Mapping the QCD radiation spectrum

Keith Pedersen
(kpeders1@hawk.iit.edu)

ILLINOIS INSTITUTE OF TECHNOLOGY

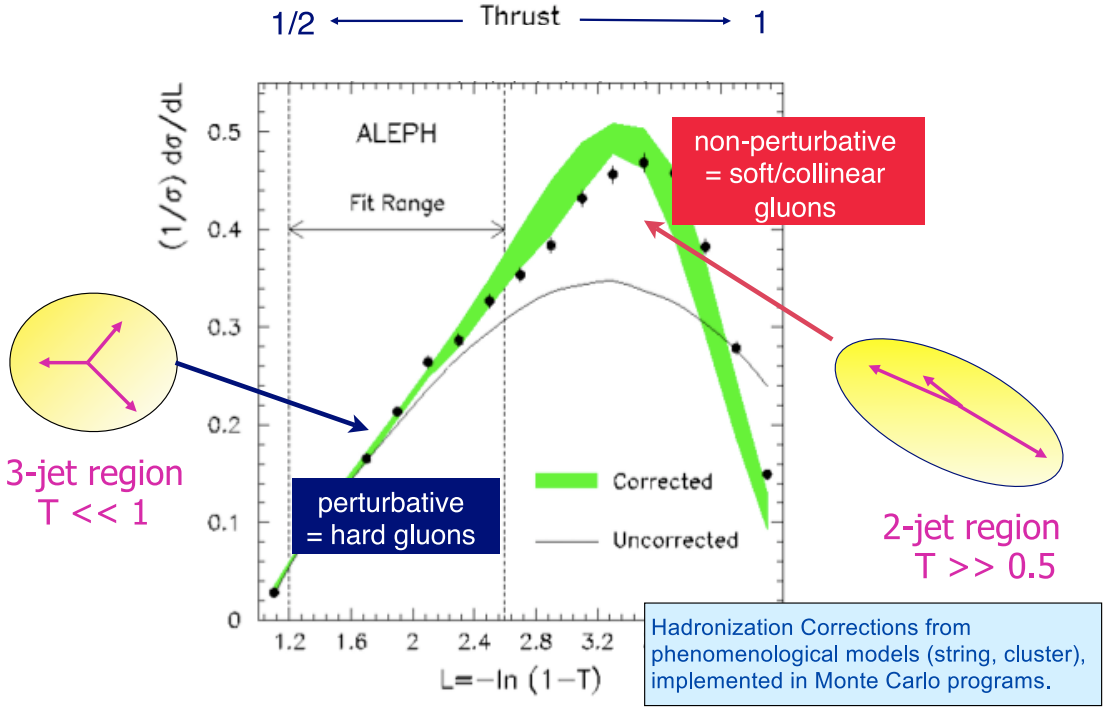
In collaboration with **Zack Sullivan**

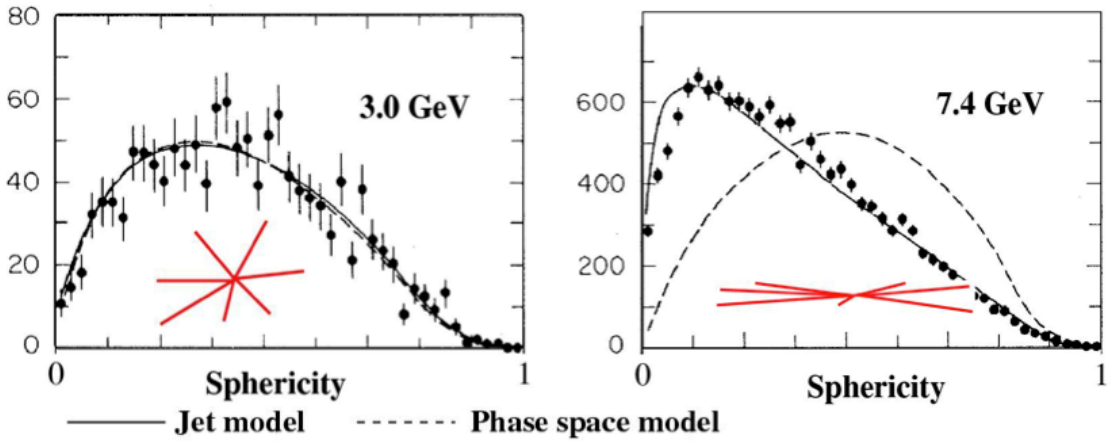
Pheno 2017

Outline

1. **The shape of QCD**
   - Can we probe QCD like the CMB?
   - Can we suppress pileup?
2. **A multipole expansion**
   - The Fox-Wolfram moments (FWM)
   - Why do the FWM fail?
   - A better multipole expansion
3. **Applications in a high-pileup environment**
   - Three jet kinematics without jets
   - Moving to a proton collider

Event shape variables





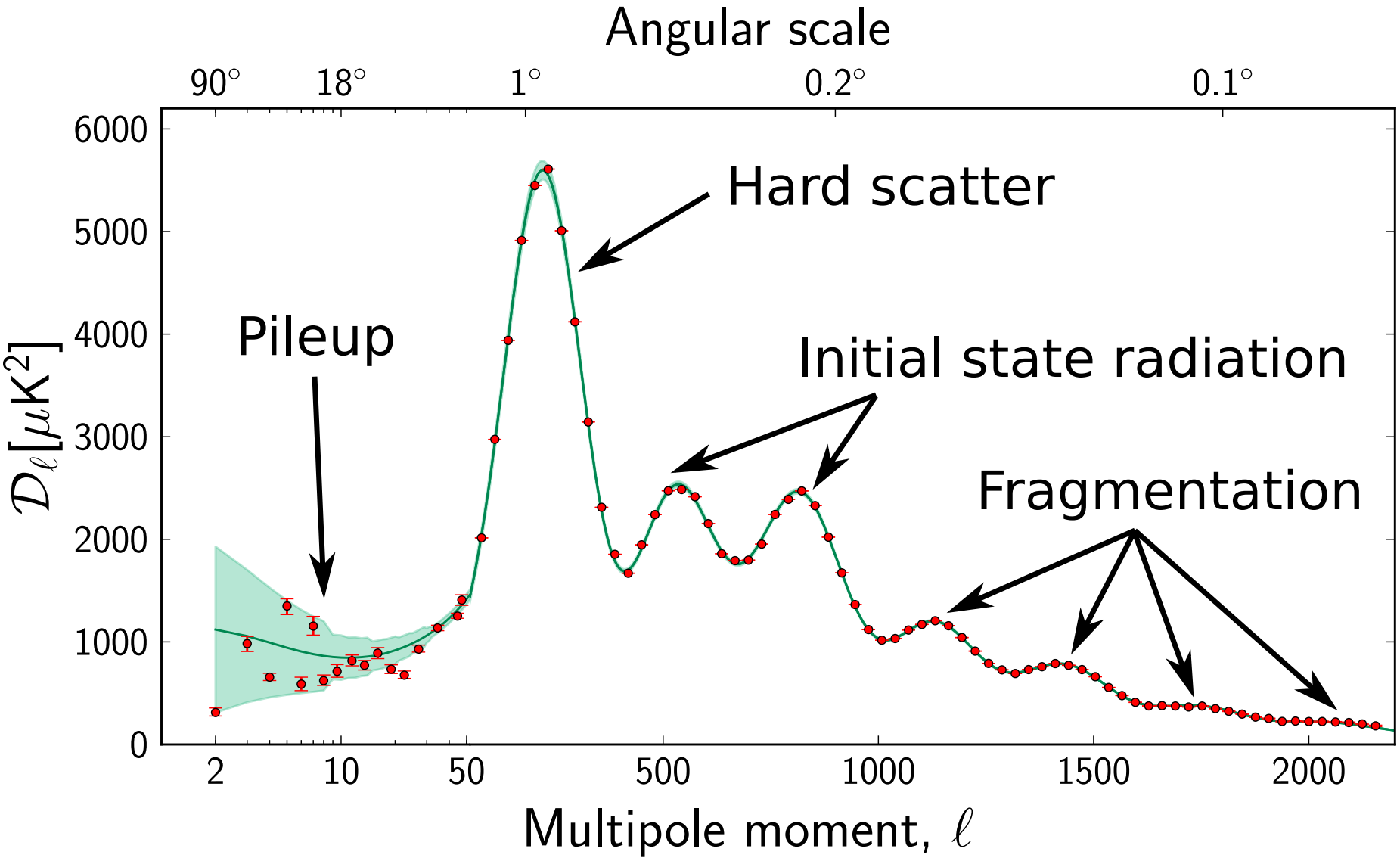
Event shape variables (thrust, sphericity, . . .) have explored:

- Quark and gluon spin.
- The strong coupling α_s.
- Jet structure (versus isotropic phase space).

Can we extract a shape curve from **a single event**? Why not do a multipole expansion?

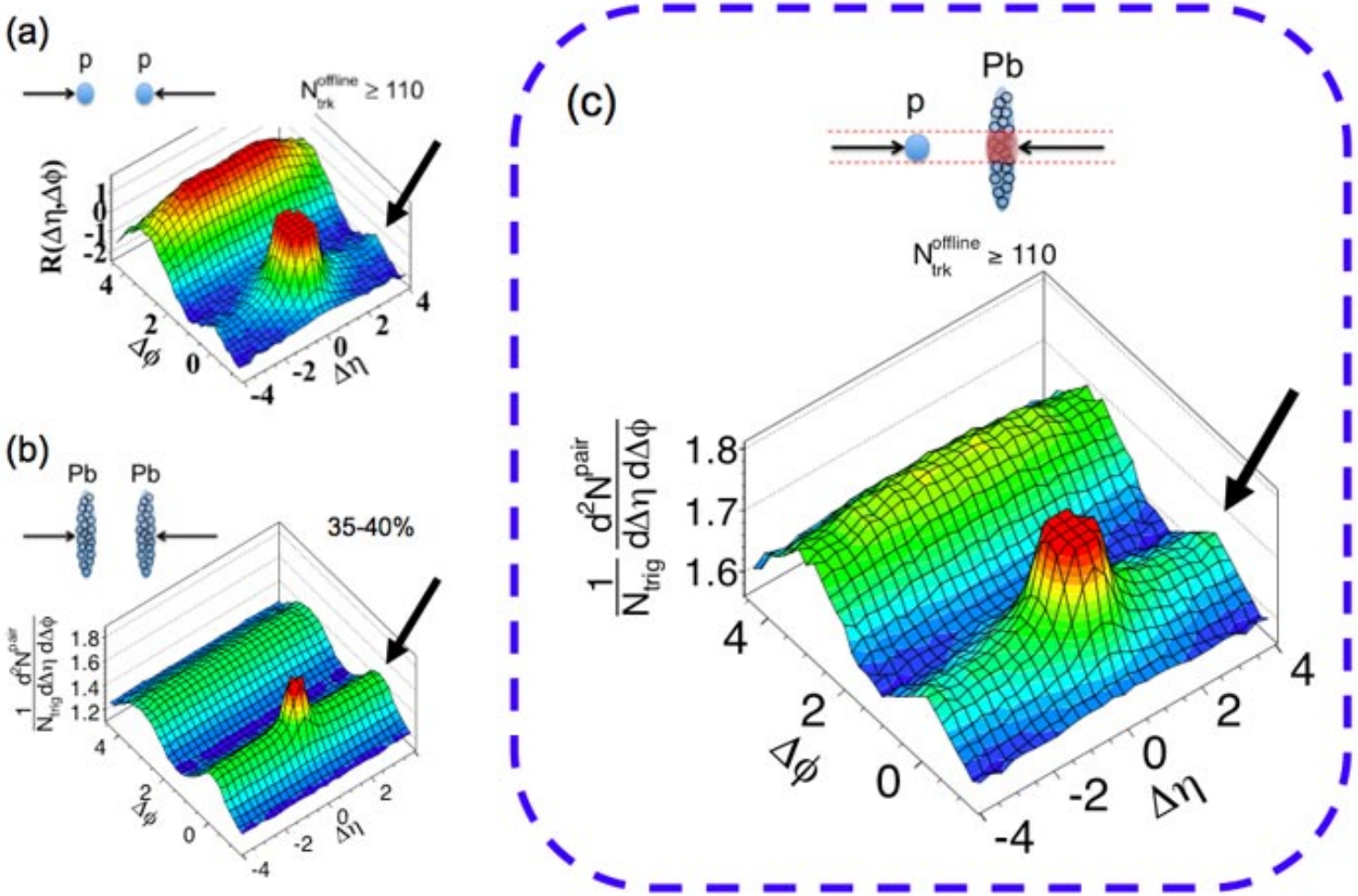
$$\rho_l^m = \int d\Omega \, Y_l^{m*}(\theta, \phi) \, \rho(\theta, \phi)$$

Can we probe QCD like the CMB?



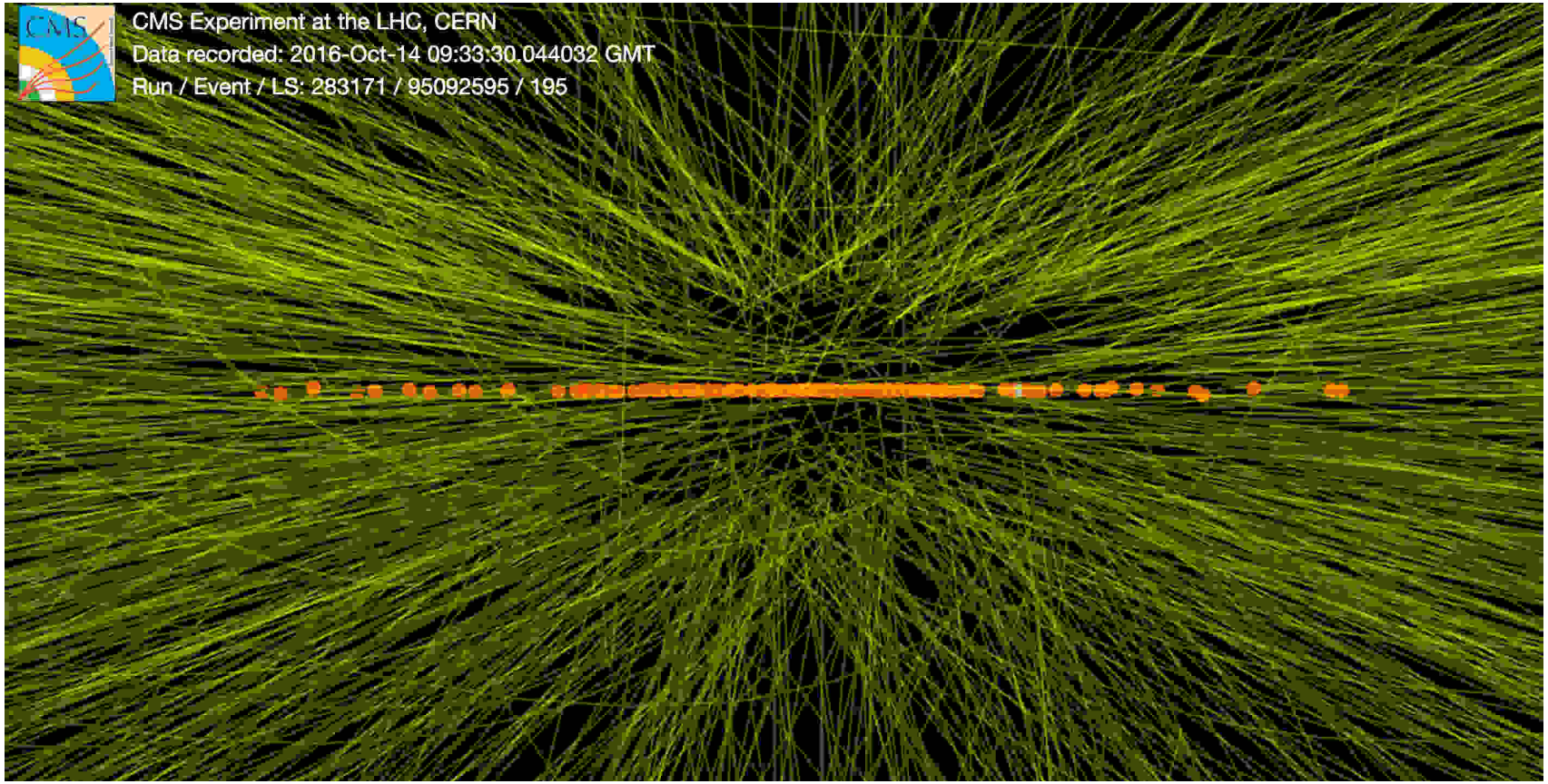
Can we identify broad, universal shapes with known physics?

The Pb-Pb ridge versus the p-p ridge



Is the ridge from **quark-gluon plasma** if we see it in *pp* collisions?

Pileup at the HL-LHC



Pileup at the HL-LHC. We will need to:

- Remove pileup energy from jets and identify pileup-only jets.
- Distinguish **boosted top** from **QCD jets + pileup**.
- Find $W \to q\bar{q}$ for precision electroweak measurements.

Outline

1. The shape of QCD
   - Can we probe QCD like the CMB?
   - Can we suppress pileup?
2. A multipole expansion
   - The Fox-Wolfram moments (FWM)
   - Why do the FWM fail?
   - A better multipole expansion
3. Applications in a high-pileup environment
   - Three jet kinematics without jets
   - Moving to a proton collider

The Fox-Wolfram moments (FWM)

FWM expand a single event's energy distribution $\rho(\theta, \phi)$ into $Y_l^m(\theta, \phi)$

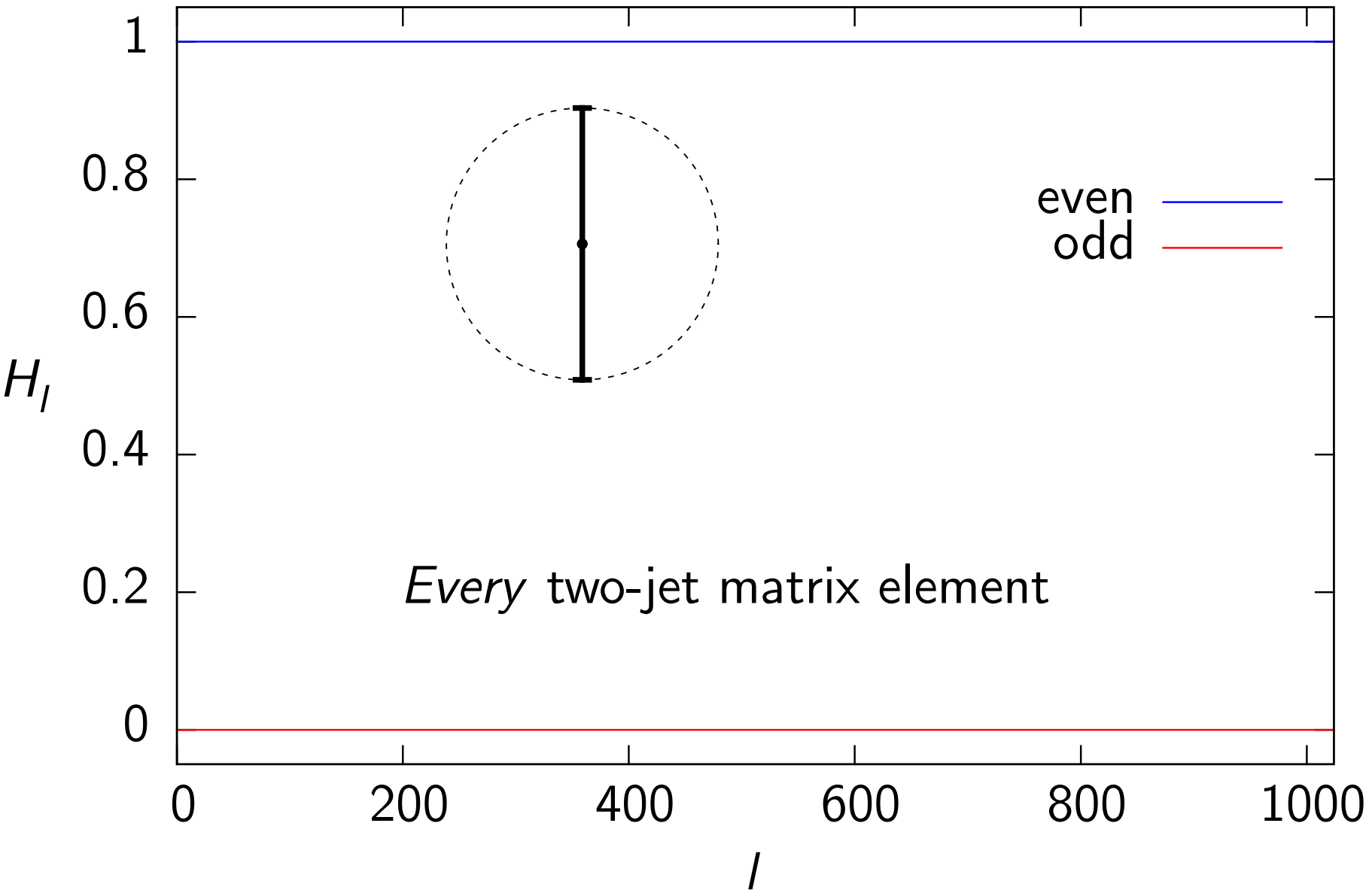
$$H_l = \sum_{i,j} f_i \, f_j \; P_l(\cos\theta_{ij}) \qquad \left(\text{energy fraction } f_i \equiv \frac{|\vec{p}_i|}{E_{\text{vis}}} \text{ and interior angle } \theta_{ij}\right)$$

Limitations:

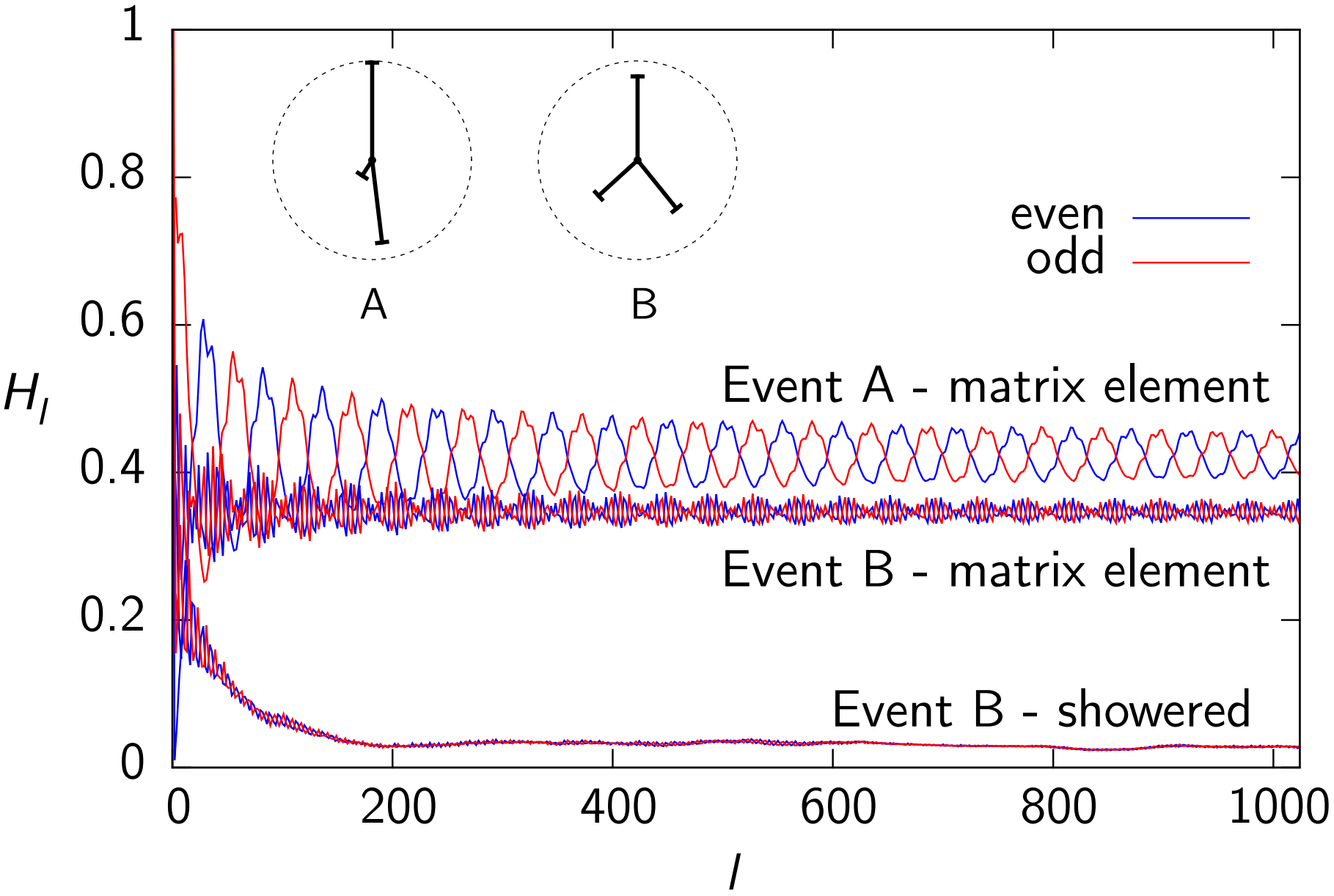
- Rotational invariance loses absolute orientation
- Conflates correlations from across the collider.
- Not Lorentz invariant . . . works best at lepton colliders.
- Must have high particle multiplicity (hobbled by shot noise).

Particle multiplicity at $\underbrace{13\text{ TeV}}_{2017} \gg \underbrace{19\text{ GeV}}_{1978}$ . . . it's time to revisit the FWM.

H_l of simple matrix elements



H_l of simple matrix elements

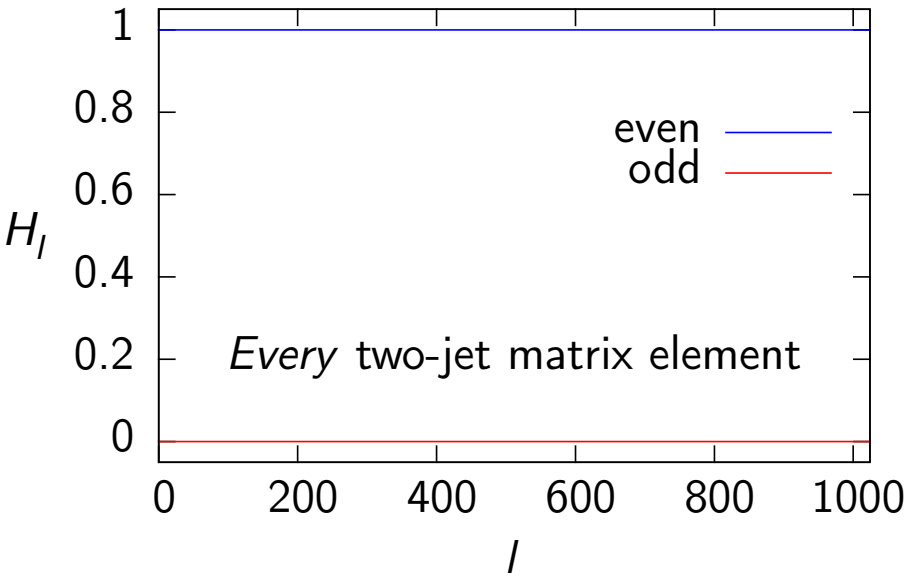


The Nyquist frequency of angular sampling

$$\int_{-\infty}^{\infty} \delta(x) e^{-i\,k\,x} \mathrm{d}x = 1$$

$$\rho(\theta, \phi) = \sum_i f_i\, \delta^3(\hat{p}_i - \hat{r})$$

- $\delta(x)$ has unlimited angular power
- The FWM are the multipole expansion of a *discrete* sample!



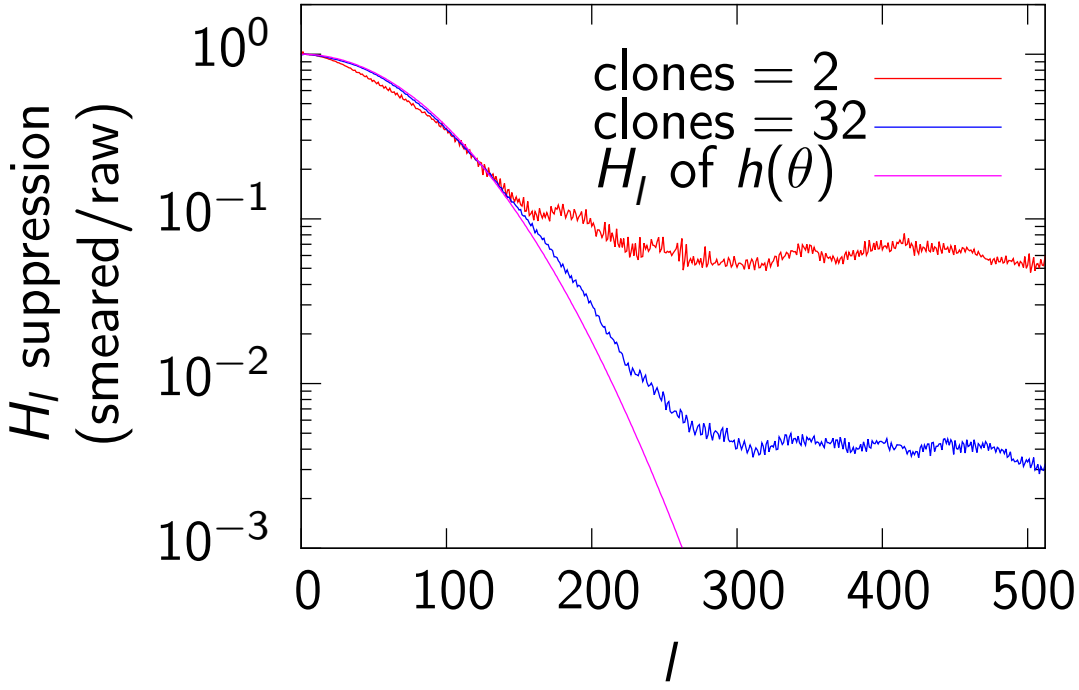
- Convert $\vec{p}_i$ into extensive objects smeared by a "Nyquist" angle σ.
  - Suppresses **high-frequencies**.
- Nyquist angle σ is determined by event multiplicity and structure, not detector resolution.

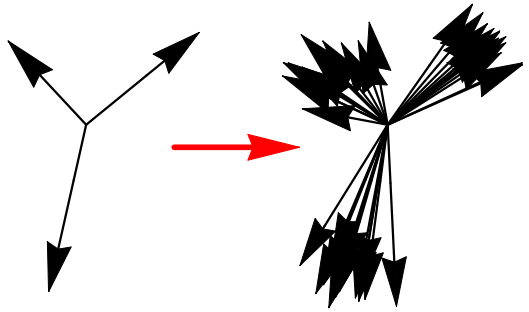
The smearing function

Ansatz for smearing function $h(\theta)$:

- Depends of polar angle θ to observed $\hat{p}_i$.
- Gaussian at small angle θ.
- No cusp as $\theta \to \pi$.

$$h(\theta) = \frac{\exp(-\sin^2(\theta/2)/2\sigma^2)}{\sigma^2(1-\exp(-2/\sigma^2))}$$

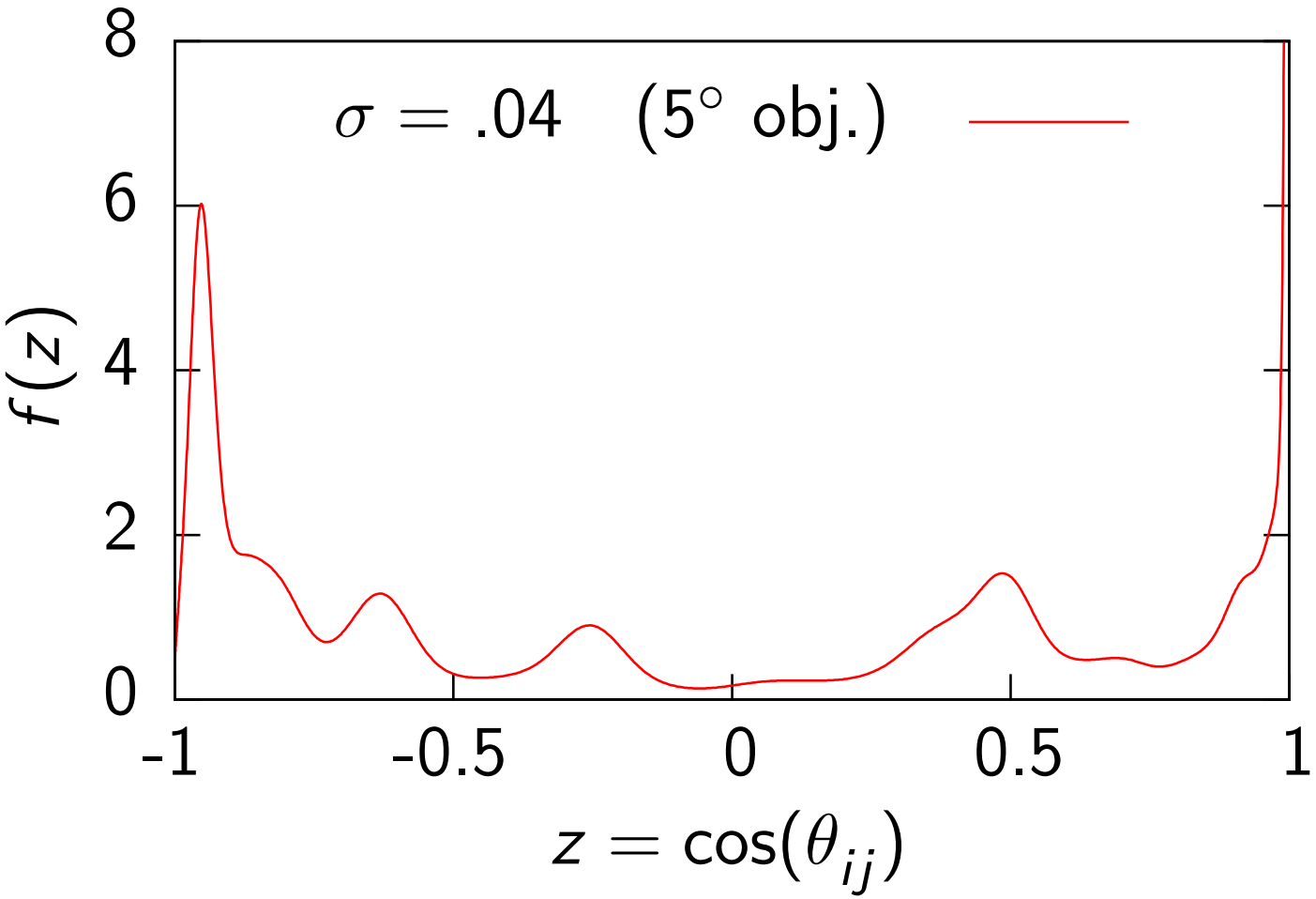


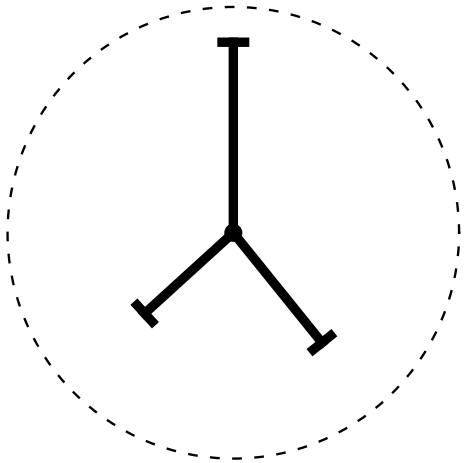


- **Convolution theorem**
  Scale the power spectrum H_l of observed $\hat{p}_i$ by the power spectrum of the smearing function.

Showered, smeared power spectra

$$H_l = \sum_{i,j} f_i f_j P_l(\cos\theta_{ij}) \qquad f(z) = \sum_l (2l+1) H_l P_l(z)$$

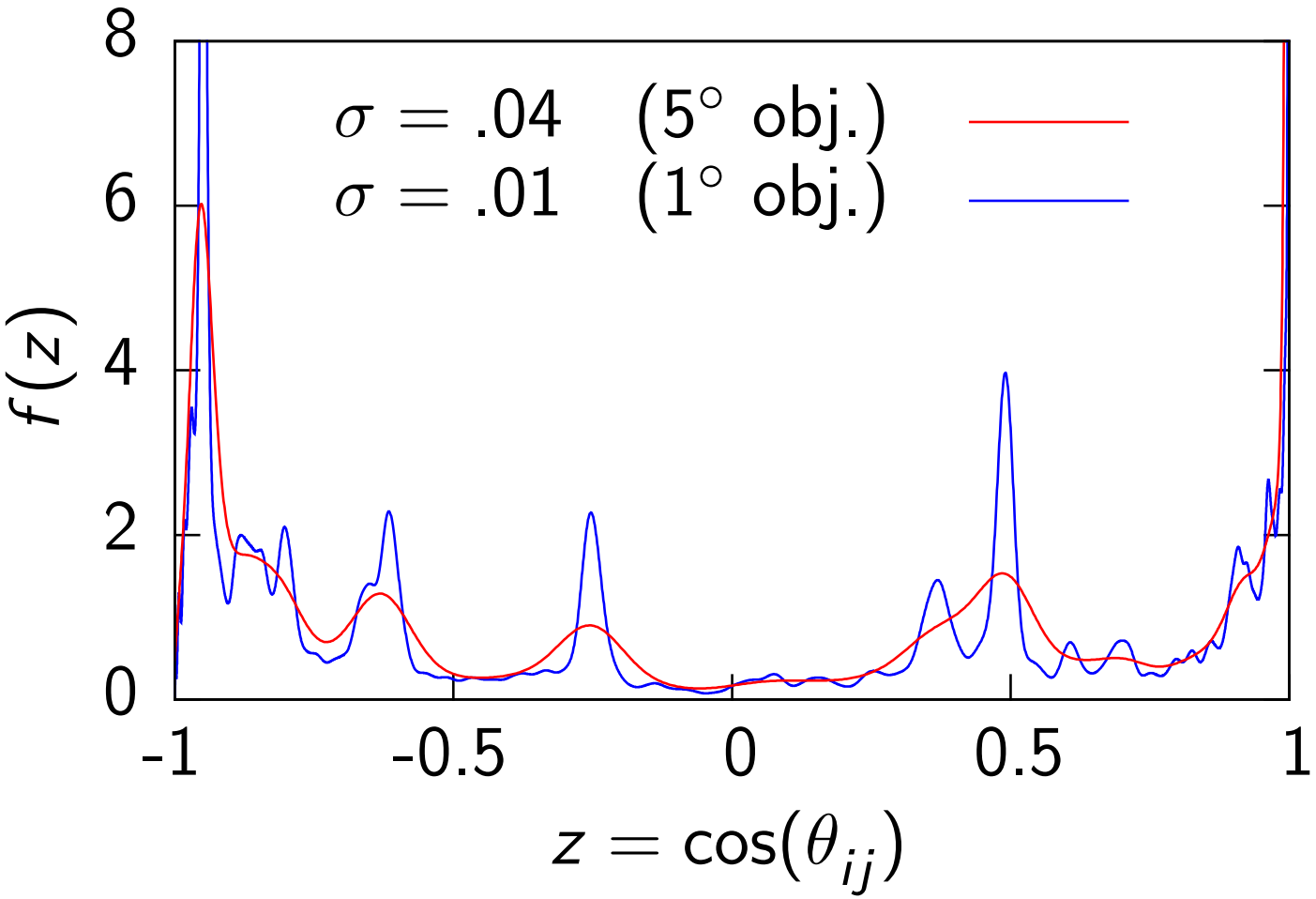


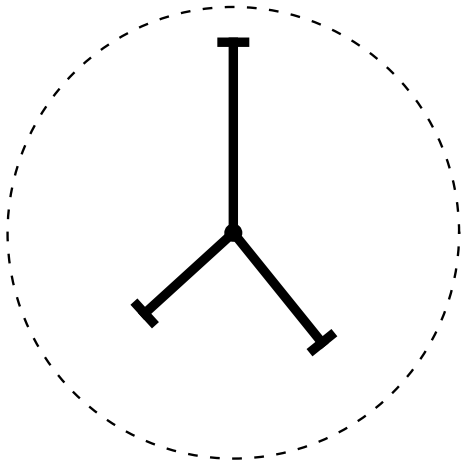


Area of peaks:
$A = f_A f_B$.

Showered, smeared power spectra

$$H_l = \sum_{i,j} f_i f_j P_l(\cos\theta_{ij}) \qquad f(z) = \sum_l (2l+1) H_l P_l(z)$$

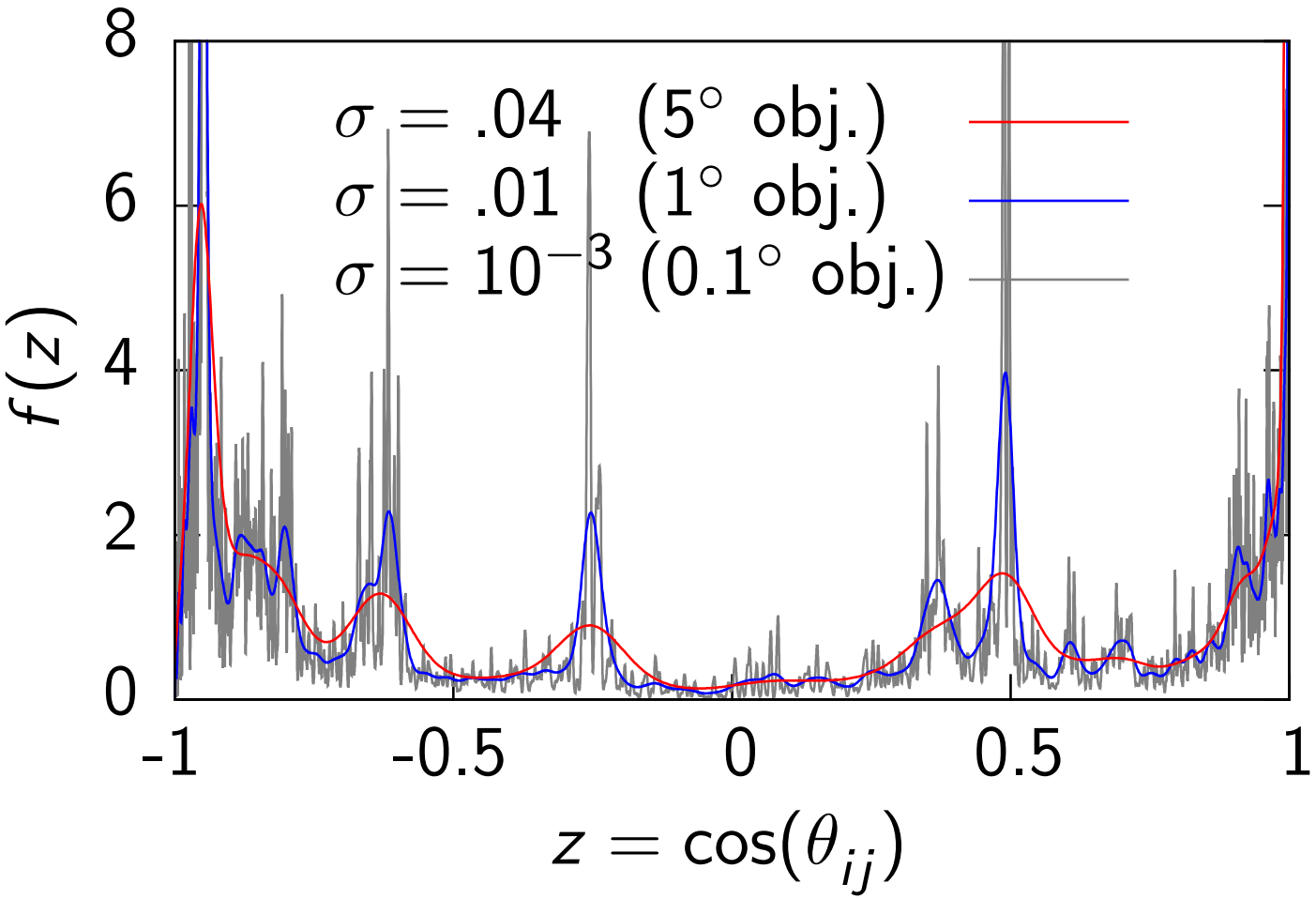


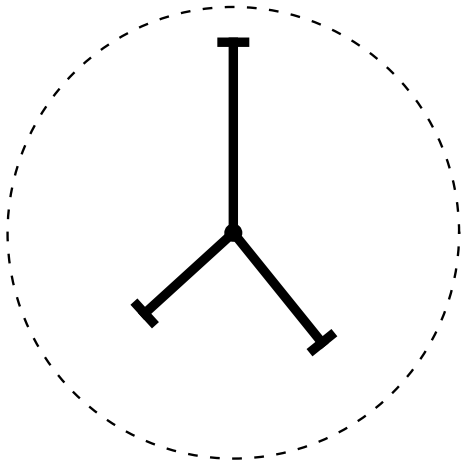


Area of peaks:
$A = f_A f_B$.

Showered, smeared power spectra

$$H_l = \sum_{i,j} f_i f_j P_l(\cos\theta_{ij}) \qquad f(z) = \sum_l (2l+1) H_l P_l(z)$$



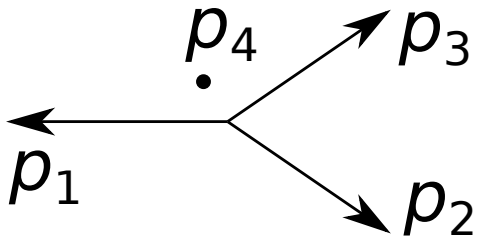


Area of peaks:
$A = f_A f_B$.

Outline

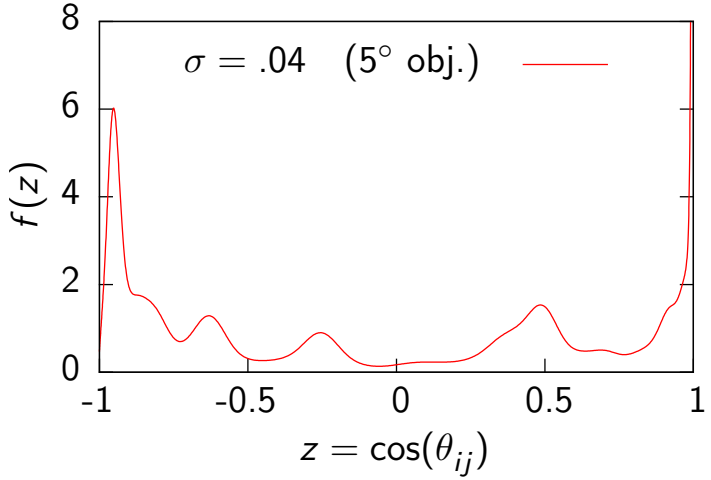
1. The shape of QCD
   - Can we probe QCD like the CMB?
   - Can we suppress pileup?
2. A multipole expansion
   - The Fox-Wolfram moments (FWM)
   - Why do the FWM fail?
   - A better multipole expansion
3. Applications in a high-pileup environment
   - Three jet kinematics without jets
   - Moving to a proton collider

3-jet kinematics at $e+e-$ ($\sqrt{s} = 1$ TeV)



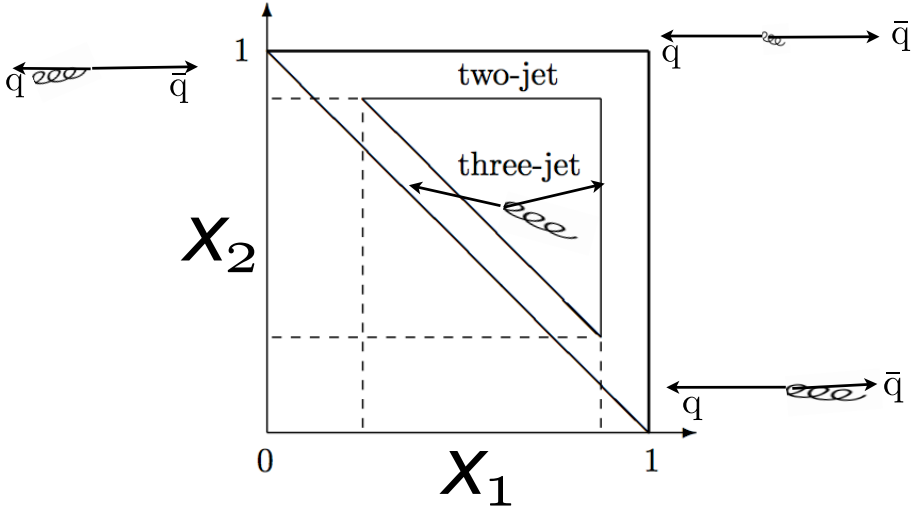
$$x_i \equiv \frac{2}{s} p_i p_0 = 2 f_i$$

$$0 \leq x_i \leq 1$$



p_4 is *heavy* ($\gamma \approx 1$), absorbing

- Wide-angle showering
- Isotropic pileup



8 observables in angular basis $f(\theta)$:

Three angles θ_{12}, θ_{13}, θ_{23}

Four areas $A_{ij} \sim x_i x_j$

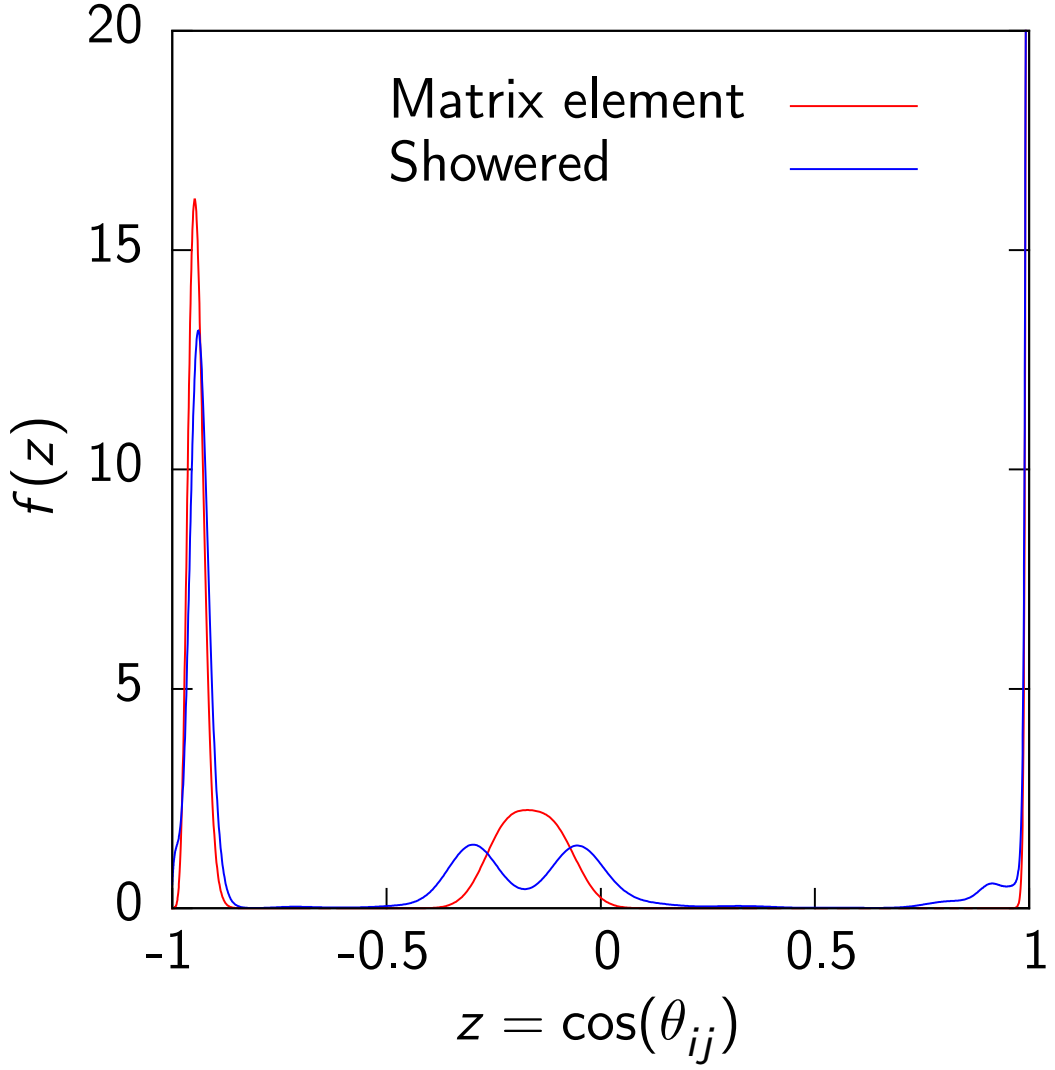
Isotropic baseline $C \sim x_4$

Kinematics without jet definitions!

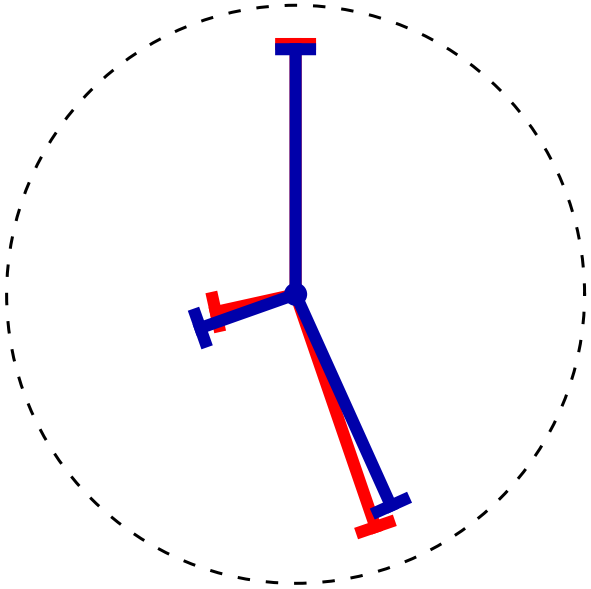
$$(p_1 + p_2)^2 = \frac{s}{4}\left((x_1 + x_2)^2 - x_3^2\right)$$

Showered versus matrix element

Fit the peaks with a Gaussian.



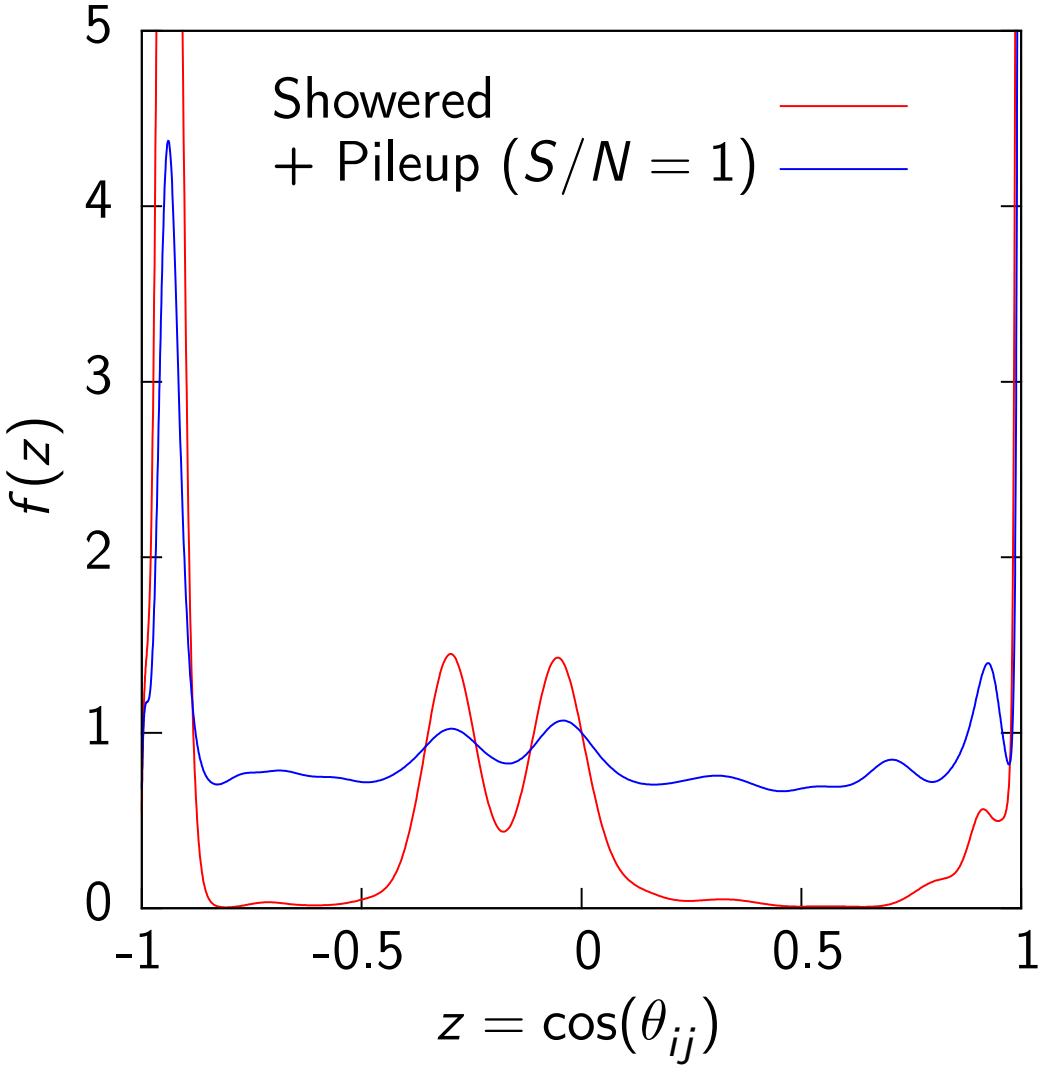
Smear at $\sigma = 0.4$ (5° obj.).



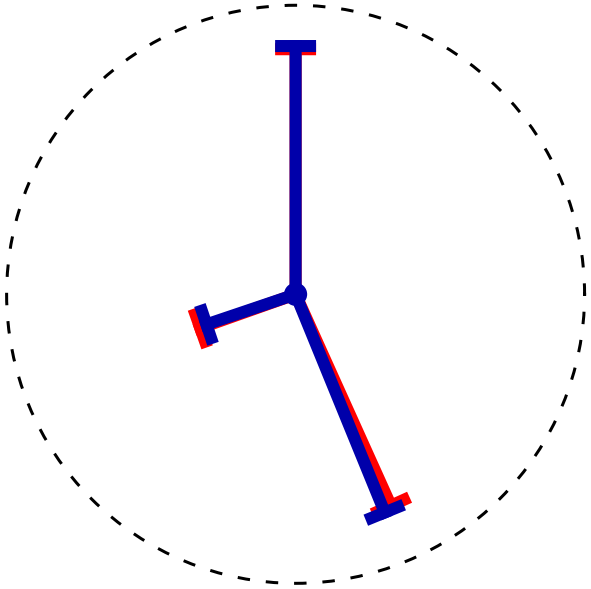
	ME	Showered
x_1	0.87	0.85 (-2%)
x_2	0.85	0.80 (-5%)
x_3	0.28	0.35 ($+23\%$)

Pseudo-detector with high pileup ($S/N = 1$)

Isotropic pileup adds a constant C.



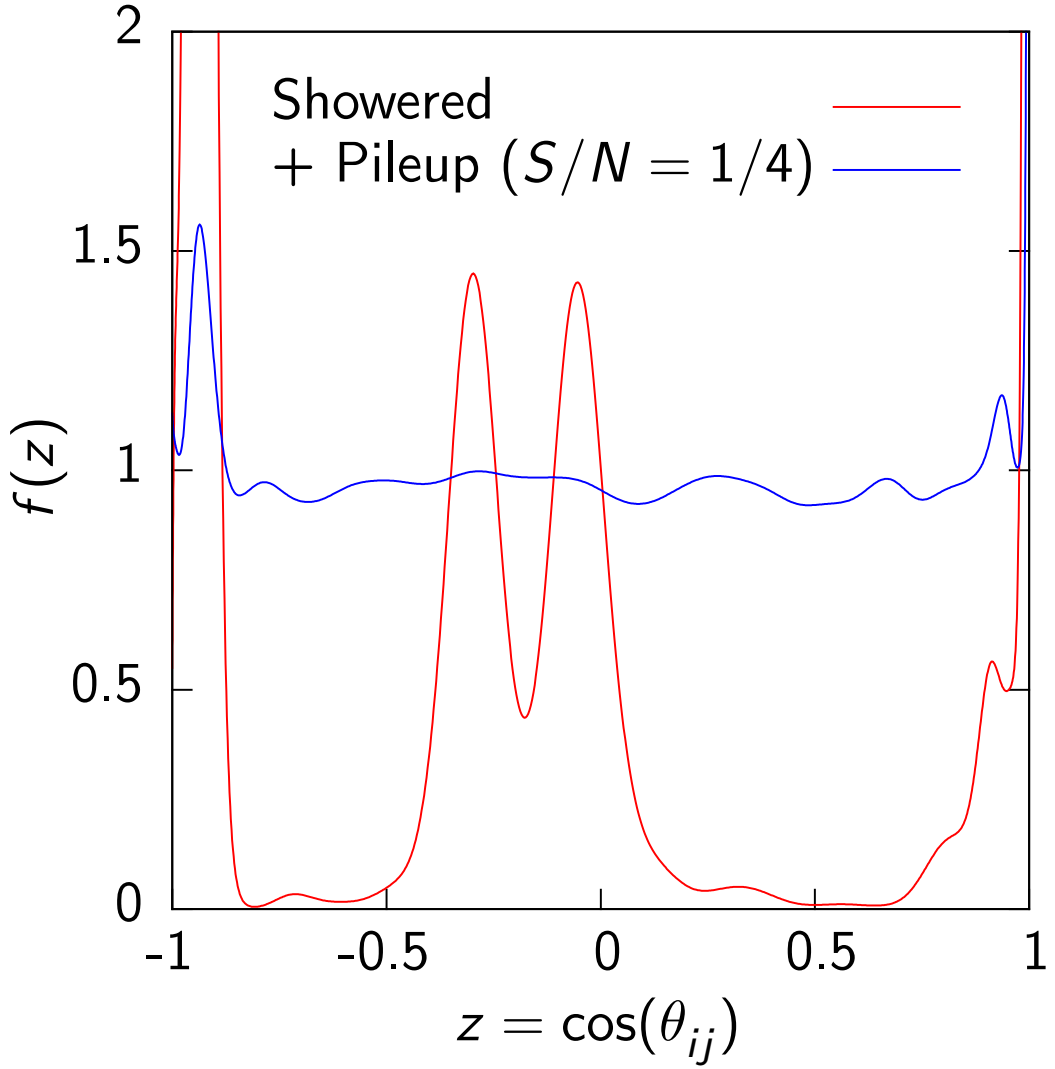
Smear at $\sigma = 0.4$ (5° obj.).



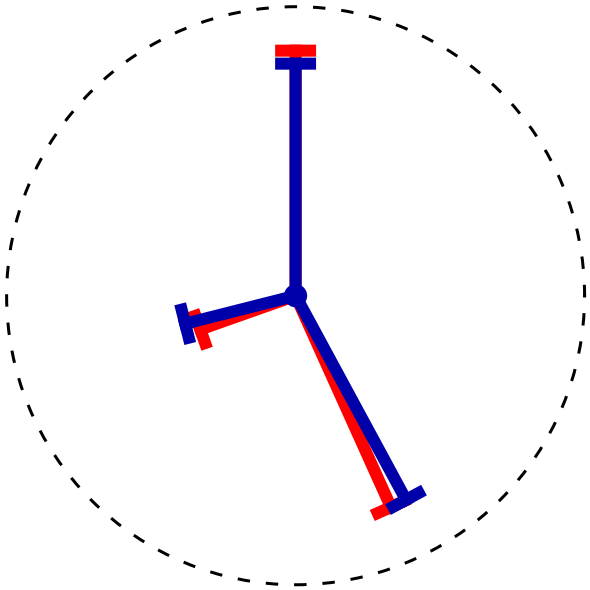
	Showered	Observed
x_1	0.85	0.86 (1%)
x_2	0.80	0.82 (2%)
x_3	0.35	0.32 (−7%)

Pseudo-detector with excessive pileup ($S/N = 1/4$)

Pileup overwhelms x_3 ... observables remain.



Smear at $\sigma = 0.4$ (5° obj.).



	Showered	Observed
x_1	0.85	0.80 (-5%)
x_2	0.80	0.80 ($\pm 0\%$)
x_3	0.35	0.40 ($+13\%$)

Conclusion

- **Angular smearing** permits a useful multipole expansion of e^+e^-.
- **Event-wise kinematics**, without a jet definition, in high pileup.
- **Next:** investigate the QCD radiation spectrum:
  - Use finer smearing angle ($\sigma \ll 5^\circ$) near $z = 1$.
  - Can we find universal shapes in individual events?
- **At the LHC**, multipole expansion must address *longitudinal boost*.