Recent Developments in Lattice Gauge Theory

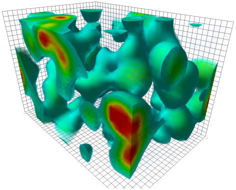
Christoph Lehner (BNL)

May 9, 2017 – Pheno 2017

A one-slide introduction to LGT

- Discretize Euclidean space-time on a lattice to regulate QFT and define the path integral
- Matter fields live on lattice sites and gauge fields on links between them (Wilson's explicit gauge invariance)
- Perform integral over fermions analytically, integral over bosons using Monte-Carlo methods on large computers

- In the following we therefore draw diagrams only with respect to quarks, photons, and leptons; gluons and their effects are generated by the statistical average.

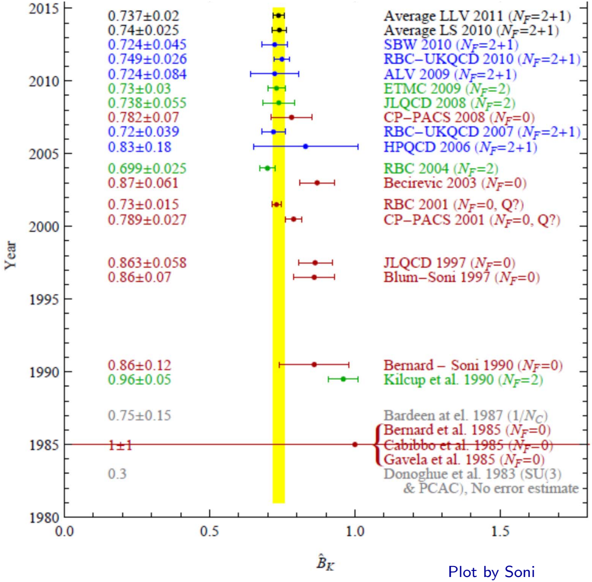


Lattice QCD action density, Leinweber, CSSM, Adelaide, 2003

Exponentially growing capabilities

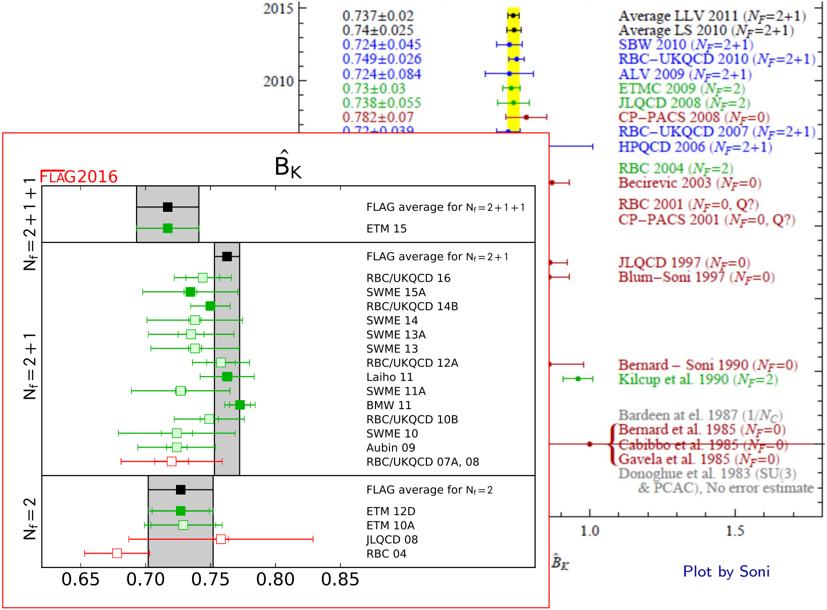
- The field tracks Moore's law of exponential growth of computing power
  - Mike Creutz's first computation of $\langle F_{\mu\nu}^2 \rangle$ in QCD on a 6^4 lattice almost 40 years ago took about 10^6 floating-point operations to complete
  - The state-of-the-art lattice QCD calculation of the $(g-2)_\mu$ hadronic light-by-light contribution by RBC/UKQCD on a 10^5 larger volume at physical pion mass in a 6.4fm box took 10^{22} floating-point operations to complete
- Combined with theory advances: precision first-principles hadronic physics directly from the QCD Lagrangian now possible

A historical example: kaon mixing



Plot by Soni

A historical example: kaon mixing



Plot by Soni

Recent developments (a selection)

- The muon g-2
- Flavor physics
- BSM/Strong dynamics

The muon g-2

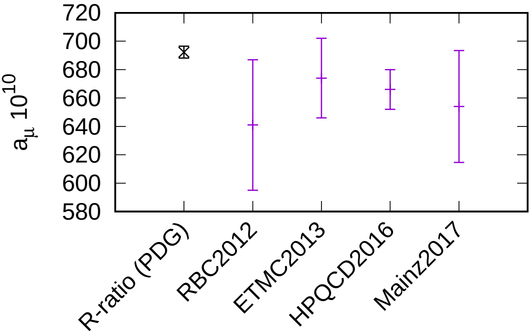
Theory status – summary

Contribution	Value $\times 10^{10}$	Uncertainty $\times 10^{10}$
QED (5 loops)	11 658 471.895	0.008
EW	15.4	0.1
HVP LO	692.3	**4.2**
HVP NLO	-9.84	0.06
HVP NNLO	1.24	0.01
Hadronic light-by-light	10.5	**2.6**
Total SM prediction	11 659 181.5	4.9
BNL E821 result	11 659 209.1	6.3
FNAL E989/J-PARC E34 goal		$\approx$ **1.6**

A reduction of uncertainty for HVP and HLbL is needed. A systematically improvable first-principles calculation is desired.

First-principles approach to HVP LO

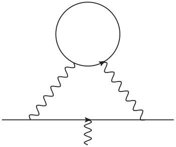
Recent lattice QCD results:



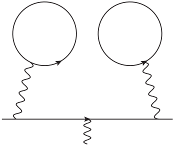
On-going efforts by BMWc, ETMC, HPQCD+FNAL/MILC, Mainz, RBC+UKQCD, ...

Related new result by BMWc (arXiv:1612.02364): published data not sufficient to compute a_μ, however, indicative of total uncertainty of an a_μ calculation: $\delta a_\mu = O(20 \times 10^{-10})$

First-principles approach to HVP LO

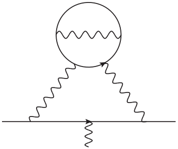


Quark-connected piece with by far dominant part from up and down quark loops, $\mathcal{O}(700 \times 10^{-10})$

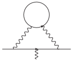


Quark-disconnected piece, $-9.6(4.0) \times 10^{-10}$

Phys.Rev.Lett. 116 (2016) 232002



QED corrections, $\mathcal{O}(10 \times 10^{-10})$

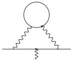


HVP quark-connected contribution

Biggest challenge to direct calculation at physical pion masses is to control statistics and potentially large finite-volume errors.

Statistics: for strange and charm solved issue, for up and down quarks existing methodology less effective

Finite-volume errors are exponentially suppressed in the simulation volume but may be sizeable



HVP quark-connected contribution

Starting from the vector current

$$J_\mu(x) = i \sum_f Q_f \overline{\Psi}_f(x) \gamma_\mu \Psi_f(x)$$

we may write

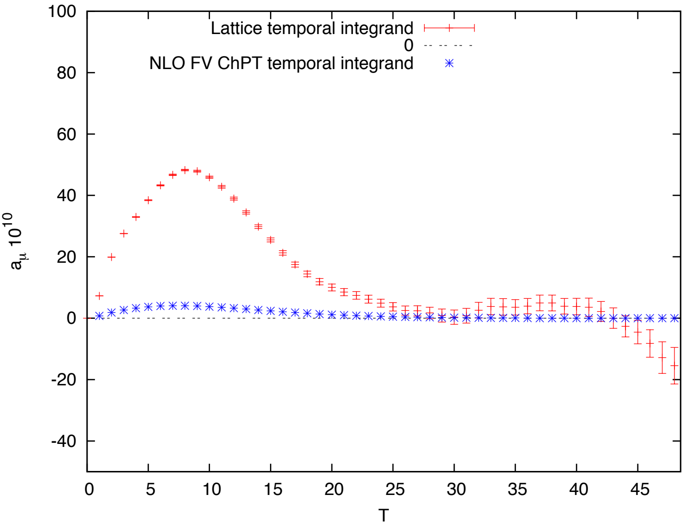
$$a_\mu^{\mathrm{HVP}} = \sum_{t=0}^{\infty} w_t C(t)$$

with

$$C(t) = \frac{1}{3} \sum_{\vec{x}} \sum_{j=0,1,2} \langle J_j(\vec{x}, t) J_j(0) \rangle$$

and w_t capturing the photon and muon part of the diagram (Bernecker-Meyer 2011).

Integrand $w_T C(T)$ for the light-quark connected contribution:



$m_\pi = 140$ MeV, $a = 0.11$ fm (RBC/UKQCD 48^3 ensemble)

Statistical noise from long-distance region

Complete first-principles analysis

- Currently the statistical uncertainty for a pure first-principles analysis in the continuum limit is at the $\Delta a_\mu \approx 15 \times 10^{-10}$ level

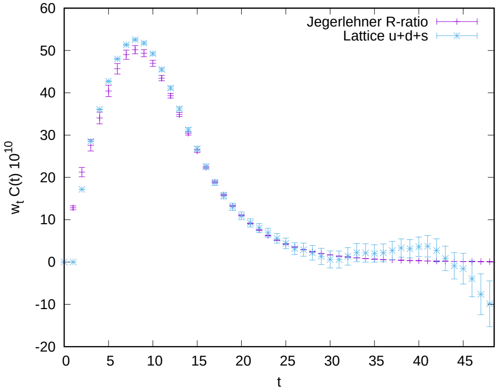
Contribution	Value $\times 10^{10}$	Uncertainty $\times 10^{10}$
QED (5 loops)	11 658 471.895	0.008
EW	15.4	0.1
HVP LO	692.3	**4.2**
HVP NLO	-9.84	0.06
HVP NNLO	1.24	0.01
Hadronic light-by-light	10.5	**2.6**
Total SM prediction	11 659 181.5	4.9
BNL E821 result	11 659 209.1	6.3
Fermilab E989 target		$\approx$ **1.6**

- Sub-percent statistical error achievable with a few more months of running

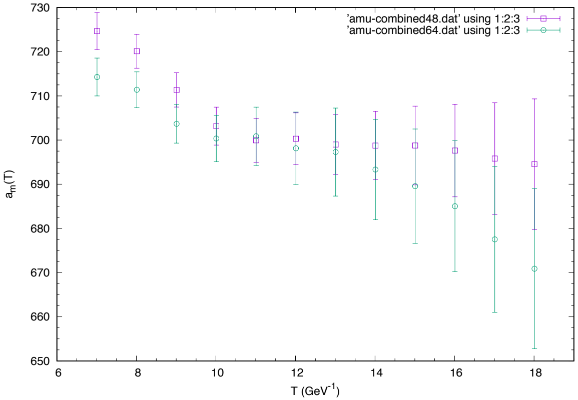
- While we are waiting for more statistics ...

Combined lattice and dispersive analysis

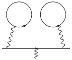
We can use the dispersion relation to overlay experimental e^+e^- scattering data (Bernecker, Meyer 2011). Below the experimental result is taken from Jegerlehner 2016:



The lattice data is precise at shorter distances and the experimental data is precise at longer distances. We can do a combined analysis with lattice and experimental data:
$a_\mu = \sum_{t=0}^{T} w_t C^{\text{lattice}}(t) + \sum_{t=T+1}^{\infty} w_t C^{\text{exp}}(t)$



Errors range from $\sim$ 0.5 to 1.2 % for T $\lesssim$ 12 (GeV^{-1});
this is a promising way to reduce the overall uncertainty on a short time-scale (C.L. ICHEP2016).
Complementary suggestion in terms of Mellin moments by de Rafael (arXiv:1702.06783).

 HVP quark-disconnected contribution

First results at physical pion mass with a statistical signal
Phys.Rev.Lett. 116 (2016) 232002

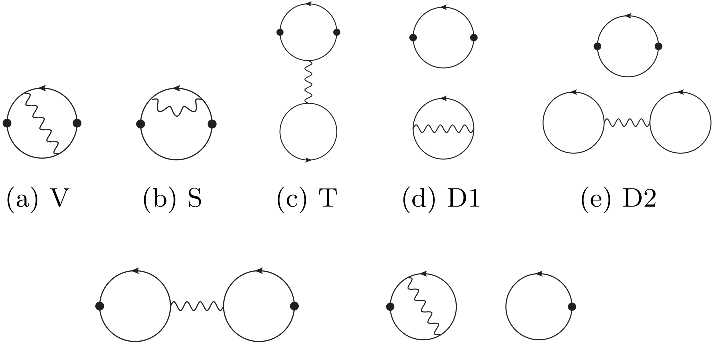
Statistics is clearly the bottleneck; calculation was a potential road-block of a first-principles calculation for a long time; due to very large pion-mass dependence calculation at physical pion mass is crucial.

New stochastic estimator allowed RBC/UKQCD to get result

$$a_\mu^{\mathrm{HVP\ (LO)\ DISC}} = -9.6(3.3)_{\mathrm{stat}}(2.3)_{\mathrm{sys}} \times 10^{-10}$$

from a modest computational investment ($\approx$ 1M core hours).

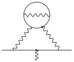
HVP QED contribution



(a) V (b) S (c) T (d) D1 (e) D2

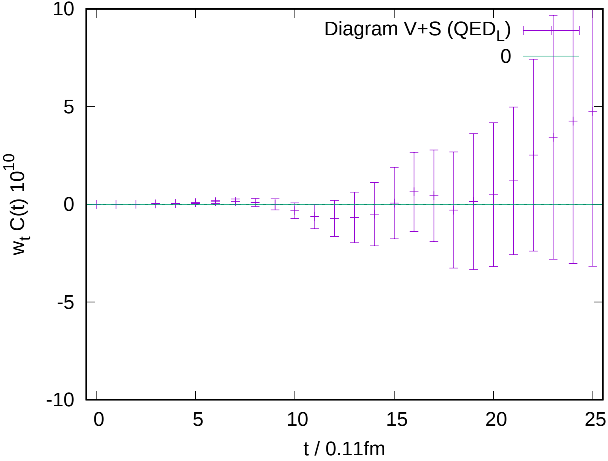
(f) F (g) D3

New method: use importance sampling in position space and local vector currents

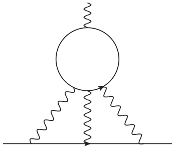


HVP QED+strong IB contributions

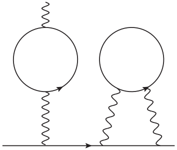
Example: HVP QED diagram V+S



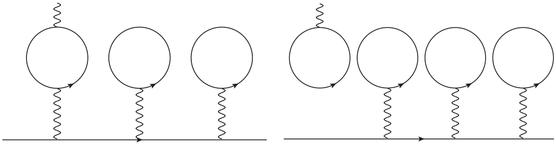
The Hadronic Light-by-Light contribution



Quark-connected piece (charge factor of up/down quark contribution: $\frac{17}{81}$)



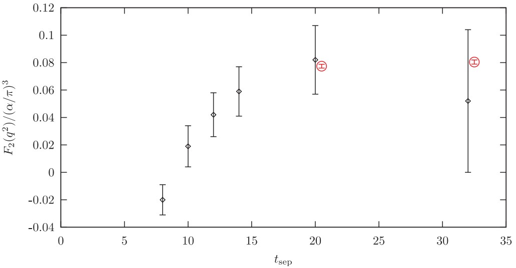
Dominant quark-disconnected piece (charge factor of up/down quark contribution: $\frac{25}{81}$)



Sub-dominant quark-disconnected pieces (charge factors of up/down quark contribution: $\frac{5}{81}$ and $\frac{1}{81}$)

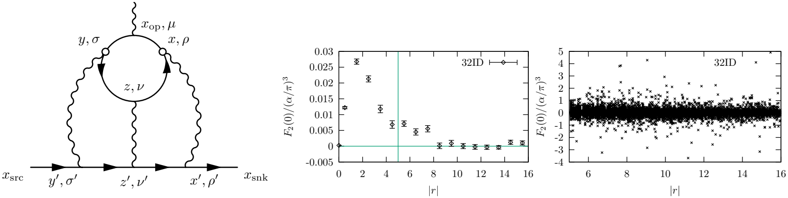
All results below are from: T. Blum, N. Christ, M. Hayakawa, T. Izubuchi, L. Jin, and C.L., Phys. Rev. D 93, 014503 (2016)

Compute quark-connected contribution with new computational strategy



yields more than an order-of-magnitude improvement (red symbols) over previous method (black symbols) for a factor of ≈ 4 smaller cost.

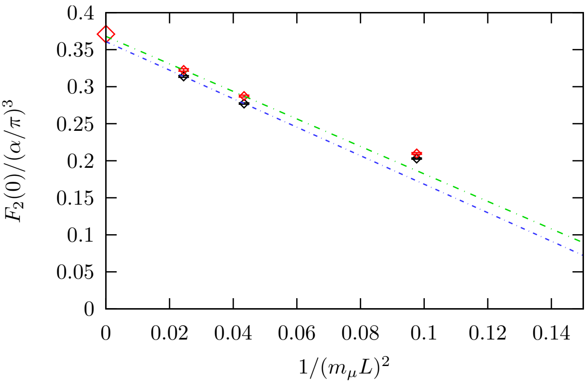
New stochastic sampling method



Stochastically evaluate the sum over vertices x and y:

- Pick random point x on lattice
- Sample all points y up to a specific distance $r = |x - y|$, see vertical red line
- Pick y following a distribution $P(|x - y|)$ that is peaked at short distances

Cross-check against analytic result where quark loop is replaced by muon loop



Current status of the HLbL

T. Blum, N. Christ, M. Hayakawa, T. Izubuchi, L. Jin, and C.L., PRL118(2017)022005

$$a_\mu^{\text{cHLbL}} = \left.\frac{g_\mu - 2}{2}\right|_{\text{cHLbL}} = (0.0926 \pm 0.0077)\left(\frac{\alpha}{\pi}\right)^3 = (11.60 \pm 0.96) \times 10^{-10} \quad (11)$$

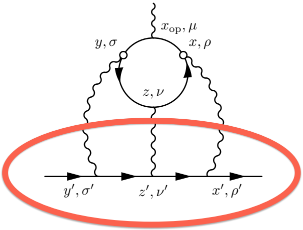
$$a_\mu^{\text{dHLbL}} = \left.\frac{g_\mu - 2}{2}\right|_{\text{dHLbL}} = (-0.0498 \pm 0.0064)\left(\frac{\alpha}{\pi}\right)^3 = (-6.25 \pm 0.80) \times 10^{-10} \quad (12)$$

$$a_\mu^{\text{HLbL}} = \left.\frac{g_\mu - 2}{2}\right|_{\text{HLbL}} = (0.0427 \pm 0.0108)\left(\frac{\alpha}{\pi}\right)^3 = (5.35 \pm 1.35) \times 10^{-10} \quad (13)$$

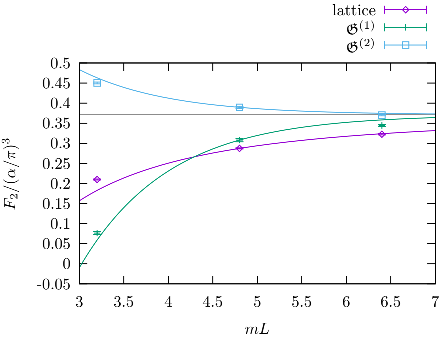
Makes HLbL an unlikely candidate to explain the discrepancy!

Next: finite-volume and lattice-spacing systematics; sub-leading diagrams

Finite-volume errors of the HLbL



Remove power-law like finite-volume errors by computing the muon-photon part of the diagram in infinite volume (C.L. talk at lattice 2015 and Green et al. 2015, PRL115(2015)222003; Asmussen et al. 2016, PoS,LATTICE2016 164)



Now completed arXiv:1705.01067 with improved weighting function.

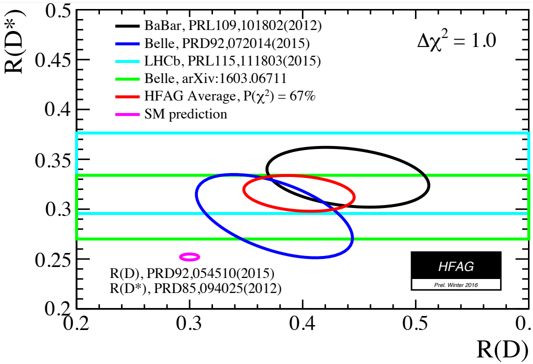
Next step: combine weighting function with existing QCD data

Flavor physics

Lepton flavor universality violation

$$R(D^{(*)}) \equiv \frac{\mathcal{B}(B \to D^{(*)}\tau\nu)}{\mathcal{B}(B \to D^{(*)}\ell\nu)}$$

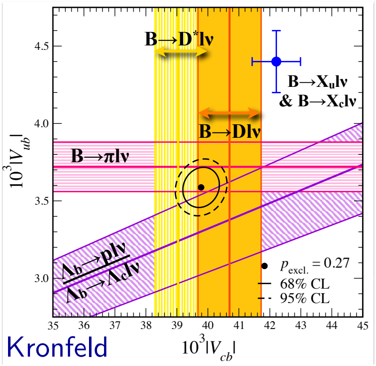
$\ell = e, \mu$ for BaBar and Belle and $\ell = \mu$ for LHCb



$R(D^*)$ using HQET, $R(D)$ from lattice

Lattice inputs:

- $B \to D\ell\nu$ near zero recoil FNAL/MILC: 1503.07237, HPQCD:1505.03925, RBC/UKQCD: Lattice2016
- $B \to D^*\ell\nu$ at zero recoil FNAL/MILC:1403.0635, HPQCD: Lattice2016; at non-zero recoil now being computed! However, D^* at physical pion mass unstable



Kronfeld

Inclusive versus exclusive tensions:

Combine with $\Lambda_b \to \Lambda_c\ell\nu$ Detmold, Lehner, Meinel 1503.01421:

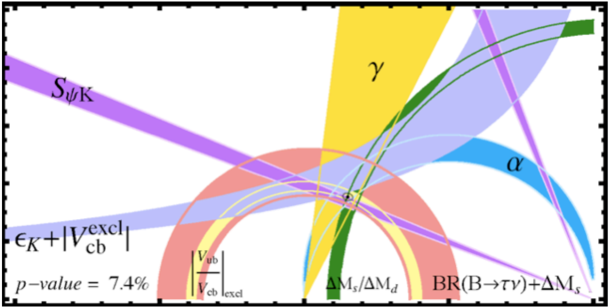
Also anomaly in $B \to K^{(*)}\mu^+\mu^-/B \to K^{(*)}e^+e^-$? 1704.06659, Soni talk this afternoon

Improved unitarity constraints

3× more precise determination of

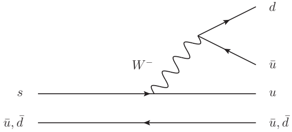
$$\xi \equiv \frac{|V_{td}|\Delta M_{B_s^0} M_{B^0}}{|V_{ts}|\Delta M_{B^0} M_{B_s^0}} = 1.211(19)$$

in FNAL/MILC PRD 93, 113016 (2016) yields new CKM constraint



Plot and fit by E. Lunghi

K$\rightarrow \pi\pi$



Brief history of lattice effort by RBC/UKQCD:

- 2011: threshold (pions at rest) computation at $m_\pi > m_\pi^{\rm phys}$
- 2012: quantitative explanation of $\Delta I = 1/2$ rule ($I = 0$ final state dominance)
- 2012: $I = 2$ final state at $m_\pi^{\rm phys}$ no $a \rightarrow 0$ limit
- 2015: $I = 2$ final state at $m_\pi^{\rm phys}$ with $a \rightarrow 0$ limit
- 2015: $I = 0$ final state at $m_\pi^{\rm phys}$ no $a \rightarrow 0$ limit

$$\eta_{00} = \frac{A(K_{\rm L} \rightarrow \pi^0\pi^0)}{A(K_{\rm S} \rightarrow \pi^0\pi^0)}, \qquad \eta_{+-} = \frac{A(K_{\rm L} \rightarrow \pi^+\pi^-)}{A(K_{\rm S} \rightarrow \pi^+\pi^-)}.$$

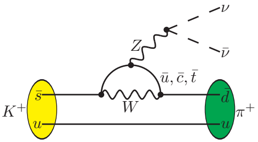
$$\mathrm{Re}(\epsilon'/\epsilon) \approx \frac{1}{6}\left(1 - \left|\frac{\eta_{00}}{\eta_{\pm}}\right|^2\right) = 16.6(2.3) \times 10^{-4} \quad \text{(experiment)}$$

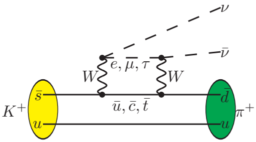
measure of direct CPV (ϵ') — measure of indirect CPV (ϵ)

RBC/UKQCD result PRL 115 (2015) 212001: $\varepsilon'/\varepsilon = 1.38(5.15)(4.43) \times 10^{-4}$;
Now reduced systematic error and more statistics, work on including IB

New methods

- Rare kaons: NA62 at CERN aims at $O(10\%)$ measurement of $\text{Br}(K^+ \to \pi^+ \nu \bar{\nu})$; Xu Feng et al. working on long-distance methodology and a first-principles computation





- Inclusive $\tau \to \nu X$ decay from lattice QCD (RBC/UKQCD 2017)

- Inclusive semi-leptonic B decays from lattice QCD (1703.01881)

BSM/Strong dynamics

Generic features of strong dynamics models emerging

- A 0^{++} "sigma" meson, much lighter than all of the resonances except pions seen in SU(3) N_f=8 (LSD and LatKMI) and for SU(2) with N_f=2 in the sextet representation by LatHC. Now efforts underway to create a two-state EFT with parameters tuned from lattice.
- Vector meson dominance works: The relation $f_V/f_\pi = \sqrt{2}$, e.g., has been confirmed in SU(3) with N_f=8 and in SU(4) with two fermion irreps (Jay and Neil).

Progress on gauge-gravity duality and SUSY

- Catterall et al. now have a lattice formulation of $N = 4$ SYM in $D = 4$; work on lower-dimensional theory, testing gauge-gravity duality, and SUSY breaking to be presented at Lattice 2017

Thanks to E. Neil for helpful discussions

Summary and outlook

- New methods allow for a substantial reduction in uncertainty of the theory calculation of the $(g-2)_\mu$; a reduction of uncertainty over the currently most precise value within the next year seems possible and over five years should track Fermilab E989 precision.

- Lattice QCD continues to play a crucial role in understanding flavor anomalies

- New methods to compute inclusive decays from lattice QCD and long-distance effects in rare decays are being developed and will be implemented in precision calculations in the next few years.

Thank you

The setup:

$$C(t) = \frac{1}{3V} \sum_{j=0,1,2} \sum_{t'} \langle \mathcal{V}_j(t+t')\mathcal{V}_j(t') \rangle_{\mathrm{SU(3)}} \tag{1}$$

where V stands for the four-dimensional lattice volume, $\mathcal{V}_\mu = (1/3)(\mathcal{V}_\mu^{u/d} - \mathcal{V}_\mu^s)$, and

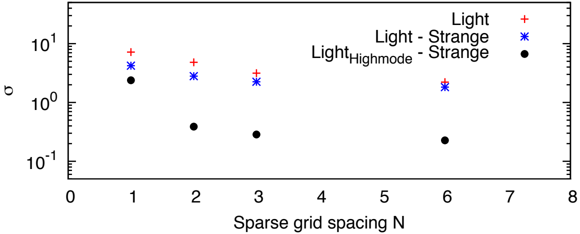
$$\mathcal{V}_\mu^f(t) = \sum_{\vec{x}} \mathrm{Im}\,\mathrm{Tr}[D^{-1}_{\vec{x},t;\vec{x},t}(m_f)\gamma_\mu]\,. \tag{2}$$

We separate 2000 low modes (up to around m_s) from light quark propagator as $D^{-1} = \sum_n v^n (w^n)^\dagger + D^{-1}_{\mathrm{high}}$ and estimate the high mode stochastically and the low modes as a full volume average Foley 2005.

We use a sparse grid for the high modes similar to Li 2010 which has support only for points x_μ with $(x_\mu - x_\mu^{(0)}) \bmod N = 0$; here we additionally use a random grid offset $x_\mu^{(0)}$ per sample allowing us to stochastically project to momenta.

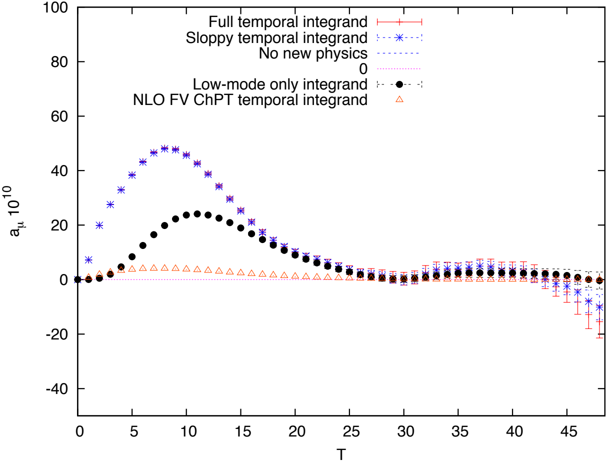
Combination of both ideas is crucial for noise reduction at physical pion mass!

Fluctuation of $\mathcal{V}_\mu$ (σ):

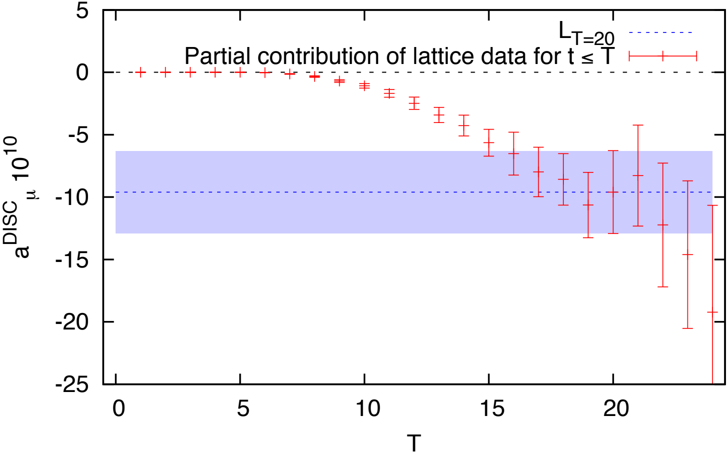


Since $C(t)$ is the autocorrelator of $\mathcal{V}_\mu$, we can create a stochastic estimator whose noise is potentially reduced linearly in the number of random samples, hence the normalization in the lower panel

Low-mode saturation for physical pion mass (here 2000 modes):

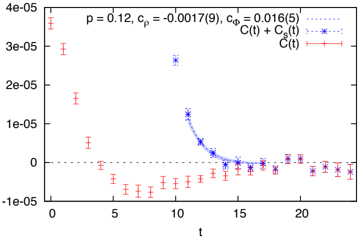


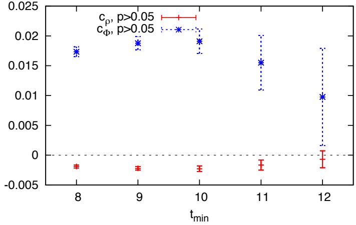
Result for partial sum $L_T = \sum_{t=0}^{T} w_t C(t)$:



For $t \geq 15$ $C(t)$ is consistent with zero but the stochastic noise is t-independent and $w_t \propto t^4$ such that it is difficult to identify a plateau region based only on this plot

Resulting correlators and fit of $C(t) + C_s(t)$ to $c_\rho e^{-E_\rho t} + c_\phi e^{-E_\phi t}$ in the region $t \in [t_{\min}, \ldots, 17]$ with fixed energies $E_\rho = 770$ MeV and $E_\phi = 1020$. $C_s(t)$ is the strange connected correlator.

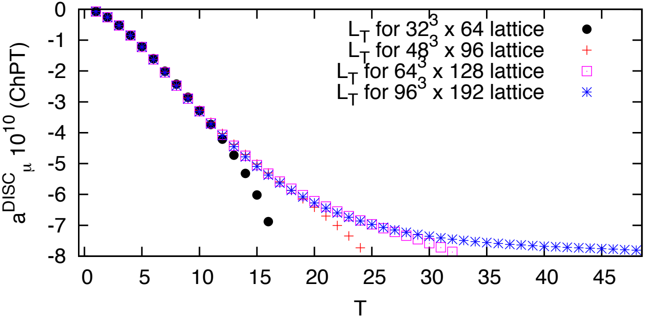




We fit to $C(t) + C_s(t)$ instead of $C(t)$ since the former has a spectral representation.

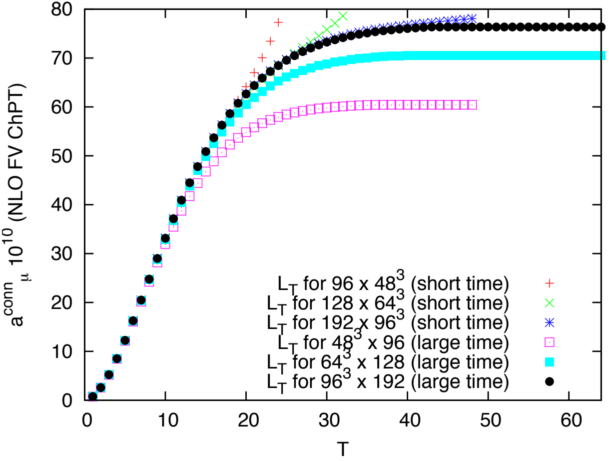
We could use this model alone for the long-distance tail to help identify a plateau but it would miss the two-pion tail

We therefore additionally calculate the two-pion tail for the disconnected diagram in ChPT:



A closer look at the NLO FV ChPT prediction (1-loop sQED):

We show the partial sum $\sum_{t=0}^{T} w_t C(t)$ for different geometries and volumes:



The dispersive approach to HVP LO

The dispersion relation

$$\Pi_{\mu\nu}(q) = i\left(q_\mu q_\nu - g_{\mu\nu}q^2\right)\Pi(q^2)$$
$$\Pi(q^2) = -\frac{q^2}{\pi}\int_{4m_\pi^2}^{\infty}\frac{ds}{s}\frac{\mathrm{Im}\Pi(s)}{q^2-s}\,.$$

allows for the determination of a_μ^{HVP} from experimental data via

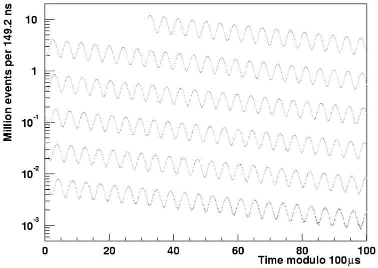
$$a_\mu^{\mathrm{HVP\ LO}} = \left(\frac{\alpha m_\mu}{3\pi}\right)^2\left[\int_{4m_\pi^2}^{E_0^2} ds\,\frac{R_\gamma^{\mathrm{exp}}(s)\hat{K}(s)}{s^2} + \int_{E_0^2}^{\infty} ds\,\frac{R_\gamma^{\mathrm{pQCD}}(s)\hat{K}(s)}{s^2}\right],$$
$$R_\gamma(s) = \sigma^{(0)}(e^+e^- \to \gamma^* \to \mathrm{hadrons})/\frac{4\pi\alpha^2}{3s}$$

Experimentally with or without additional hard photon (ISR: $e^+e^- \to \gamma^*(\to \mathrm{hadrons})\gamma$)

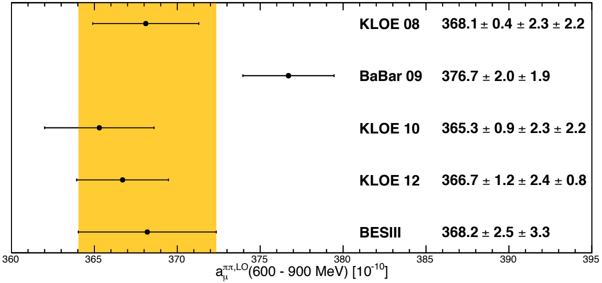
Experimental setup: muon storage ring with tuned momentum of muons to cancel leading coupling to electric field

$$\vec{\omega}_a = -\frac{q}{m}\left[a_\mu \vec{B} - \left(a_\mu - \frac{1}{\gamma^2 - 1}\right)\frac{\vec{\beta}\times\vec{E}}{c}\right]$$

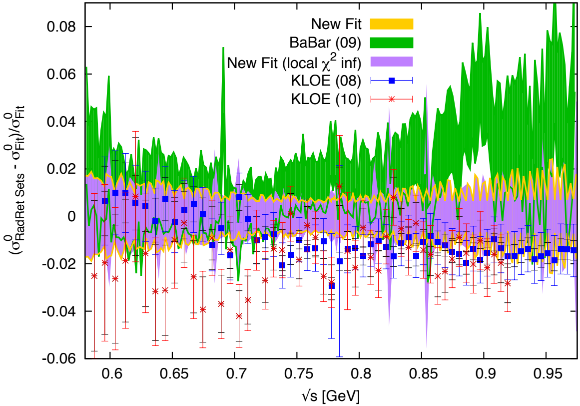
Because of parity violation in weak decay of muon, a correlation between muon spin and decay electron direction exists, which can be used to measure the anomalous precession frequency ω_a:

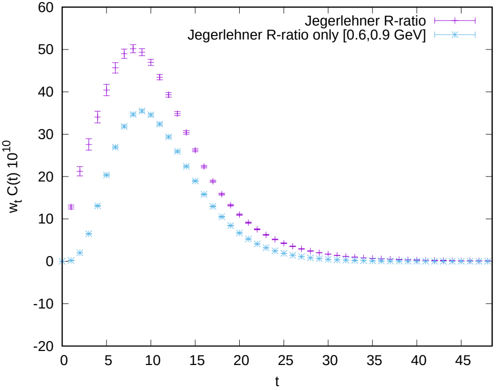


BESIII 2015 update:



Hagiwara et al. 2011:





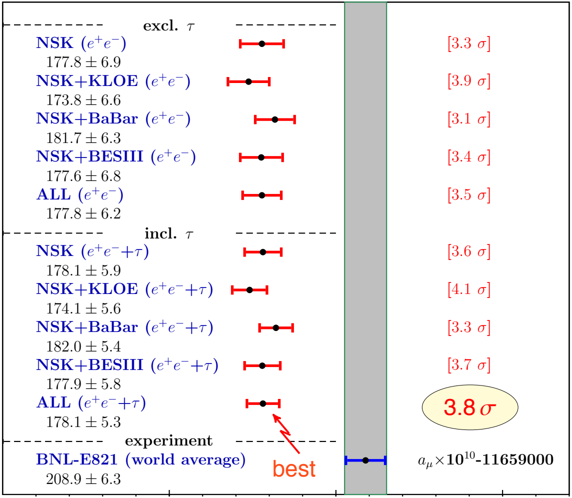
Problematic experimental region can readily be replaced by precise lattice data. Lattice also can be arbiter regarding different experimental data sets.

Jegerlehner FCCP2015 summary:

final state	range (GeV)	$a_\mu^{\mathrm{had}(1)} \times 10^{10}$ (stat) (syst) [tot]	rel	abs
ρ	(0.28, 1.05)	507.55 (0.39) (2.68)[2.71]	0.5%	39.9%
ω	(0.42, 0.81)	35.23 (0.42) (0.95)[1.04]	3.0%	5.9%
ϕ	(1.00, 1.04)	34.31 (0.48) (0.79)[0.92]	2.7%	4.7%
J/ψ		8.94 (0.42) (0.41)[0.59]	6.6%	1.9%
Υ		0.11 (0.00) (0.01)[0.01]	6.8%	0.0%
had	(1.05, 2.00)	60.45 (0.21) (2.80)[2.80]	4.6%	42.9%
had	(2.00, 3.10)	21.63 (0.12) (0.92)[0.93]	4.3%	4.7%
had	(3.10, 3.60)	3.77 (0.03) (0.10)[0.10]	2.8%	0.1%
had	(3.60, 9.46)	13.77 (0.04) (0.01)[0.04]	0.3%	0.0%
had	(9.46,13.00)	1.28 (0.01) (0.07)[0.07]	5.4%	0.0%
pQCD	(13.0,∞)	1.53 (0.00) (0.00)[0.00]	0.0%	0.0%
data	(0.28,13.00)	687.06 (0.89) (4.19)[4.28]	0.6%	0.0%
total		688.59 (0.89) (4.19)[4.28]	0.6%	100.0%

Results for $a_\mu^{\mathrm{had}(1)} \times 10^{10}$. Update August 2015, incl SCAN[NSK]+ISR[KLOE10,KLOE12,BaBar,BESIII]

Jegerlehner FCCP2015 summary ($\tau \leftrightarrow e^+e^-$):



Our setup:

$$C(t) = \frac{1}{3V} \sum_{j=0,1,2} \sum_{t'} \langle \mathcal{V}_j(t+t')\mathcal{V}_j(t') \rangle_{\mathrm{SU(3)}} \quad (3)$$

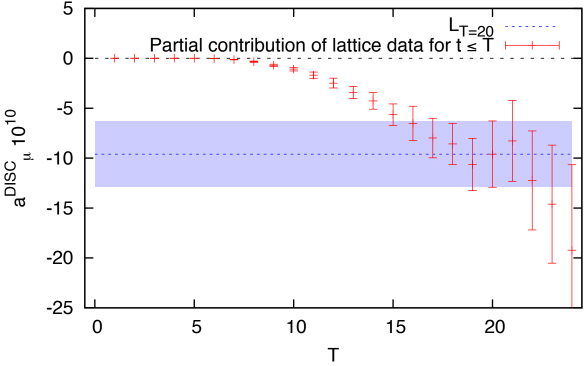
where V stands for the four-dimensional lattice volume, $\mathcal{V}_\mu = (1/3)(\mathcal{V}_\mu^{u/d} - \mathcal{V}_\mu^s)$, and

$$\mathcal{V}_\mu^f(t) = \sum_{\vec{x}} \mathrm{Im}\,\mathrm{Tr}[D^{-1}_{\vec{x},t;\vec{x},t}(m_f)\gamma_\mu]\,. \quad (4)$$

We separate 2000 low modes (up to around m_s) from light quark propagator as $D^{-1} = \sum_n v^n (w^n)^\dagger + D^{-1}_{\mathrm{high}}$ and estimate the high mode stochastically and the low modes as a full volume average Foley 2005.

We use a sparse grid for the high modes similar to Li 2010 which has support only for points x_μ with $(x_\mu - x_\mu^{(0)}) \bmod N = 0$; here we additionally use a random grid offset $x_\mu^{(0)}$ per sample allowing us to stochastically project to momenta.

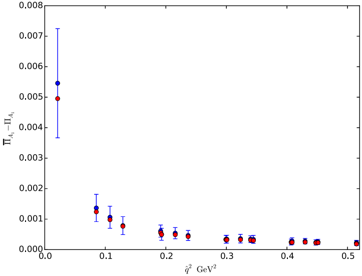
Study $L_T = \sum_{t=T+1}^{\infty} w_t C(t)$ and use value of T in plateau region (here $T = 20$) as central value. Use a combined estimate of a resonance model and the two-pion tail to estimate systematic uncertainty.

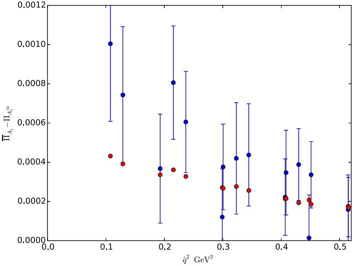


Combined with an estimate of discretization errors, we find

$$a_\mu^{\mathrm{HVP\ (LO)\ DISC}} = -9.6(3.3)_{\mathrm{stat}}(2.3)_{\mathrm{sys}} \times 10^{-10} \,. \qquad (5)$$

From Aubin et al. 2015 (arXiv:1512.07555v2)



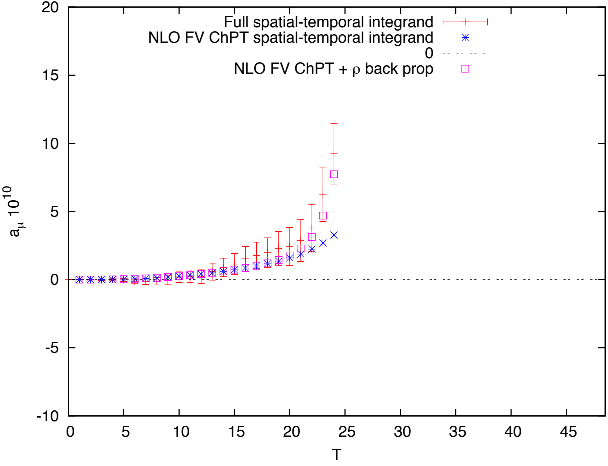


MILC lattice data with $m_\pi L = 4.2$, $m_\pi \approx 220$ MeV; Plot difference of $\Pi(q^2)$ from different irreps of 90-degree rotation symmetry of spatial components versus NLO FV ChPT prediction (red dots)

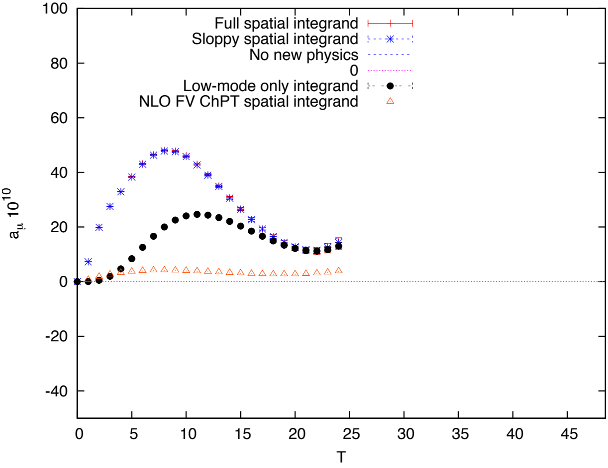
While the absolute value of a_μ is poorly described by the two-pion contribution, the volume dependence may be described sufficiently well to use ChPT to control FV errors at the 1% level; this needs further scrutiny

Aubin et al. find an $O(10\%)$ finite-volume error for $m_\pi L = 4.2$ based on the $A_1 - A_1^{44}$ difference (right-hand plot)

Compare difference of integrand of $48 \times 48 \times 96 \times 48$ (spatial) and $48 \times 48 \times 48 \times 96$ (temporal) geometries with NLO FV ChPT $(A_1 - A_1^{44})$:



$m_\pi = 140$ MeV, $p^2 = m_\pi^2/(4\pi f_\pi)^2 \approx 0.7\%$



Full spatial integrand
Sloppy spatial integrand
No new physics
0
Low-mode only integrand
NLO FV ChPT spatial integrand
$a_\mu\ 10^{10}$
T
100
80
60
40
20
0
-20
-40
0
5
10
15
20
25
30
35
40
45