

On HQET and NRQCD Operators of Mass Dimension 8 and Above

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Based on: A.G and G. Paz [[arXiv:1702.08904](https://arxiv.org/abs/1702.08904)]

Outline

Introduction

Construction of HQET Matrix elements

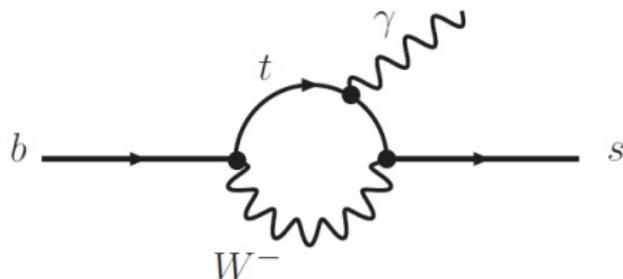
New Results

INTRODUCTION

Introduction

- Our Universe is mostly made out of matter instead of anti-matter.
- CP Violation: particle behaves differently than anti-particles
- For the radiative decays $\bar{B} \rightarrow X_s \gamma$ and $B \rightarrow X_s \gamma$ the CP asymmetry defined as

$$A_{X_s} = \frac{\Gamma(\bar{B} \rightarrow X_s \gamma) - \Gamma(B \rightarrow X_s \gamma)}{\Gamma(\bar{B} \rightarrow X_s \gamma) + \Gamma(B \rightarrow X_s \gamma)}$$



Introduction

- The Leading power correction to the decay rate is given by,

$$\Gamma = \sum_{n=0}^{\infty} \frac{1}{M_H} \sum_k c_{k,n} \langle O_{k,n} \rangle$$

- The $c_{k,n}$ are the Wilson Coefficients that can be calculated from perturbative QCD.
- $O_{k,n}$ are non-perturbative HQET matrix elements measured, e.g., in
[P. Gambino, K. J. Healey, S. Turczyk PLB 763, 60 (2016)]
- These Matrix elements have the form

$$O_{k,n} \sim \bar{h} i D^{\mu_1} i D^{\mu_2} \dots i D^{\mu_n} (s^\lambda) h T_{\mu_1 \mu_2 \dots \mu_n}(\lambda)$$

$$D^\mu = \partial^\mu + ig A^{\mu a} T^a$$

Introduction

- HQET matrix elements are important
 - To calculate decay rates accurately
 - ⇒ A probe of new physics
 - To analyze the data from Belle-II experiment

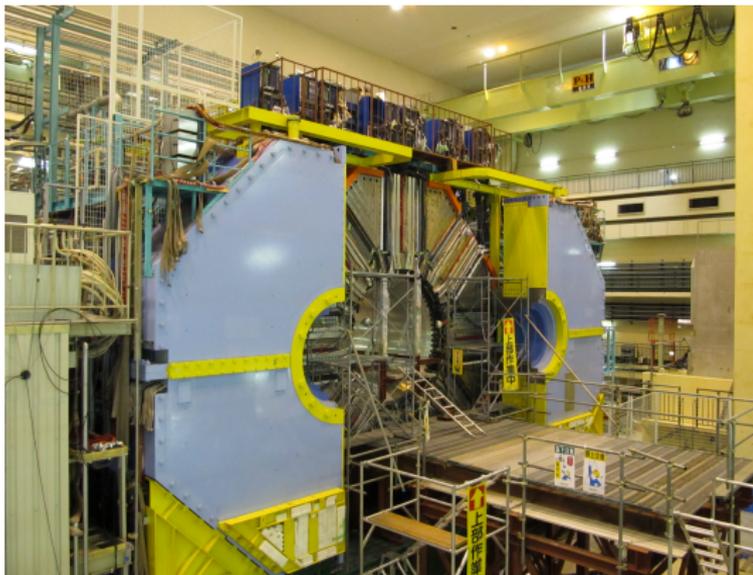


Figure: Belle-II experiment

Introduction

- List of these matrix elements up to mass dimension 8 was provided in
[\[Mannel, Turczyk, Uraltsev JHEP 1011, 109\(2010\)\]](#)
- In our work we provide:
[\[A. Gunawardana and G. Paz, arXiv:1702.08904\[hep-ph\]\]](#)
 - A general method for a systematic construction of these matrix elements to any order of $1/M$.
 - Construction of matrix elements up to mass dimension 9
 - Connection between the matrix elements and NRQCD/HQET

CONSTRUCTION OF MATRIX ELEMENTS

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- In the rest frame of Heavy quark $v^\mu = (1, 0, 0, 0) \therefore v \cdot v = 1$.

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- From PT symmetry:

$$\langle B(v) | \bar{h} iD^{\mu_1} \dots iD^{\mu_n} h | B(v) \rangle \stackrel{PT}{=} \langle B(v) | \bar{h} iD^{\mu_1} \dots iD^{\mu_n} h | B(v) \rangle^*,$$

$$\langle B(v) | \bar{h} iD^{\mu_1} \dots iD^{\mu_n} s^\lambda h | B(v) \rangle \stackrel{PT}{=} -\langle B(v) | \bar{h} iD^{\mu_1} \dots iD^{\mu_n} s^\lambda h | B(v) \rangle^*.$$

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- Hermitian conjugation:

$$\langle B | \bar{h} iD^{\mu_1} \dots iD^{\mu_n} (s^\lambda) h | B \rangle = \langle B | \bar{h} iD^{\mu_n} \dots iD^{\mu_1} (s^\lambda) h | B \rangle^*.$$

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- PT+ Hermitian Conjugation \Rightarrow Inversion Symmetry
 - Independent : Symmetric under inversion of indices.
 - Spin Dependent : Anti-symmetric under inversion of indices

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 - Independent : Symmetric under inversion of indices.
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- From HQET equation of Motion:
 - $iv \cdot Dh = 0, \bar{h} iv \cdot D = 0$
 - $v \cdot s = 0$

Construction of Matrix elements

- Matrix elements only depend on:
 - v^μ
 - $g^{\mu_i \mu_j} \Rightarrow \Pi^{\mu_i \mu_j} = g^{\mu_i \mu_j} - v^{\mu_i} v^{\mu_j}$: Parity even
 - $\epsilon^{\mu_i \mu_j \mu_k \mu_l}$: Parity odd

Spin Independent Matrix elements

- The spin independent matrix elements are invariant under parity.
- Dimension 3

$$\frac{1}{2M_B} \langle B | \bar{h}h | B \rangle = 1.$$

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- Dimension 5

$$\frac{1}{2M_B} \langle B | \bar{h} i D^{\mu_1} i D^{\mu_2} h | B \rangle = a^{(5)} \Pi^{\mu_1 \mu_2}$$

Spin Independent Matrix elements

- Dimension 6

$$\frac{1}{2M_B} \langle B | \bar{h} iD^{\mu_1} iD^{\mu_2} iD^{\mu_3} h | B \rangle = a^{(6)} \Pi^{\mu_1 \mu_3} v^{\mu_2}.$$

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- Dimension 7

$$\begin{aligned} \frac{1}{2M_B} \langle B | \bar{h} iD^{\mu_1} iD^{\mu_2} iD^{\mu_3} iD^{\mu_4} h | B \rangle &= a_{12}^{(7)} \Pi^{\mu_1 \mu_2} \Pi^{\mu_3 \mu_4} + a_{13}^{(7)} \Pi^{\mu_1 \mu_3} \Pi^{\mu_2 \mu_4} \\ &+ a_{14}^{(7)} \Pi^{\mu_1 \mu_4} \Pi^{\mu_2 \mu_3} + b^{(7)} \Pi^{\mu_1 \mu_4} v^{\mu_2} v^{\mu_3}. \end{aligned}$$

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- Dimension 8

$$\begin{aligned} \frac{1}{2M_B} \langle B | \bar{h} iD^{\mu_1} iD^{\mu_2} iD^{\mu_3} iD^{\mu_4} iD^{\mu_5} h | B \rangle &= a_{12}^{(8)} (\Pi^{\mu_1 \mu_2} \Pi^{\mu_3 \mu_5} v^{\mu_4} + \Pi^{\mu_1 \mu_3} \Pi^{\mu_5 \mu_4} v^{\mu_2}) + \\ a_{13}^{(8)} (\Pi^{\mu_1 \mu_3} \Pi^{\mu_2 \mu_5} v^{\mu_4} + \Pi^{\mu_5 \mu_3} \Pi^{\mu_4 \mu_1} v^{\mu_2}) &+ a_{15}^{(8)} (\Pi^{\mu_1 \mu_5} \Pi^{\mu_3 \mu_4} v^{\mu_2} + \Pi^{\mu_1 \mu_5} \Pi^{\mu_3 \mu_2} v^{\mu_4}) + \\ b_{12}^{(8)} \Pi^{\mu_1 \mu_2} \Pi^{\mu_4 \mu_5} v^{\mu_3} + b_{14}^{(8)} \Pi^{\mu_1 \mu_4} \Pi^{\mu_2 \mu_5} v^{\mu_3} &+ b_{15}^{(8)} \Pi^{\mu_1 \mu_5} \Pi^{\mu_2 \mu_4} v^{\mu_3} + \\ c^{(8)} \Pi^{\mu_1 \mu_5} v^{\mu_2} v^{\mu_3} v^{\mu_4}. & \end{aligned}$$

(Symmetric under inversion of indices)

Spin Dependent Matrix elements

- Spin Dependent matrix elements are parity odd
- Dimension 3

$$\frac{1}{2M_B} \langle B | \bar{h} s^\lambda h | B \rangle = 0 \quad (\because v \cdot s = 0)$$

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$$\frac{1}{2M_B} \langle B | \bar{h} iD^{\mu_1} iD^{\mu_2} s^\lambda h | B \rangle = i \tilde{a}^{(5)} \epsilon^{\rho \mu_1 \mu_2 \lambda} v_\rho.$$

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$$\frac{1}{2M_B} \langle B | \bar{h} i D^{\mu_1} i D^{\mu_2} i D^{\mu_3} s^\lambda h | B \rangle = i \tilde{a}^{(6)} v^{\mu_2} \epsilon^{\rho \mu_1 \mu_3 \lambda} v_\rho.$$

Spin Dependent Matrix elements

- Dimension 7

$$\begin{aligned} & \frac{1}{2M_B} \langle B | \bar{h} iD^{\mu_1} iD^{\mu_2} iD^{\mu_3} iD^{\mu_4} s^\lambda h | B \rangle = \\ & i\tilde{a}_{12}^{(7)} \left(\Pi^{\mu_1\mu_2} \epsilon^{\rho\mu_3\mu_4\lambda} v_\rho - \Pi^{\mu_4\mu_3} \epsilon^{\rho\mu_2\mu_1\lambda} v_\rho \right) + i\tilde{a}_{13}^{(7)} \left(\Pi^{\mu_1\mu_3} \epsilon^{\rho\mu_2\mu_4\lambda} v_\rho - \Pi^{\mu_4\mu_2} \epsilon^{\rho\mu_3\mu_1\lambda} v_\rho \right) + \\ & + i\tilde{a}_{14}^{(7)} \Pi^{\mu_1\mu_4} \epsilon^{\rho\mu_2\mu_3\lambda} v_\rho + i\tilde{a}_{23}^{(7)} \Pi^{\mu_2\mu_3} \epsilon^{\rho\mu_1\mu_4\lambda} v_\rho + i\tilde{b}^{(7)} v^{\mu_2} v^{\mu_3} \epsilon^{\rho\mu_1\mu_4\lambda} v_\rho, \end{aligned}$$

(Anti Symmetric under inversion of indices)

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$$+ i\tilde{a}_{14}^{(7)} \Pi^{\mu_1\mu_4} \epsilon^{\rho\mu_2\mu_3\lambda} v_\rho + i\tilde{a}_{23}^{(7)} \Pi^{\mu_2\mu_3} \epsilon^{\rho\mu_1\mu_4\lambda} v_\rho + i\tilde{b}^{(7)} v^{\mu_2} v^{\mu_3} \epsilon^{\rho\mu_1\mu_4\lambda} v_\rho,$$

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- Dimension 8

$$\frac{1}{2M_B} \langle B | \bar{h} iD^{\mu_1} iD^{\mu_2} iD^{\mu_3} iD^{\mu_4} iD^{\mu_5} s^\lambda h | B \rangle =$$

$$i\tilde{a}_{12}^{(8)} \left(v^{\mu_3} \Pi^{\mu_1\mu_2} \epsilon^{\rho\mu_4\mu_5\lambda} v_\rho - v^{\mu_3} \Pi^{\mu_4\mu_5} \epsilon^{\rho\mu_2\mu_1\lambda} v_\rho \right) + i\tilde{a}_{14}^{(8)} \left(v^{\mu_3} \Pi^{\mu_1\mu_4} \epsilon^{\rho\mu_2\mu_5\lambda} v_\rho - v^{\mu_3} \Pi^{\mu_5\mu_2} \epsilon^{\rho\mu_4\mu_1\lambda} v_\rho \right) +$$

$$+ i\tilde{a}_{15}^{(8)} v^{\mu_3} \Pi^{\mu_1\mu_5} \epsilon^{\rho\mu_2\mu_4\lambda} v_\rho + i\tilde{a}_{24}^{(8)} v^{\mu_3} \Pi^{\mu_2\mu_4} \epsilon^{\rho\mu_1\mu_5\lambda} v_\rho +$$

$$+ i\tilde{b}_{13}^{(8)} \left(v^{\mu_2} \Pi^{\mu_1\mu_3} \epsilon^{\rho\mu_4\mu_5\lambda} v_\rho - v^{\mu_4} \Pi^{\mu_5\mu_3} \epsilon^{\rho\mu_2\mu_1\lambda} v_\rho \right) + i\tilde{b}_{14}^{(8)} \left(v^{\mu_2} \Pi^{\mu_1\mu_4} \epsilon^{\rho\mu_3\mu_5\lambda} v_\rho - v^{\mu_4} \Pi^{\mu_5\mu_2} \epsilon^{\rho\mu_3\mu_1\lambda} v_\rho \right) +$$

$$+ i\tilde{b}_{15}^{(8)} \left(v^{\mu_2} \Pi^{\mu_1\mu_5} \epsilon^{\rho\mu_3\mu_4\lambda} v_\rho - v^{\mu_4} \Pi^{\mu_1\mu_5} \epsilon^{\rho\mu_3\mu_2\lambda} v_\rho \right) + i\tilde{b}_{34}^{(8)} \left(v^{\mu_2} \Pi^{\mu_3\mu_4} \epsilon^{\rho\mu_1\mu_5\lambda} v_\rho - v^{\mu_4} \Pi^{\mu_3\mu_2} \epsilon^{\rho\mu_5\mu_1\lambda} v_\rho \right) +$$

$$+ i\tilde{b}_{35}^{(8)} \left(v^{\mu_2} \Pi^{\mu_3\mu_5} \epsilon^{\rho\mu_1\mu_4\lambda} v_\rho - v^{\mu_4} \Pi^{\mu_3\mu_1} \epsilon^{\rho\mu_5\mu_2\lambda} v_\rho \right) + i\tilde{b}_{45}^{(8)} \left(v^{\mu_2} \Pi^{\mu_4\mu_5} \epsilon^{\rho\mu_1\mu_3\lambda} v_\rho - v^{\mu_4} \Pi^{\mu_2\mu_1} \epsilon^{\rho\mu_5\mu_3\lambda} v_\rho \right) +$$

$$+ i\tilde{c}^{(8)} v^{\mu_2} v^{\mu_3} v^{\mu_4} \epsilon^{\rho\mu_1\mu_5\lambda} v_\rho.$$

- Results agree with [\[Mannel, Turczyk, Uraltsev JHEP 1011, 109\(2010\)\]](#)

New Results

New Result: Dimension 9 HQET matrix element

$$\begin{aligned}
 & \frac{1}{2M_H} \langle H | \bar{h} i D^{\mu_1} i D^{\mu_2} i D^{\mu_3} i D^{\mu_4} i D^{\mu_5} i D^{\mu_6} h | H \rangle = a_{12,34}^{(9)} \Pi^{\mu_1 \mu_2} \Pi^{\mu_3 \mu_4} \Pi^{\mu_5 \mu_6} + \\
 & + a_{12,35}^{(9)} (\Pi^{\mu_1 \mu_2} \Pi^{\mu_3 \mu_5} \Pi^{\mu_4 \mu_6} + \Pi^{\mu_1 \mu_3} \Pi^{\mu_2 \mu_4} \Pi^{\mu_5 \mu_6}) + a_{12,36}^{(9)} (\Pi^{\mu_1 \mu_2} \Pi^{\mu_3 \mu_6} \Pi^{\mu_4 \mu_5} + \Pi^{\mu_1 \mu_4} \Pi^{\mu_2 \mu_3} \Pi^{\mu_5 \mu_6}) + \\
 & + a_{13,25}^{(9)} \Pi^{\mu_1 \mu_3} \Pi^{\mu_2 \mu_5} \Pi^{\mu_4 \mu_6} + a_{13,26}^{(9)} (\Pi^{\mu_1 \mu_3} \Pi^{\mu_2 \mu_6} \Pi^{\mu_4 \mu_5} + \Pi^{\mu_1 \mu_5} \Pi^{\mu_2 \mu_3} \Pi^{\mu_4 \mu_6}) + \\
 & + a_{14,25}^{(9)} \Pi^{\mu_1 \mu_4} \Pi^{\mu_2 \mu_5} \Pi^{\mu_3 \mu_6} + a_{14,26}^{(9)} (\Pi^{\mu_1 \mu_4} \Pi^{\mu_2 \mu_6} \Pi^{\mu_3 \mu_5} + \Pi^{\mu_1 \mu_5} \Pi^{\mu_2 \mu_4} \Pi^{\mu_3 \mu_6}) + \\
 & + a_{15,26}^{(9)} \Pi^{\mu_1 \mu_5} \Pi^{\mu_2 \mu_6} \Pi^{\mu_3 \mu_4} + a_{16,23}^{(9)} \Pi^{\mu_1 \mu_6} \Pi^{\mu_2 \mu_3} \Pi^{\mu_4 \mu_5} + a_{16,24}^{(9)} \Pi^{\mu_1 \mu_6} \Pi^{\mu_2 \mu_4} \Pi^{\mu_3 \mu_5} + \\
 & + a_{16,25}^{(9)} \Pi^{\mu_1 \mu_6} \Pi^{\mu_2 \mu_5} \Pi^{\mu_3 \mu_4} + b_{12,36}^{(9)} (\Pi^{\mu_1 \mu_2} \Pi^{\mu_3 \mu_6} v^{\mu_4} v^{\mu_5} + \Pi^{\mu_1 \mu_4} \Pi^{\mu_5 \mu_6} v^{\mu_2} v^{\mu_3}) + \\
 & + b_{12,46}^{(9)} (\Pi^{\mu_1 \mu_2} \Pi^{\mu_4 \mu_6} v^{\mu_3} v^{\mu_5} + \Pi^{\mu_1 \mu_3} \Pi^{\mu_5 \mu_6} v^{\mu_2} v^{\mu_4}) + b_{12,56}^{(9)} \Pi^{\mu_1 \mu_2} \Pi^{\mu_5 \mu_6} v^{\mu_3} v^{\mu_4} + \\
 & + b_{13,26}^{(9)} (\Pi^{\mu_1 \mu_3} \Pi^{\mu_2 \mu_6} v^{\mu_4} v^{\mu_5} + \Pi^{\mu_1 \mu_5} \Pi^{\mu_4 \mu_6} v^{\mu_2} v^{\mu_3}) + \\
 & + b_{13,46}^{(9)} \Pi^{\mu_1 \mu_3} \Pi^{\mu_4 \mu_6} v^{\mu_2} v^{\mu_5} + b_{14,26}^{(9)} (\Pi^{\mu_1 \mu_4} \Pi^{\mu_2 \mu_6} v^{\mu_3} v^{\mu_5} + \Pi^{\mu_1 \mu_5} \Pi^{\mu_3 \mu_6} v^{\mu_2} v^{\mu_4}) + \\
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 & b_{16,23}^{(9)} (\Pi^{\mu_1 \mu_6} \Pi^{\mu_2 \mu_3} v^{\mu_4} v^{\mu_5} + \Pi^{\mu_1 \mu_6} \Pi^{\mu_4 \mu_5} v^{\mu_2} v^{\mu_3}) + \\
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 & + b_{16,34}^{(9)} \Pi^{\mu_1 \mu_6} \Pi^{\mu_3 \mu_4} v^{\mu_2} v^{\mu_5} + c^{(9)} \Pi^{\mu_1 \mu_6} v^{\mu_2} v^{\mu_3} v^{\mu_4} v^{\mu_5}.
 \end{aligned}$$

Summary

- Higher dimensional HQET matrix elements are important in measuring the decay rates of radiative B decays.
- Presented a new method for systematic construction of these matrix elements.
- For the first time we present the tensor decomposition of spin independent matrix element at mass dimension 9 and HQET/NRQCD Lagrangian at mass dimension 8

Summary

- Applications:
 - Calculation of the moments of Leading order shape function.
 - Estimation of CP asymmetry for radiative B decays.
- Issues of extra color factor pointed out in
[A. Kobach and S. Pal [arXiv:1704.00008]]
and will be discussed in an updated version of
[A. Gunawardana and G. Paz, arXiv:1702.08904[hep-ph]]

THANK YOU

BACKUP SLIDES

- From the color matrices:

$$[T^a, T^b] = if^{abc} T_c,$$

$$\{T^a, T^b\} = \frac{1}{3}\delta^{ab} + d^{abc} T_c.$$

- $E_a^i T^a = \frac{i}{g}[iv \cdot D, iD^i]$
- Operator: E^2

$$E_a^i E_b^j \delta_{ij} T^a T^b \rightarrow E_a^i E_b^j \delta_{ij} \delta^{ab} \text{ and } E_a^i E_b^j \delta_{ij} d^{abc} T_c$$

[A. Kobach and S. Pal [arXiv:1704.00008]]

- CP asymmetry(experimental)= 0.015 ± 0.02
- $\mathcal{B}(B \rightarrow X_s \gamma) (E_\gamma > 1.6 \text{ GeV}) = (3.36 \pm 0.16) \times 10^{-4}$
[Heavy Flavor Averaging Group (2016)]
- $\mathcal{B}(B \rightarrow X_s \gamma) (E_\gamma > 1.6 \text{ GeV}) = 3.15 \pm 0.23 \times 10^{-4}$
[Misiak et al]
- $\mathcal{A}_{X_s \gamma}^{SM} = (0.44_{-0.10}^{+0.15} \pm 0.03_{-0.09}^{+0.19})\%$
- $\mathcal{A}_{X_s \gamma} = -(1.2 \pm 2.8)\%$ [Y. Amhis et al.[HFAG], arXiv:1207.1158[hep-ex]]