Kaluza–Klein Towers in the Early Universe:
Phase Transitions, Relic Abundances, and Applications to Axion Cosmology

[arXiv:1612.08950]
[arXiv:1509.00470]

Jeff Kost
with Keith Dienes (Arizona) and Brooks Thomas (Lafayette)
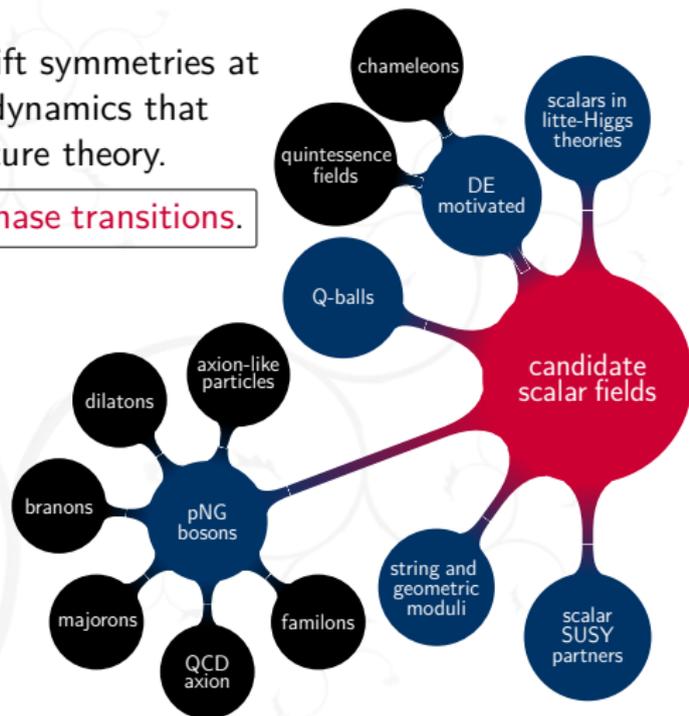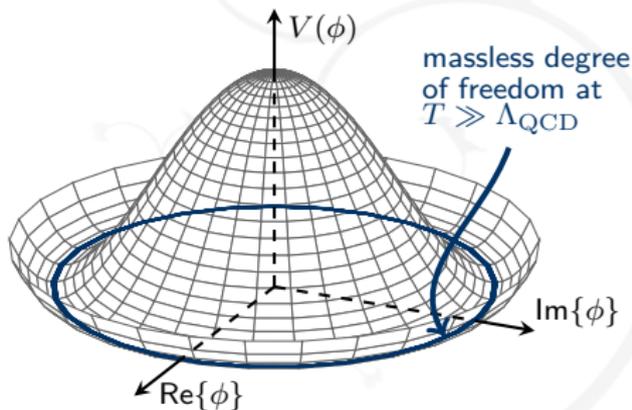
PHENO 2017
Tuesday, May 9th, 2017

Scalars in the Early Universe
Impact of Mass-Generating Phase Transitions

- Additional scalar fields commonly appear in extensions of the SM, and tend to play an important role in early-universe cosmology.

- These fields are often light due to shift symmetries at high scales, but are broken by some dynamics that enters in the effective lower-temperature theory.

  i.e., they undergo mass-generating phase transitions.

example: QCD Axion

massless degree
of freedom at
$T \gg \Lambda_{\mathrm{QCD}}$

chameleons

scalars in litte-Higgs theories

quintessence fields

DE motivated

Q-balls

candidate scalar fields

dilatons

axion-like particles

branons

pNG bosons

string and geometric moduli
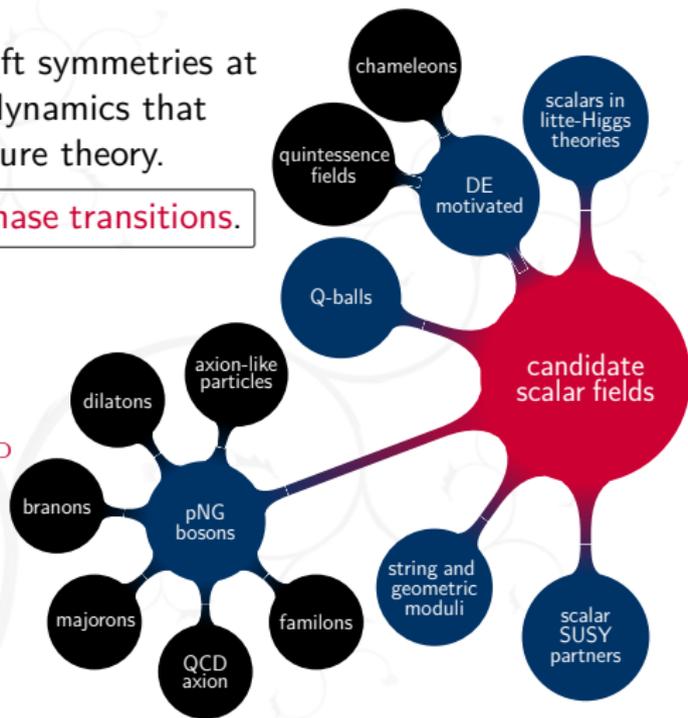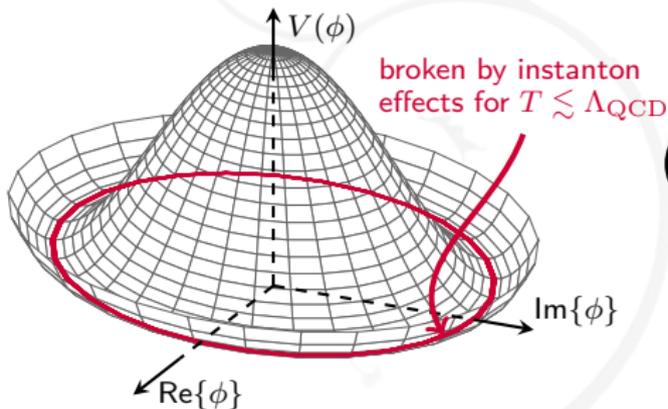
scalar SUSY partners

majorons

famlions

QCD axion

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broken by instanton effects for $T \lesssim \Lambda_{\mathrm{QCD}}$

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Scalars in the Early Universe
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- All of this can be <u>very important for model building</u>: the energy density ρ carried by these scalar(s) at late times (used to compute abundances, overclosure bounds, *etc.*) is generally **sensitive to the timescale δ_G over which such a phase transition unfolds**.

- With ***multiple fields*** $\{\phi_\lambda\}$, such transitions can generate off-diagonal elements in the mass matrix $\mathcal{M}^2$, and thus **mixing is also generated** amongst the fields in a dynamical, time-dependent way.

- The interplay of mixing/mass-generation has a **surprisingly rich influence**, even in the context of simple generic two-field models [arXiv:1509.00470].
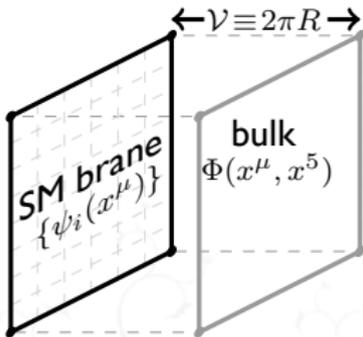
  focus of this talk:

  $\Rightarrow$ what happens with **large collections** of scalar fields, such as those furnished by models with extra dimensions?

Mass Generation in a KK Tower
The Framework

- Consider a spacetime geometry $\mathcal{M} \times S^1/\mathbb{Z}_2$, *i.e.* an extra dimension compactified on a line segment, with a bulk scalar $\Phi(x^\mu, x^5)$:

$$\mathcal{S} = \int d^4x\, dx^5 \left[\underbrace{\frac{1}{2}\partial_M \Phi^* \partial^M \Phi}_{\substack{\Phi \text{ shift symmetry} \\ \text{forbids bulk mass}}} + \delta(x^5)\mathcal{L}_{\text{brane}}(\psi_i, \Phi) \right]$$

$\leftarrow \mathcal{V} \equiv 2\pi R \rightarrow$

SM brane $\{\psi_i(x^\mu)\}$

bulk $\Phi(x^\mu, x^5)$

- The 4D mass matrix then mixes the fields:

$$\mathcal{M}^2 = m^2(t) \begin{bmatrix} 1 & \sqrt{2} & \sqrt{2} & \cdots \\ \sqrt{2} & 2 + \frac{M_c^2}{m^2(t)} & 2 & \cdots \\ \sqrt{2} & 2 & 2 + \frac{4M_c^2}{m^2(t)} & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix}$$
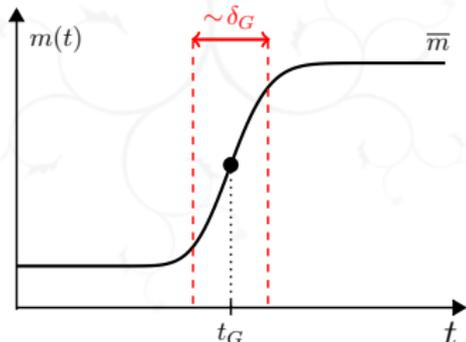
$$M_c \equiv 1/R \geq 4.49 \cdot 10^{-12}\text{GeV}$$

- $\overline{m} \gg M_c$ indicates highly mixed ensemble

Interactions with fields on brane can lead to an effective 4D mass $m(t)$:

$$\frac{1}{\mathcal{V}}\mathcal{L}_{\text{brane}}(\Phi) = -\frac{1}{2}m^2(t)\,|\Phi|^2 + \ldots$$

and we parameterize our ignorance:

Evolving the System

In a flat FRW cosmology the KK modes $\{\phi_k\}$ evolve as

$$\ddot{\phi}_k + 3H(t)\dot{\phi}_k + \sum_{\ell=0}^{\infty} \mathcal{M}_{k\ell}^2(t)\phi_\ell = 0 \ ,$$
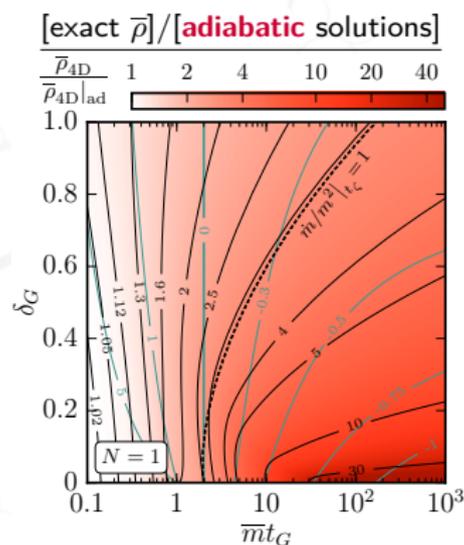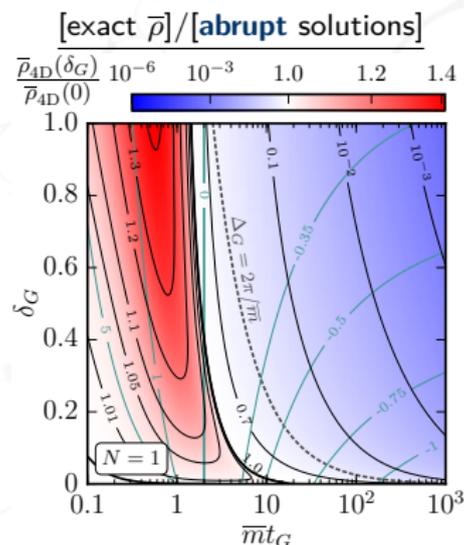
which in general cannot be solved analytically due to the time-dependence in $\mathcal{M}_{k\ell}^2$ near the phase transition.

$\Rightarrow$ perform numerics on *truncated* tower of N modes, and recover features through $N \to \infty$ limiting behavior.

Survey of Four-Dimensional ($N = 1$) Limit
Standard Approximations

- Two approximations are commonly use in the literature to compute late-time abundances in single-field models that undergo such phase transitions:
  - **abrupt** approximation $\overline{\rho}_{4D}$ (where $\delta_G \to 0$)
  - **adiabatic** approximation $\overline{\rho}_{4D}|_{ad}$ (where $\dot{m}/m^2 \ll 1$)

[exact $\overline{\rho}$]/[**abrupt** solutions] (left plot)

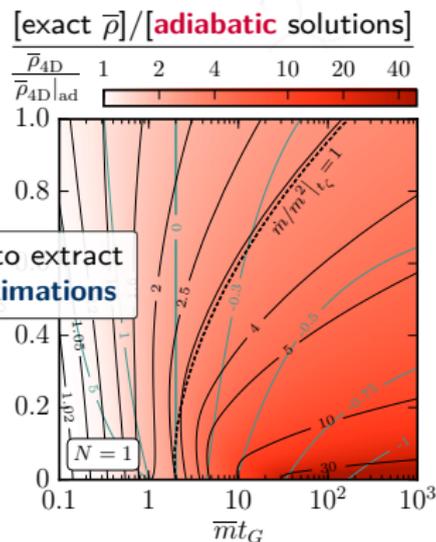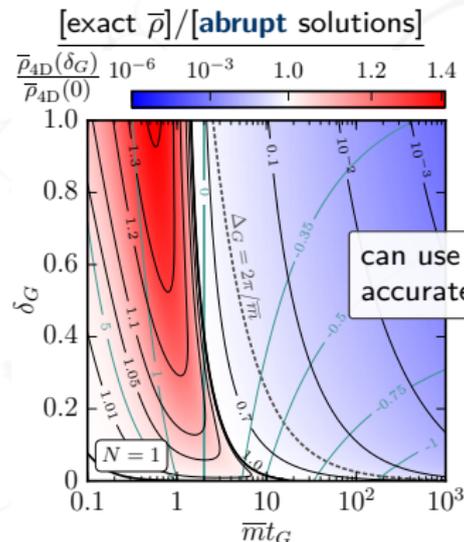[exact $\overline{\rho}$]/[**adiabatic** solutions] (right plot)

- Even for $N = 1$, there are regions of parameter space that are **inaccesible to the standard approximation**, particularly in the $m \gg 1/t_G$ regime.

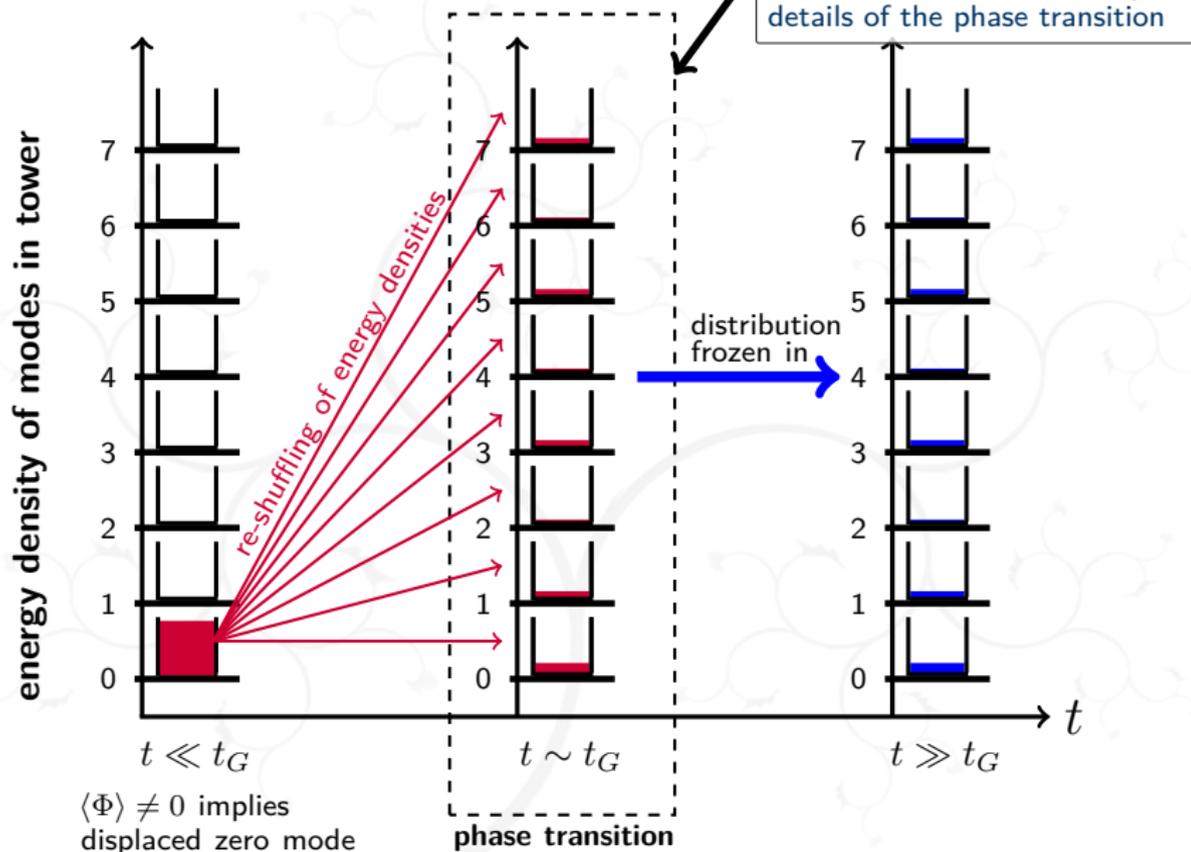# Survey of Four-Dimensional ($N = 1$) Limit
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[exact $\overline{\rho}$]/[**abrupt** solutions]

[exact $\overline{\rho}$]/[**adiabatic** solutions]

can use numerical results to extract accurate **analytical approximations**

- Even for $N = 1$, there are regions of parameter space that are **inaccessible to the standard approximation**, particularly in the $m \gg 1/t_G$ regime.
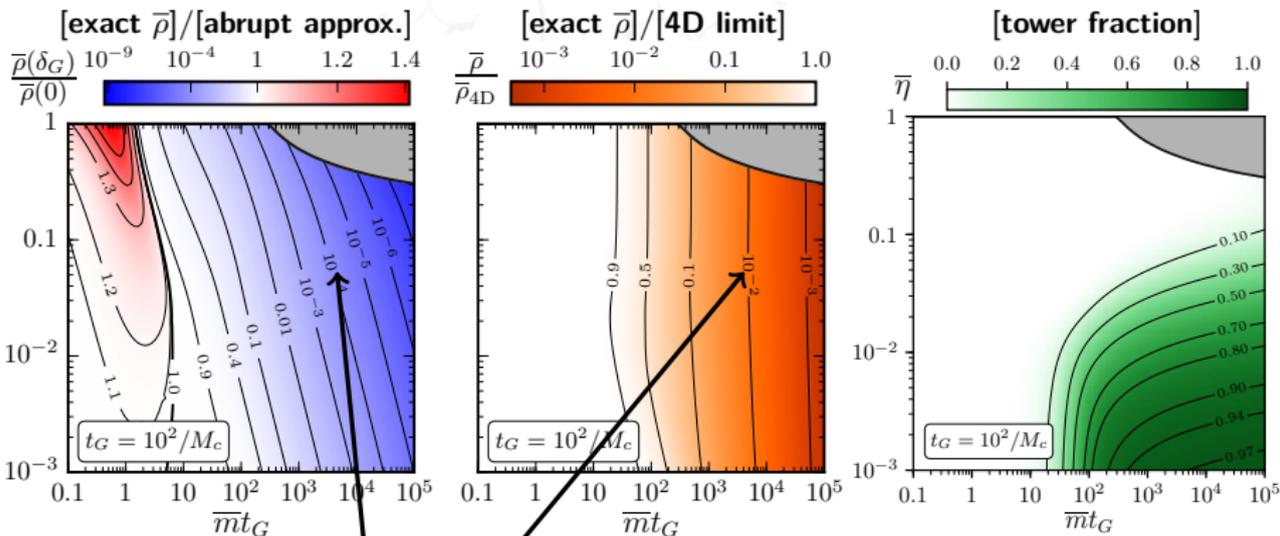
Dynamics of the $N > 1$ Tower
A Qualitative Description

suppresses/enhances modes by different amounts according to details of the phase transition

energy density of modes in tower

re-shuffling of energy densities

distribution frozen in

$t \ll t_G$

$\langle \Phi \rangle \neq 0$ implies displaced zero mode

$t \sim t_G$

phase transition

$t \gg t_G$

t

- Equipped with a method to efficiently compute asymptotia for large N, we now have the ability to compute results effectively for the **full KK tower**.
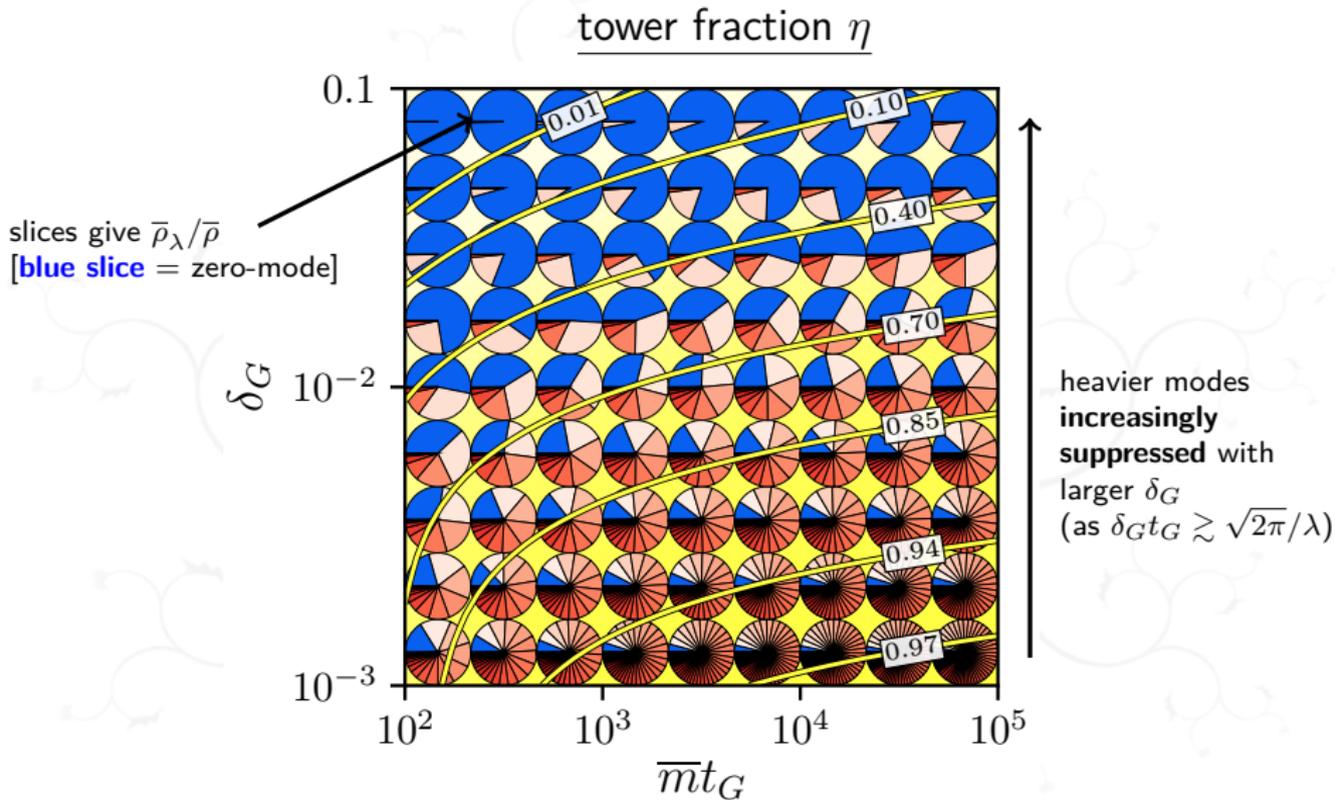

again extract general **analytical approximations** in different regions
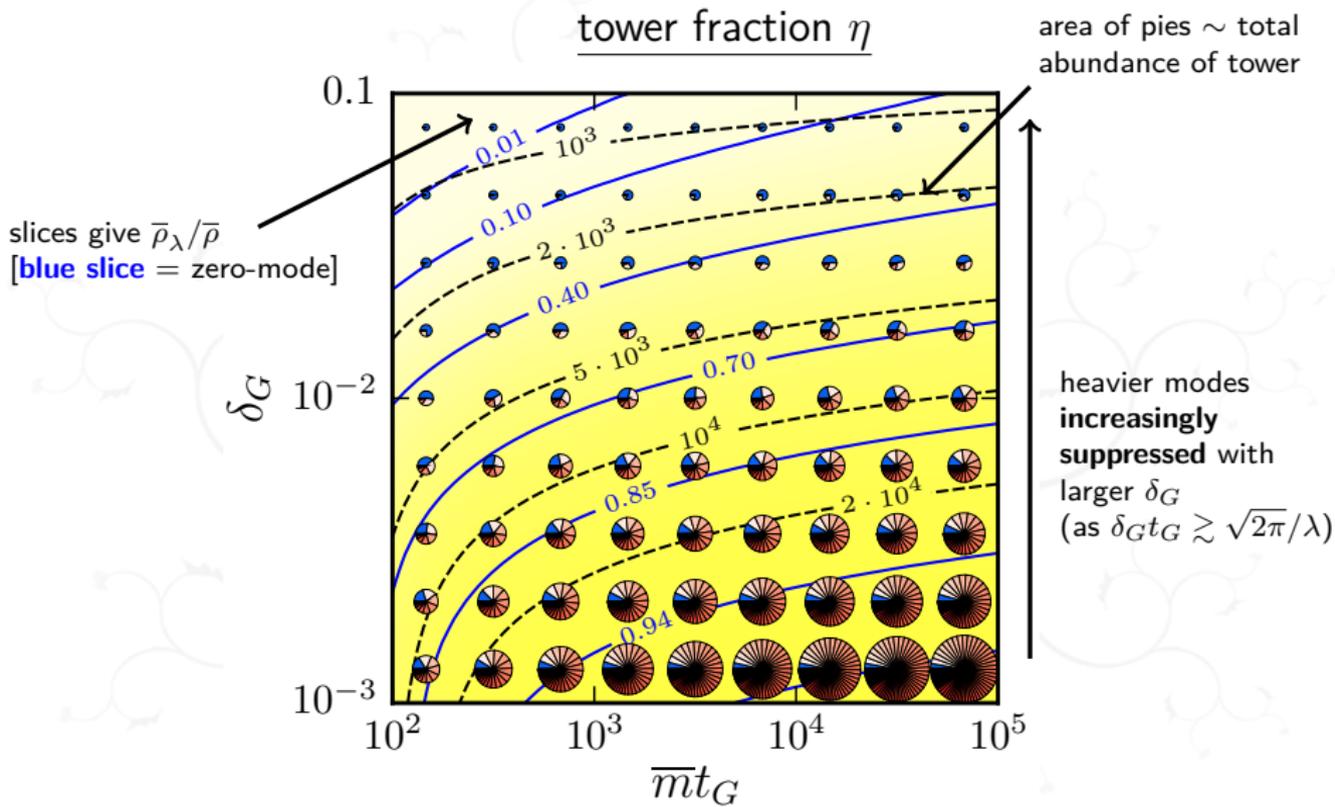
$$\eta \equiv 1 - \max_\lambda \left\{ \frac{\overline{\rho}_\lambda}{\overline{\rho}} \right\}$$

fraction of abundance in subdominant modes

Tower Fractions and Distributions

tower fraction η

slices give $\overline{\rho}_\lambda / \overline{\rho}$
[**blue slice** = zero-mode]

δ_G

$\overline{m} t_G$

heavier modes
**increasingly
suppressed** with
larger δ_G
(as $\delta_G t_G \gtrsim \sqrt{2\pi}/\lambda$)

Tower Fractions and Distributions

tower fraction η

area of pies $\sim$ total abundance of tower

slices give $\overline{\rho}_\lambda / \overline{\rho}$
[**blue slice** = zero-mode]

heavier modes **increasingly suppressed** with larger δ_G
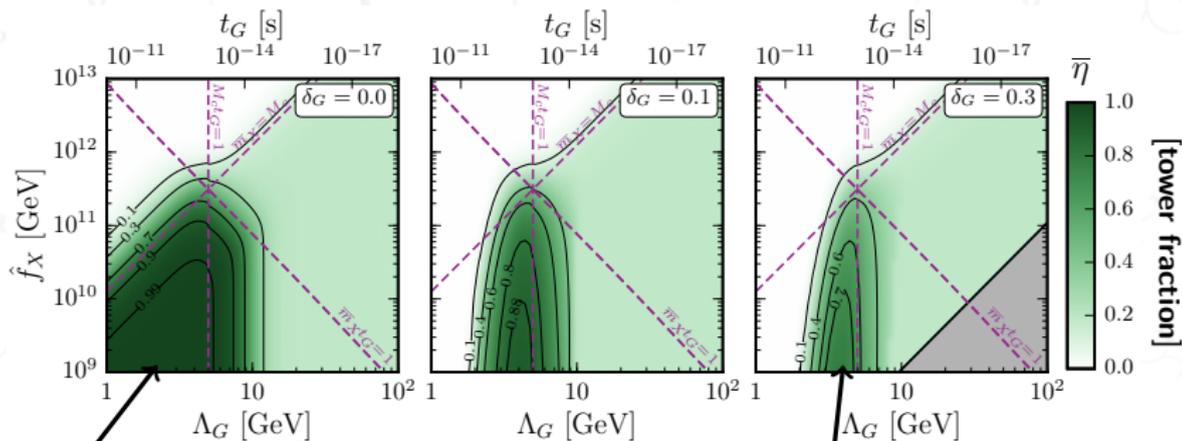(as $\delta_G t_G \gtrsim \sqrt{2\pi}/\lambda$)

Example: Axion in the Bulk

- At this point we can drop the generality of Φ and apply our machinery to a specific model: for example a **bulk axion-like particle (ALP)**.

- Our $\{t_G, \overline{m}_X, M_c\}$ parameter space is mapped onto $\{\Lambda_G, \hat{f}_X, M_c\}$

associated confinement scale

effective 4D decay constant

$$t_G = \sqrt{\frac{45 g_*(T_{\mathrm{RH}})}{2\pi^2}} \frac{T_{\mathrm{RH}}^2 M_p}{g_*(\Lambda_G)\Lambda_G^4} \qquad \overline{m}_X^2 = \frac{\mathcal{C}^2 g^2}{32\pi^2}\frac{\Lambda_G^4}{\hat{f}_X^2}$$
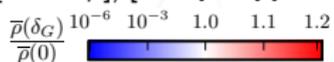

maximum tower fraction in $1 \lesssim M_c t_G \lesssim \overline{m}_X t_G$
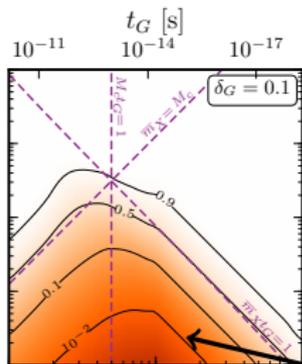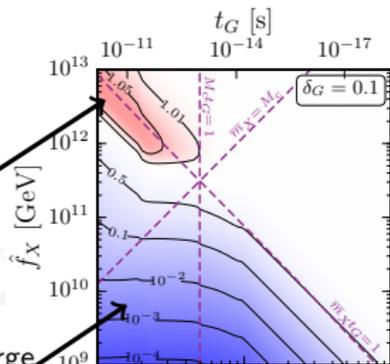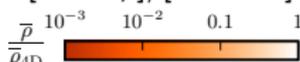
transition suppresses heavier modes, confining maximum to $1 \sim M_c t_G \lesssim \overline{m}_X t_G$

Example: Axion in the Bulk

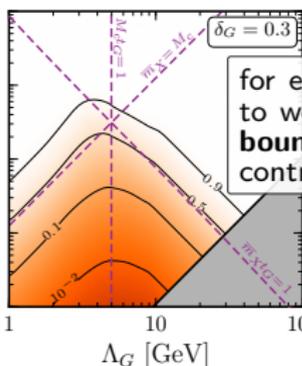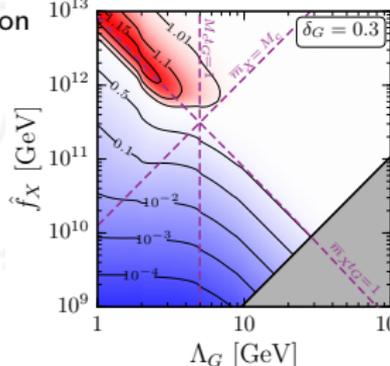[exact $\bar{\rho}$]/[abrupt approx.]   [exact $\bar{\rho}$]/[4D limit]

$\frac{\bar{\rho}(\delta_G)}{\bar{\rho}(0)}$ 10^{-6} 10^{-3} 1.0 1.1 1.2 $\frac{\bar{\rho}}{\bar{\rho}_{4D}}$ 10^{-3} 10^{-2} 0.1 1

enhancements and large suppressions relative to abrupt approximation

presence of extra dimension produces significant additional suppression of $\bar{\rho}$ for $\bar{m} \gtrsim M_c \cup \bar{m} t_G \gtrsim 1$

for example, this could lead to weakening in overclosure bounds if topological defects contribute to abundance

Conclusions

- We have investigated the effects of **dynamical mass generation** on the cosmological abundances of the KK modes associated with a bulk scalar:

  - both the **total energy density** and its **distribution** across individual modes in the ensemble are sensitive to the **details of the phase transition**

  - derived a variety of asymptotic scaling behaviors and **analytic expressions** for the energy densities of the tower as functions of relevant model parameters

  - applied the general machinery of our framework to the **example of a bulk axion**, allowing us to determine where the standard approximations succeed/fail — suggest weakening of overclosure bounds in certain regions

- **There are many possible future directions**:

  - we assumed a single flat extra dimension, but what phenomena arise with a **warped geometry** and/or multiple extra spatial dimensions?

  - implications of large suppressions for **cosmological moduli problem**?

  - we operated under assumption that the KK tower is $\ll \rho_{\mathrm{crit}}$ during the mass-generation epoch, but what is the effect of the **backreaction on H** away from this regime (*i.e.*, where scalars play role during inflation/(p)reheating)?

  - implications of large suppressions for **cosmological moduli problem** that generically appears in string theory?