Yukawa Sector of Minimal $SO(10)$ Unification

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Phenomenology 2017 Symposium
May 8, 2017

JHEP 1702 (2017) 136 [K.S. Babu, Borut Bajc, Shaikh Saad]

Outline of the talk

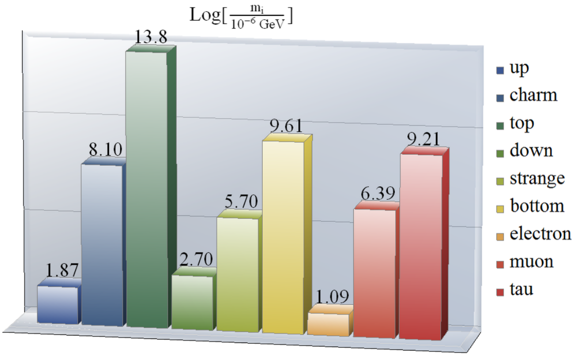
- Observed Hierarchy in Fermion masses and mixings.
- The Standard Model (SM) and the "Flavor Puzzle".
- Why $SO(10)$ Grand Unified Theory (GUT)?
- Yukawa Sector of Minimal $SO(10)$ Unification.
- Symmetry Breaking, Gauge Coupling Unification, Proton Decay

Charged Fermion Mass Spectrum and Hierarchies

- **up-type quarks**
- $m_u \sim 6.5 \times 10^{-6}$
- $m_c \sim 3.3 \times 10^{-3}$
- $m_t \sim 1$

- **down-type quarks**
- $m_d \sim 1.5 \times 10^{-5}$
- $m_s \sim 3 \times 10^{-4}$
- $m_b \sim 1.5 \times 10^{-2}$

- **charged leptons**
- $m_e \sim 3 \times 10^{-6}$
- $m_\mu \sim 6 \times 10^{-4}$
- $m_\tau \sim 1 \times 10^{-2}$



charged fermion mass spectrum strongly hierarchical

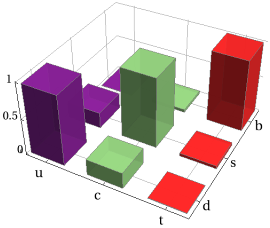
Neutrino mass differences and CKM and PMNS matrices

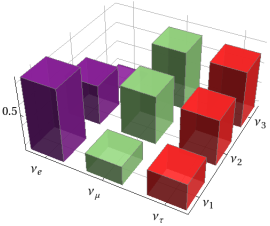
- **neutrinos** (assuming normal hierarchy)
- $\Delta m^2_{sol} \sim 7.5 \times 10^{-5} eV^2$; $m_2 \sim 8.5 \times 10^{-12}$ GeV
- $\Delta m^2_{atm} \sim 2.5 \times 10^{-3} eV^2$; $m_3 \sim 5 \times 10^{-11}$ GeV

Neutrino mass spectrum shows mild hierarchy

$$V_{CKM} \sim \begin{bmatrix} 0.976 & 0.22 & 0.004 \\ -0.22 & 0.98 & 0.04 \\ 0.007 & -0.04 & 1 \end{bmatrix}$$

$$U_{PMNS} \sim \begin{bmatrix} 0.85 & -0.54 & 0.16 \\ 0.33 & 0.62 & -0.72 \\ -0.40 & -0.59 & -0.70 \end{bmatrix}$$





Quark mixing angles are small and Leptonic mixing angles are large

Standard Model of Particle Physics and "Flavor Puzzle"

- 19 free parameters, while 14 of them are associated with the flavor sector. 6 quark masses, 3 charged lepton masses, 4 mixing parameters in the CKM matrix and 1 strong CP violating parameters, $\overline{\theta}$.

- Including neutrino masses and mixings into the SM, additional 9 parameters. 3 masses, 3 mixing angles and 3 phases.

- Are all these parameters arbitrary or inter-connected? Why the charged fermion masses are strongly hierarchical but neutrino masses are not? Why neutrino masses are so small? why quark mixing angles are so small but the leptonic mixing angles are large?

  Fundamental understanding of the origin of these parameters is currently lacking.

Why $SO(10)$ GUT?

- Unification of the three SM gauge couplings can be achieved.
- Quarks and Leptons of each family are unified into a single irreducible 16 dimensional representation.
- Predicts the existence of right-handed neutrino, ν_R already contained in 16-plet.
- Presence of ν_R naturally explains extremely light neutrino mass via seesaw mechanism.
- $SO(10)$ gauge symmetry is automatically anomaly free.
- Electric charge quantization is understood due to non-Abelian nature of the gauge group.

Flavor puzzle can be understood in $SO(10)$ due to unifying all fermions into a single multiplet.

Search for Economic Yukawa Sector of $SO(10)$ GUT

- Existing literature: Most constructions use complex Higgs in the fundamental representation (10-plet).

- This Work: Detail analysis of different possible Higgs set in search of most economic Yukawa sector.

- No new fermions beyond the three families of chiral 16s.
- No additional symmetry, only $SO(10)$ gauge symmetry.
- Non-supersymmetric framework.
- Economy $\neq$ least number of fields with least number of parameters.

Yukawa Sector of $SO(10)$ Unification

- Fermion bilinear: $16 \times 16 = 10_s + 120_a + 126_s$.
- 10 and 120 are real representations of $SO(10)$.
- 126 is complex representation of $SO(10)$.
- The most general Yukawa sector

$$\mathcal{L}_{yuk} = 16_F(Y_{10}^i 10_H^i + Y_{120}^j 120_H^j + Y_{126}^k \overline{126}_H^k)16_F.$$

$i = 1, 2, ..n_{10}$, $j = 1, 2, ..n_{120}$ and $k = 1, 2, ..n_{126}$.

What is smallest possible set of $\{n_{10}, n_{120}, n_{126}\}$ for a realistic model?

Basic Requirement

- To generate flavor mixing, atleast two fields are required to be present.

$$n_{10} + n_{120} + n_{126} \geq 2$$

- One of them must be $\overline{126}_H$ to provide large Majorana masses to ν_R.

$$n_{126} \geq 1$$

Model: $\{n_{10}, n_{120}, n_{126}\} = \{0, 0, 1 + 1\}$

- At the GUT scale: $m_\tau = -3m_b, m_\mu = -3m_s$ and $m_e = -3m_d$.
- Observed value gives: $m_\tau \sim 1.7m_b$ at the GUT scale.
- With intermediate threshold effect: $m_\tau \sim (1.4 - 1.7)m_b$ at the GUT scale.

Not a realistic model.

Model: $\{n_{10}, n_{120}, n_{126}\} = \{1, 0, 1\}$

- $10 = (2,2,1) + (1,1,6)$ under Pati-Salam group $SU(2)_L \times SU(2)_R \times SU(4)_C$.
- Bi-doublet (2,2,1) can have two independent vacuum expectation values (v_u and v_d).
- Recall: 10_H is real.
- So, Higgs doublet in 10_H is self-conjugate.
- Reality of 10_H implies: $v_u = v_d^*$, i.e, $r = |v_u/v_d| = 1$.
- Phenomenological requirement $r \sim m_t/m_b \sim 70 \gg 1$.

Not a realistic model.

Model: $\{n_{10}, n_{120}, n_{126}\} = \{0, 1, 1\}$

- Again, due to reality of 120_H, VEV ratios appearing in mass matrices are all of same order.
- Hierarchy among these ratios are needed to explain the observed hierarchy of the charged fermion masses.

  Bajc, Melfo, Senjanovic, Vissani 05

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Not a realistic model.

Summary for $n_{10} + n_{120} + n_{126} = 2$

Models with $n_{10} + n_{120} + n_{126} = 2$ are not viable.

Model: $\{n_{10}, n_{120}, n_{126}\} = \{0, 2, 1\}$

- Model predicts: $m_\tau \sim 3m_b$.
- With deviation within 5% of m_s/m_b, which is a tiny number.
- Phenomenological requirement $m_\tau \sim (1.4 - 1.7)m_b$
- This conclusion is valid for arbitrary number of 120_H.

Not a realistic model.

Model: $\{n_{10}, n_{120}, n_{126}\} = \{1, 1, 1\}$ *or* $\{2, 0, 1\}$

- Our analysis shows, either of the models can be realistic.
- $\{n_{10}, n_{120}, n_{126}\} = \{1, 1, 1\}$: Two symmetric and one anti-symmetrix Yukawa coupling matrices.
- $\{n_{10}, n_{120}, n_{126}\} = \{2, 0, 1\}$: Three symmetric Yukawa coupling matrices.

Model with $\{n_{10}, n_{120}, n_{126}\} = \{1, 1, 1\}$:
(i) less number of parameters.
(ii) more attractive (natural?), since no replication of same multiplet.

Conventional Models: $\{n_{10}, n_{120}, n_{126}\} = \{2, 0, 1\}$

- A complex 10 = two real 10s : $10_c = \frac{10_1 + i\,10_2}{\sqrt{2}}$
- Two independent Yukawa coupling matrices

$$\mathcal{L}_{yuk} \supset 16_F Y_{10} 10_c 16_F + 16_F Y_{10}^c 10_c^* 16_F .$$

- $10_c + 10_c^* + 126$: Too many parameters.
- Charge assignment exterior to $SO(10)$ can forbid one of the terms.
- 13 magnitudes+8 phases: realistic model.
- $SO(10) \to SO(10) \times G$ (such as $U(1)$ Peccei-Quinn).

K.S. Babu, R. Mohapatra, 92; T. Fukuyama and N. Okada, 02; H. S. Goh, R. N. Mohapatra and S. P. Ng, 03

S. Bertolini, M. Frigerio and M. Malinsky, 04; K. S. Babu and C. Macesanu, 05; A. S. Joshipura and K. M. Patel, 11

G. Altarelli and D. Meloni, 13; A. Dueck and W. Rodejohann, 13

Not true grand unified symmetry.

Yukawa Sector of Minimal $SO(10)$ Unification

- real 10_H+ real 120_H+ complex 126_H:

  $\mathcal{L}_{yuk} = 16_F(Y_{10}10_H + Y_{120}120_H + Y_{126}\overline{126}_H)16_F$.

- The mass matrices:

$$
\begin{aligned}
M_U &= D + S + A, \\
M_D &= D + r_1 S + e^{i\phi} A, \\
M_E &= D - 3r_1 S + r_2 A, \\
M_{\nu_D} &= D - 3S + r_2^* e^{i\phi} A, \\
M_{\nu_{R,L}} &= c_{R,L} S.
\end{aligned}
$$

- The neutrino mass matrix from seesaw mechanism:

$$M_N = M_{\nu_L} - M_{\nu_D} M_{\nu_R}^{-1} M_{\nu_D}^T.$$

- 16 magnitudes+13 phases.

Best Fit Values

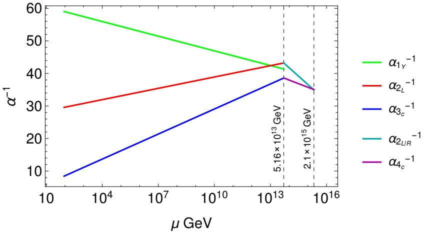
Masses (in GeV) and Mixing parameters	Inputs (at $\mu = M_{GUT}$)	Fitted values (**type-I**) (at $\mu = M_{GUT}$)	pulls (**type-I**)	Fitted values (**type-I+II**) (at $\mu = M_{GUT}$)	pulls (**type-I+II**)
$m_u/10^{-3}$	0.442±0.149	0.444	0.009	0.442	-0.0002
m_c	0.238±0.007	0.238	-0.002	0.238	0.0001
m_t	74.51±0.65	74.52	0.009	74.52	-0.005
$m_d/10^{-3}$	1.14±0.11	1.14	-0.0002	1.14	-0.00006
$m_s/10^{-3}$	21.58±1.14	21.60	0.007	21.59	0.0001
m_b	0.994±0.009	0.994	0.002	0.994	0.000005
$m_e/10^{-3}$	0.470692±0.000470	0.470674	-0.03	0.470675	-0.003
$m_\mu/10^{-3}$	99.3658±0.0993	99.3618	-0.04	99.3621	-0.003
m_τ	1.68923±0.00168	1.68925	0.01	1.68925	0.001
$\lvert V_{us}\rvert/10^{-2}$	22.54±0.06	22.54	0.002	22.54	0.00008
$\lvert V_{cb}\rvert/10^{-2}$	4.856±0.06	4.856	0.001	4.856	0.0007
$\lvert V_{ub}\rvert/10^{-2}$	0.420±0.013	0.420	-0.007	0.420	-0.0001
δ_{CKM}	1.207±0.054	1.207	0.01	1.207	0.005
$\Delta m^2_{sol}/10^{-4}(\mathrm{eV}^2)$	1.29±0.04 (1×10^{15}GeV) 1.27±0.04 (7.3×10^{12}GeV)	1.27	-0.48	1.27	0.04
$\Delta m^2_{atm}/10^{-3}(\mathrm{eV}^2)$	4.12±0.13 (1×10^{15}GeV) 4.05±0.13 (7.3×10^{12}GeV)	4.06	- 0.46	4.06	0.04
$\sin^2\theta_{12}^{\mathrm{PMNS}}$	0.308±0.017	0.308	-0.01	0.308	0.00001
$\sin^2\theta_{23}^{\mathrm{PMNS}}$	0.387±0.0225	0.387	-0.01	0.387	-0.00006
$\sin^2\theta_{13}^{\mathrm{PMNS}}$	0.0241±0.0025	0.0241	0.01	0.0241	-0.0003
δ^{PMNS}	-	2.81°	-	-150.82°	-
$m_1(meV)$	-	0.15	-	10.2	-
χ^2	-	-	0.45	-	0.004

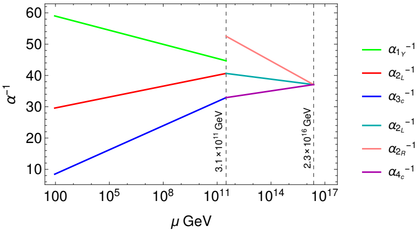
no solution for Type-II dominance case is found.

Symmetry Breaking and Gauge Coupling Unification

Can be achieved by adding 45_H or 54_H or 210_H.

- $SO(10) \xrightarrow{45_H} SU(2)_L \times SU(2)_R \times SU(3)_C \times U(1)_{B-L}$.
- $SO(10) \xrightarrow{54_H} SU(2)_L \times SU(2)_R \times SU(4)_C \times D$.
- $SO(10) \xrightarrow{210_H} SU(2)_L \times SU(2)_R \times SU(4)_C$.





Proton Decay Branching Ratios

p decay modes	**type-I**	**type-I+II**
$p \to \overline{\nu} + \pi^+$	49.07%	48.77%
$p \to e^+\pi^0$	42.57%	35.16%
$p \to \mu^+K^0$	4.13%	5.12%
$p \to \mu^+\pi^0$	1.60%	5.62%
$p \to \overline{\nu}K^+$	1.19%	2.64%
$p \to e^+K^0$	0.99%	2.28%
$p \to e^+\eta$	0.40%	0.33%
$p \to \mu^+\eta$	0.01%	0.05%

Summary

- We have performed a detail analysis in search of the most economic Yukawa sector of models based on $SO(10)$ gauge symmetry without the presence of any exterior symmetry.
- We prove $\{n_{10}, n_{120}, n_{126}\} = \{1, 1, 1\}$ is the minimal realistic set.
- Symmetry breaking and gauge coupling unification is studied.
- Proton decay branching rations are computed.
- The dominant decays of the proton are found to be $p \to \overline{\nu}\pi^{+}$ and $p \to e^{+}\pi^{0}$.

THANKS!