THE OHIO STATE UNIVERSITY

U.S. DEPARTMENT OF ENERGY

Zero-Range Effective Field Theory for Resonant Wino Dark Matter

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Talk Outline

- Wino dark matter and annihilation
  - Direct pair annihilation and conventional techniques
  - Bound-state annihilation
    - Can bound-state annihilation provide **tighter constraints** on dark matter models?
    - New theoretical tools needed to better understand bound-state effects
- Zero-Range Effective Field Theory
  - See 1611.06212 and upcoming paper for more details
- Example bound-state formation calculation
- Conclusion and outlook

Wino WIMP dark matter

Motivation: 'wimp miracle'

- TeV scale weakly interacting particle naturally produces correct DM relic density

Fundamental theory is either

- SM with one additional electroweak triplet: $\tilde{w} = \begin{pmatrix} \tilde{w}^+ & \tilde{w}^0 & \tilde{w}^- \end{pmatrix}$
- MSSM where the Lightest Supersymmetric Particle is a wino-like neutralino
- Refer to the DM candidate as a 'wino'

Wino masses:

- Neutral wino mass $M\sim$ few TeV, charged winos $M+\delta$
  - Radiative corrections give $\delta = 170$ MeV, insensitive to M

(Pierce et al. NPB 1997)

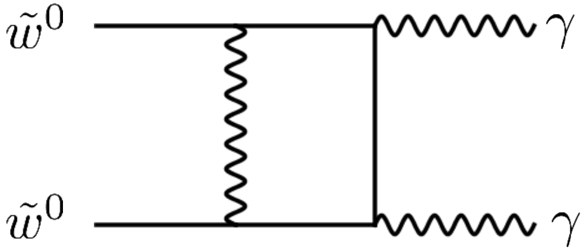
Wino interactions and nonperturbative effects

A pair of neutral winos can annihilate into a pair of electroweak gauge bosons

$$\left.\begin{aligned}\tilde{w}^0\tilde{w}^0 &\to Z^0Z^0\\ &\to W^+W^-\end{aligned}\right\}\text{Continuous }\gamma\text{-ray and positron signals}$$

$$\left.\begin{aligned}\tilde{w}^0\tilde{w}^0 &\to \gamma\gamma\\ &\to \gamma Z^0\end{aligned}\right\}\text{Monochromatic }\gamma\text{-ray signals}$$

Leading-order (LO) annihilation cross-section for a pair of photons:



$$(v\sigma_{\rm ann})_{\rm LO} \sim \alpha^2\alpha_2^2/m_W^2$$

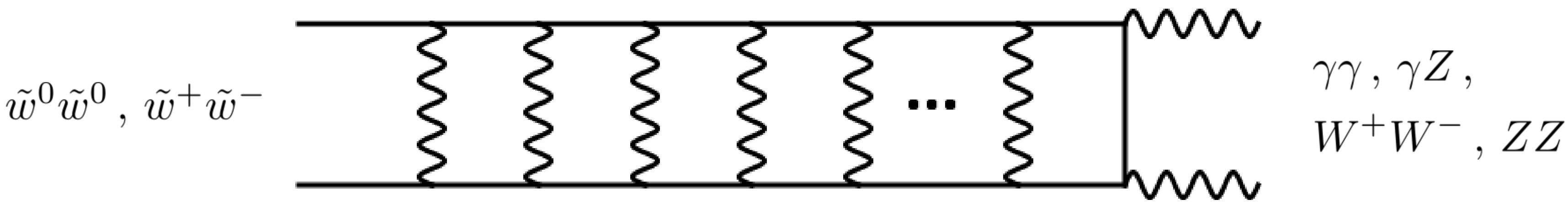
(Hisano et al. PRD 2005)

$(v\sigma_{\rm ann})_{\rm LO}$ **exceeds unitarity bound** $4\pi/vM^2$ for sufficiently large M

Higher order diagrams must be included to calculate the annihilation rate

Wino interactions and nonperturbative effects

Higher order diagrams for $\tilde{w}^0\tilde{w}^0$ or $\tilde{w}^+\tilde{w}^-$ pair annihilation involve exchange of EW gauge bosons:



Ladder diagrams must be summed to all orders to compute $v\sigma_{\rm ann}$

- Each 'rung' of the ladder gives a factor of $\alpha_2 M/m_W$ **(Hisano et al. PRD 2005)**
- For large enough M, $\alpha_2 M/m_W \sim 1$
- The annihilation cross sections can receive enhancements: the "Sommerfeld enhancements"

Difficult to calculate in the fundamental field theory (summing up diagrams to all orders)

- Instead, calculate with a Nonrelativistic Effective Field Theory (solving the Schrödinger equation)

Nonrelativistic Effective Field Theory (NREFT)

Numerically solve the Schrödinger equation

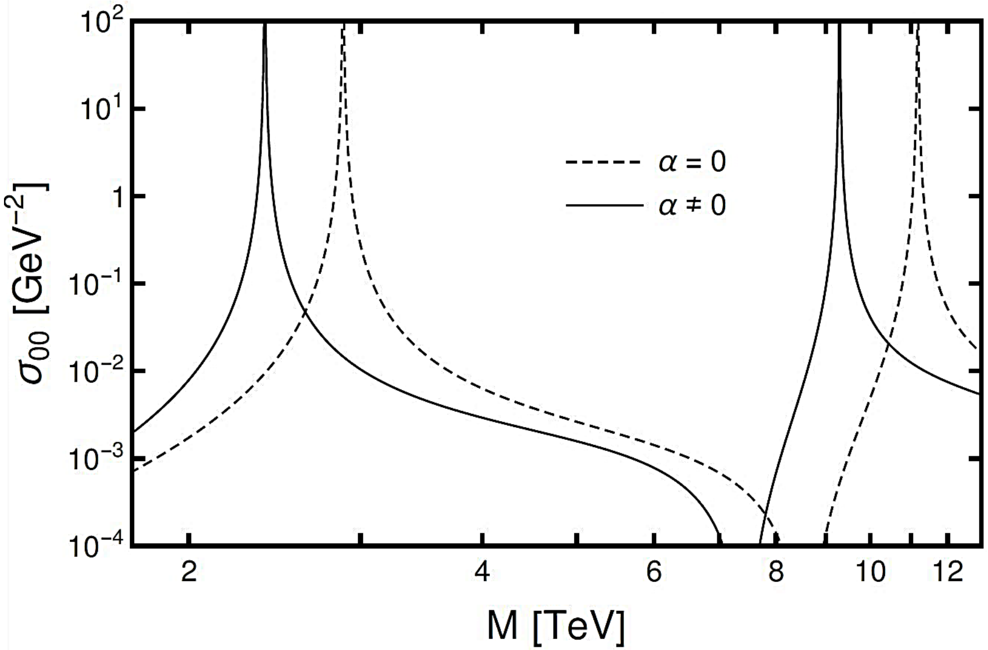
- Wino pairs $\tilde{w}^0\tilde{w}^0$ and $\tilde{w}^+\tilde{w}^-$ form coupled channels interacting with potential

$$V(r) = \begin{pmatrix} 0 & -\sqrt{2}\,\alpha_2\,e^{-m_W r}/r \\ -\sqrt{2}\,\alpha_2\,e^{-m_W r}/r & -\alpha/r - \alpha_2\,c_W^2\,e^{-m_Z r}/r \end{pmatrix}$$

Channel	Threshold Energy
$\tilde{w}^0\tilde{w}^0$	0
$\tilde{w}^+\tilde{w}^-$	2δ

W , γ , Z exchange

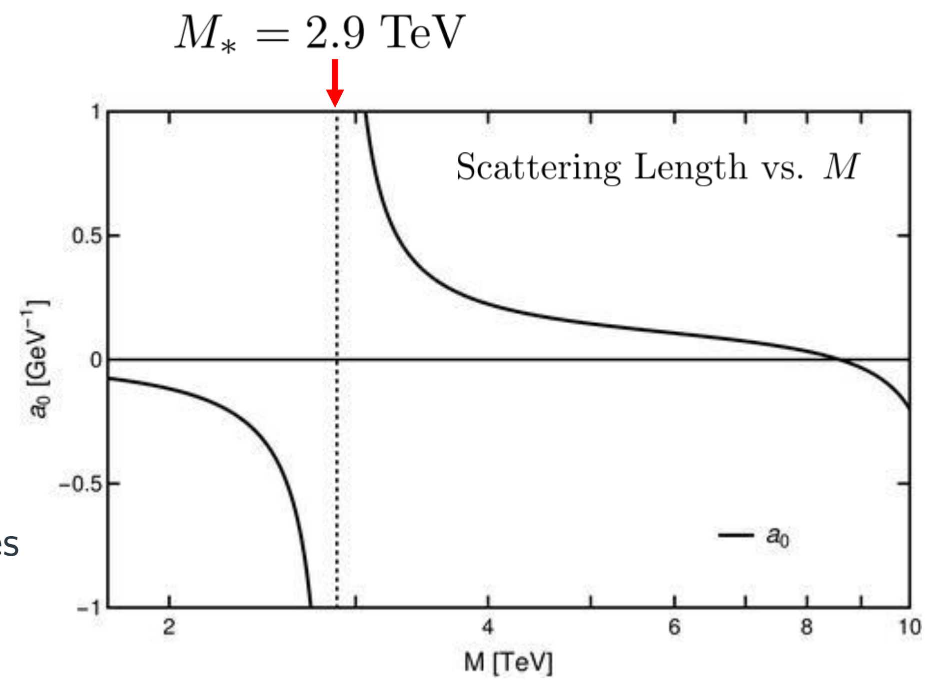
- Sequence of critical masses where a resonance exists at the $\tilde{w}^0\tilde{w}^0$ threshold
  - Cross section is resonantly enhanced: **very large** Sommerfeld enhancements
  - First critical mass at 2.4 TeV with Coulomb potential included
  - Shifts to 2.9 TeV without the Coulomb potential



Resonant Wino DM and bound states

The Coulomb potential can be treated as a perturbation

- Remaining potentials have **short ranges** of $1/m_W$
- Guarantees low energy scattering has universal properties (determined by the scattering length)



For neutral wino **S-wave** scattering:

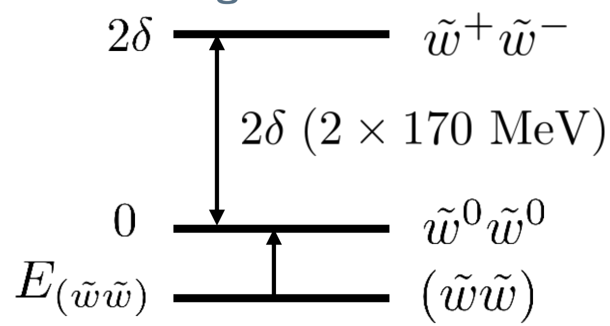
$$\tilde{w}^0\tilde{w}^0 \to \tilde{w}^0\tilde{w}^0$$

- Scattering length diverges at the critical mass M_*
- Cross section diverges as a_0^2 at small energies and saturates the unitarity bound

When $M > M_*$, the resonance is a real S-wave bound state, denoted $(\tilde{w}\tilde{w})$

- Binding energy determined by inverse scattering length near the resonance
- Vanishes at the critical mass as $1/a_0 \to 0$

Relative energies:

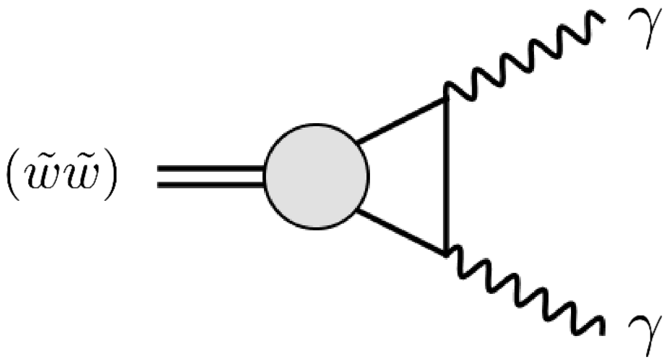


Bound state annihilation

Bound states can form in the scattering of winos

Bound-state production can be followed by annihilation of the bound state, sometimes into two monochromatic photons:

$$(\tilde{w}\tilde{w}) \rightarrow \gamma\gamma$$



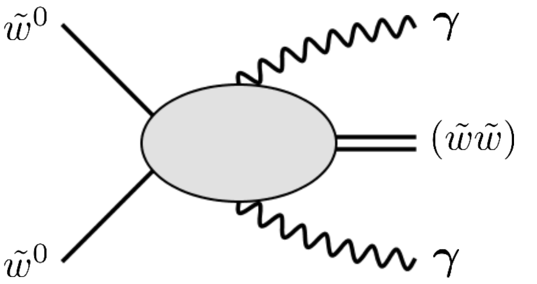
Direct annihilation rates and bound state annihilation rates **add together**

- Theoretical predictions for monochromatic photons are **enhanced** by additional production mechanisms
- May allow **tighter constraints** to be placed on models

Example bound-state formation mechanisms

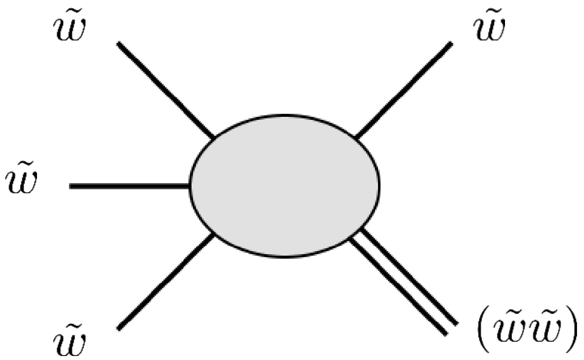
S-wave bound states can form in neutral wino scattering:

- Through a double radiative transition: $\tilde{w}^0\tilde{w}^0 \rightarrow (\tilde{w}\tilde{w}) + \gamma\gamma$



(See Baumgart et al. 1610.07617 for single photon emission, p-wave case)

- Through three body recombination: $\tilde{w}\tilde{w}\tilde{w} \rightarrow (\tilde{w}\tilde{w}) + \tilde{w}$



Difficult to calculate by solving the Schrödinger equation

- Calculate with a new tool: **Zero-Range Effective Field Theory**

Zero-Range Effective Field Theory (ZREFT)

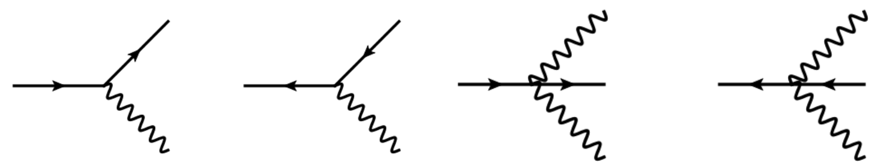
Lagrangian:

$$\mathcal{L} = \tilde{w}^{0\dagger}\left(i\partial_0 + \frac{\boldsymbol{\nabla}^2}{2M}\right)\tilde{w}^0 + \sum\nolimits_{\pm} \tilde{w}^{\pm\dagger}\left(iD_0 + \frac{\boldsymbol{D}^2}{2M} - \delta\right)\tilde{w}^{\pm} + \mathcal{L}_{\mathrm{EM}} + \mathcal{L}_{\mathrm{zero-range}}$$

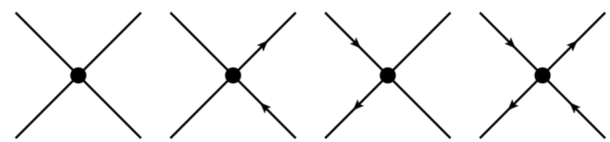
- Photon interactions arise from covariant derivatives for charged winos:

$$D_0\tilde{w}^{\pm} = (\partial_0 \pm ieA_0)\tilde{w}^{\pm}\boldsymbol{D}\tilde{w}^{\pm} = (\boldsymbol{\nabla} \mp ie\boldsymbol{A})\tilde{w}^{\pm}$$

- Single and double photon vertices:

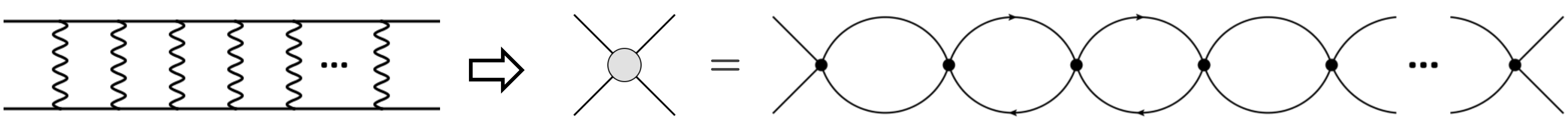


- Zero-range contact interactions for pairs of winos:



(Arrow on line: charged wino)

- Non-perturbative electroweak interactions included by summing bubbles to all orders



- Must renormalize contact vertices to reproduce correct behavior from the fundamental theory

The renormalization and power counting of the Zero-Range EFT is governed by its RG fixed points

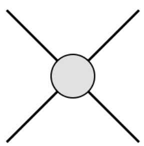
(Lensky and Birse, EPJ 2011)

- Three fixed points for a two-channel theory correspond to the number of fine-tuned parameters:
- 0, 1, or 2 resonances at the scattering threshold requiring 0, 1, or 2 fine-tunings

If only the wino mass M is tuned to its critical value M_*, expect a **single resonance** at the neutral-wino threshold

- Single resonance channel becomes a linear combination of $\tilde{w}^0\,\tilde{w}^0$ and $\tilde{w}^+\,\tilde{w}^-$ with mixing angle ϕ
- No scattering in the orthogonal channel

Analytic elastic scattering amplitude at leading order (LO):



$$= \boldsymbol{\mathcal{T}}(E) = \frac{8\pi\cos^2\phi/M}{-\gamma + \sqrt{M(2\delta - E)}\,\sin^2\phi - \boldsymbol{i}\sqrt{ME}\,\cos^2\phi}$$

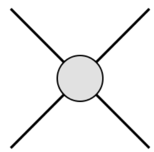
- Fine-tuning of physical quantity M becomes a fine-tuning of scattering parameter γ
- Saturate unitarity bound: Denominator of $\mathcal{T}(E)$ must vanish at $E = 0$ when $M = M_*$
- Fixes $\gamma = \sqrt{2M_*\delta}\,\sin^2\phi$
- The ZREFT at LO has **one free parameter**: ϕ
- Matching analytic results to numerical results from solving the Schrödinger equation gives $\phi = 39.8°$

Comparing ZREFT with NREFT

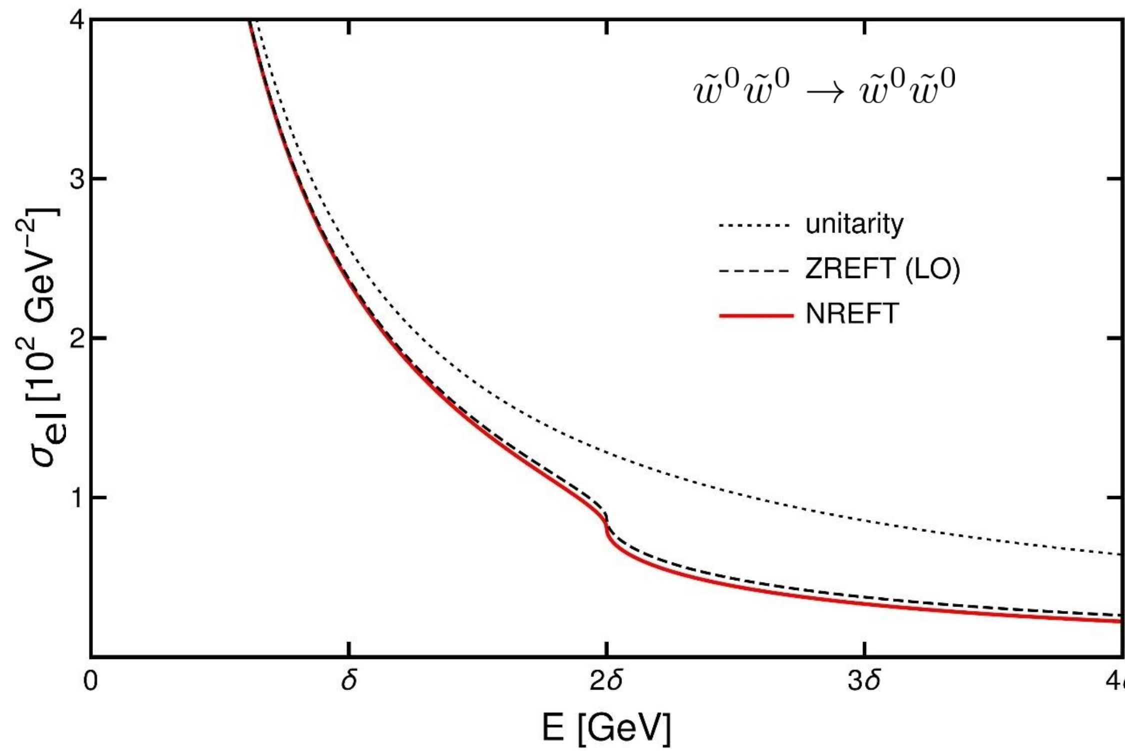
NREFT result obtained by solving the Schrödinger equation numerically with potential

$$V(r) = \begin{pmatrix} 0 & -\sqrt{2}\,\alpha_2\, e^{-m_W r}/r \\ -\sqrt{2}\,\alpha_2\, e^{-m_W r}/r & -\alpha_2\, c_W^2\, e^{-m_Z r}/r \end{pmatrix}$$

ZREFT result obtained analytically at LO:

$\sigma_{\rm el}(E) = \dfrac{M^2}{8\pi}\Big|$  $\Big|^2$

$$= \frac{M^2}{8\pi}|\boldsymbol{\mathcal{T}}(E)|^2$$



Quantitative and qualitative behavior reproduced by LO result

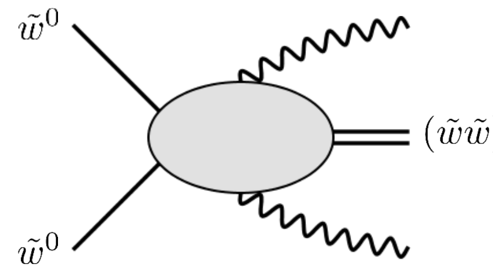
- Unitarity bound saturated at low energy
- Non-trivial behavior reproduced at charged wino threshold $E = 2\delta$
- Excellent fit at low energy

Example calculation in ZREFT: Production of wino-pair bound state via radiative transition

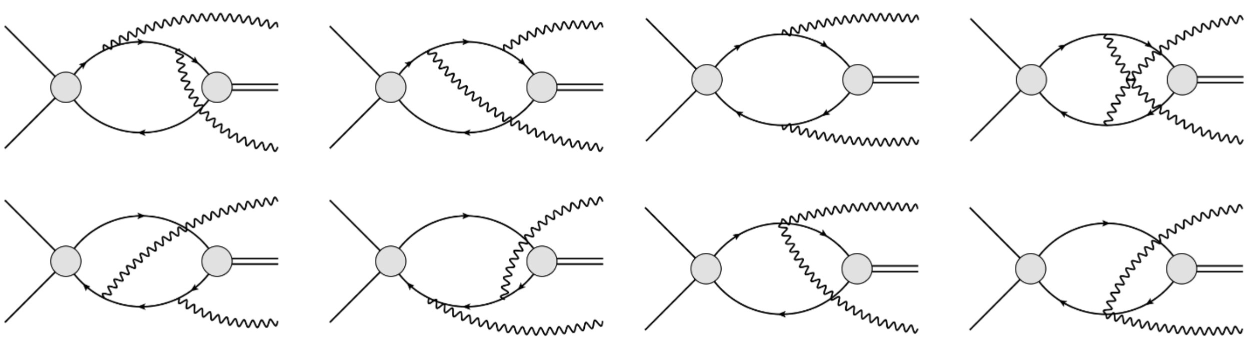
To form an **S-wave** bound state, the wino pair loses energy by radiating photons

Single-photon emission: forbidden by parity

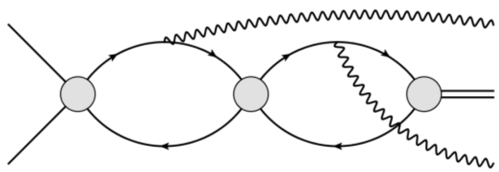
$$\tilde{w}^0 \tilde{w}^0 \to (\tilde{w}\tilde{w}) + \text{soft photons}$$



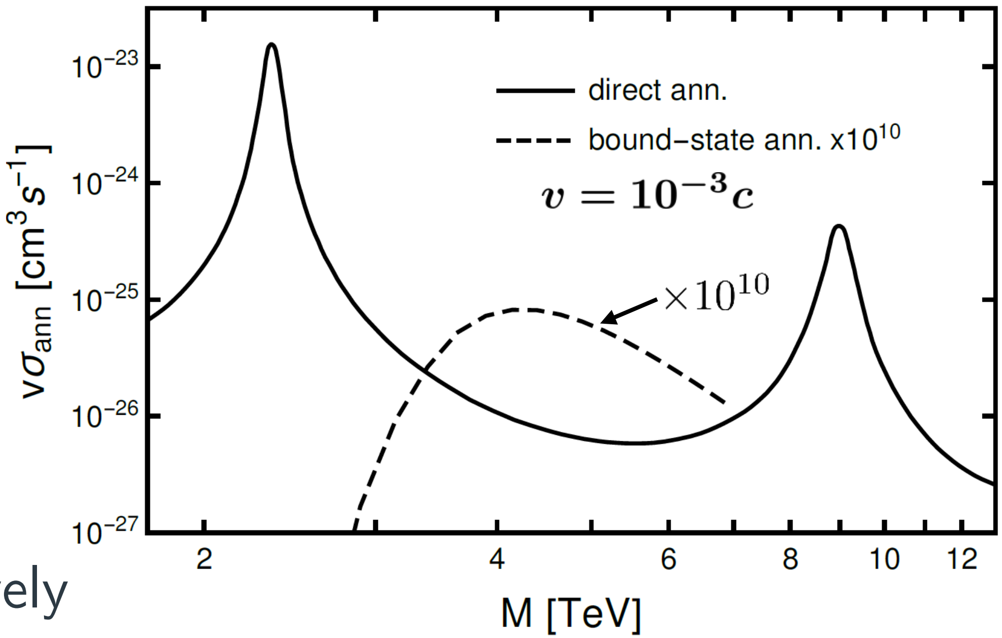
At leading order in α, there are eight contributing diagrams:



Eight more two-bubble diagrams with one photon attached to each bubble vanish by parity:

 $= 0$

Annihilation rate of bound states produced via a double radiative transition is highly suppressed compared to direct annihilation rate



Analytic result from leading order ZREFT at small energies is relatively simple:

$$v\sigma_{\rm ann} \approx \frac{\tan^4\phi\, \alpha^2\, M^2\, \hbar^3}{53760\, a_0\, \delta^5\, c^2}\,(E/Mc^2)^6$$

Rate scales as $\boldsymbol{v^{12}}$ vs. $\boldsymbol{v^{-2}}$ for direct pair annihilation, near critical mass

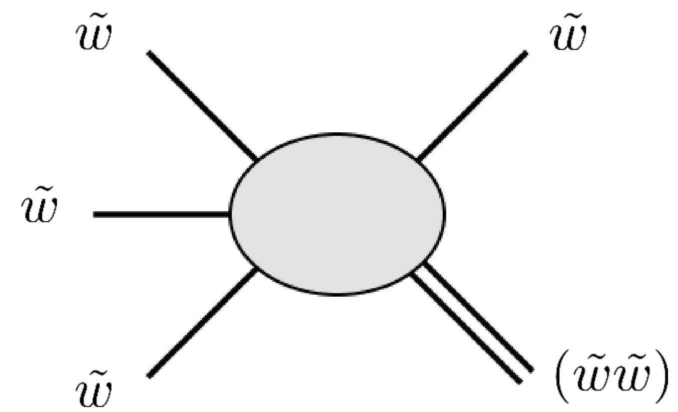
- Highly suppressed at low wimp velocities
- This particular process is not of phenomenological interest, but illustrates the power of the ZREFT to calculate a process that is very difficult by solving the Schrödinger equation

Three body recombination

Annihilation rate of bound states formed through three body recombination (a number changing process) can be calculated in ZREFT

Qualitative properties from cold atom physics:

$$\frac{\mathrm{d}n}{\mathrm{d}t} \sim K\, n^3 \quad , \quad K \sim \frac{1}{M^5}\langle v^{-4}\rangle$$



- n : wino number density
- K : rate constant

Scales as $\boldsymbol{v^{-4}}$ vs. $\boldsymbol{v^{-2}}$ for direct pair annihilation

- Could become more important at **small wimp velocity** such as in dwarf galaxies

Proportional to $\boldsymbol{n^3}$ vs $\boldsymbol{n^2}$ for direct pair annihilation

- Could become more important at **higher wimp density**, such as at centers of dark matter halos

Conclusion and outlook

Zero-Range EFT describes low energy wino **scattering** **very well**

- **Single parameter** at LO reproduces the results from conventional method of solving the Schrödinger equation
- Can be systematically improved with two more parameters at NLO

ZREFT can also straightforwardly describe **bound state production** and annihilation

- Difficult to calculate using the conventional method of solving the Schrödinger equation
- ZREFT provides a framework to explore which processes are of phenomenological interest
- Can be used to study early universe conditions of neutral and charged winos

In the future:

- Compute the three-body recombination rate in ZREFT
- Develop ZREFTs for other forms of dark matter, such as a higgsino-like wimp candidate

Thank you!