The minimal flipped SU(5)

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Some Motivations

- Fine tuning problems

(i) The cosmological constant problem, (ii) The gauge hierarchy problem, (iii) The strong CP problem, (iv) The SM fermion mass hierarchy problem

- Other theoretical problems

(i) Stability problem, (ii) No explanation for structure of fermion mass structure, (iii) No explanation for charge quantization, (iv) No realization of gauge coupling unification → GUTs provides one of possible solutions to explain (iii) & (iv)

The Particle Content of Model

- Non-supersymmetric flipped $SU(5)\times U(1)_X$ which can also be constructed in SO(10) model.
- one pair of vector-like fermions $Q_x + Q_x^c$ and $D_x + D_x^c$ which forms a complete $SU(5)\times U(1)_X$ representation.

The fermions:

$$F_i = (Q_i \quad D_i^c \quad N_i^c), \qquad \overline{f_i} = (U_i^c \quad L_i), \qquad \overline{l_i} = E_i^c$$
$$F_x = (Q_x \quad D_x^c \quad N_x^c) \quad \overline{F_x} = (Q_x^c \quad D_x \quad N_x)$$

The scalars:

$$\Phi = (Q_\Phi \quad D_\Phi^c \quad N_\Phi^c), \qquad \phi = (D_\phi \quad H)$$

The quantum numbers of fermions in $SU(5)\times U(1)_X$:

$$F_i/\Phi/F_x = (10 \quad 1), \qquad \bar{f}_i = (\bar{5} \quad -3), \qquad \bar{l}_i = (1 \quad 5)$$
$$\bar{F}_x = (\overline{10} \quad 1), \qquad \phi = (5 \quad -2)$$

The Yukawa coupling and vector-like mass terms:

$$\begin{aligned} -\mathcal{L} = {} & M_V F_x \bar{F}_x + \mu_i F_i \bar{F}_x + y_{ij}^D F_i F_j \phi + y_{ij}^{U_\nu} F_i \bar{f}_j \bar{\phi} \\ & + y_{ij}^E \bar{l}_i \bar{f}_j \phi + y_x^D F_x F_x \phi + y_{xi}^D F_x F_i \phi + y_{xi}^{U_\nu} F_x \bar{f}_i \bar{\phi} \\ & + y_{ij}^N \frac{1}{M_{PL}} \bar{\Phi}\bar{\Phi} F_i F_j + y_x^N \frac{1}{M_{PL}} \bar{\Phi}\bar{\Phi} F_x F_x \\ & + y_{xi}^N \frac{1}{M_{PL}} \bar{\Phi}\bar{\Phi} F_x F_i + y_x^{\bar{N}} \frac{1}{M_{PL}} \bar{\Phi}\bar{\Phi} \bar{F}_x \bar{F}_x + \text{H.C.} \end{aligned}$$

- Once Φ field develops VEV, the N_i^c, N_x^c and N_x can obtain a mass around 10^{14} GeV.
- Assuming $M_V \approx \mu_i \approx 1$ TeV, we could have at low energy $Q_x + Q_x^c$ and $D_x + D_x^c$ quarks without involving any fine tuning!

- Neutrino masses and mixing could be explained through seesaw mechanism. The baryon number asymmetry could be explained via thermal leptogenesis.

Gauge Coupling Unification

Gauge coupling unification could be achieved when adding one pair of vector-like fermions. Taking $M_V = 1$ TeV, we run two loop RGE,

$$\frac{dg_i}{dln\mu} = \frac{b_i}{16\pi^2} g_i^3 + \frac{g_i^3}{(16\pi^2)^2}\left(\sum_{j=1}^{3} B_{ij} g_j^2 - C_i^t y_t^2\right)$$

If the renomalization scale is ≥ 1 TeV, we need to consider the beta functions for gauge couplings which receives an additional contribution from vector-like fermions

$$b_i^{SM} = \begin{pmatrix} \frac{41}{10} & -\frac{19}{6} & -7 \end{pmatrix}$$

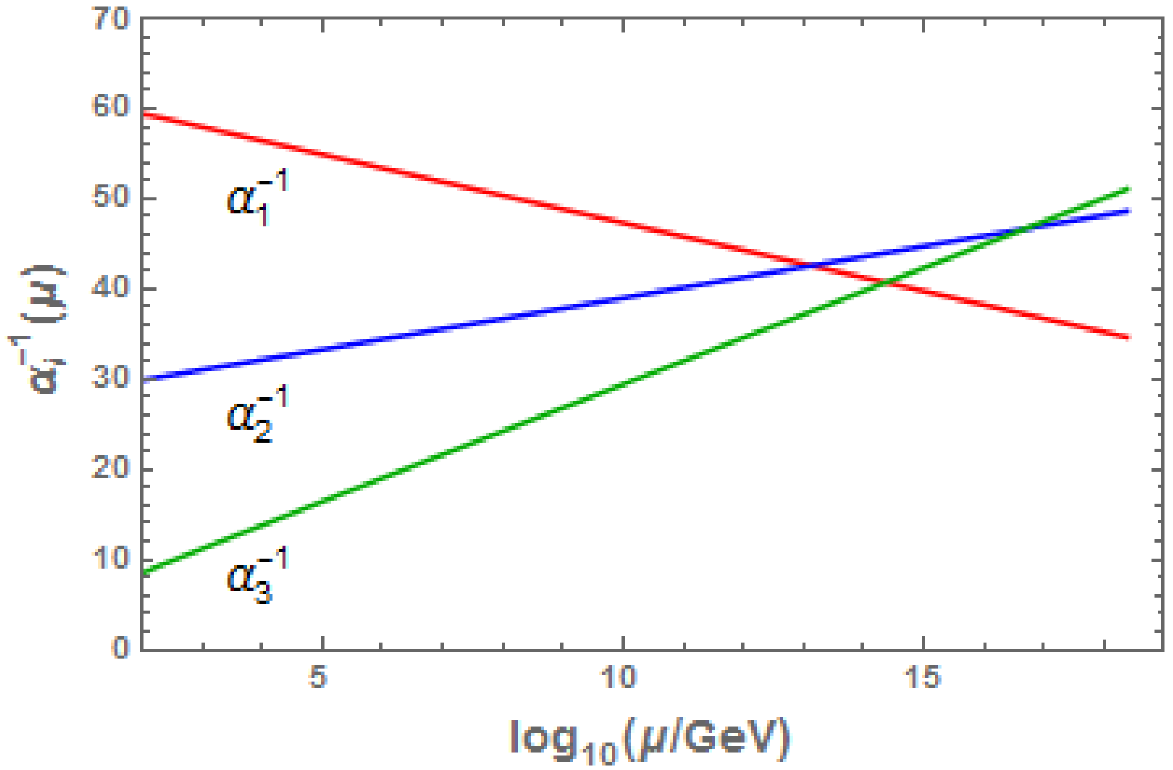
$$B_{ij}^{SM} = \begin{pmatrix} \frac{199}{50} & \frac{27}{10} & \frac{44}{5} \\ \frac{9}{10} & \frac{35}{6} & 12 \\ \frac{11}{10} & \frac{9}{2} & -26 \end{pmatrix}$$

$$C_i^t = \begin{pmatrix} \frac{17}{10} & \frac{3}{2} & 2 \end{pmatrix}$$

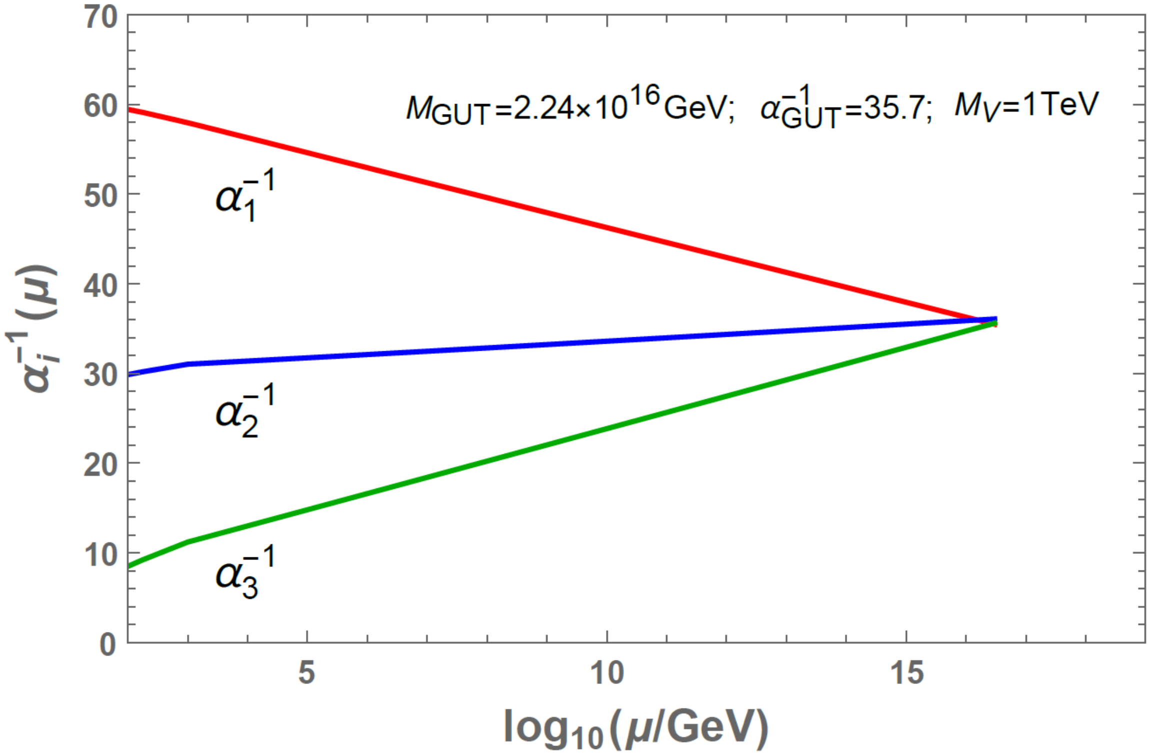
$$b_i' = \begin{pmatrix} \frac{2}{5} & 2 & 2 \end{pmatrix}$$

$$B_{ij}' = \begin{pmatrix} \frac{3}{50} & \frac{3}{10} & \frac{8}{5} \\ \frac{1}{10} & \frac{49}{2} & 8 \\ \frac{1}{5} & 3 & \frac{114}{3} \end{pmatrix}$$

$$C_i^{D_1} = C_i^{D_2} = \begin{pmatrix} \frac{1}{2} & \frac{3}{2} & 2 \end{pmatrix}$$



Gauge coupling evolution in the SM



Gauge coupling evolution in the our model. Defining the gauge coupling unification condition as $\alpha_{GUT}^{-1} \equiv \alpha_1^{-1} = \frac{(\alpha_2^{-1}+\alpha_3^{-1})}{2}$.

Stability Problem

For the stability problem, the Higgs quartic coupling is very sensitive to the top quark mass.

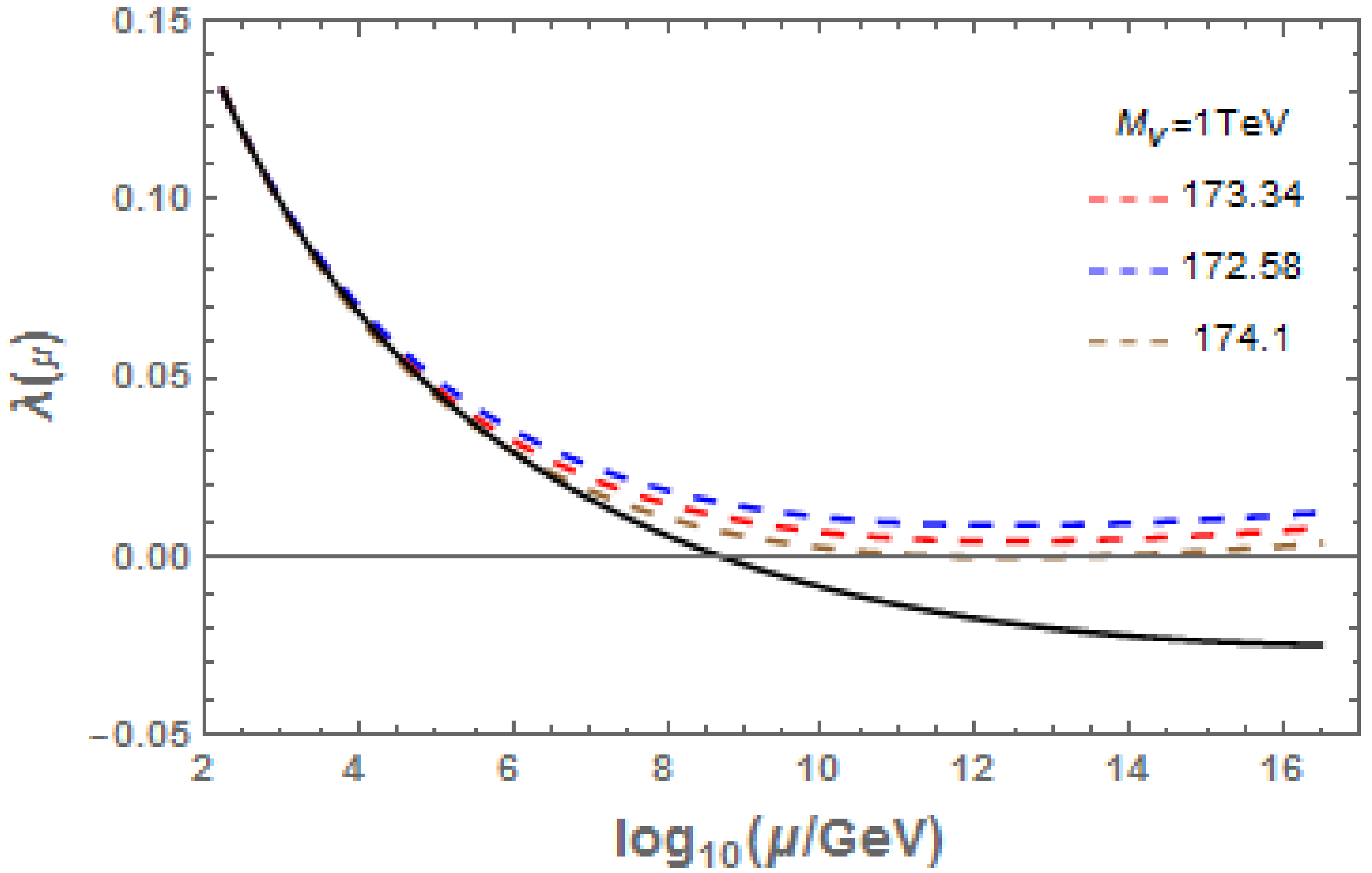
$$\frac{d\lambda}{dln\mu} = \frac{1}{16\pi^2}\beta_\lambda^{(1)} + \frac{1}{(16\pi^2)^2}\beta_\lambda^{(2)}$$

where $\beta_\lambda^{(1)}$ and $\beta_\lambda^{(2)}$ one loop and two loop beta functions.

$$\beta_\lambda^{(1)} = 12\lambda^2 - \left(\frac{9}{5}g_1^2 + 9g_2^2\right)\lambda + \frac{9}{4}\left(\frac{3}{25}g_1^4 + \frac{2}{5}g_1^2 g_2^2 + g_2^4\right) + 12y_t^2\lambda - 12y_t^4$$

Additional contributions from vector-like fermions to the one- and two- loop beta function for λ [1]. To evade the stability problem, we predict m_t should be smaller than one sigma upper bound.

[1]: I. Gogoladze, B. He and Q. Shafi, Phys. Lett. B 690, 495 (2010)



The two-loop RGE evaluation for the Higgs quartic couplings.
The dashed lines stand for the Higgs quartic couplings in our model with the central value and one sigma deviations of top quark mass.
The black solid line corresponds to the SM case with $m_t = 173.34$ GeV

CKM Mixing

In our model, the CKM mixing is generated through the mixture of vector-like quarks and SM quarks. Assuming the 3 × 3 mass matrices of SM up and down type quarks are

$$m_u = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & m_t \end{pmatrix}, \quad m_d = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & m_b \end{pmatrix},$$

The general 4×4 mass matrix of up and general 5×5 mass matrix of down type quarks are

$$(u_i, U_Q)\begin{pmatrix} m_u & \Lambda' \\ \Lambda & M_U \end{pmatrix}\begin{pmatrix} u_j^c \\ U_Q^c \end{pmatrix} \qquad (d_i, D, D_Q)\begin{pmatrix} m_d & \Sigma' \\ \Sigma & M_D \end{pmatrix}\begin{pmatrix} d_j^c \\ D^c \\ D_Q^c \end{pmatrix}$$

After block diagonalization, the 3 × 3 mass matrices of SM up and down quarks are

$$M_u = m_u A + \Lambda' C, \qquad M_d = m_d A' + \Sigma' C'$$

Using a bi-unitary transformation to diagonalize the SM mass matrix we got, we could parametrize the CKM matrix as

$$V_{CKM} \equiv U_L^{u\dagger} U_L^d \sim \begin{pmatrix} \frac{m_1}{m_u}\frac{m_4}{m_d} & \frac{m_1}{m_u}\frac{m_4}{m_d}\frac{m_d}{m_s}K_1 & \frac{m_1}{m_u}\frac{m_4}{m_d}\frac{m_d}{m_b}K_2 \\ \frac{m_1}{m_u}\frac{m_4}{m_d}\frac{m_u}{m_c}H_1 & \frac{m_2}{m_c}\frac{m_5}{m_s} & \frac{m_2}{m_c}\frac{m_5}{m_s}\frac{m_s}{m_b}K_3 \\ \frac{m_1}{m_u}\frac{m_4}{m_d}\frac{m_u}{m_t}H_2 & \frac{m_2}{m_c}\frac{m_5}{m_s}\frac{m_c}{m_t}H_3 & 1 \end{pmatrix}$$

In our model, adding vector-like quarks will contribute to the SM prediction of neutral B meson oscillation.

$$\Delta M_d^{SM} = (0.534 \pm 0.091) \times 10^{12}\ \hbar s^{-1}, \qquad \Delta M_d^{Exp} = (0.510 \pm 0.003) \times 10^{12}\ \hbar s^{-1}$$

$$\Delta M_s^{SM} = (17.30 \pm 2.6) \times 10^{12}\ \hbar s^{-1}, \qquad \Delta M_d^{Exp} = (17.761 \pm 0.022) \times 10^{12}\ \hbar s^{-1}$$

The main contribution to theoretical uncertainties are from the non-perturbative effect at low energy QCD.

Meson Oscillation

The neutral meson mixing are dominated by the box diagram in which top quark and vector-like up type quark running in the loop. Following the approach in [2], we define a general 5×5 non-unitary V_{CKM}. As one of realistic examples in our model, the moduli of 5×5 non-unitary $(V_{CKM})_{ab}$ is

$$|V| = \begin{pmatrix} 0.9741 & 0.2254 & 0.004109 & 1.42 \cdot 10^{-5} & 1.22 \cdot 10^{-5} \\ 0.2215 & 0.995 & 0.040414 & 2.34 \cdot 10^{-4} & 1.87 \cdot 10^{-4} \\ 0.008177 & 0.04004 & 1.009 & 0.01226 & 0.00981 \\ 2.6 \cdot 10^{-6} & 0.00132 & 0.380141 & 2.2 \cdot 10^{-6} & 1.1 \cdot 10^{-5} \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

We define $\lambda^a_{qq'} \equiv V^*_{aq'} V_{aq}$ for mesons with down quarks. The correction to the mixing $(M_{12})_{qq'}$ is

$$\frac{(M_{12})_{qq'}}{(M_{12}^{SM})_{qq'}} \sim 1 + \left(\frac{\lambda^U_{qq'}}{\lambda^t_{qq'}}\right)^2 \frac{S(x_U)}{S(x_t)} + 2\frac{\lambda^U_{qq'}}{\lambda^t_{qq'}} \frac{S(x_U, x_t)}{S(x_t)} + \cdots\cdots$$

[2]: F. J. Botella, G. C. Branco, M. Nebot, M. N. Rebelo and J. I. Silva-Marcos, arXiv:1610.03018 [hep-ph].

Following their convention, the corrections with respect to the SM predictions are defined as

$$\Delta(P_0) \equiv \left|\left(\frac{M_{12}}{M_{12}^{SM}}\right)_{P_0}\right| - 1$$

where P_0 could be K^0, B_d^0 and B_s^0. Using the 5 × 5 non-unitary V_{CKM} shown in the previous slide, we obtain the corrections in our model

$$\begin{pmatrix} \Delta(K^0) \\ \Delta(B_d^0) \\ \Delta(B_s^0) \end{pmatrix} = \begin{pmatrix} 5.5 \times 10^{-5} \\ 6.3 \times 10^{-4} \\ 0.066 \end{pmatrix}$$

We found the correction is not negligible in the B_s^0 oscillation system.

Summary

- With one pair of vector-like fermions in non-susy flipped SU(5), we could achieve gauge coupling unification.
- The stability problem is solved
- We can apply seesaw mechanism to explain the neutrino mass and mixing.
- The quark masses and CKM mixing could be described correctly
- The possible signals from neutral meson mixing might not be negligible.

Dark Matter and Inflation

Adding a real scalar S with a Z_2 symmetry so that it is stable, the potential for S and ϕ is

$$V = \lambda_\phi(|\phi|^2 - v^2)^2 + \frac{1}{2}m_S^2 S^2 + \frac{k}{2}|\phi|^2 S^2 + V_I(S),$$
$$V_I(S) = A tanh^4(S/f)$$

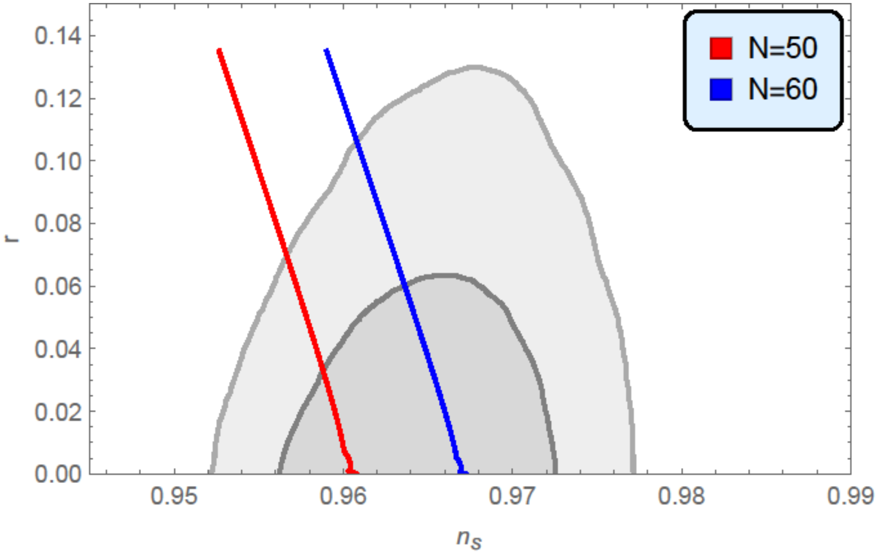
Where m_s is around the electroweak scale. Thus inflation potential is given by $V_I(S)$. In terms of the well known slow-roll parameters,

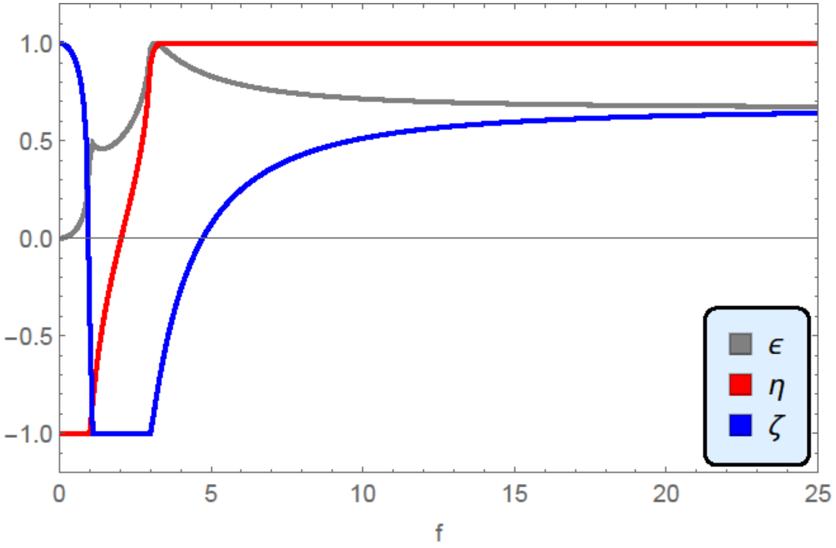
$$\epsilon = \frac{M_{PL}^2}{2}\left(\frac{V_I'}{V_I}\right)^2, \eta = M_{PL}^2\left(\frac{V_I''}{V_I}\right), \zeta = M_{PL}^4\left(\frac{V_I' V_I''}{V_I^2}\right)$$

$$n_s = 1 - 6\epsilon + 2\eta, r = 16\epsilon$$
$$\alpha_s = \frac{dn_s}{dlnk} = -24\epsilon^2 + 16\epsilon\eta - 2\zeta$$
$$P_s = \frac{V}{24\pi^2\epsilon}$$

$$n_s = 0.968 \pm 0.006, r = 0.028^{+0.026}_{-0.025}$$
$$\alpha_s = -0.003 \pm 0.007$$
$$P_s = 2.20 \times 10^{-9}$$

Here we present the numerical results for *r* versus n_s





Because *f* is the only parameter determining the inflationary observable $n_s, r\ and\ \alpha_s$, we present the slow-roll parameters ϵ, η, and ξ versus *f* in the upper right figure.

To have (n_s, r) within the 1σ region of the Planck 2015 results for TT,TE,EE+lowP in the upper left figure, we obtain that f should be lie in the ranges $0 < f \leq 13.4$ for *N*=60 and $0 < f \leq 7.3$ for *N*=50. After inflation, $V_I(S)$ is negligible at low energy.