Black holes in expanding universe

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Introduction:

- The existence of Primordial black holes(PBH) is critical regarding the evolution of early universe.

- Primary process behind their formation is the gravitational collapse of density inhomogeneity in the early universe.

- According to Hoop conjecture, particles collision can lead to the formation of black holes if they are confined within their own Schwarzschild radius in the center of mass energy frame.

- We applied hoop conjecture in the Friedman- Robertson-Walker(FRW) space time to determine the number density of black hole(n_{bh}) produced and compared it with number density of the radiation(n_{rad}) at a given energy scale defined by the temperature of the system.

Introduction(contd.):

- Under present conditions, black hole formation through particle collision is sub dominant due to the lack of highly energetic particles.

- Hot and dense nature led to significantly higher probability of black hole formation through particle collision in the early universe.

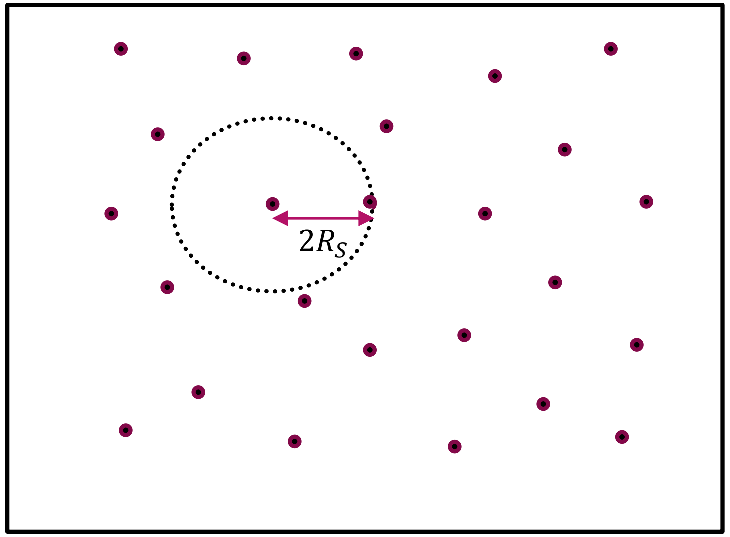
- However, due to large Hubble constant, black hole formation should have been more difficult in early universe than in flat space-time.

- Do black holes produced by particle collisions become significant at high energy scale ?

Formalism:

- Consider a massless bosonic gas at a given volume and temperature. The center of mass energy of two particles colliding with momentum k_1 and k_2 is given by

$$M = \sqrt{2k_1k_2\,(1 - \cos\theta)}$$



- The probability of particles colliding within impact parameter R is

$$dP = \frac{4\,\pi}{3}(R)^3 f(k_2)\frac{d^3k_2}{(2\pi\,)^3}$$

- To form a black hole through collision, the impact parameter $R < 2\,R_S$.

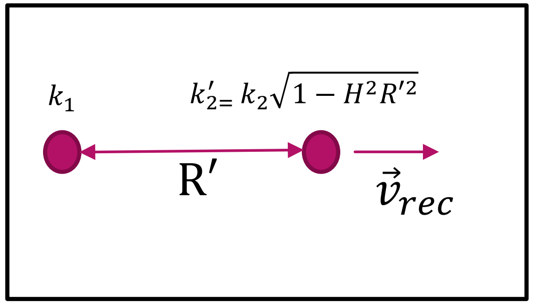
Formalism(contd.):

- The FRW metric can be written as

$$ds^2 = -(1 - H^2 r^2)dt^2 + dr^2 - (2Hr)drdt$$

Where $\vec{r} = a(t)\,\vec{x}$

- The expansion of space-time leads to the redshift in energy of particles.



- Modified hoop conjecture takes the form

$$R' = 2\,R_S\sqrt{\left(\sqrt{1 + H^4 R_S^4} - 2\,H^2 R_S^2\right)}$$

Slow expansion limit

$$R' = 2\,R_s\sqrt{1 - 2H^2 R_S^2}$$

$$\frac{M'}{R'} = \frac{M}{2\,R_S}$$

Formalism(contd.):

- The number density of the black hole can be written as

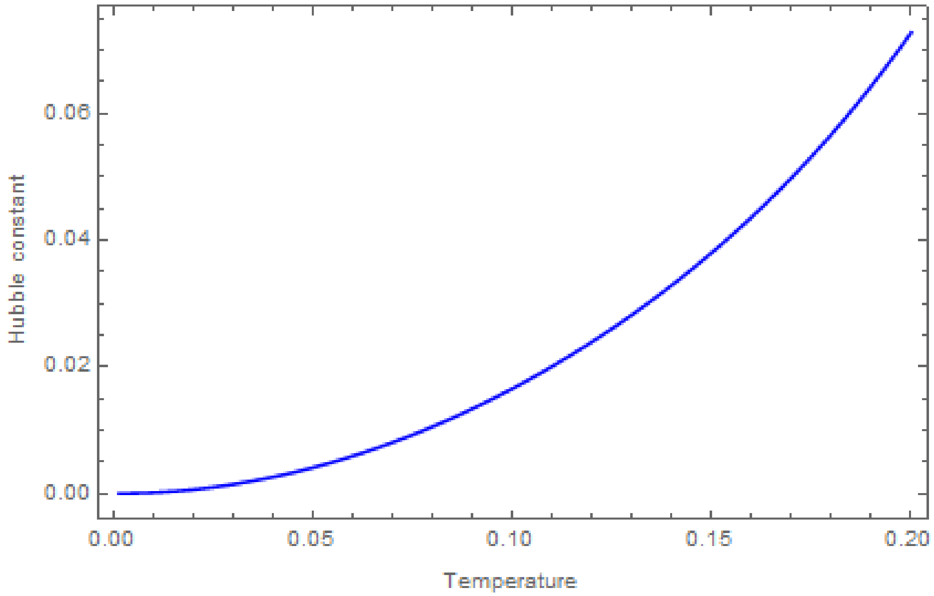
$$n_{bh} = \frac{1}{(2\,\pi)^6} \int \frac{4\,\pi}{3} (R)^3 f(k_1) f(k_2) d^3k_1\, d^3k_2$$

- In slow expansion limit, this integral can be done exactly to analytic form of n_{bh} as

$$n_{bh} = \frac{1470\,\zeta\,({}^{9}/{}_{2})^2}{\pi^2}\,T^9 \;-\; \frac{1020600\,\zeta({}^{11}/{}_{2})^2}{\pi^2}\,H^2\,T^{11}$$

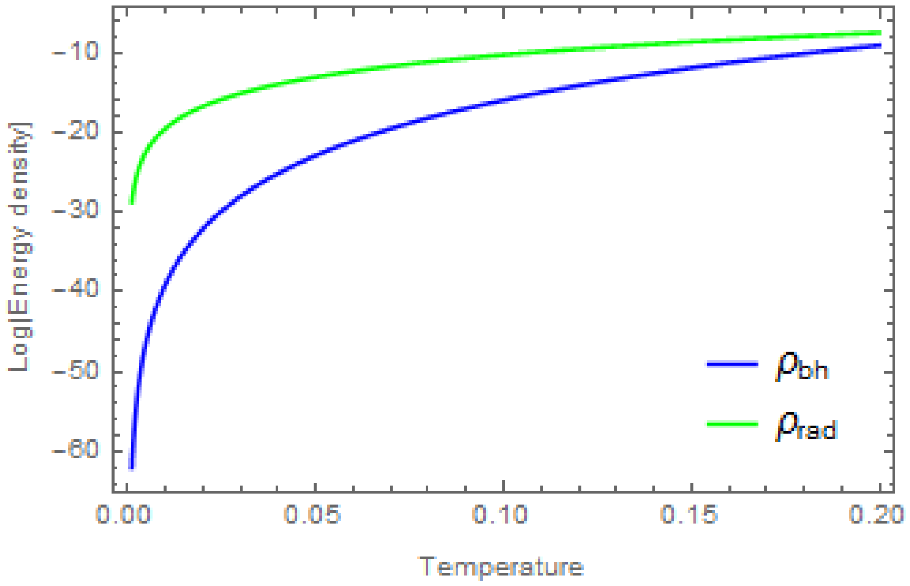
- Friedman Equation can be used to eliminate H leaving temperature as the only free variable of the system.

$$H^2 = \frac{8\,\pi}{3}(\rho_{bh} + \rho_{rad}) \quad \longrightarrow \quad H = \sqrt{\frac{a_1\,T^4 + a_2\,T^{10}}{\frac{3}{8\pi} - b_1 T^{12}}}$$

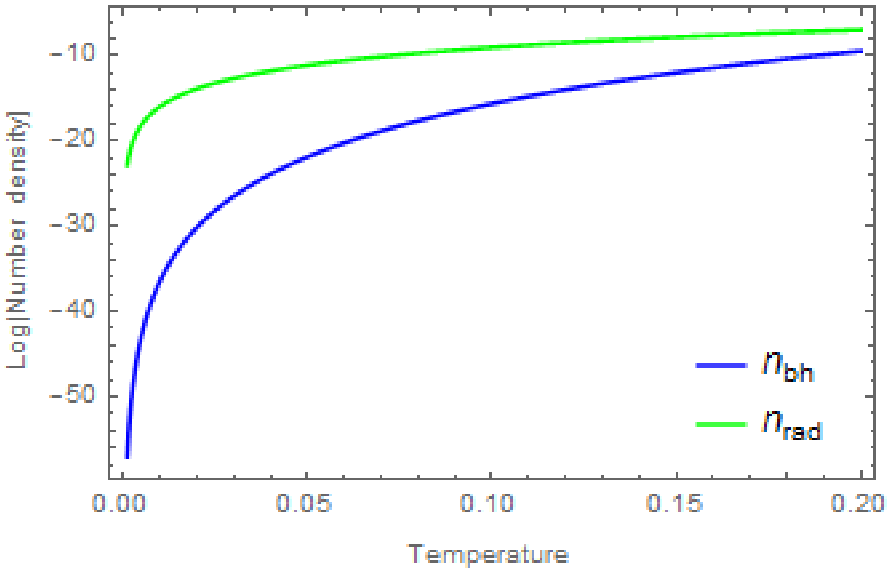


Results:

Log[Energy density] vs Temperature



Log[Number density] vs Temperature



Constraints on black holes:

- We forbade the formation of sub-Planckian black holes, because formalism cannot be applied at Planck scale.

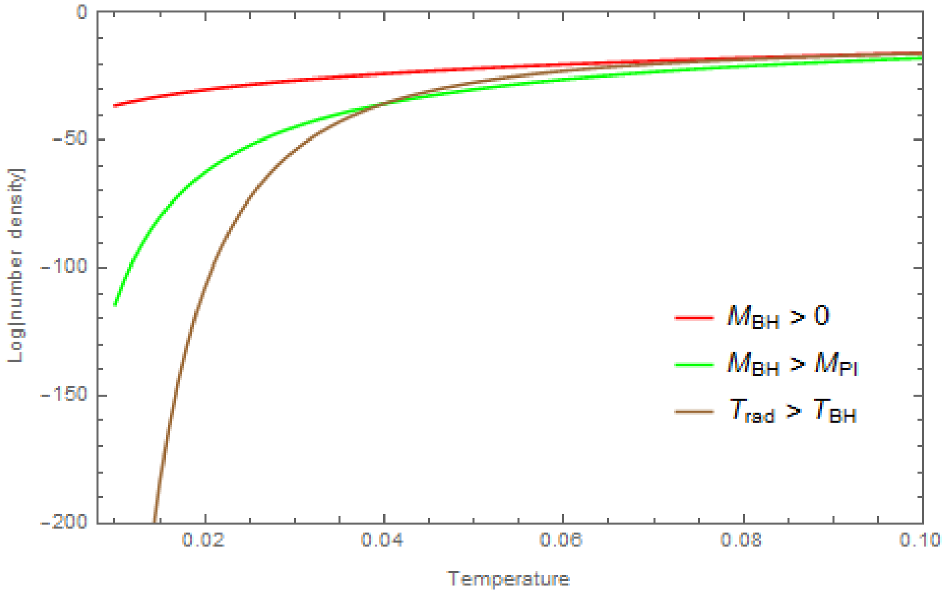
- To create sustainable black holes, temperature of the black hole due to Hawking radiation should be lower than the temperature of background radiation.

$$M_{bh} > M_{pl} \qquad\qquad T_{bh} < T_{rad}$$

- These constraints translate to the constraints on the variable k_1, k_2 and θ as

$$M_{bh} > M_{pl}: \quad 1 - \cos\theta > \frac{1}{2\,k_1 k_2} \quad \text{and} \quad k_2 > \frac{1}{4\,k_1}$$

$$T_{bh} < T_{rad}: \quad 1 - \cos\theta > \frac{1}{128\,\pi^2 k_1 k_2 T^2} \quad \text{and} \quad k_2 > \frac{1}{256\pi^2\,k_1 T^2}$$



Conclusion:

- In early universe, number density of black hole through hoop conjecture is significant.
- But black holes never dominate the system at any temperature in the explored regime due to the rapid expansion of the space-time.
- As universe cooled down, thermodynamically stable black holes became unfeasible.

Thank You!