The Proton Radius Puzzle

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Form Factors

- Matrix element of EM current between nucleon states give rise to two form factors ($q = p_f - p_i$)

$$\langle N(p_f)| \sum_q e_q\, \bar{q}\gamma^\mu q |N(p_i)\rangle = \bar{u}(p_f)\left[\gamma^\mu F_1(q^2) + \frac{i\sigma_{\mu\nu}}{2m} F_2(q^2) q^\nu\right] u(p_i)$$

- Sachs electric and magnetic form factors

$$G_E(q^2) = F_1(q^2) + \frac{q^2}{4m_p^2} F_2(q^2) \qquad G_M(q^2) = F_1(q^2) + F_2(q^2)$$

$$G_E^p(0) = 1 \qquad G_M^p(0) = \mu_p \approx 2.793$$

- The slope of G_E^p

$$\boxed{\langle r^2\rangle_E^p = 6 \frac{dG_E^p}{dq^2}\bigg|_{q^2=0}}$$

determines the charge radius $r_E^p \equiv \sqrt{\langle r^2\rangle_E^p}$

- The proton *magnetic* radius

$$\langle r^2\rangle_M^p = \frac{6}{G_M^p(0)} \frac{dG_M^p(q^2)}{dq^2}\bigg|_{q^2=0}$$

Charge radius from atomic physics

- Lamb shift in muonic hydrogen [Pohl et al. Nature **466**, 213 (2010)]
  $r_E^p = 0.84184(67)$ fm
  more recently $r_E^p = 0.84087(39)$ fm [Antognini et al. Science **339**, 417 (2013)]
- CODATA value [Mohr et al. RMP **80**, 633 (2008)]
  $r_E^p = 0.87680(690)$ fm
  more recently $r_E^p = 0.87750(510)$ fm [Mohr et al. RMP **84**, 1527 (2012)]
  extracted mainly from (electronic) hydrogen
- **5σ discrepancy!**

- This is the proton radius puzzle

What could be the reason for the discrepancy?

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1) Problem with the electronic extraction?

2) Hadronic Uncertainty?

3) New Physics?

Problem with the electronic extraction?

- Recent development: use of the z expansion
  based on known analytic properties of form factors

- **The** method for **meson** form factors
  [Flavor Lattice Averaging Group, EPJ C **74**, 2890 (2014)]

- Now applied successfully to **baryon** form factors
  to extract r_E^p, r_M^p, r_M^n, m_A...

PDG 2016: r_E^p

Citation: C. Patrignani *et al.* (Particle Data Group), Chin. Phys. C, **40**, 100001 (2016)

p CHARGE RADIUS

This is the rms electric charge radius, $\sqrt{\langle r_E^2 \rangle}$.

VALUE (fm)			*DOCUMENT ID*		*TECN*	*COMMENT*
0.8751	**±0.0061**		MOHR	16	RVUE	2014 CODATA value
0.84087	**±0.00026**	**±0.00029**	ANTOGNINI	13	LASR	μp-atom Lamb shift
• • • We do not use the following data for averages, fits, limits, etc. • • •						
0.895	±0.014	±0.014	[1] LEE	15	SPEC	Just 2010 Mainz data
0.916	±0.024		LEE	15	SPEC	World data, no Mainz
0.8775	±0.0051		MOHR	12	RVUE	2010 CODATA, $e p$ data
0.875	±0.008	±0.006	ZHAN	11	SPEC	Recoil polarimetry
0.879	±0.005	±0.006	BERNAUER	10	SPEC	$e p \to e p$ form factor
0.912	±0.009	±0.007	BORISYUK	10		reanalyzes old $e p$ data
0.871	±0.009	±0.003	HILL	10		z-expansion reanalysis
0.84184	±0.00036	±0.00056	POHL	10	LASR	See ANTOGNINI 13
0.8768	±0.0069		MOHR	08	RVUE	2006 CODATA value
0.844	$^{+0.008}_{-0.004}$		BELUSHKIN	07		Dispersion analysis
0.897	±0.018		BLUNDEN	05		SICK 03 + 2γ correction
0.8750	±0.0068		MOHR	05	RVUE	2002 CODATA value
0.895	±0.010	±0.013	SICK	03		$e p \to e p$ reanalysis

[Hill, GP PRD **82** 113005 (2010)]

[Lee, Arrington, Hill, PRD **92**, 013013 (2015)]

PDG 2016: r_M^p

Citation: C. Patrignani *et al.* (Particle Data Group), Chin. Phys. C, **40**, 100001 (2016)

p MAGNETIC RADIUS

This is the rms magnetic radius, $\sqrt{\langle r_M^2 \rangle}$.

VALUE (fm)	*DOCUMENT ID*		*TECN*	*COMMENT*
0.776±0.034±0.017	[1] LEE	15	SPEC	Just 2010 Mainz data
• • • We do not use the following data for averages, fits, limits, etc. • • •				
0.914±0.035	LEE	15	SPEC	World data, no Mainz
0.87 ±0.02	EPSTEIN	14		Using ep, en, $\pi\pi$ data
0.867±0.009±0.018	ZHAN	11	SPEC	Recoil polarimetry
0.777±0.013±0.010	BERNAUER	10	SPEC	$ep \rightarrow ep$ form factor
0.876±0.010±0.016	BORISYUK	10		Reanalyzes old $ep \rightarrow ep$ data
0.854±0.005	BELUSHKIN	07		Dispersion analysis

[1] Authors also provide values for a combination of all available data.

[Epstein, GP, Roy PRD **90**, 074027 (2014)]
[Lee, Arrington, Hill, PRD **92**, 013013 (2015)]

PDG 2016: r_M^n

Citation: C. Patrignani *et al.* (Particle Data Group), Chin. Phys. C, **40**, 100001 (2016)

n MAGNETIC RADIUS

This is the rms magnetic radius, $\sqrt{\langle r_M^2 \rangle}$.

VALUE (fm)	*DOCUMENT ID*		*COMMENT*
$0.864^{+0.009}_{-0.008}$ OUR AVERAGE			
0.89 ± 0.03	EPSTEIN	14	Using $e\,p$, $e\,n$, $\pi\pi$ data
$0.862^{+0.009}_{-0.008}$	BELUSHKIN	07	Dispersion analysis

[Epstein, GP, Roy PRD **90**, 074027 (2014)]

The bottom line

- Scattering:
  - World $e-p$ data [Lee, Arrington, Hill '15]
    $r_E^p = 0.918 \pm 0.024$ fm
  - Mainz $e-p$ data [Lee, Arrington, Hill '15]
    $r_E^p = 0.895 \pm 0.020$ fm
  - Proton, neutron and π data [Hill , GP '10]
    $r_E^p = 0.871 \pm 0.009 \pm 0.002 \pm 0.002\,\mathrm{fm}$

- Muonic hydrogen
  - [Pohl et al. Nature **466**, 213 (2010)]
    $r_E^p = 0.84184(67)$ fm
  - [Antognini et al. Science **339**, 417 (2013)]
    $r_E^p = 0.84087(39)$ fm

- The bottom line:
  using z expansion scattering disfavors muonic hydrogen

- Is there a problem with muonic hydrogen *theory*?

Muonic hydrogen theory

- Is there a problem with muonic hydrogen *theory*?

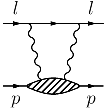
- Potentially yes!
  [Hill, GP PRL **107** 160402 (2011)]

- Muonic hydrogen measures ΔE and translates it to r_E^p

- [Pohl et al. Nature **466**, 213 (2010) Supplementary information]
  $\Delta E = 206.0573(45) - 5.2262(r_E^p)^2 + 0.0347(r_E^p)^3\ \mathrm{meV}$

- [Antognini et al. Science **339**, 417 (2013), Ann. of Phy. **331**, 127]
  $\Delta E = 206.0336(15) - 5.2275(10)(r_E^p)^2 + 0.0332(20)\ \mathrm{meV}$

- In both cases apart from r_E^p need two-photon exchange



Two photon exchange

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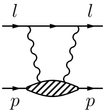

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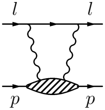

$$
\begin{aligned}
W^{\mu\nu} &= \frac{1}{2}\sum_s i\int d^4x\, e^{iq\cdot x}\langle \boldsymbol{k}, s|\,T\{J^\mu_{\rm e.m.}(x)J^\nu_{\rm e.m.}(0)\}|\boldsymbol{k}, s\rangle \\
&= \left(-g^{\mu\nu}+\frac{q^\mu q^\nu}{q^2}\right)W_1+\left(k^\mu-\frac{k\cdot q\, q^\mu}{q^2}\right)\left(k^\nu-\frac{k\cdot q\, q^\nu}{q^2}\right)W_2
\end{aligned}
$$

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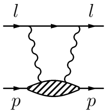

$$W^{\mu\nu} = \frac{1}{2}\sum_s i\int d^4x\, e^{iq\cdot x}\langle \boldsymbol{k}, s|T\{J^\mu_{\rm e.m.}(x)J^\nu_{\rm e.m.}(0)\}|\boldsymbol{k}, s\rangle$$

$$= \left(-g^{\mu\nu}+\frac{q^\mu q^\nu}{q^2}\right)W_1+\left(k^\mu-\frac{k\cdot q\, q^\mu}{q^2}\right)\left(k^\nu-\frac{k\cdot q\, q^\nu}{q^2}\right)W_2$$

- Dispersion relations ($\nu = 2k\cdot q,\ Q^2=-q^2$)

$$W_1(\nu, Q^2) = W_1(0, Q^2)+\frac{\nu^2}{\pi}\int_{\nu_{\rm cut}(Q^2)^2}^{\infty} d\nu'^2\,\frac{{\rm Im}W_1(\nu', Q^2)}{\nu'^2(\nu'^2-\nu^2)}$$

$$W_2(\nu, Q^2) = \frac{1}{\pi}\int_{\nu_{\rm cut}(Q^2)^2}^{\infty} d\nu'^2\,\frac{{\rm Im}W_2(\nu', Q^2)}{\nu'^2-\nu^2}$$

- W_1 requires subtraction...

Two photon exchange

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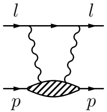

- Imaginary part of TPE related to data:
  form factors, structure functions

Two photon exchange

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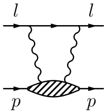


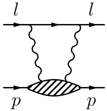
- Imaginary part of TPE related to data:
  form factors, structure functions

- Cannot reproduce it from its imaginary part:
  Dispersion relation requires subtraction

- Need poorly constrained non-perturbative function $W_1(0, Q^2)$

- Calculable in small Q^2 limit using NRQED
  [Hill, GP, PRL **107** 160402 (2011)]

Two Photon Exchange: large Q^2 limit



- Calculable in *large* Q^2 limit using Operator Product Expansion (OPE) [J. C. Collins, NPB **149**, 90 (1979)]

  The photon "sees" the quarks and gluons inside the proton

$$W_1(0, Q^2) = c/Q^2 + \mathcal{O}\left(1/Q^4\right)$$

- Result was used to estimate two photon exchange effects
- c calculated in [J. C. Collins, NPB **149**, 90 (1979)]

RENORMALIZATION OF THE COTTINGHAM FORMULA

John C. COLLINS *
Joseph Henry Laboratories, Princeton University, Princeton, New Jersey 08540, USA

Received 23 October 1978

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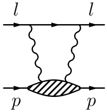

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- Was it?

Large Q^2 behavior

- In 1978 Collins calculated EM corrections to the nucleon mass
  The mass only depends on spin-0 operators (q quark, $G^{\mu\nu}$ gluon)

$$\langle P|m_q\bar{q}q|P\rangle, \qquad \langle P|G^{\mu\nu}G_{\mu\nu}|P\rangle$$

	Quark	Gluon
Spin-0	Collins '78	Collins '78

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- For $W_1(0, Q^2)$ you need also spin-2 operators

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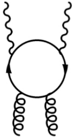
- Need to calculate the spin-2 contribution
  [Hill, GP arXiv:1611.09917, to appear in PRD]

	Quark	Gluon
Spin-0	Collins '78	Collins '78
Spin-2	Hill, GP '16	Hill, GP '16

- Collins's result is not enough for muonic hydrogen!

Large Q^2 behavior

- Requires 1-loop calculation



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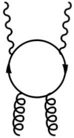

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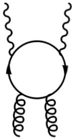

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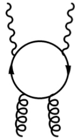

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- For three light quark u, d, s
  Correct result: $\sum_q e_q^2 = (\frac{2}{3})^2 + (\frac{1}{3})^2 + (\frac{1}{3})^2 = \frac{2}{3}$
  Collins: $\sum_q = 3$
  Too large by a factor of 4.5...

Large Q^2 behavior

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- Even worse, quark spin-0 and gluon spin-0 come with opposite signs
  After correcting the mistake they largely cancel
  $W_1(0, Q^2)$ is **dominated** by spin-2 contribution

- Lesson: It is important to do a full calculation

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- Flip side: You cannot use $m_n - m_p$ to constrain muonic hydrogen

Two Photon Exchange: Modeling

- "Aggressive" modeling: use OPE for $Q^2 \geq 1$ GeV^2
  Use NRQED for $Q^2 \leq 0.1$ GeV^2
  Model unknown Q^4 and $1/Q^4$

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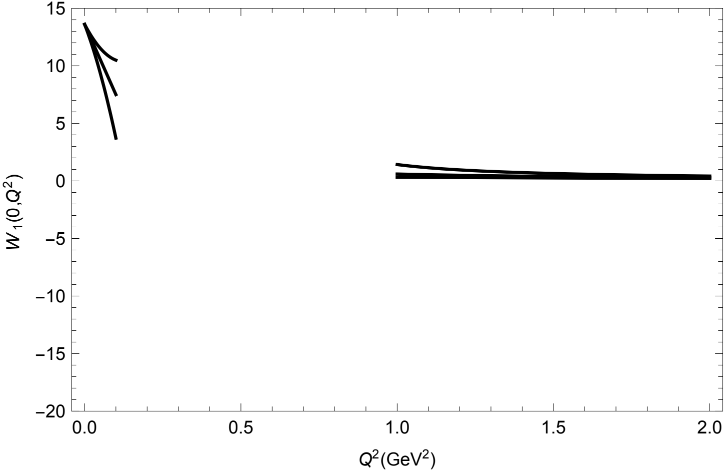
  Model unknown Q^4 and $1/Q^4$
- How to connect the curves?



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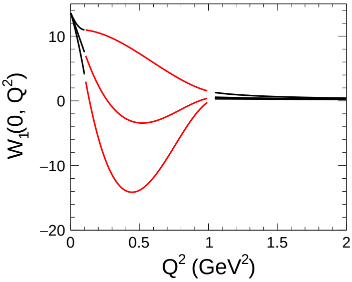

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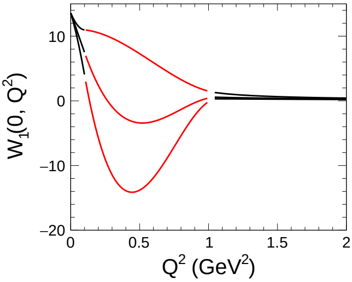

- Energy contribution: $\delta E(2\mathrm{S})^{W_1(0,Q^2)} \in [-0.046\,\mathrm{meV},\ -0.021\,\mathrm{meV}]$
  To explain the puzzle need this to be ~ -0.3 meV
- Caveats: OPE might be only valid for larger Q^2
  $W_1(0, Q^2)$ might be different than the interpolated lines

Experimental test

- How to test?
- New experiment: $\mu - p$ scattering
  MUSE (MUon proton Scattering Experiment) at PSI
  [R. Gilman et al. (MUSE Collaboration), arXiv:1303.2160]

- Need to connect muon-proton scattering and muonic hydrogen can use a new effective field theory: QED-NRQED
  [Hill, Lee, GP, Mikhail P. Solon, PRD **87** 053017 (2013)]
  [Steven P. Dye, Matthew Gonderinger, GP, PRD **94** 013006 (2016)]

Conclusions

Conclusions

- Proton radius puzzle: $> 5\sigma$ discrepancy between
  - r_E^p from muonic hydrogen
  - r_E^p from hydrogen and $e - p$ scattering

- Recent muonic deuterium results find similar discrepancies
  [Pohl et al. Science **353**, 669 (2016)]

- After 6 years the origin is still not clear

1) Is it a problem with the electronic extraction?
2) Is it a hadronic uncertainty?
3) is it new physics?

- Motivates a reevaluation of our understanding of the proton

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- Motivates a direct connection between muon-proton scattering (MUSE experiment) and muonic hydrogen
using a new effective field theory: QED-NRQED
[Steven P. Dye, Matthew Gonderinger, GP, *in progress*]

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- Much more work to do!

- Thank you

Backup

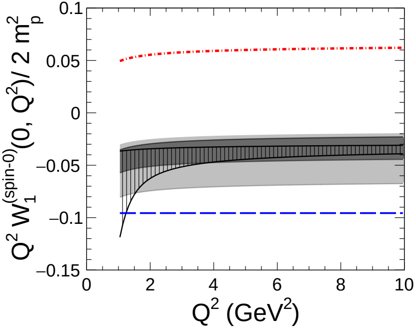
Large Q^2 behavior: Spin 0 contribution

- The *correct* spin 0 result

$$\frac{Q^2}{2m_p^2} W_1^{(\mathrm{spin}-0)}(0, Q^2) = -2\sum_q e_q^2 f_q^{(0)} + \left(\sum_q e_q^2\right) \frac{\alpha_s}{12\pi} \tilde{f}_g^{(0)}$$

- quark and gluon matrix elements related by

$$1 = (1-\gamma_m)\sum_q f_q^{(0)} - \frac{\beta}{2g}\tilde{f}_g^{(0)}$$



- Dashed blue: quark
- Dash-dotted red: gluon
- Vertical stripes: perturbative uncertainty $Q/2 < \mu < 2Q$
- Solid bands: hadronic uncertainties on matrix elements

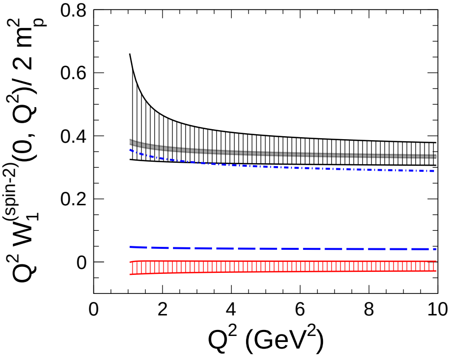
Large Q^2 behavior: Spin 2 contribution

- The *new* spin 2 result

$$\frac{Q^2}{2m_p^2} W_1^{(\text{spin}-2)}(0,Q^2) = 2\sum_q e_q^2 f_q^{(2)}(\mu) + \left(\sum_q e_q^2\right) \frac{\alpha_s}{4\pi}\left(-\frac{5}{3} + \frac{4}{3}\log\frac{Q^2}{\mu^2}\right) f_g^{(2)}(\mu)$$

- quark and gluon matrix elements related by

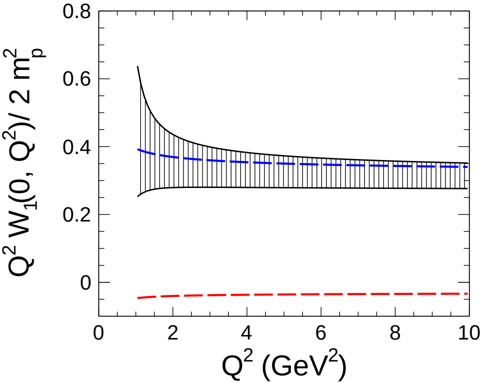
$$\sum f_q^{(2)}(\mu) + f_g^{(2)}(\mu) = 1$$



- Dashed blue: down quark
- Dash-dotted blue: up quark
- Red: Gluon contribution
- Vertical stripes: perturbative uncertainty $Q/2 < \mu < 2Q$
- Solid bands: hadronic uncertainty

Large Q^2 behavior: Total contribution

- The total contribution



- Dashed red: spin 0
- Dashed blue: spin 2
- Vertical stripes: total contribution with perturbative and hadronic errors added in quadrature

Two Photon exchange: small Q^2 and large Q^2

- Using NRQED we have control over low Q^2

$$W_1(0,Q^2) = 2a_p(2+a_p) + \frac{Q^2}{m_p^2}\left\{\frac{2m_p^3\bar{\beta}}{\alpha} - a_p - \frac{2}{3}\left[(1+a_p)^2 m_p^2 (r_M^p)^2 - m_p^2 (r_E^p)^2\right]\right\} + \mathcal{O}\left(Q^4\right)$$

- Using OPE we have control over the high Q^2

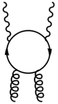
$$\frac{Q^2}{2m_p^2} W_1^{(\mathrm{spin}-0)}(0,Q^2) = -2\sum_q e_q^2 f_q^{(0)} + \left(\sum_q e_q^2\right)\frac{\alpha_s}{12\pi}\tilde{f}_g^{(0)}$$

$$\frac{Q^2}{2m_p^2} W_1^{(\mathrm{spin}-2)}(0,Q^2) = 2\sum_q e_q^2 f_q^{(2)}(\mu) + \left(\sum_q e_q^2\right)\frac{\alpha_s}{4\pi}\left(-\frac{5}{3} + \frac{4}{3}\log\frac{Q^2}{\mu^2}\right) f_g^{(2)}(\mu)$$

- The problem, like the joke, is how to make a whole fish from a head and a tail...

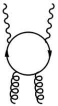
- Before this work we had only the low Q^2
  knowing the large Q^2 allows to connect the dots

Gluon operators



- Gluons: must be color singlet $G_a^{\alpha\beta} G_a^{\rho\sigma}$
  A product of (E^i, B^i) and (E^j, B^j) has $7 \times 6/2 = 21$ components:

Gluon operators



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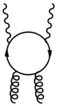
Gluon operators



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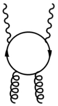
Gluon operators



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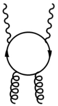
- 9 components of traceless symmetric tensor: $G^{\mu\alpha} G_\alpha^{\ \nu} - \frac{1}{4} G^{\alpha\beta} G_{\alpha\beta} g^{\mu\nu}$
  chromomagnetic stress-energy tensor
- What else? 10 components of

Gluon operators



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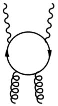
$$O^{\mu\alpha\nu\beta} = -\frac{1}{4}\left(\epsilon^{\mu\alpha\rho\sigma}\epsilon^{\nu\beta\kappa\lambda} + \epsilon^{\mu\beta\rho\sigma}\epsilon^{\nu\alpha\kappa\lambda}\right) G_{\rho\kappa} G_{\sigma\lambda} - \text{all possible traces}$$

For example $O^{0123} = G^{01} G^{23} + G^{03} G^{21} = E^1 B^1 - E^3 B^3$

Gluon operators



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  For example $O^{0123} = G^{01} G^{23} + G^{03} G^{21} = E^1 B^1 - E^3 B^3$
- For protons: $\langle \text{Proton} | O^{\mu\alpha\nu\beta} | \text{Proton} \rangle = 0$
  What about $\langle \text{Medium} | O^{\mu\alpha\nu\beta} | \text{Medium} \rangle$?
  Solution looking for a problem...