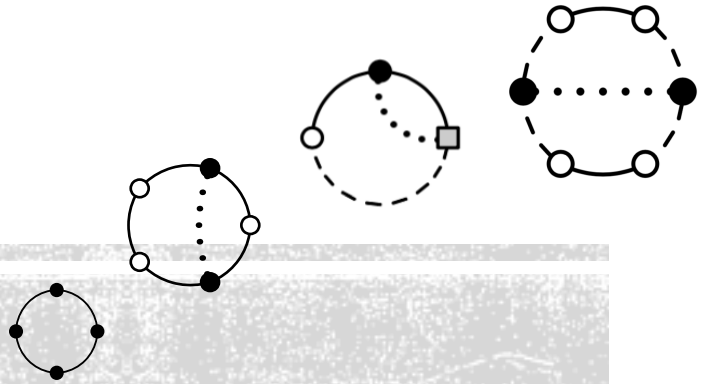

Covariant diagrams for one-loop matching & SUSY threshold corrections

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Zhengkang Zhang (University of Michigan)

Based on:

- ***ZZ [arXiv: 1610.00710]***
- ***Wells, ZZ [work in progress]***

See also:

- ***Ellis, Quevillon, You, ZZ, Phys.Lett.B762,166 [arXiv: 1604.02445] and to appear***

The Effective Field Theory (EFT) matching problem

- Given $\mathcal{L}_{\rm UV}[\varphi_H, \varphi_L]$ with Heavy fields φ_H and Light fields φ_L satisfying $m^2_{\varphi_H} \gg |m^2_{\varphi_L}|$, at low energy $E \ll m_{\varphi_H}$,

$$\boxed{\mathcal{L}_{\rm EFT}[\varphi_L] = ?}$$

- Conventional approach: Feynman diagrams.
  - Calculate scattering amplitudes/correlation functions with Feynman diagrams in both UV theory and EFT.
  - Equate the results and solve for EFT operator coefficients.
- But there is a *more direct (and more elegant) approach*!

EFT matching from path integral — tree level

- Let's look at the path integral.

$$\int [D\varphi_H][D\varphi_L]\, e^{i\int d^d x\, \mathcal{L}_{\rm UV}[\varphi_H,\varphi_L]} = \int [D\varphi_L]\, e^{i\int d^d x\, \mathcal{L}_{\rm EFT}[\varphi_L]}$$

- Tree-level = stationary point approximation.

$$\mathcal{L}_{\rm EFT}^{\rm tree}[\varphi_L] = \mathcal{L}_{\rm UV}\big[\varphi_{H,\rm c}[\varphi_L], \varphi_L\big]$$

where $\varphi_{H,\rm c}$ solves classical equations of motion:

$$\left.\frac{\delta \mathcal{L}_{\rm UV}}{\delta \varphi_H}\right|_{\varphi_H=\varphi_{H,\rm c}} = 0$$

EFT matching from path integral — one-loop level

- Background field method: $\varphi_H = \varphi_{H,\mathrm{b}} + \varphi'_H\,, \quad \varphi_L = \varphi_{L,\mathrm{b}} + \varphi'_L$

$$\begin{aligned}\mathcal{L}_{\mathrm{UV}}[\varphi_H, \varphi_L] + J_L \varphi_L \quad &= \quad \mathcal{L}_{\mathrm{UV}}\big[\varphi_{H,\mathrm{c}}[\varphi_{L,\mathrm{b}}], \varphi_{L,\mathrm{b}}\big] + J_L \varphi_{L,\mathrm{b}} \\ &\quad -\frac{1}{2}\big(\varphi_H'^T \;\; \varphi_L'^T\big)\, \boxed{\mathcal{Q}_{\mathrm{UV}}\big[\varphi_{H,\mathrm{c}}[\varphi_{L,\mathrm{b}}], \varphi_{L,\mathrm{b}}\big]} \begin{pmatrix}\varphi'_H \\ \varphi'_L\end{pmatrix} \\ &\quad +\mathcal{O}(\varphi'^3)\end{aligned}$$

- If we were to compute the 1(L)PI effective action (Legendre transform of the path integral),

$$\Gamma^{\text{1-loop}}_{\mathrm{L,UV}}[\varphi_{L,\mathrm{b}}] = i\, c_s\, \log\det \boxed{\mathcal{Q}_{\mathrm{UV}}\big[\varphi_{H,\mathrm{c}}[\varphi_{L,\mathrm{b}}], \varphi_{L,\mathrm{b}}\big]}$$

  - c_s is spin factor (= +1/2 for real scalar, -1/2 for Weyl fermion)

EFT matching from path integral — one-loop level

- But instead, we are interested in the low-energy ***local*** effective Lagrangian.
- After careful manipulations of the functional determinant, we can show ***(ZZ [1610.00710])*** —

$$\boxed{\int d^d x\, \mathcal{L}_{\rm EFT}^{\text{1-loop}}[\varphi_L] = \Gamma_{\rm L,UV}^{\text{1-loop}}[\varphi_L]\Big|_{\rm hard} = i\, c_s\ \log\det \mathcal{Q}_{\rm UV}\left[\varphi_{H,\rm c}[\varphi_L], \varphi_L\right]\Big|_{\rm hard}}$$

- Expansion by regions: expand integrand before integrating.
- Full integral = hard region + soft region contributions.
  - Hard region: $|q^2| \sim m_{\varphi_H}^2 \gg |m_{\varphi_L}^2|$ *(q = loop momentum)*
  - Soft region: $|q^2| \sim |m_{\varphi_L}^2| \ll m_{\varphi_H}^2$

See e.g. Beneke, Smirnov, hep-ph/9711391; Jantzen, 1111.2589.

EFT matching from path integral — one-loop level

$$\int d^d x\, \mathcal{L}_{\rm EFT}^{\text{1-loop}}[\varphi_L] = \Gamma_{\rm L,UV}^{\text{1-loop}}[\varphi_L]\Big|_{\rm hard} = i\, c_s\ \log\det \mathcal{Q}_{\rm UV}\left[\varphi_{H,\rm c}[\varphi_L], \varphi_L\right]\Big|_{\rm hard}$$

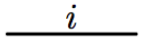
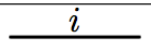
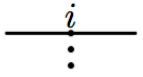
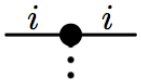
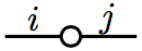
- How to compute this functional determinant (in a nice way)?
  - **Covariant derivative expansion (CDE)**: expansion in $m_{\varphi_H}^{-1}$ that is ***gauge-covariant*** — we never separate D_μ into ∂_μ and $-igA_\mu$.
    - *Gaillard [Nucl.Phys.B268,669 (1986)]; Chan [Phys.Rev.Lett.57,1199 (1986)]; Cheyette [Nucl.Phys.B297,183 (1988)];*
    - ***Henning, Lu, Murayama [1412.1837]**; Chiang, Huo [1505.06334]; Huo [1506.00840, 1509.05942]; **Drozd, Ellis, Quevillon, You [1512.03003]**.*
  - Debate: ***Is this CDE approach general enough?***
    - *Del Aguila, Kunszt, Santiago [1602.00126]; Boggia, Gomez-Ambrosio, Passarino [1603.03660]; Henning, Lu, Murayama [1604.01019]; Ellis, Quevillon, You, **ZZ** [1604.02445]; **Fuentes-Martin, Portoles, Ruiz-Femenia [1607.02142]**.*
  - Answer: ***Yes** (with a **systematic diagrammatic formulation**)!* — ***ZZ [1610.00710]***

CDE series = Σcovariant diagrams

(ZZ [1610.00710])

$$\int d^d x\, \mathcal{L}_{\rm EFT}^{\text{1-loop}}[\varphi_L] = \Gamma_{\rm L,UV}^{\text{1-loop}}[\varphi_L]\Big|_{\rm hard} = i\, c_s\ \log\det \mathcal{Q}_{\rm UV}\left[\varphi_{H,\rm c}[\varphi_L], \varphi_L\right]\Big|_{\rm hard}$$

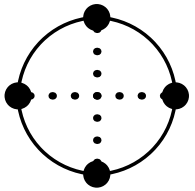
- Assume general form: $\mathcal{Q}_{\rm UV} = \mathbf{K} + \mathbf{X}$
  - **K**: kinetic operator (diagonal, $-P^2 + m^2 \,/\, -\not{P} + m$ for bosons/fermions)
  - **X**: interaction matrix $\mathbf{X} = \mathbf{U}[\varphi] + P_\mu\, \mathbf{Z}^\mu[\varphi] + \mathbf{Z}^{\dagger\mu}[\varphi]\, P_\mu + \mathcal{O}(P^2)$
    - Notation: $P_\mu \equiv iD_\mu$ (hermitian operator).
- Basic rules:

Building block	Bosonic	Fermionic
Propagator	 $= 1$	 $= M_i$ (heavy only)  $= -\gamma^\mu$
P insertion	 $= 2P_\mu$	 $= -\not{P}$
U insertion	 $= U_{ij}$	

draw dotted lines to contract Lorentz indices in pairs

Example: integrating out a complex scalar (e.g. squark)

(ZZ [1610.00710])



$$= -i \cdot \frac{1}{4} \cdot \mathcal{I}[q^4]^4_i \cdot \mathrm{tr}(2P^\mu \cdot 2P^\nu \cdot 2P_\mu \cdot 2P_\nu)$$

- **Spin factor**: - $i/2$ for each real scalar.
- **Symmetry factor**: $1/S$ if diagram has Z_S symmetry.
- **Loop integral**: $\mathcal{I}[q^{2n_c}]^{n_i n_j \cdots}_{ij\ldots}$ if diagram has
  - n_i propagators with mass M_i and
  - n_c Lorentz contractions

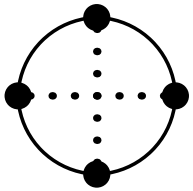
master integrals

$$\int \frac{d^d q}{(2\pi)^d} \frac{q^{\mu_1} \cdots q^{\mu_{2n_c}}}{(q^2 - M_i^2)^{n_i} (q^2 - M_j^2)^{n_j} \cdots} \equiv g^{\mu_1 \cdots \mu_{2n_c}} \mathcal{I}[q^{2n_c}]^{n_i n_j \cdots}_{ij\ldots}$$

completely symmetric tensor, e.g. $g^{\mu\nu\rho\sigma} = g^{\mu\nu} g^{\rho\sigma} + g^{\mu\rho} g^{\nu\sigma} + g^{\mu\sigma} g^{\nu\rho}$

Example: integrating out a complex scalar (e.g. squark)

(ZZ [1610.00710])



$$= \boxed{-i} \cdot \boxed{\frac{1}{4}} \cdot \boxed{\mathcal{I}[q^4]_i^4} \cdot \boxed{\mathrm{tr}(2P^\mu \cdot 2P^\nu \cdot 2P_\mu \cdot 2P_\nu)}$$

$$-\frac{i}{16\pi^2}\frac{1}{24}\log\frac{M_i^2}{Q^2} \qquad \text{part of } -\frac{1}{2}g^2\,\mathrm{tr}(G^{\mu\nu}G_{\mu\nu}) = -\,\mathrm{tr}(P^2P^2) + \mathrm{tr}(P^\mu P^\nu P_\mu P_\nu)$$

- This diagram gives rise to

$$\mathcal{L}_{\text{eff}} \supset \left(1 - \boxed{\frac{g^2}{48\pi^2}\log\frac{M_i^2}{Q^2}}\right)\left[-\frac{1}{4}\mathrm{tr}(G^{\mu\nu}G_{\mu\nu})\right]$$

  - => gauge coupling **threshold correction** (by rescaling gauge fields and couplings s.t. kinetic terms are canonically normalized).

$$\frac{g_{\text{eff}}^2(\mu)}{g^2(\mu)} = 1 + \frac{g^2}{48\pi^2}T(R_i)\log\frac{M_i^2}{Q^2}$$

Dynkin index *matching scale*

Threshold corrections from EFT matching: general formulation

(Wells, ZZ [work in progress])

- Integrate out heavy BSM fields from path integral

$$\int [D\varphi_{\rm BSM}][D\varphi_{\rm SM}]\, e^{i\int d^dx\, \mathcal{L}[\varphi_{\rm BSM},\varphi_{\rm SM}]} = \int [D\varphi_{\rm SM}]\, e^{i\int d^dx\, \mathcal{L}_{\rm SMEFT}[\varphi_{\rm SM}]}$$

$$\begin{aligned}\mathcal{L}_{\rm SMEFT} \;=\; & \mathcal{L}_{\rm SM} + \delta Z_\phi |D_\mu\phi|^2 + \sum_{f=q,u,d,l,e} \bar f\, \boldsymbol{\delta Z_f}\, i\not{D} f - \frac{1}{4}\delta Z_G G^A_{\mu\nu}G^{A\mu\nu} - \frac{1}{4}\delta Z_W W^I_{\mu\nu}W^{I\mu\nu} - \frac{1}{4}\delta Z_B B_{\mu\nu}B^{\mu\nu} \\ & + \delta m^2|\phi|^2 + \delta\lambda|\phi|^4 + \left(\bar u\, \boldsymbol{\delta y_u}\, q\cdot\epsilon\cdot\phi + \bar d\, \boldsymbol{\delta y_d}\, q\cdot\phi^* + \bar e\, \boldsymbol{\delta y_e}\, l\cdot\phi^* + \text{h.c.}\right) + \text{dimension 6 ...}\end{aligned}$$

$$\begin{aligned}
& g_3 - g_3^{\rm eff} = \frac{1}{2}\, g_3\, \delta Z_G\,, \quad g_2 - g_2^{\rm eff} = \frac{1}{2}\, g_2\, \delta Z_W\,, \quad g_1 - g_1^{\rm eff} = \frac{1}{2}\, g_1\, \delta Z_B\,, \\
& m^2 - m^2_{\rm eff} = \delta m^2 + m^2\, \delta Z_\phi\,, \quad \lambda - \lambda_{\rm eff} = \delta\lambda + 2\,\lambda\, \delta Z_\phi\,, \\
& \boldsymbol{y_u} - \boldsymbol{y_u}^{\rm eff} = \boldsymbol{\delta y_u} + \frac{1}{2}(\boldsymbol{y_u}\, \boldsymbol{\delta Z_q} + \boldsymbol{\delta Z_u}\, \boldsymbol{y_u} + \boldsymbol{y_u}\, \delta Z_\phi)\,, \\
& \boldsymbol{y_d} - \boldsymbol{y_d}^{\rm eff} = \boldsymbol{\delta y_d} + \frac{1}{2}(\boldsymbol{y_d}\, \boldsymbol{\delta Z_q} + \boldsymbol{\delta Z_d}\, \boldsymbol{y_d} + \boldsymbol{y_d}\, \delta Z_\phi)\,, \\
& \boldsymbol{y_e} - \boldsymbol{y_e}^{\rm eff} = \boldsymbol{\delta y_e} + \frac{1}{2}(\boldsymbol{y_e}\, \boldsymbol{\delta Z_l} + \boldsymbol{\delta Z_e}\, \boldsymbol{y_e} + \boldsymbol{y_e}\, \delta Z_\phi)\,.
\end{aligned}$$

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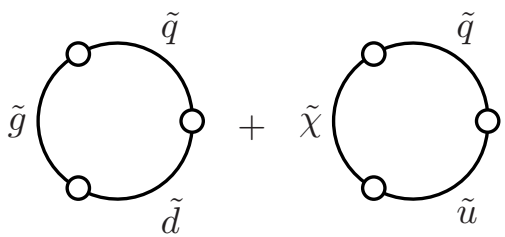
  - These renormalizable terms are invisible in low-energy experiment, but are important for inferring high-scale physics.
  - In the specific case of matching the MSSM onto the SMEFT,
    - What values of these renormalizable operator coefficients are needed to achieve gauge and Yukawa coupling unification?
    - ***Are they correlated with dim-6 operators (which we can see)?***

Computing 1-loop SUSY threshold corrections with covariant diagrams

(reproducing Bagger, Matchev, Pierce, Zhang [hep-ph/9606211])

- Example: dominant contributions to $\boldsymbol{y_d} - \boldsymbol{y_d^{\rm eff}}$
  - Squark-gluino loop + squark-Higgsino loop



$$+\,\text{h.c.} = -\frac{i}{2} M_3\, \mathcal{I}^{111}_{\tilde{q}\tilde{d}\tilde{g}}\, \text{tr}\big(U_{\tilde{d}\tilde{g}} U_{\tilde{g}\tilde{q}} U_{\tilde{q}\tilde{d}}\big) - \frac{i}{2}\mu\, \mathcal{I}^{111}_{\tilde{q}\tilde{u}\tilde{\chi}}\, \text{tr}\big(U_{\tilde{u}\tilde{\chi}} U_{\tilde{\chi}\tilde{q}} U_{\tilde{q}\tilde{u}}\big) + \text{h.c.}$$

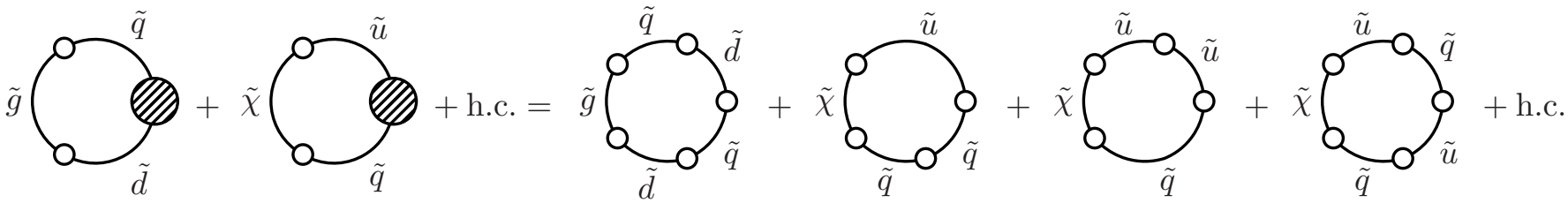
$$\simeq \frac{\tan\beta}{16\pi^2}\left[\frac{8}{3} g_3^2\, \mu\, M_3\, \tilde{\mathcal{I}}^{111}_{\tilde{q}\tilde{d}\tilde{g}}\, (\bar{d}\, \boldsymbol{y_d}\, q\cdot\phi^* + \text{h.c.}) + s_\beta^{-2}\, \mu\, A_u\, \tilde{\mathcal{I}}^{111}_{\tilde{q}\tilde{u}\tilde{\chi}}\, (\bar{d}\, \boldsymbol{y_d} \boldsymbol{y_u^\dagger} \boldsymbol{y_u}\, q\cdot\phi^* + \text{h.c.})\right]$$

where $\tilde{\mathcal{I}}^{111}_{ijk} \equiv \mathcal{I}^{111}_{ijk} / \frac{i}{16\pi^2} = \frac{M_i^2 \log M_i^2}{(M_k^2 - M_i^2)(M_i^2 - M_j^2)} + \frac{M_j^2 \log M_j^2}{(M_i^2 - M_j^2)(M_j^2 - M_k^2)} + \frac{M_k^2 \log M_k^2}{(M_j^2 - M_k^2)(M_k^2 - M_i^2)}$

- $\frac{y_b - y_b^{\rm eff}}{y_b} \sim \mathcal{O}(10\%)$ needed for b-tau Yukawa unification.
  - *See e.g. Tobe, Wells [hep-ph/0301015]; Elor, Hall, Pinner, Ruderman [1206.5301].*

Relating bottom Yukawa threshold correction to hbb coupling

(Wells, ZZ [work in progress])

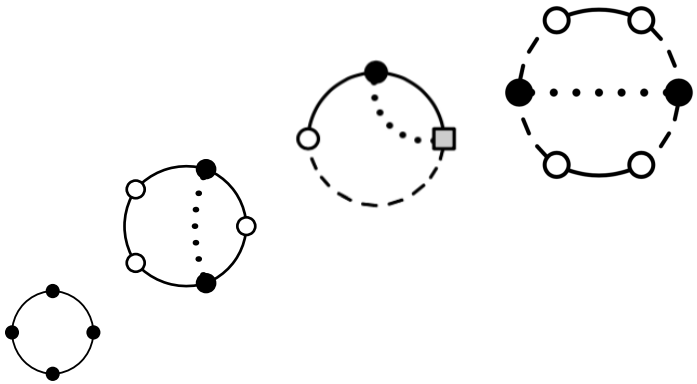


- These diagrams give rise to dim-6 operator $|\phi|^2 \left(\bar{d}\, \boldsymbol{C_{d\phi}}\, q \cdot \phi^* + \text{h.c.}\right)$
- hbb coupling modification

$$\Delta\kappa_b = -\frac{v^2}{y_b} C_{b\phi} = -\frac{2\, m_b^2 \tan^2\beta}{\mathcal{I}^{111}_{\tilde{q}\tilde{d}\tilde{g}} / \left(\mu^2\, \mathcal{I}^{221}_{\tilde{q}\tilde{d}\tilde{g}}\right)} \boxed{\delta_b^{\tilde{q}\tilde{d}\tilde{g}}} - \frac{2\, m_t^2}{\mathcal{I}^{111}_{\tilde{q}\tilde{u}\tilde{\chi}} / \left(\mathcal{I}^{211}_{\tilde{q}\tilde{u}\tilde{\chi}} + \mathcal{I}^{121}_{\tilde{q}\tilde{u}\tilde{\chi}} + A_u^2\, \mathcal{I}^{221}_{\tilde{q}\tilde{u}\tilde{\chi}}\right)} \boxed{\delta_b^{\tilde{q}\tilde{u}\tilde{\chi}}}$$

where $$\frac{y_b - y_b^{\text{eff}}}{y_b} \simeq \boxed{\delta_b^{\tilde{q}\tilde{d}\tilde{g}}} + \boxed{\delta_b^{\tilde{q}\tilde{u}\tilde{\chi}}} = \boxed{\frac{g_3^2 \tan\beta}{16\pi^2} \cdot \frac{8}{3}\, \mu\, M_3\, \tilde{\mathcal{I}}^{111}_{\tilde{q}\tilde{d}\tilde{g}}} + \boxed{\frac{y_t^2 \tan\beta}{16\pi^2} \cdot s_\beta^{-2}\, \mu\, A_u\, \tilde{\mathcal{I}}^{111}_{\tilde{q}\tilde{u}\tilde{\chi}}}$$

Summary



- A new systematic approach to 1-loop EFT matching, which (unlike Feynman diagrams) preserves gauge covariance:

$$\int d^d x\, \mathcal{L}_{\rm EFT}^{\text{1-loop}}[\varphi_L] = i\, c_s\ \log\det \mathcal{Q}_{\rm UV}\left[\varphi_{H,\rm c}[\varphi_L], \varphi_L\right]\Big|_{\rm hard}$$
$$= \text{CDE series (where derivatives are covariant)}$$
$$= \text{sum of 1-loop covariant diagrams}$$

  - ***ZZ [1610,00710].***

- We have re-computed the full 1-loop MSSM threshold corrections using covariant diagrams. EFT formulation helps build connection between high-scale physics e.g. unification (dim-4) and low-energy phenomenology (dim-6).
  - ***Wells, ZZ [work in progress].***