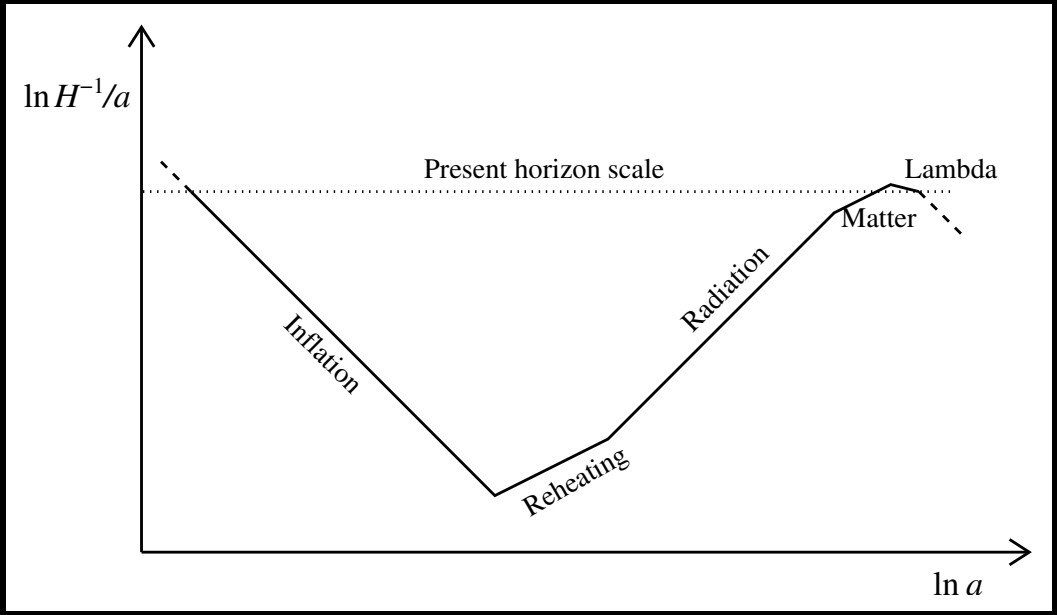
$$\epsilon = \frac{g^2}{16\pi^2}\frac{1}{N_{CMB}} \qquad \eta = -\frac{1}{2N_{CMB}}$$

- For small g, ϵ small enough

$$A_{\mathcal{R}}^{1/2} = 5\times 10^{-5}\left(\frac{N_{CMB}}{40}\right)\left(\frac{H_I}{10^5\ \mathrm{GeV}}\right)\left(\frac{7.3\times 10^{-9}}{g}\right)$$

N_{CMB} and the History of the Universe

- N_{CMB} depends on expansion of universe after inflation
  - Lower $\rho_{reh} \rightarrow$ decrease radiation domination
  - Slower expansion during RD less e-folds needed



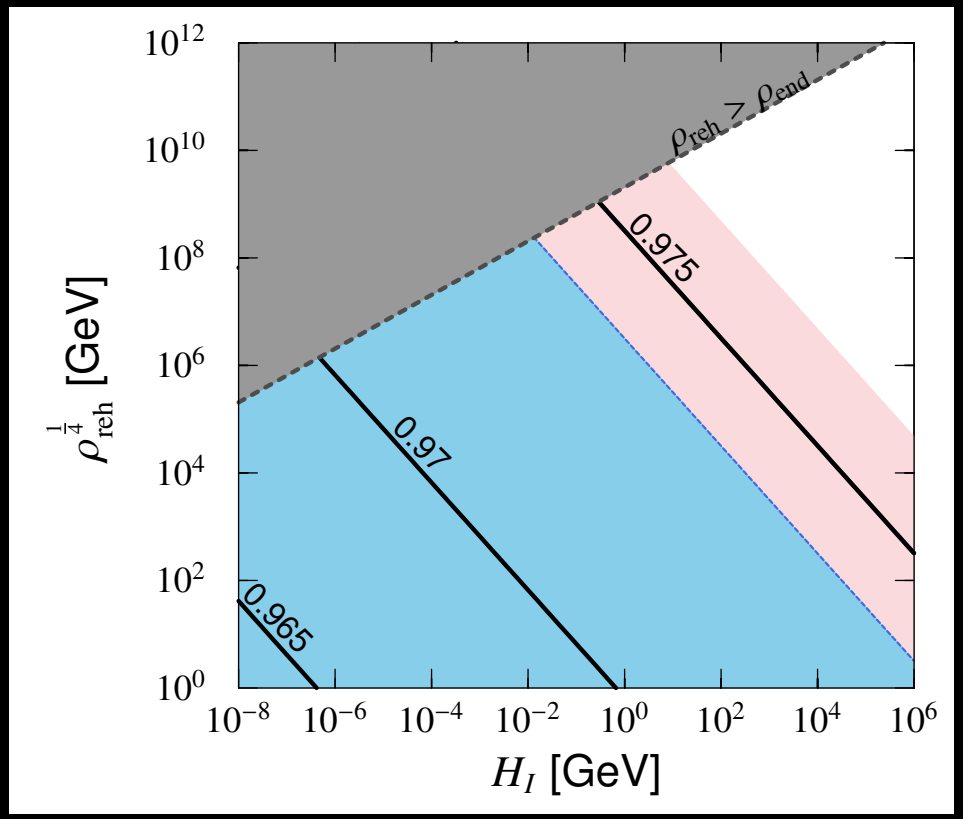
Liddle, Leach

- Instantaneous reheat/low-scale inflation

$$N_{CMB} = 39 + \frac{1}{3}\ln\left(\frac{H_I}{10^5\ \mathrm{GeV}}\right) + \frac{1}{3}\ln\left(\frac{\rho_{reh}^{1/4}}{100\ \mathrm{GeV}}\right)$$

The Spectral Tilt

- Reduced N_{CMB} puts n_s in $1-\sigma$ lines



– Spectral tilt

$$n_s - 1 = 2\eta = -0.026\left(\frac{39}{N_{CMB}}\right)$$

– Number of e-folds

$$N_{CMB} = 39 + \frac{1}{3}\ln\left(\frac{H_I}{10^5\ \mathrm{GeV}}\right) + \frac{1}{3}\ln\left(\frac{\rho_{reh}^{1/4}}{100\ \mathrm{GeV}}\right)$$

Why Dynamical D-Terms

- D-term generated by strongly interacting theory
  - Breaks U(1) during inflation
  - Breaking prevents string formation
- Energy of inflation in ϕ_+
  - ϕ_+ has a vev much larger than its mass
  - ϕ_+ gives large mass to fields it couples too
- Use dynamical sector to couple ϕ_+
  - Dynamical sector field allow coupling of ϕ_+ with singlet
  - Large vev small couplings $\rightarrow$ short lifetime of ϕ_+
  - Additional Singlet cancels $\langle\phi_+\rangle$ mass contribution

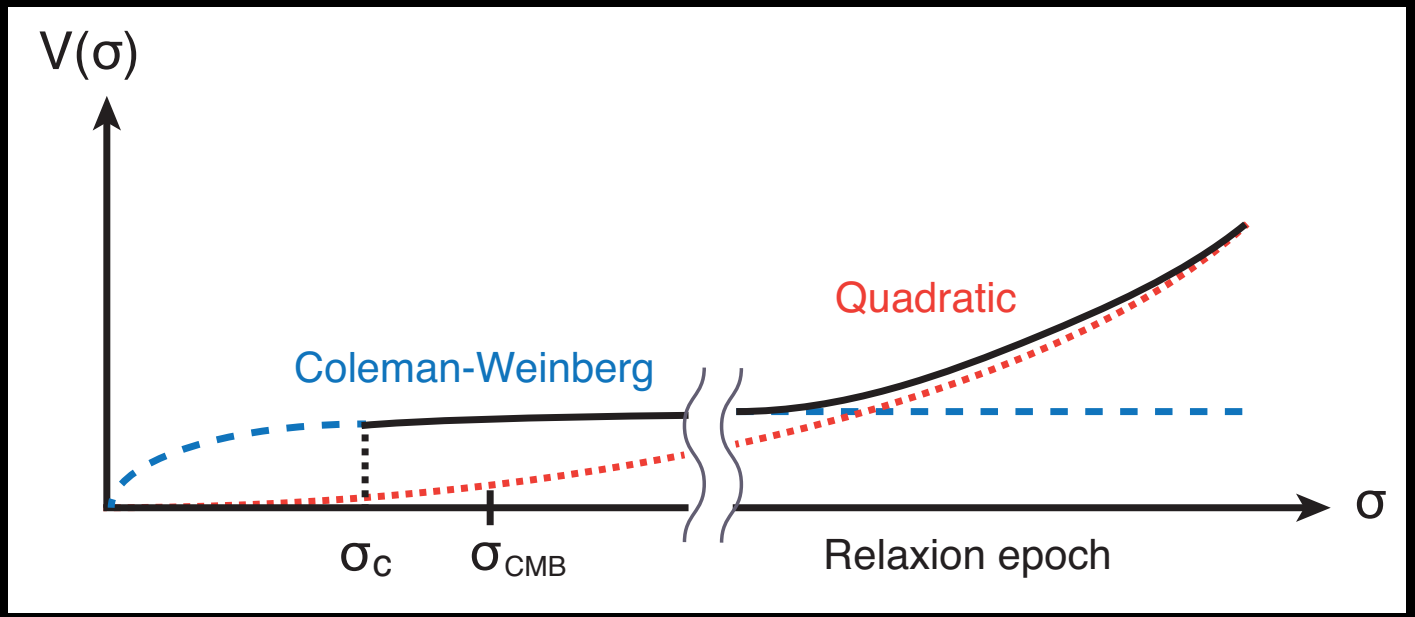
$$\Delta V \supset |\kappa_1 M_- \phi_+ + \kappa_2 H_u H_d + m_R \bar{R}|^2$$

Inflaton as the Second Field

- The inflaton can play the roll of the amplitudon ($T = \tau + i\sigma$)

$$W_{S,T} = \frac{m_S}{2} S^2 + \frac{m_T}{2} T^2 \qquad W_{inf} = \kappa T \phi_+ \phi_-$$

- D-term and relaxion have very different energies

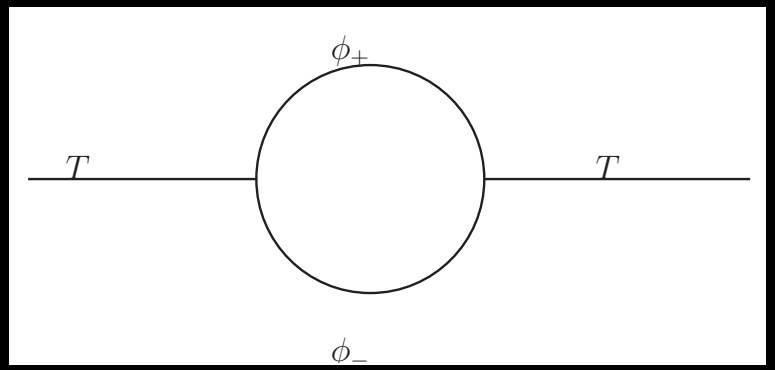


Shift Symmetry Breaking of Inflaton Sector

- Inflaton has relatively large shift symmetry breaking

$$W = \kappa T \phi_+ \phi_- \qquad \kappa \gtrsim 10^{-2}$$

- Loop correction transmit shift symmetry breaking to Kähler



$$K \supset \frac{|\kappa|^2}{16\pi^2}|T|^2$$

- SUGRA corrections to scalar potential generate mass for T

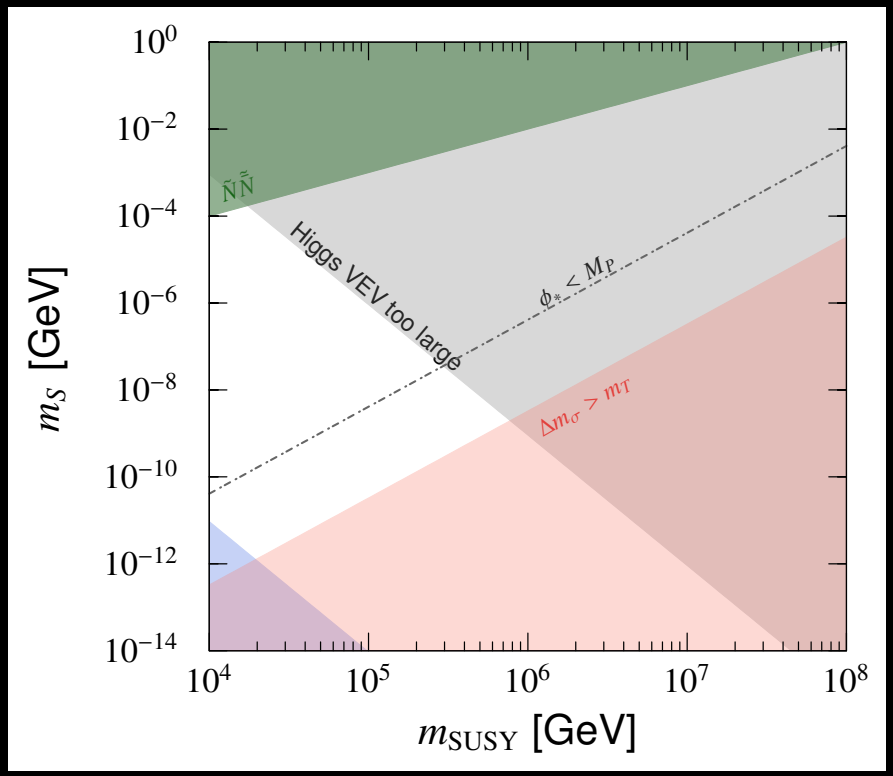
$$V \supset e^{\frac{K}{M_P}}|F_S|^2 + \ldots \qquad \rightarrow \qquad V \supset \frac{|\kappa|^2}{16\pi^2}\frac{|F_S|^2}{M_P^2}|T|^2$$

- Kähler corrections give lower bound on m_T

$$m_T \gtrsim \frac{\kappa}{4\pi}\frac{|F_S|}{M_P} = \frac{\kappa}{4\pi}\frac{m_{SUSY} f}{M_P}$$

Constraint Summary

- $\zeta = 10^{-8} \quad r_{TS} = 0.1 \quad r_\Lambda = 1 \quad r_{\rm SUSY} = 1.$



- Parameters

$$g_S = \zeta \frac{m_S}{f_\phi} \quad g_T = \zeta \frac{m_T}{f_\sigma}$$

$$f \equiv f_\phi = f_\sigma \quad r_{TS} \equiv \frac{m_T}{m_S}$$

$$r_\Lambda \equiv \frac{\Lambda_N}{f} \quad r_{\rm SUSY} \equiv \frac{m_{\rm SUSY}}{f}$$

$$M_L = m_{\rm SUSY} \ ,$$

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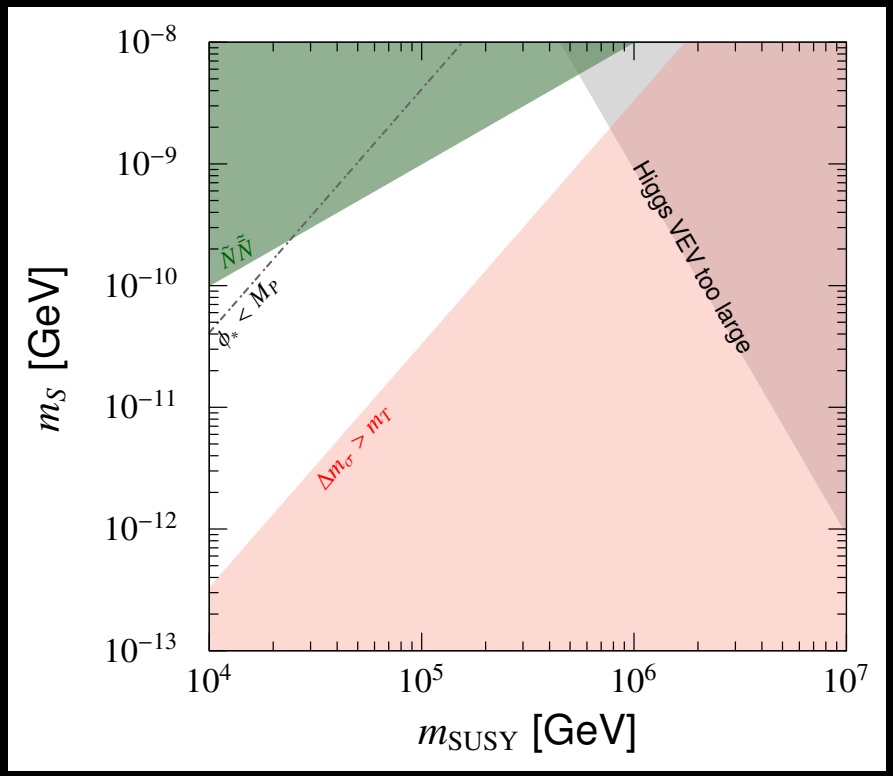

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Conclusions

- CMB fluctuations relate potential slope to Hubble
- Constraints on r push us towards low-scale inflation
- Relaxion and axions prefer very low-scale inflation
- Very low-scale D-term Inflation possible
  - ϵ and η of correct size (from loops)
  - Low-scale inflation reduces N_{CMB} giving correct n_s
- Inflaton and amplitudon can be combined

Dynamical Relaxation

$$-\mathcal{L} \supset \left(-M^2 + g\phi\right)|H|^2 + \tfrac{1}{2}(g\phi)^2 + \Lambda^4(H)\cos\left(\tfrac{\phi}{f}\right)$$

- Two distinct contributions to Higgs mass
  - M^2: All radiative corrections plus tree-level piece
  - $g\phi$: shift symmetry breaking field dependent Higgs mass
- $g\phi = M^2$ special dynamically
  - $g\phi > M^2 \rightarrow \langle H \rangle = 0$
  - $g\phi < M^2 \rightarrow \langle H \rangle \neq 0$
- Shift symmetry breaking $\rightarrow$ slowly relax back to minimum
  - Rolling ϕ scans Higgs mass
- Higgs dependent potential to stop relaxion
  - $\langle H \rangle \neq 0 \rightarrow \Lambda^4(H) \neq 0$

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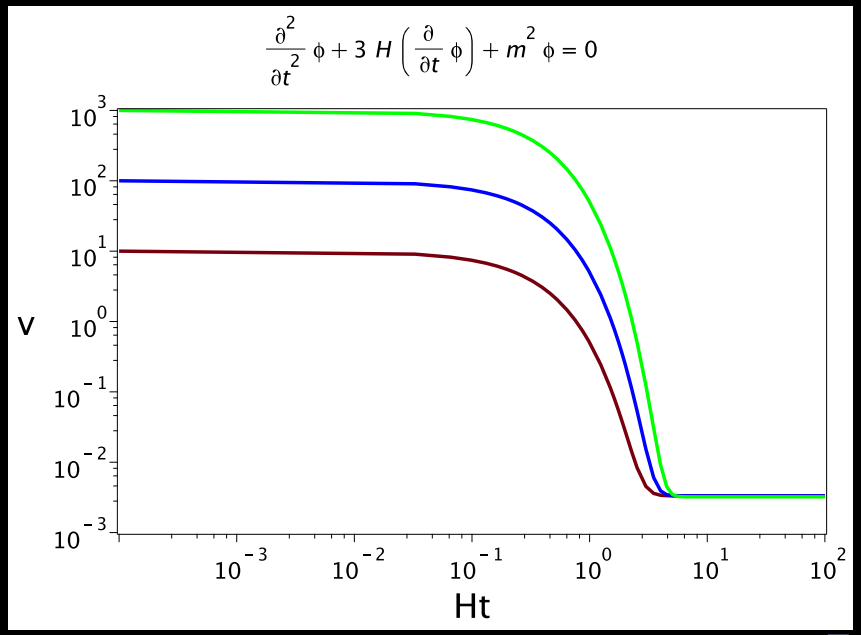
Insensitivity to Initial Conditions

- $\langle H \rangle$ sensitive to ϕ's approach to $m_H^2 = 0$
  - ϕ must stop with $(-M^2 + g\phi)/M^2 \ll 1$
- Hubble friction makes approach to $m_H^2 = 0$ similar
  - $\dot{\phi} \simeq v_{term}$ (Independent of IC)
  - Slow $v_{term} \to (-M^2 + g\phi)/M^2 \ll 1 \qquad (\phi \sim M^2/g)$



Simple Dynamical Relaxation Model

$$\mathcal{L} \supset \left(-M^2 + g\phi\right)|H|^2 + \tfrac{1}{2}(g\phi)^2 + \Lambda_{QCD}^4 \cos\left(\frac{\phi}{f}\right)$$

- Low scale theory: SM + axion Graham, Kaplan, Rajendran
- Axion Nambu-Goldston boson of $U(1)_{PQ}$
- "Explicit" breaking of $U(1)_{PQ}$
  - Gives axion dependent mass for Higgs
  - Gives mass to axion $\rightarrow$ evolution
- Quark condensate further break $U(1)_{PQ}$

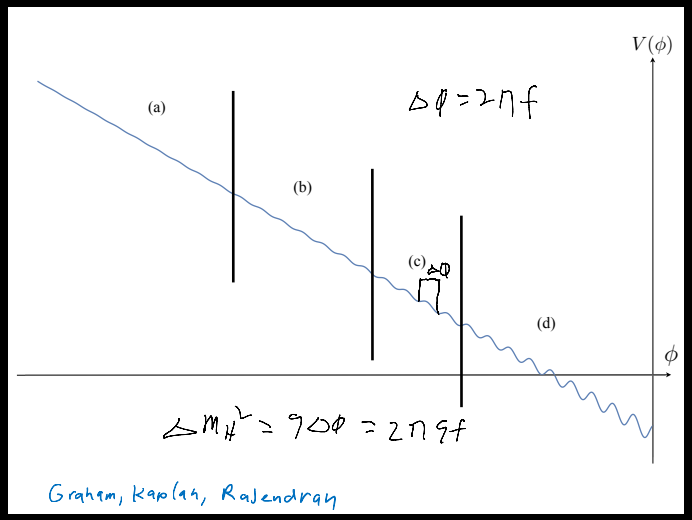
$$-\mathcal{L} \supset y\langle H\rangle e^{i\frac{\phi}{f}}\langle\bar{q}_L q_R\rangle + y^\dagger\langle H^\dagger\rangle e^{-i\frac{\phi}{f}}\langle\bar{q}_R q_L\rangle = 2y\langle H\rangle\langle\bar{q}_L q_R\rangle\cos\left(\frac{\phi}{f}\right)$$

Relaxion Evolution

$$\mathcal{L} \supset \left(-M^2 + g\phi\right)|H|^2 + \tfrac{1}{2}(g\phi)^2 + \Lambda_{QCD}^4 \cos\left(\tfrac{\phi}{f}\right)$$

a. $g\phi > M^2$
   - Higgs mass positive, $\langle H \rangle = 0$
   - ϕ rolling driven by $(g\phi)^2$ term
   - Instanton potential vanishes

b. $g\phi < M^2$
   - $\langle H \rangle \neq 0$, instanton potential grows with Higgs vev
   - Instaton potential begins to affect relaxion



c. Classical movement of ϕ stops

$$\frac{\partial V}{\partial \phi} = 0$$

   - tunneling allowed $t_t < 1/H_0$

d. Minimum stable for $1/H_0$

Strong CP Problem

- Instanton potential stops relaxion

$$\frac{\partial V}{\partial \phi} = g^2 \phi + \frac{m_\pi^2(\langle H \rangle) f_\pi^2}{f} \sin\left(\frac{\phi}{f}\right) + .. \sim 0$$

- Relaxion stopping point

$$\sin\left(\frac{\phi}{f}\right) \qquad \rightarrow \qquad \theta_{QCD} = \frac{\phi}{f} \sim 1$$

- Neutron EDM constraints

$$\theta_{QCD} \lesssim 10^{-11}$$

Relaxion Constraints

- Inflation dominates vacuum energy
  - Inflation unaffected by relaxion

$$H_I > \frac{M^2}{M_P}$$

- Relaxion classical rolls ($\dot{\phi}\Delta t > H$)

$$H_I < (gM^2)^{1/3} \simeq \left(\frac{m_\pi^2 f_\pi^2}{f}\right)^{1/3} = 6 \times 10^{-5} \left(\frac{10^9 \text{ GeV}}{f}\right)^{1/3}$$

- Upper bound on allowed radiative corrections to $m_H^2 \ll M_P^2$
  - $\theta_{QCD} \sim 1$

$$M < \left(\frac{m_\pi^2 f_\pi^2 M_P^3}{f}\right)^{1/6} \sim 10^7 \text{GeV} \left(\frac{10^9 \text{ GeV}}{f}\right)^{1/6}$$

The Numbers

- Shift symmetry breaking parameter

$$g \sim \frac{m_\pi^2 f_\pi^2}{M^2 f} = 10^{-27}\ \mathrm{GeV} \left(\frac{10^7\ \mathrm{GeV}}{M}\right)^2 \left(\frac{10^9\ \mathrm{GeV}}{f}\right)$$

- Super-Planckian relaxion excursion

$$\phi \sim \frac{M^2}{g} = \frac{M^4 f}{m_\pi^2 f_\pi^2} \sim 10^{41}\ \mathrm{GeV} \left(\frac{M}{10^7\ \mathrm{GeV}}\right)^4 \left(\frac{f}{10^9\ \mathrm{GeV}}\right)$$

- Many e-folds of inflation

$$N \gtrsim \frac{H^2}{g} \gtrsim \frac{M^4}{g^2 M_P^2} = \frac{M^8 f^2}{m_\pi^4 f_\pi^4 M_P^2} \sim 10^{44} \left(\frac{M}{10^7\ \mathrm{GeV}}\right)^8 \left(\frac{f}{10^9\ \mathrm{GeV}}\right)^2$$

Strong CP Problem

- Non-QCD axion can have $\theta \sim 1$
  - Introduce new strongly interacting SU(N)
  - N, SM singlet, coupled to Higgs
  - N condensate generates stopping potential

$$\mathcal{L} \supset \left(m_N + \lambda \frac{|H|^2}{M_L}\right) NN^c \to \left(m_N + \boxed{\lambda \frac{|H|^2}{M_L}}\right) \Lambda^3 \cos\left(\frac{\phi}{f}\right)$$

- Radiative corrections constrain model
  - Higgs loops generate mass correction to m_N

$$\Delta m_N \sim \frac{\lambda f_\pi^2}{M_L} \lesssim \frac{\lambda v^2}{M_L} \qquad \to \qquad f_\pi \lesssim v$$

  - Lightest m_N must be smaller than Λ

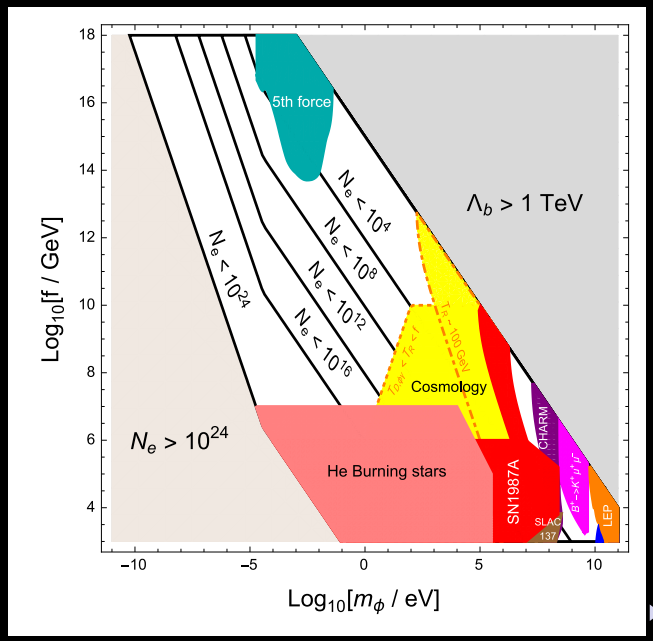
$$\frac{\lambda v^2}{m_L} \lesssim 4\pi f_\pi$$

Consequences of SU(N) θ_{QCD}

- Relaxion has order one mixing with Higgs

$$\left\langle \frac{\partial^2 V}{\partial\phi\partial H} \right\rangle = \lambda \frac{v\Lambda^3}{M_L f} \sin(\theta_{s'}) \sim v^2 \qquad \rightarrow \qquad \theta_{\phi H} \sim \lambda \frac{\Lambda^2}{M_L f} \frac{v\Lambda}{m_H^2 - m_\phi^2}$$

- Mixing leads to large coupling with SM (Choi, Im)
  - LEP bounds on $e^+e^- \rightarrow Z\phi$ (Mixing)
  - Electron/Proton EDM Bounds (Mixing/CP Violation)
  - B-meson decays



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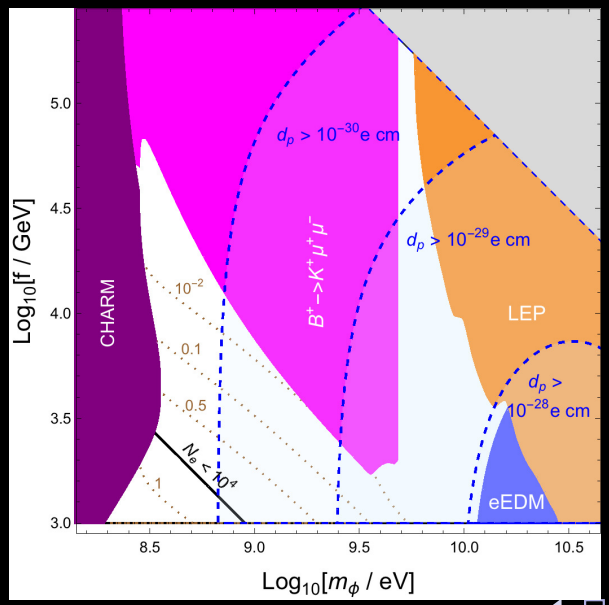


UV Completion

- Upper limit on M for Non-QCD axion

$$M < 2 \times 10^8 \text{ GeV} \left(\frac{f_{\pi'}}{30 \text{ GeV}}\right)^{4/7} \left(\frac{M}{f}\right)^{1/7}$$

- UV completion needed
  - Supersymmetry with $m_{SUSY} \sim M$
  - Composite Higgs with $\Lambda \sim M$
- Supersymmetric UV completion
  - Theory perturbative to Planck scale
  - Is the source of SUSY breaking
  - Relaxion naturalizes up to m_{SUSY}
  - Supersymmetry naturalizes beyond m_{SUSY}

Supersymmetric Axion Relaxion

- Shift symmetric Scalar particle

$$K \supset (S + S^\dagger)^2 + \ldots$$

- Shift symmetry/SUSY breaking in superpotential

$$\frac{1}{2}(g\phi)^2 \quad \rightarrow \quad W \supset \frac{m}{2}S^2 \quad \& \quad F_S = im\phi \neq 0$$

- SUSY breaking generates relaxion dependent Higgs soft masses

$$g\phi|H|^2 \quad \rightarrow \quad \int d^4\theta \frac{(S+S^\dagger)^2}{M^{*2}}|H_{u,d}|^2 = \frac{m^2\phi^2}{M^{*2}}|H_{u,d}|^2$$

- Supersymmetric Higgs mass sets natural realxion scale

$$-M^2|H|^2 \quad \rightarrow \quad W = \mu_0 H_u H_d \quad \mu = \frac{m\phi}{f} - \mu_0$$

- Instanton potential from gauge kinetic function (Axion)

$$\frac{\phi}{32\pi f_\phi} G^{a\mu\nu} G^a_{\mu\nu} \quad \rightarrow \quad \int d\theta^2 c_a \frac{S}{16\pi^2 f_\phi} \mathrm{Tr}(\mathcal{W}_a \mathcal{W}_a) + \mathrm{h.c.}$$

Higgs Mass of SUSY Relaxion

- Relaxion dependent Higgs sector parameters

$$m^2_{H_{u,d}} = c_{u,d}\frac{m^2\phi^2}{f^2} \qquad \mu = \mu_0 - c_\mu\frac{m\phi}{f} \qquad B_\mu = c_0\mu\frac{m\phi}{f} + c_B\frac{m^2\phi^2}{f^2}$$

- $\mathrm{Det}(M_H^2) < 0$ signifies EWSB

$$\mathrm{Det}(M_H^2) = \left(m^2_{H_u} + |\mu|^2\right)\left(m^2_{H_d} + |\mu|^2\right) - |B_\mu|^2$$

- $m\phi \gg \mu_0 \rightarrow \mathrm{Det}(M_H^2) > 0$

$$\left(c_u + |c_\mu|^2\right)\left(c_d + |c_\mu|^2\right) - |c_B|^2 > 0$$

- $\mathrm{Det}(M_H^2) < 0$ EWSB occurs
  - for $\mu \simeq 0$ and $c_u c_d < c_B^2$

The SUSY Scale

- Relaxion stopping potential determined by QCD potential

$$V' = \frac{m^2}{2}\phi + \frac{1}{f}\Lambda_{QCD}^4 = 0$$

- Inflaton dominates energy $\rightarrow$ upper bound on m_{SUSY}

$$H_I \gtrsim \frac{m\phi}{M_P} \sim \frac{m_{SUSY} f}{M_P}$$

- Relaxion Classically rolls

$$H_I < (m m_{SUSY} f)^{1/3} \simeq \Lambda_{QCD} \left(\frac{\Lambda_{QCD}}{f}\right)^{1/3} = 2 \times 10^{-4}\ \mathrm{GeV} \left(\frac{10^9\ \mathrm{GeV}}{f}\right)^{1/3}$$

- Upper bound on SUSY scale ($\mu_0 \sim m_{SUSY}$)

$$m_{SUSY} < \left(\frac{\Lambda_{QCD}^4 M_P^3}{f^4}\right)^{1/3} \sim 5 \times 10^5 \mathrm{GeV} \left(\frac{10^9\ \mathrm{GeV}}{f}\right)^{4/3}$$

- Susy complicates solving strong CP problem
  - Inflaton relaxion coupling constrained by holomorphy

The Numbers: SUSY

- Shift symmetry breaking parameter

$$m \simeq \frac{\Lambda_{QCD}^4}{f^2 m_{SUSY}} \sim 10^{-26}\ \mathrm{GeV}$$

- Super-Planckian relaxion excursion

$$\Delta\phi \sim \phi_* = \frac{m_{SUSY}}{m c_\mu} = 10^{39}\ \mathrm{GeV} \left(\frac{300\mathrm{MeV}}{\Lambda_{QCD}}\right)^4 \left(\frac{f}{10^9\mathrm{GeV}}\right)^2 \left(\frac{m_{SUSY}}{10^5\mathrm{GeV}}\right)^2$$

- Many e-folds of inflation

$$N = \frac{3H^2\Delta\phi}{V'(\phi)} > 10^{42} \left(\frac{300\mathrm{MeV}}{\Lambda}\right)^8 \left(\frac{f}{10^9\mathrm{GeV}}\right)^6 \left(\frac{m_{SUSY}}{10^5\mathrm{GeV}}\right)^4$$

Two-Field Supersymmetric Relaxion Model

- Two axion-like particles, i.e. $K \supset (S + S^\dagger)^2 + (T + T^\dagger)^2$
- Superpotential explicit breaks shift symmetry($T = \tau + i\sigma, S = s + i\phi$)

$$W_{S,T} = \frac{m_S}{2} S^2 + \frac{m_T}{2} T^2$$

- SUSY breaking still from F_S
- EWSB order parameter the same (Det(M_H^2))
- $H_{(u,d)}, T, S$ coupling to SU(N) charged $N, \bar{N}$

$$W_N = \boxed{\left(m_N + i g_S S + i g_T T + \frac{\lambda}{M_L} H_u H_d\right)} N\bar{N}$$

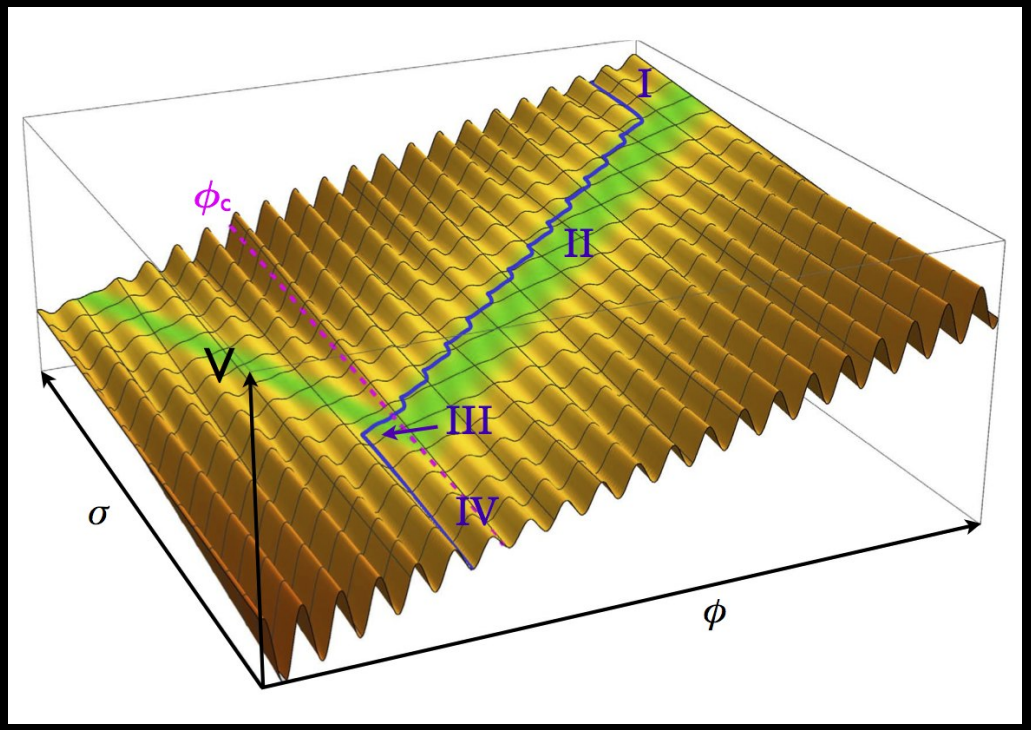
- $N, \bar{N}$ giving stopping potential

$$V_{\phi,\sigma}(\phi, \sigma, H_u H_d) = \frac{1}{2}|m_S|^2 \phi^2 + \frac{1}{2}|m_T|^2 \sigma^2 + \boxed{\mathcal{A}(\phi, \sigma, H_u H_d)} \Lambda_N^3 \cos\left(\frac{\phi}{f_\phi}\right)$$

Phase I: σ Rolls

- I. σ rolls ϕ is stuck

$$V = \frac{1}{2}|m_S|^2\phi^2 + \frac{1}{2}|m_T|^2\sigma^2 + \left(m_N - g_S\phi - g_T\sigma + \frac{\lambda}{M_L}H_uH_d\right)\Lambda_N^3\cos\left(\frac{\phi}{f_\phi}\right)$$



- ϕ trapped

$$|m_S|^2\phi \ll \frac{\Lambda_N^3}{f_\phi}|\mathcal{A}(\phi,\sigma,0)|$$

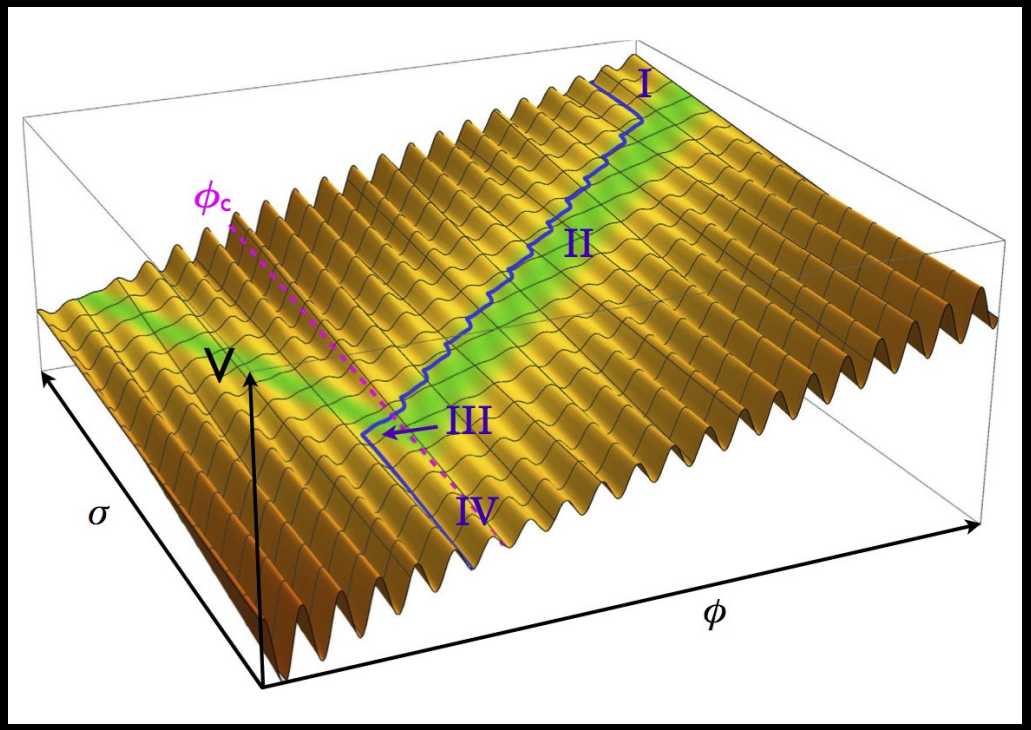
- σ slow rolls

$$\frac{\partial V_{\phi,\sigma}}{\partial\sigma} \simeq |m_T|^2\sigma$$

Phases II: ϕ and σ Track Each Other

- II. ϕ tracks σ ($\mathcal{A} \simeq 0$)

$$V = \frac{1}{2}|m_S|^2\phi^2 + \frac{1}{2}|m_T|^2\sigma^2 + \left(m_N - g_S\phi - g_T\sigma + \frac{\lambda}{M_L}H_uH_d\right)\Lambda_N^3\cos\left(\frac{\phi}{f_\phi}\right)$$



- ϕ rolls

$$|m_S|^2\phi \gtrsim \frac{\Lambda_N^3}{f_\phi}|\mathcal{A}(\phi,\sigma,0)|$$

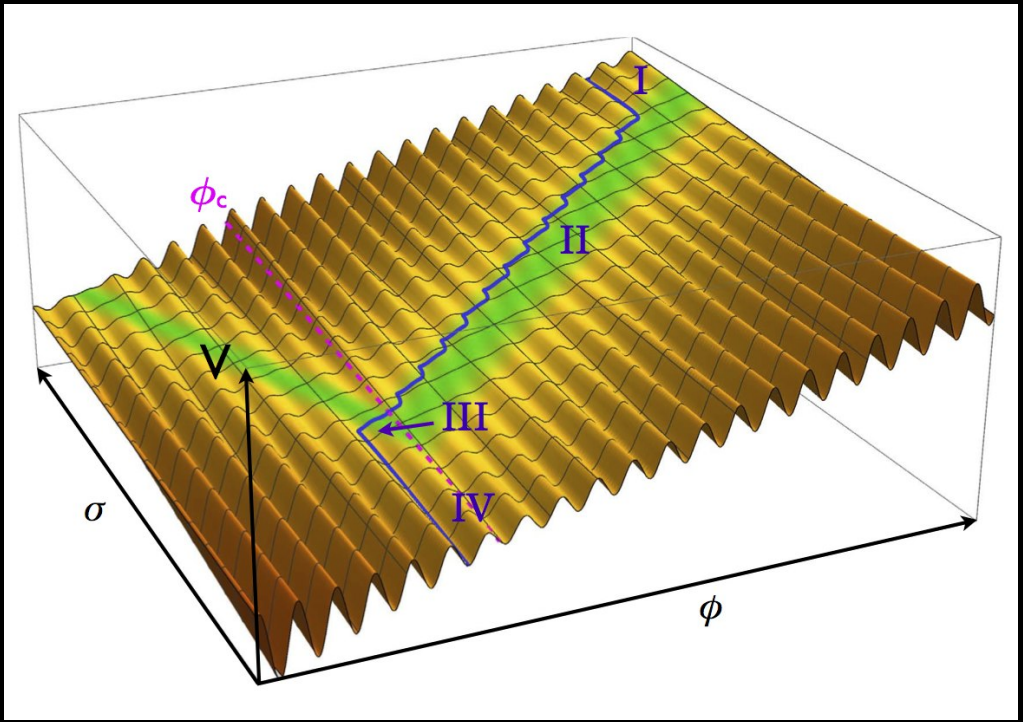
- ϕ tracks σ

$$\frac{g_S}{\sqrt{2}}\frac{d\phi}{dt} < -\frac{g_T}{\sqrt{2}}\frac{d\sigma}{dt}$$

Phases III, IV: EWSB $\rightarrow \phi$ Trapped

- III. m_H^2 becomes negative
- IV. ϕ is trapped

$$V = \frac{1}{2}|m_S|^2\phi^2 + \frac{1}{2}|m_T|^2\sigma^2 + \left(m_N - g_S\phi - g_T\sigma + \frac{\lambda}{M_L}H_uH_d\right)\Lambda_N^3\cos\left(\frac{\phi}{f_\phi}\right)$$



- A grows with v

$$\frac{\partial A}{\partial t} \simeq \frac{\partial A}{\partial v}\frac{\partial v}{\partial t} > 0$$

- ϕ trapped again

$$|m_S|^2\phi \ll \frac{\Lambda_N^3}{f_\phi}|\mathcal{A}(\phi,\sigma,v)|$$

Espinosa, Grojean, Panico, Pomarol, Pujolas, Servant

The SUSY and Inflation Scale

- Shift symmetry breaking mass ($\lambda \lesssim 4M_L v^2/(\Lambda_N^3 \sin(2\beta))$)

$$V' = \frac{m^2}{2}\phi + \frac{\lambda v^2 \sin(2\beta)}{4M_L}\frac{\Lambda_N^3}{f} = 0$$

- Inflaton dominates vacuum energy

$$H_I \gtrsim \frac{m\phi}{M_P} \sim \frac{m_{SUSY} f_\phi}{M_P}$$

- Classical rolling beats quantum spreading

$$H_I < (m m_{SUSY} f_\phi)^{1/3} \simeq v\left(\frac{v}{f_\phi}\right)^{1/3} = 33\ \mathrm{GeV}\ \left(\frac{10^9\ \mathrm{GeV}}{f}\right)^{1/3}$$

- Upper bound on SUSY scale ($\mu_0 \sim m_{SUSY}$)

$$m_{SUSY} < v^{4/7} M_P^{3/7} \sim 1.4 \times 10^9 \mathrm{GeV}\left(\frac{10^9\ \mathrm{GeV}}{f}\right)^{4/3}$$

The Numbers

- Shift symmetry breaking parameter

$$m \simeq \frac{v^4}{f^2 m_{SUSY}} \sim 3 \times 10^{-6}\ \mathrm{GeV} \left(\frac{10^5\ \mathrm{GeV}}{m_{SUSY}}\right)^2 \left(\frac{10^5\ \mathrm{GeV}}{f}\right)$$

- Sub-Planckian relaxion excursion

$$\Delta\phi > \phi_* \sim 10^{17}\ \mathrm{GeV} \times \left(\frac{m_{\mathrm{SUSY}}}{10^5\ \mathrm{GeV}}\right) \left(\frac{f_\phi}{10^5\ \mathrm{GeV}}\right) \left(\frac{10^{-5}\ \mathrm{GeV}}{m_S}\right)$$

- Many e-folds of inflation

$$N_e \simeq \frac{H_I \Delta\phi}{\left|\frac{d\phi}{dt}\right|} \gtrsim \frac{H_I^2}{|m_S|^2} = 10^{12} \times \left(\frac{H_I}{100\ \mathrm{GeV}}\right)^2 \left(\frac{10^{-4}\ \mathrm{GeV}}{|m_S|}\right)^2$$

Experimental Constraints

$$V = \frac{1}{2}|m_S|^2\phi^2 + \frac{1}{2}|m_T|^2\sigma^2 + \left(m_N - g_S\phi - g_T\sigma + \frac{\lambda}{M_L}H_uH_d\right)\Lambda_N^3\cos\left(\frac{\phi}{f_\phi}\right)$$

- Relaxion minimization condition continues to evolve
  - Min initially at $\left(\frac{\phi}{f_\phi}\right) \sim 1$

$$\frac{\partial V}{\partial \phi} \sim |m_S|^2\phi + \boxed{\lambda H_u H_d \frac{\Lambda_N^3}{M_L f_\phi}} \sin\left(\frac{\phi}{f_\phi}\right)$$

Experimental Constraints

$$V = \frac{1}{2}|m_S|^2\phi^2 + \frac{1}{2}|m_T|^2\sigma^2 + \left(m_N - g_S\phi - g_T\sigma + \frac{\lambda}{M_L}H_uH_d\right)\Lambda_N^3\cos\left(\frac{\phi}{f_\phi}\right)$$

- Relaxion minimization condition continues to evolve
  - After σ reaches min, $\left(\frac{\phi}{f_\phi}\right) \ll 1$

$$\frac{\partial V}{\partial \phi} \sim |m_S|^2\phi + \boxed{\mathrm{Max}\left(\frac{g_S m_{SUSY} f_\phi}{m_s}, m_N\right)}\sin\left(\frac{\phi}{f_\phi}\right)$$

- Shift in minimum drastically reduces Higgs-relaxion mixing
- Relaxion SM model couplings suppressed
  - Experimental constraints disappear

Periodic Potential From Additional SU(N)

- QCD instanton potential too small
  - Λ_{QCD} sets relaxion mass
- Additional confining gauge symmetry ($\Lambda_N \gg \Lambda_{QCD}$)

	$SU(N)_R$	$SU(2)_W$	$U(1)_Y$
N	N	1	1
$\bar{N}$	$\bar{N}$	1	1
L	N	2	1
$\bar{L}$	$\bar{N}$	2	-1

- Couplings T and S to confining gauge theory

$$W_N = m_N N\overline{N} + i g_S S N\overline{N} + i g_T T N\overline{N} + \frac{\lambda}{\Lambda} H_u H_d N\overline{N}$$
$$+ \left(\frac{1}{2g_a^2} - i\frac{\Theta_a}{16\pi^2} - c_a \frac{S}{16\pi^2 f_\phi} \right) \mathrm{Tr}(\mathcal{W}_a \mathcal{W}_a)$$

SUGRA and the Relaxion

- Planck suppressed corrections to potential

$$V = \epsilon^{K/M_P^2}\left(D^i W D_i W - 3\frac{|W|^2}{M_P^2}\right)$$

- For $\sigma, \phi > M_P$ the $|W|^2$ term dominates
- Relaxion process allong $(m_S\phi^2)^2/M_P^2$ which does work
- Larger sequestered no-scale SUSY breaking

$$V = e^{K/M_P^2}\left(W^{*i}W_i + \frac{1}{M_P^2}(W^{*i}K_i W + \text{h.c.}) + (K^i K_i - 3M_P^2)\frac{|W|^2}{M_P^4}\right).$$

- Exact no-scale means $K^i K_i = 3M_P^2$.
  - Break no scale a little bit
  - Corrections to Flat SUSY small

Spectra of Relaxion Sector

- Real part of relaxion mass heavy

$$\int d^4\theta \frac{\left(S+S^\dagger\right)^4}{f_\phi^2} \sim \frac{|F_S|^2}{f_\phi^2} s^2$$

- Relaxion mass heavy due large confining scale

$$m_\phi^2 \simeq \frac{\Lambda_N^3}{f_\phi^2}\mathcal{A}\left(\phi_*, 0, \frac{v^2(\phi_*)}{4}\sin 2\beta\right) \simeq \frac{g_S\Lambda_N^3\phi_*}{f_\phi^2} \simeq \frac{g_S\Lambda_N^3 m_{SUSY}}{f m_S}$$

- Relaxiono mass always of order m_S
- Amplitudon mass is quite light

$$m_\sigma = \mathrm{Max}\left(\frac{|F_S|^2}{M_{TS}^2}, m_T\right)$$

- Amplitudino mass is also rather light

$$m_{\tilde{\sigma}} = \mathrm{Max}\left(\frac{F_S}{M_{ST}}, m_T\right)$$

UV Completion: Clockwork Axion

- $N+1$ U(1) with explicit breaking to a single U(1)

Choi,Hui Im; Kaplan,Ratazzi

$$W_{\rm UV} = \sum_{i=0}^{N} \lambda_i S_i \left(\phi_i \bar{\phi}_i - f_i^2\right) + \epsilon \sum_{i=0}^{N-1} \left(\bar{\phi}_i \phi_{i+1}^2 + \phi_i \bar{\phi}_{i+1}^2\right)$$

- Parameterize light superfield

$$\phi_i = f_i e^{\frac{\Pi_i}{f_i}} \ , \qquad \bar{\phi}_i = f_i e^{-\frac{\Pi_i}{f_i}}$$

- Effective superpotetial

$$W_{\rm eff} = 2\epsilon \sum_{i=0}^{N-1} f_i f_{i+1}^2 \cosh\left[\frac{\Pi_i}{f_i} - \frac{2\Pi_{i+1}}{f_{i+1}}\right]$$

- Massless mode, S, corresponding to remaining U(1)

$$S = c_N \sum_{i=0}^{N} \frac{f_i}{2^i f_0} \Pi_i \ ,$$

UV Completion: Continued

- Couple ϕ_0 to SU(N) charged field

$$W \supset \phi_0 \bar{Q} Q \quad \rightarrow \quad c_a \frac{S}{16\pi^2 f_\phi} \mathrm{Tr}(\mathcal{W}_a \mathcal{W}_a)$$

- When $N\bar{N}$ condense generates relaxion potential
- Soft masses from coupling ϕ_N to additional SU(N)

$$V_N \sim \widetilde{\Lambda}_N^4 \cos\left(\frac{\phi}{2^N f_0}\right) \quad \supset \quad \tilde{\Lambda}^4 \frac{\phi^2}{2^{N+1} f_0}$$

- g's generated from coupling in Kähler

$$i\frac{\kappa}{\widetilde{M}_N^2}\int d^4\theta N\bar{N}\Xi^*\bar{\Xi}^* + \mathrm{h.c.} \simeq i\frac{\kappa}{\widetilde{M}_N^2}\int d^2\theta\, \widetilde{\Lambda}_N^3 e^{\frac{\Pi_N}{f_N}} N\bar{N} + \mathrm{h.c.} \simeq \int d^2\theta \frac{i\kappa\widetilde{\Lambda}_N^3}{f_\phi 2^N \widetilde{M}_N^2} SN\bar{N} + \mathrm{h.c.}$$

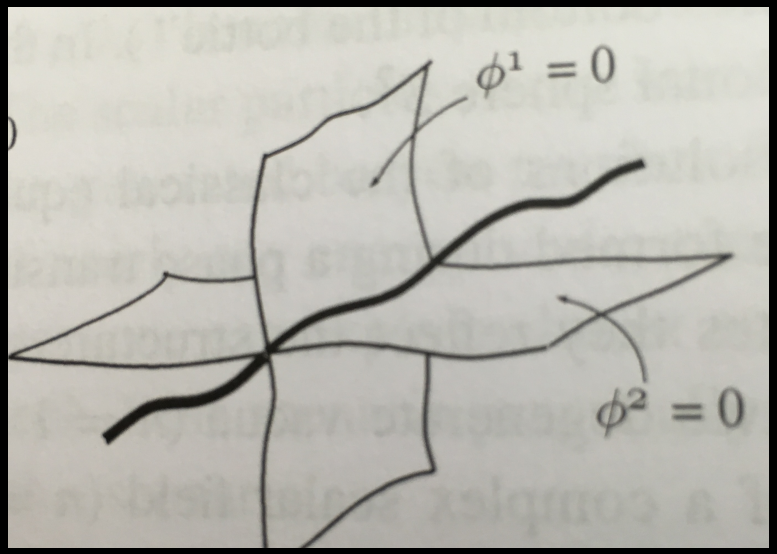
String Formation

- String formation in U(1) gauge theory
  - Phase of ϕ_+ a flat direction ($\phi_+ = v_+ e^{\theta}$

$$V = \frac{g^2}{2}\left(|\phi_+|^2 - \xi\right)^2 + |\lambda T|^2 |\phi_+|^2$$

- Vacuum manifold not simply connected

$$|\phi_+|^2 = \phi_{+r}^2 + \phi_{+i}^2 = \xi$$



String Formation: Two Sectors of U(1) Breaking

- Hidden sector breaking of U(1)

$$V = \frac{g^2}{2}\left(|\phi_+|^2 - \xi\right)^2 + |\lambda T|^2|\phi_+|^2 + C\phi_+ + C^\dagger\phi_+^\dagger \quad \supset \quad |C|v_+\cos(\theta+\theta_c)$$

- Superpotential connects phase of two sectors

$$V_F \supset |\lambda T\phi_+ + \lambda_- TM_-|^2 \quad \supset \quad \lambda\lambda_+|T|^2 M_- v_+\cos(\theta)$$

- Quantum fluctuations could still form strings

$$\langle\phi_+^2\rangle = \frac{H^3}{12\pi m} < v_+^2 \sim \left(\frac{\lambda_+}{\lambda}\right)^2 \xi$$

- Quantum fluctuation at end of inflation

$$H < 25\ \mathrm{GeV}\left(\frac{\lambda_+}{10^{-19}}\right)\left(\frac{10^{-2}}{\lambda}\right)$$

SUSY Breaking From Dynamical D-term

- Quantum modified moduli space

$$M_+ M_- - \Lambda^2 = 0$$

- Mass terms force $\langle M_+ \rangle \neq \langle M_- \rangle$

$$W = \lambda_+ \Lambda Z_+ M_+ + \lambda_- \Lambda Z_- M_-$$

- Residual contribution to D-term after DSB

$$\xi = \left(|M_-|^2 - |M_+|^2 \right) \sim \Lambda^2 \sim (10^{16}\ \mathrm{GeV})^2$$

- Supersymmetry breaking from DSB sector

$$F = \sqrt{2\lambda_- \lambda_+} \Lambda^2$$

- Superpotential interaction couple two sectors
  - Interaction constrains phase of inflaton sector fields

$$\Delta W = \lambda_- T \phi_- M_+ + \lambda_+ T \phi_+ M_-$$