A NEW APPROACH TO SCATTERING AMPLITUDES

Neil Christensen

Illinois State University

in collaboration with

Cory Russ and Santiago Pinto, ISU Students

Joshua Henderson, University High School Student

Scattering Amplitudes from Eigenstates

- With great improvements in computational power over the years, perhaps we are at a point that we can directly diagonalize the Hamiltonian and calculate the scattering amplitude directly from the eigenstates.

$$S_{\alpha,\beta} = \langle \Psi_\alpha, \Psi_\beta \rangle$$

The Hamiltonian

- In order to do this, we need to first Legendre transform the Lagrangian to the Hamiltonian, plug in the field definition in terms of the creation and annihilation operators, and discretize momentum space. For example, for $\lambda\phi^4$ theory,

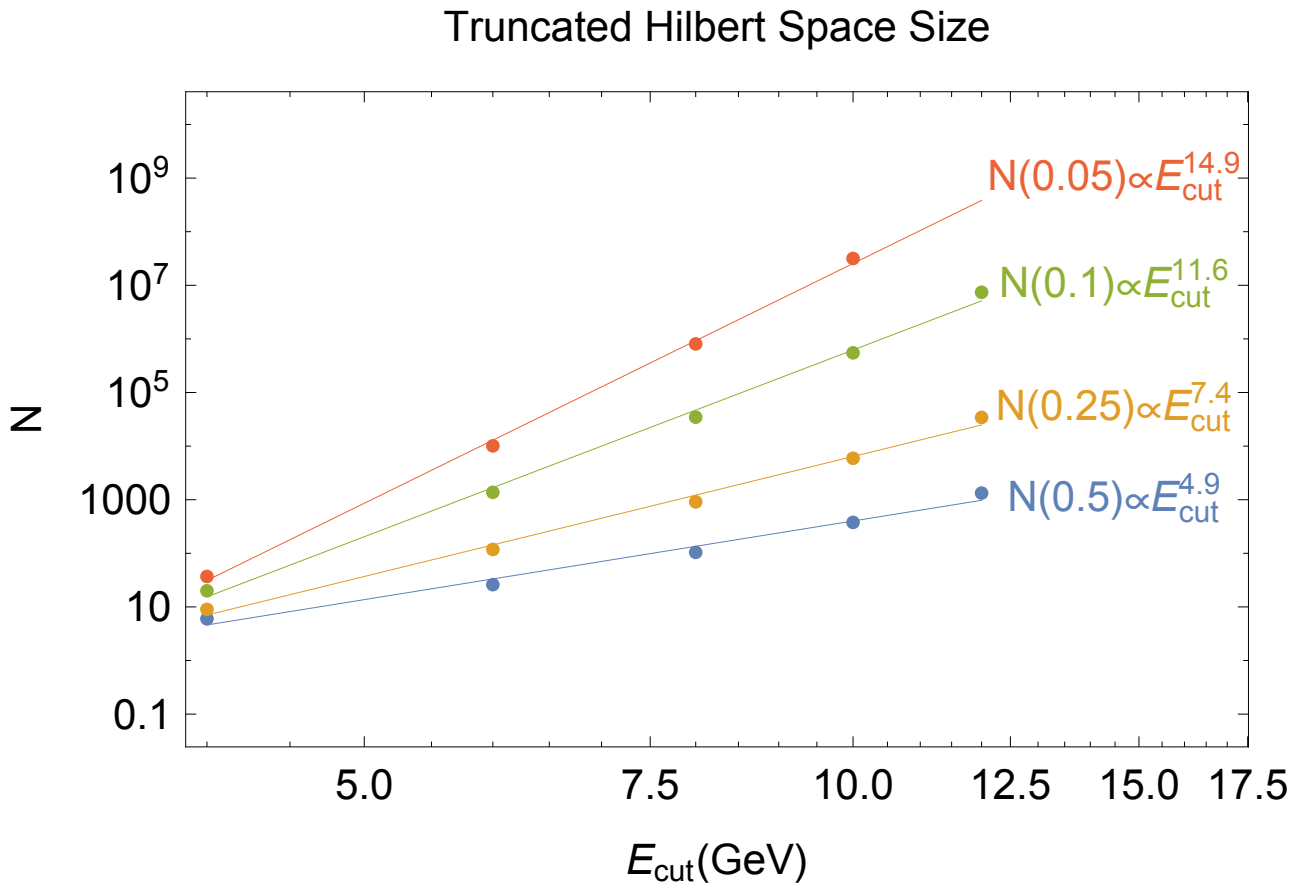
$$H = \sum_p \omega a_p^\dagger a_p + \frac{\lambda \Delta p}{96} \sum_{p_1+p_2+p_3+p_4=0} \Big[a_{p_1} a_{p_2} a_{p_3} a_{p_4}$$

$$+4a^\dagger_{-p_1} a_{p_2} a_{p_3} a_{p_4} + 6a^\dagger_{-p_1} a^\dagger_{-p_2} a_{p_3} a_{p_4}$$

$$+4a^\dagger_{-p_1} a^\dagger_{-p_2} a^\dagger_{-p_3} a_{p_4} + a^\dagger_{-p_1} a^\dagger_{-p_2} a^\dagger_{-p_3} a^\dagger_{-p_4} \Big]$$

Hilbert Space Size

- Even after discretizing momentum space, the Hilbert space is still infinite dimensional. If we used an energy cutoff, it would still grow way too quickly.



Important Basis States

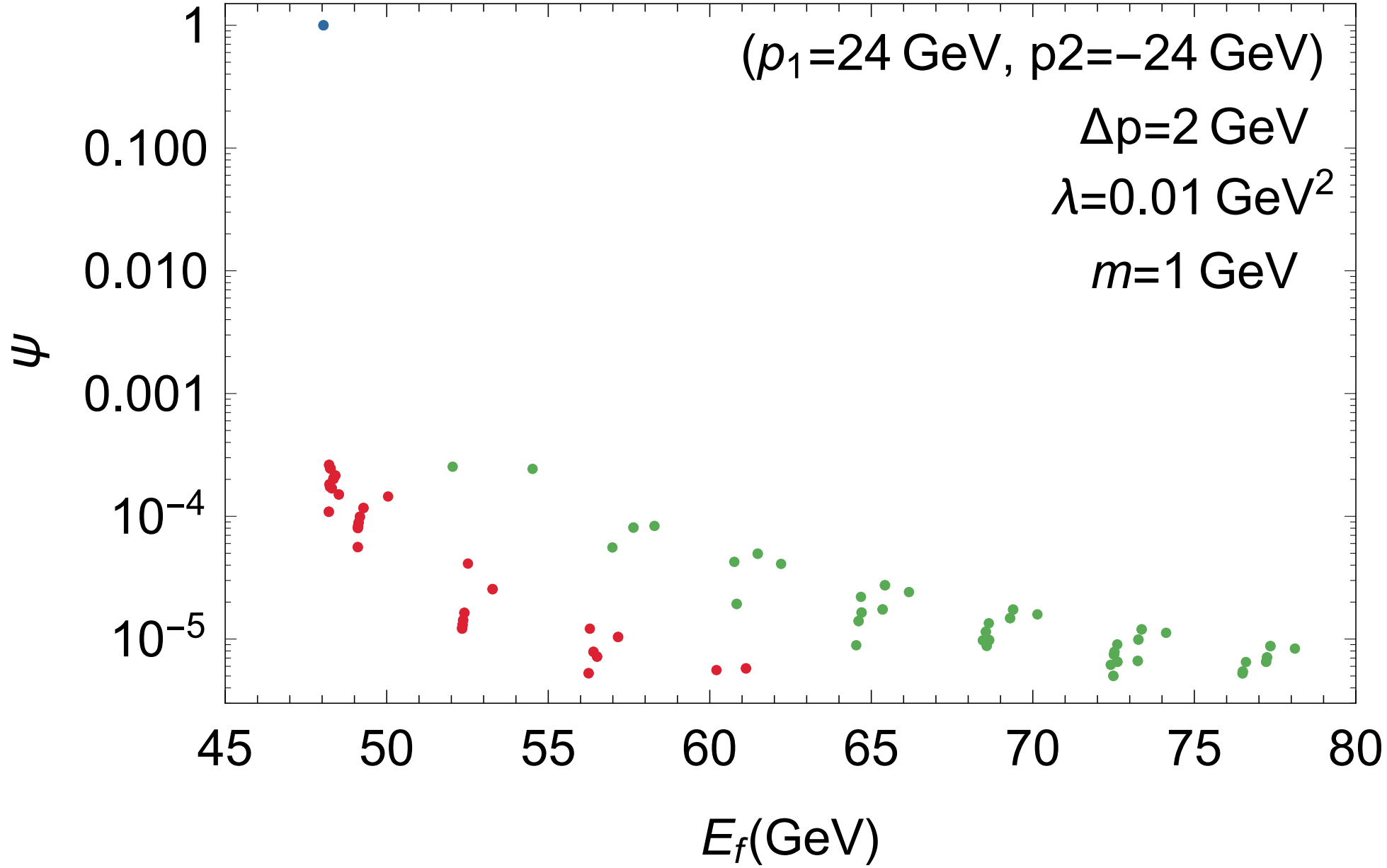
- Although the Hilbert space is way too big to diagonalize directly, only a very small handful of the basis states are important to a particular eigenstate.
- If we could determine just the important basis states for a particular scattering state, then we could diagonalize the much, much smaller reduced Hilbert space with just the important basis states.

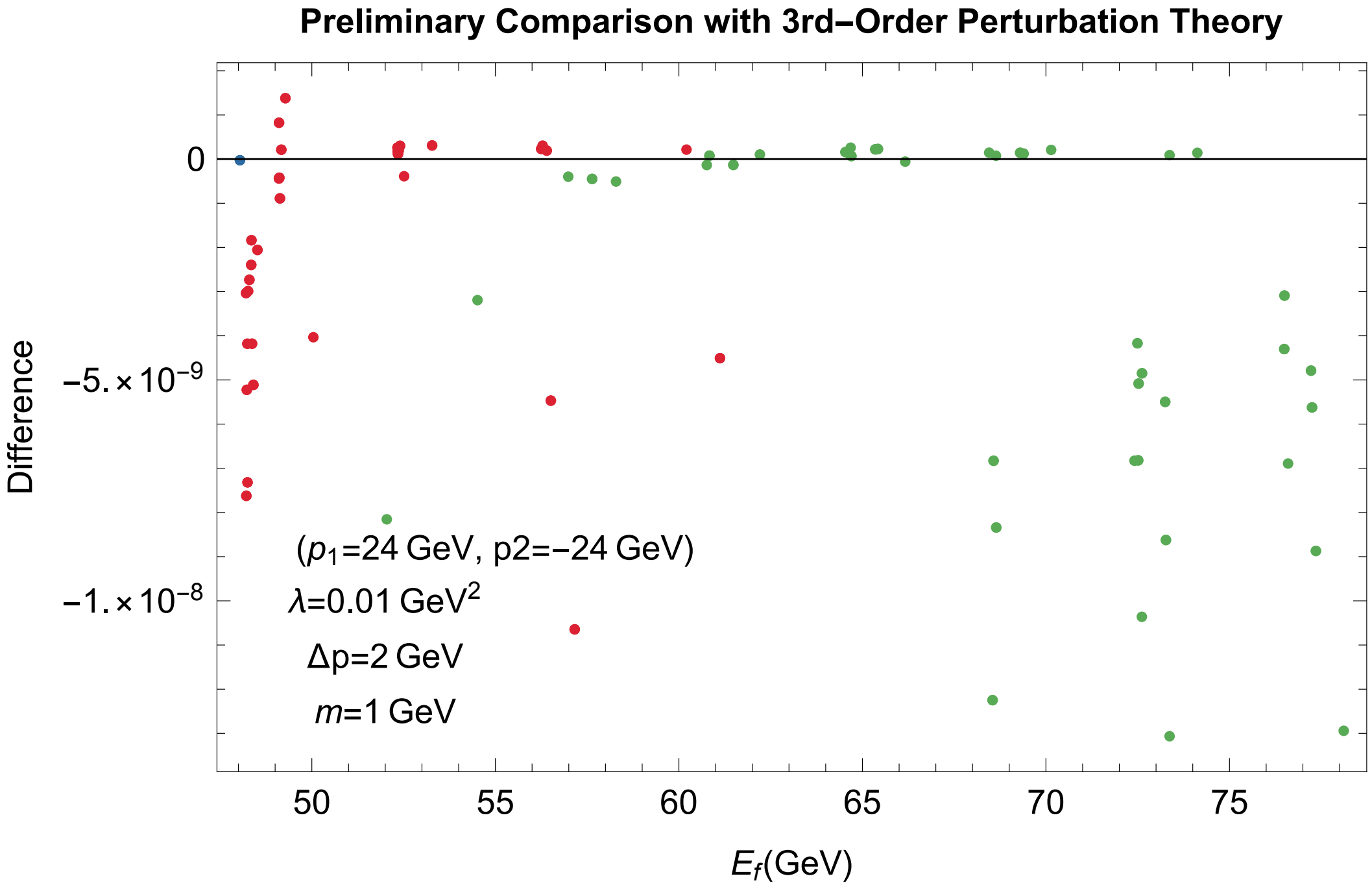
$$\Psi(12,-12) = 0.999999|12,-12\rangle$$
$$-0.000262|12,-5,-4,-3\rangle - 0.000262|-12,5,4,3\rangle$$
$$-0.000254|12,0,0,0,0,12\rangle + \cdots$$

Cyclic Random Process

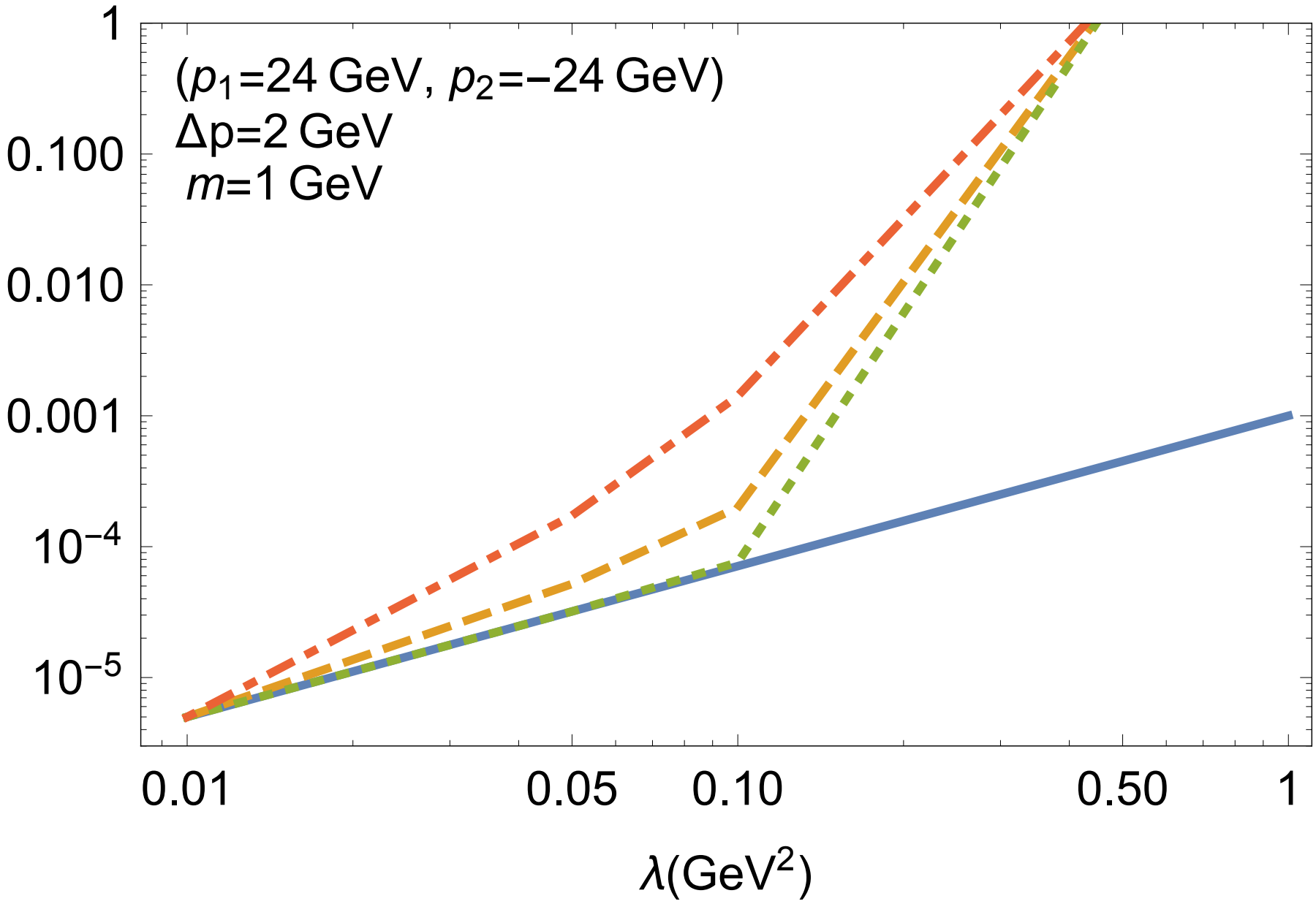
- Instead, we use a random approach based on Physics Letters B503 (2001) 223-235.
  1. Begin with the lowest order perturbative solution.
     - Quick and a good seed for our method
  2. Remove basis states below cutoff. Call remaining basis states the reduced Hilbert space.
     - The cutoff sets the precision of the calculation for that dp.
  3. Randomly add new basis states to the reduced Hilbert space.
     - Choose a random basis state from the reduced Hilbert space. Act on it with a random operator from the Hamiltonian. This produces a random new basis state "close" to the reduced Hilbert space.
  4. Diagonalize the Hamiltonian in the new reduced Hilbert space.
  5. Repeat steps 2-4 until the eigenstate is achieved.

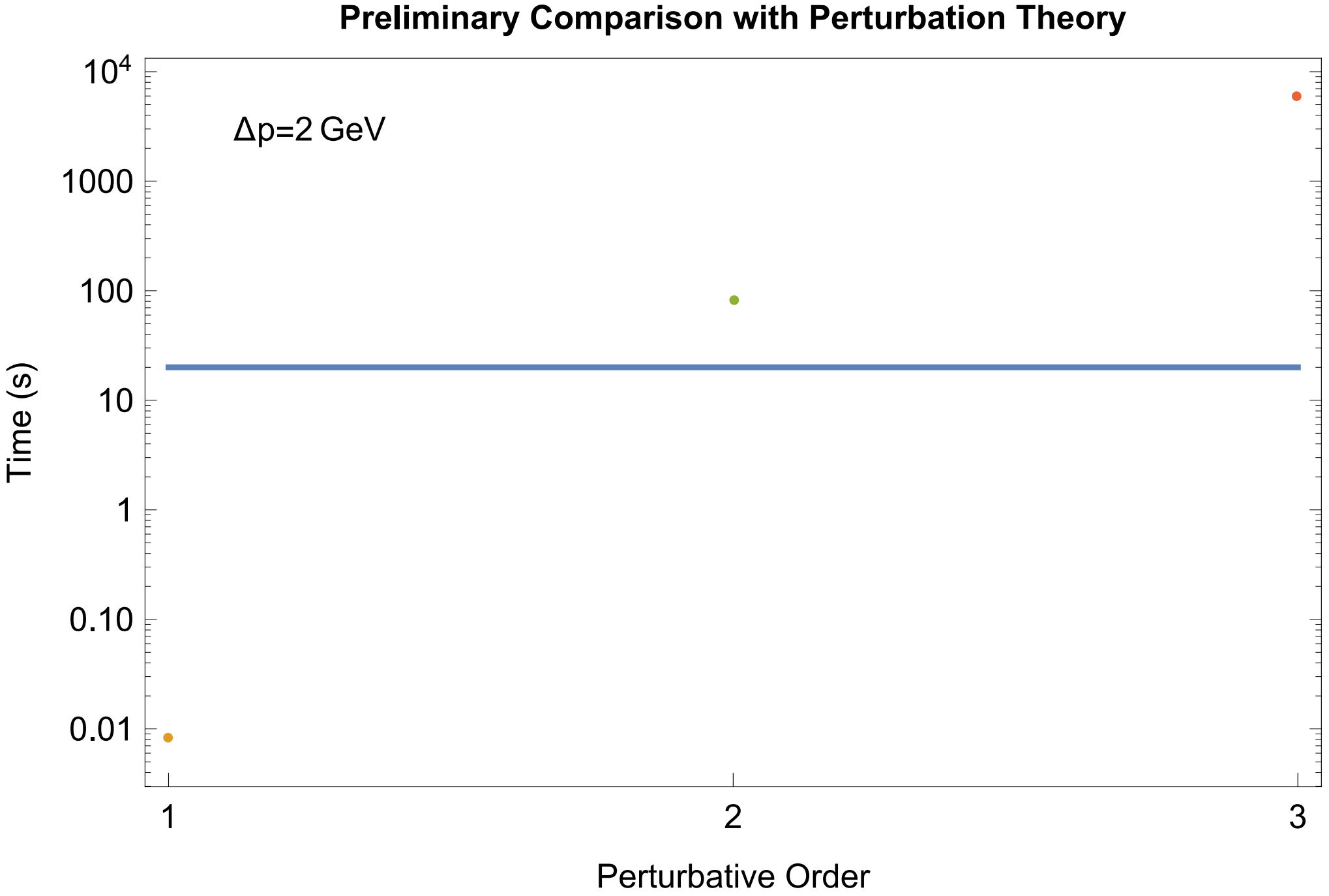
Preliminary State Wavefunction



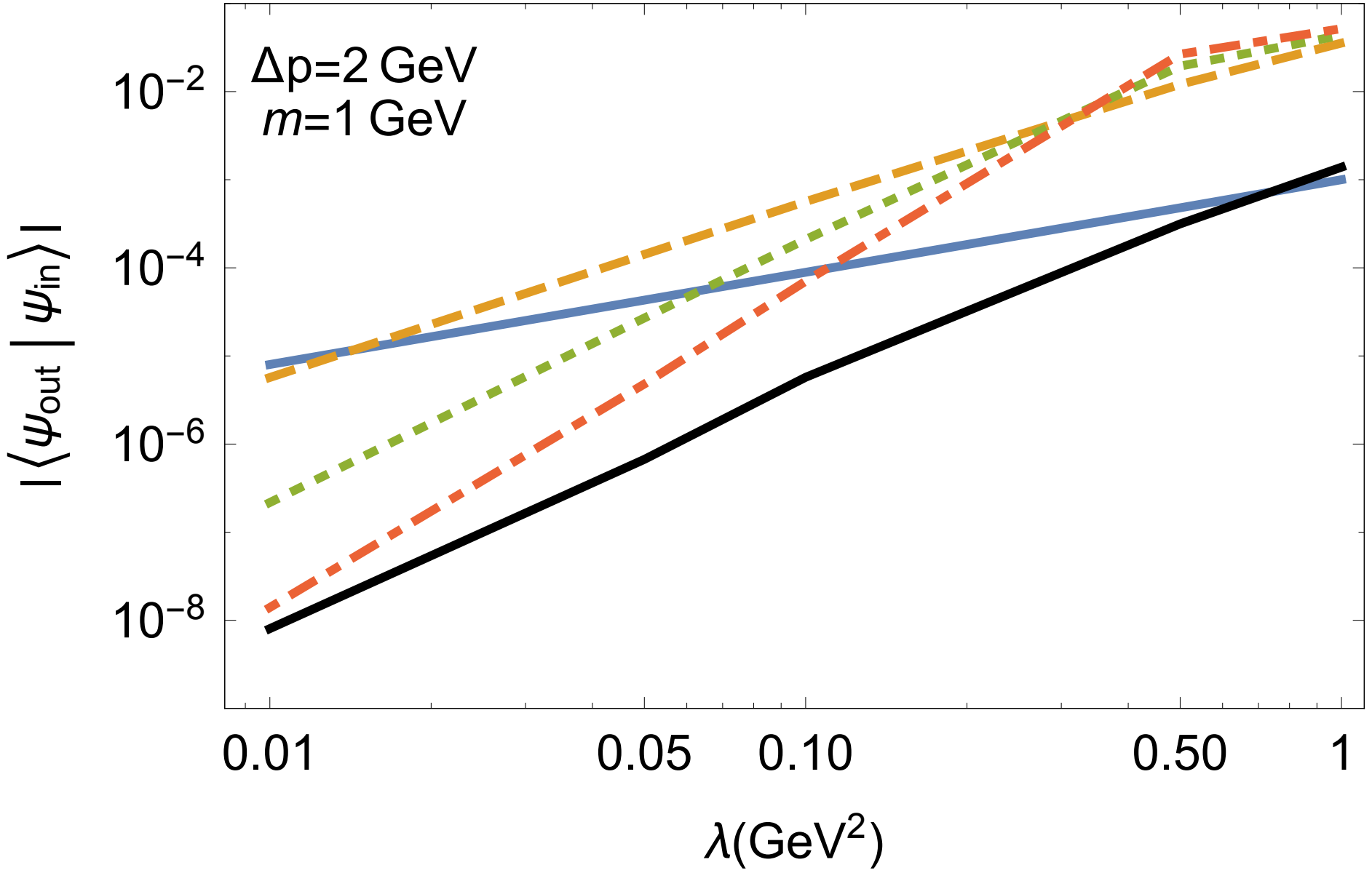

Preliminary Comparison with 3rd–Order Perturbation Theory
0
−5.×10^−9
−1.×10^−8
Difference
(p1=24 GeV, p2=−24 GeV)
λ=0.01 GeV^2
Δp=2 GeV
m=1 GeV
50
55
60
65
70
75
Ef(GeV)

Preliminary Comparison with Perturbation Theory



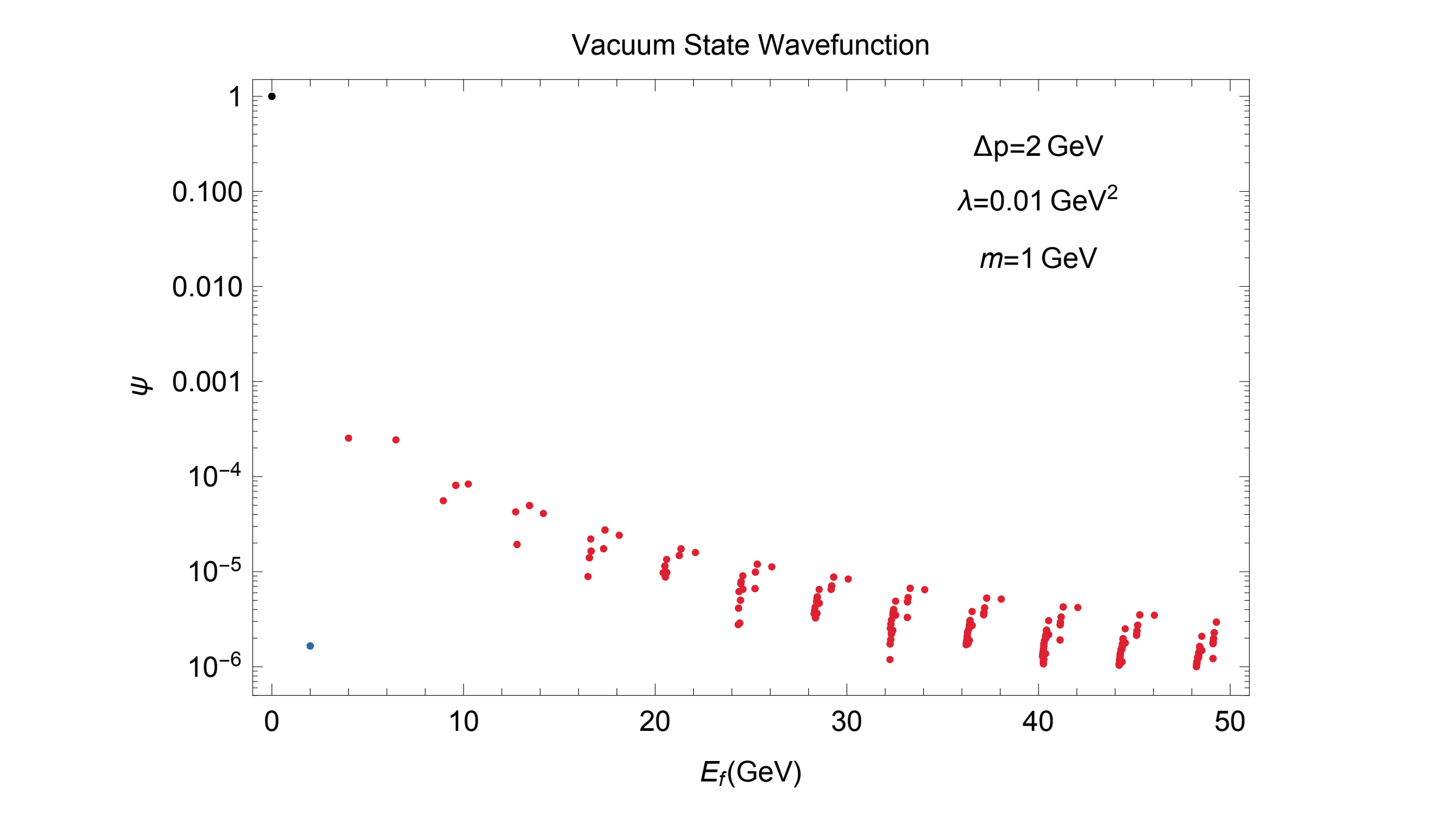

Preliminary Comparison with Perturbation Theory
Δp=2 GeV
Time (s)
10^4
1000
100
10
1
0.10
0.01
1
2
3
Perturbative Order

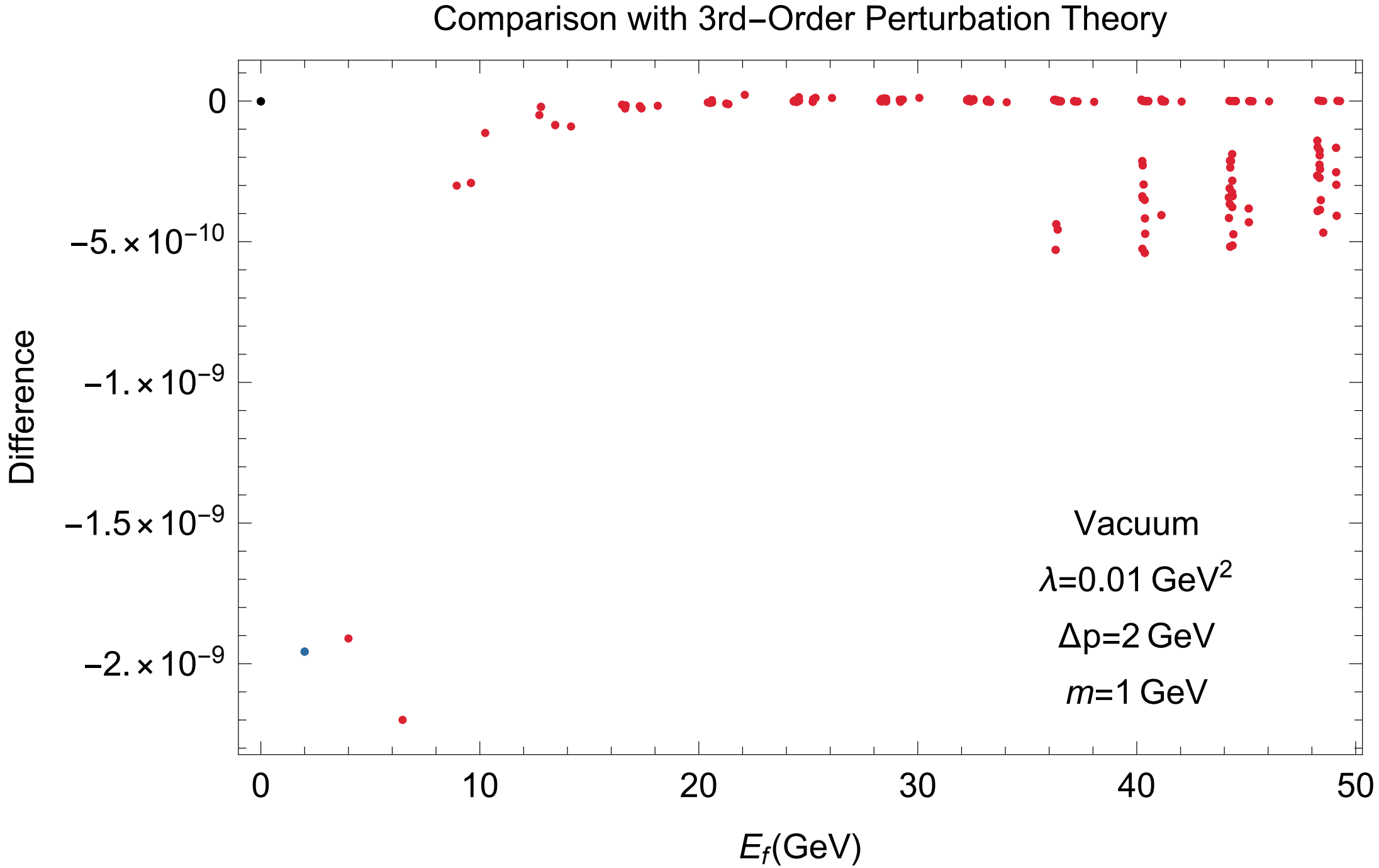
Preliminary Scattering Amplitude

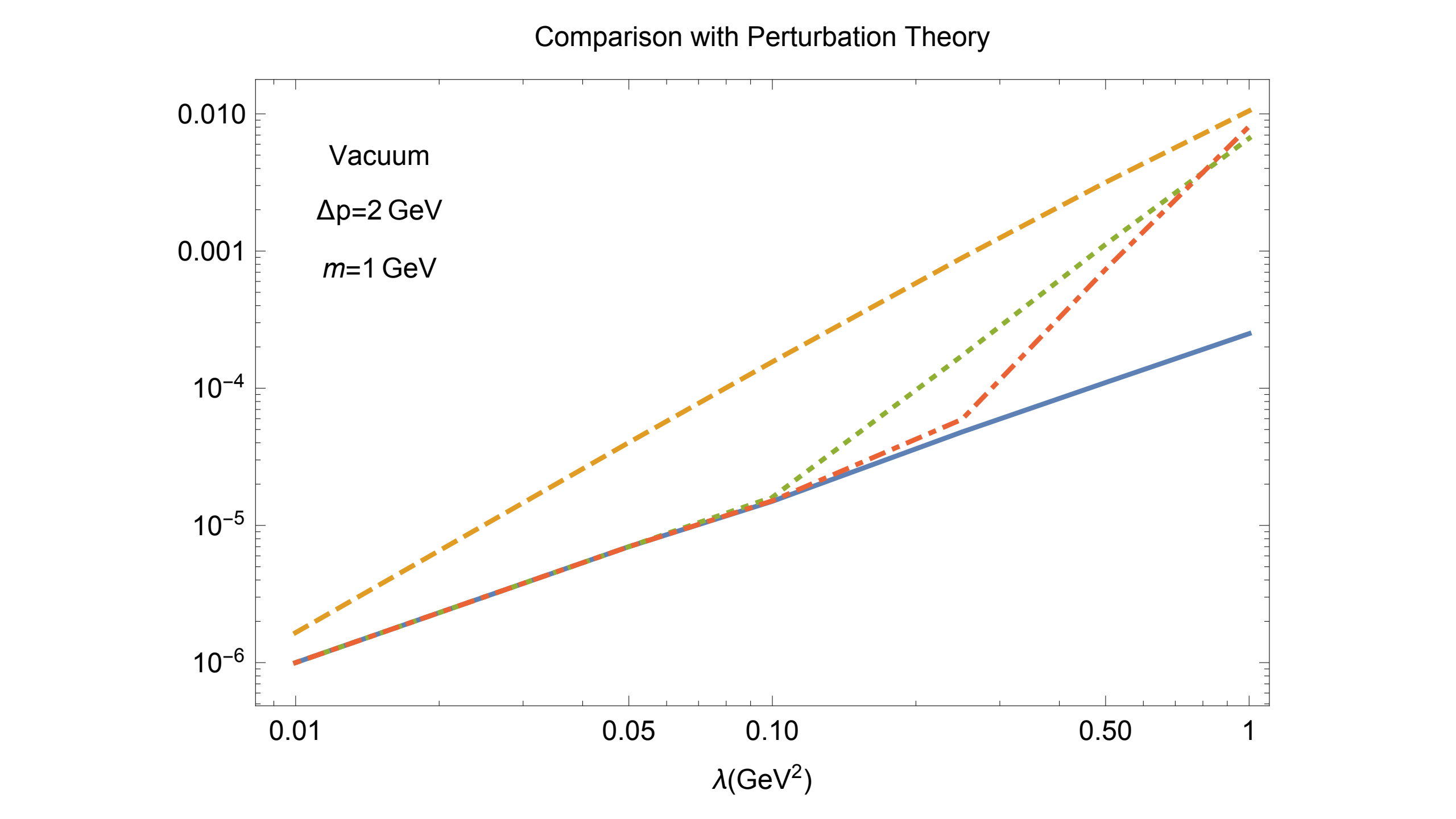


Conclusions

- We are exploring a new method for calculating the scattering amplitude.
- We use a cyclic random procedure for determining the most important basis states for a scattering eigenstate.
- We have shown preliminary results for a scattering eigenstate and for an inner product between eigenstates (the S-Matrix).
- We have compared our results with perturbation theory. Higher orders in perturbation theory get closer and closer to our results.
- Our code appears to be more efficient than higher orders of perturbation theory.


Vacuum State Wavefunction
Δp=2 GeV
λ=0.01 GeV²
m=1 GeV
ψ
1
0.100
0.010
0.001
10⁻⁴
10⁻⁵
10⁻⁶
0
10
20
30
40
50
E_f(GeV)


Comparison with 3rd−Order Perturbation Theory
0
−5.×10−10
−1.×10−9
−1.5×10−9
−2.×10−9
Difference
0
10
20
30
40
50
Ef(GeV)
Vacuum
λ=0.01 GeV2
Δp=2 GeV
m=1 GeV


Comparison with Perturbation Theory
Vacuum
Δp=2 GeV
m=1 GeV
0.010
0.001
10^−4
10^−5
10^−6
0.01
0.05
0.10
0.50
1
λ(GeV^2)