# Dipole polarizabilities of $\pi^{\pm}$-mesons

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The main experimental works, where dipole polarizabilities of charged pions have been determined, are considered. Possible reasons for the differences between the experimental data are discussed. In particular, it is shown that the account of the $\sigma$-meson gives a significant correction to the value of the polarizability obtained in the latest experiment of the COMPASS collaboration.

PACS numbers: 13.40.-f, 11.55.Fv, 11.55.Hx, 12.39.Fe, 14.40.-n

## I. INTRODUCTION

Pion polarizabilities are fundamental structure parameters values of which are very sensitive to predictions of different theoretical models. Therefore, an accurate experimental determination of these parameters is very important for testing the validity of such models. These parameters characterize the behavior of the pion in an external electromagnetic field. The dipole ($\alpha_1$ and $\beta_1$) and quadrupole ($\alpha_2$ and $\beta_2$) polarizabilities are defined [1, 2] through the expansion of the non-Born helicity amplitudes of the Compton scattering on the pion over $t$ at the fixed $s = \mu^2$:

$$\begin{aligned} M_{++}(s=\mu^2,t) &= \pi\mu\left[2(\alpha_1-\beta_1)+\frac{t}{6}(\alpha_2-\beta_2)\right]+\mathcal{O}(t^2), \\ M_{+-}(s=\mu^2,t) &= \frac{\pi}{\mu}\left[2(\alpha_1+\beta_1)+\frac{t}{6}(\alpha_2+\beta_2)\right]+\mathcal{O}(t^2), \end{aligned} \tag{1}$$

where $s$ ($t$) is the square of the total energy (momentum transfer) in the $\gamma\pi$ center of mass (c.m.) system, $\mu$ is the pion mass, $\alpha_i$ and $\beta_i$ are electric and magnetic polarizabilities correspondingly. In the following the dipole polarizabilities are given in units $10^{-4}\,\mathrm{fm}^3$.

The most of experimental data obtained for the difference of the dipole polarizabilities of the charged pions are presented in Table I.

The polarizabilities were determined by analyzing the processes of the high energy pions scattering in the Coulomb field of heavy nuclei ($\pi^- A \to \gamma\pi^- A'$) via the Primakoff effect, radiative pion photoproduction from proton ($\gamma p \to \gamma\pi^+ n$), and two-photon production of pion pairs ($\gamma\gamma \to \pi\pi$). As seen from Table I, the data vary from 4 up to 40 and are in conflict even for experiments performed with the same method. In this paper we will consider possible reasons for such disagreements.

## II. RADIATIVE PHOTOPRODUCTION OF THE $\pi^+$-MESON FROM THE PROTON

An experiment on the radiative $\pi^+$-meson photoproduction from the proton ($\gamma p \to \gamma\pi^+ n$) was carried out at the Mainz Microtron MAMI [3] in the kinematical region of 537 MeV $< E_\gamma <$ 817 MeV and $140^\circ \le \theta_{\gamma\gamma}^{cm} \le 180^\circ$, where $\theta_{\gamma\gamma}^{cm}$ is a polar angle in the c.m. system of the outgoing photon and pion.

The theoretical calculations of the cross section for the reaction $\gamma p \to \gamma\pi^+ n$ show that the contribution of nucleon resonances is suppressed for photons scattered backward in the c.m. system of the reaction $\gamma\pi \to \gamma\pi$. Moreover, integration over $\varphi$ and $\theta_{\gamma\gamma}^{cm}$ essentially decreases the contribution of nucleon resonances from the crossed channels. In addition, the difference $(\alpha_1-\beta_1)\pi^{\pm}$ gives the biggest contribution to the cross section for $\theta_{\gamma\gamma}^{cm}$ in the same region of $140^\circ - 180^\circ$. Therefore, one considered the cross section of radiative pion photoproduction integrated over $\varphi$ from $0^\circ$ to $360^\circ$ and over $\theta_{\gamma\gamma}^{cm}$ from $140^\circ$ to $180^\circ$,

$$\int_0^{360^\circ} d\varphi \int_{-1}^{-0.766} d\cos\theta_{\gamma\gamma}^{cm}\, \frac{d\sigma_{\gamma p\to\gamma\pi^+ n}}{dt\,ds\,d\Omega_{\gamma\gamma}}. \tag{2}$$

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TABLE I: Review of experimental data on $(\alpha_1 - \beta_1)_{\pi^\pm}$

| Experiments | | $(\alpha_1 - \beta_1)_{\pi^\pm}$ |
|---|---|---|
| $\gamma p \to \gamma\pi^+ n$ | MAMI (2005) [3] | $11.6 \pm 1.5_{stat} \pm 3.0_{syst} \pm 0.5_{mod}$ |
| $\gamma p \to \gamma\pi^+ n$ | Lebedev Phys. Inst. (1984) [4] | $40 \pm 20$ |
| $\pi^- A \to \gamma\pi^- A'$ | Serpukhov (1983) [5] | $13.6 \pm 2.8 \pm 2.4$ |
| $\pi^- A \to \gamma\pi^- A'$ | COMPASS (2007) [6] | $4.0 \pm 1.2 \pm 1.4$ |
| D. Babusci *et al.* (1992) [7] | | |
| | PLUTO [8] | $38.2 \pm 9.6 \pm 11.4$ |
| | DM 1 [9] | $34.4 \pm 9.2$ |
| | MARK II [10] | $4.4 \pm 3.2$ |
| J.F. Donoghue, B.R. Holstein (1993) [11] | | $5.4$ |
| | MARK II [10] | |
| A.E. Kaloshin, V.V. Serebryakov (1994) [12] | | $5.25 \pm 0.95$ |
| | MARK II [10] | |
| L.V. Fil'kov, V.L. Kashevarov (2006) [13] | | $13^{+2.6}_{-1.9}$ |
| $\gamma\gamma \to \pi^+\pi^-$ | fit to the data [10, 14–18] | |
| | from threshold to 2.5 GeV | |
| R. Garcia-Martin, B. Moussallam | | |
| | (2010) [19], $\gamma\gamma \to \pi^+\pi^-$ | $4.7$ |

The work was carried out at values of $s$ up to $15\mu^2$. It has been shown in Ref. [20], that the contribution of the $\sigma$-meson is noticeable at such high values of $s$. The contributions of other mesonic resonances $(\rho, a_1, b_1, a_2)$ are negligible here.

The values of the pion polarizabilities have been obtained from a fit of the cross section calculated by two different theoretical models to the data. In the first model the contribution of all the pion and nucleon pole diagrams was taken into account. In the second model in addition to the nucleon and the pion pole diagrams (without the anomalous magnetic moments of nucleons) the contribution of the resonances $\Delta(1232)$, $P_{11}(1440)$,$D_{13}(1520)$, and $S_{11}(1535)$ and the $\sigma$-meson were included.

It should be noted that the contribution of the sum of the pion polarizabilities is very small in the considered region of $140^\circ \lesssim \theta^{cm}_{\gamma\gamma} \lesssim 180^\circ$ . The estimate shows that the contribution of $(\alpha_1 + \beta_1)_{\pi^\pm} = 0.4$ to the value of $(\alpha_1 - \beta_1)_{\pi^\pm}$ is less than 1%.

To increase the confidence that the model dependence of the result was under control, kinematic regions were considered where the difference between the models did not exceed 3%. First, the kinematic region was considered where the contribution of the pion polarizability is negligible, i.e. the region $1.5\mu^2 < s < 5\mu^2$. As a result, it was shown that the dependence of the differential cross-section on $t$ is well reproduced. Then the kinematic region was investigated where the polarizability contribution is biggest. This is the region $5\mu^2 < s < 15\mu^2$ and $-12\mu^2 < t < -2\mu^2$. In the range $t > -2\mu^2$ the polarizability contribution is small and this region was excluded.

Analysis of these data gave the following result

$$(\alpha_1 - \beta_1)_{\pi^\pm} = 11.6 \pm 1.5_{stat} \pm 3.0_{syst} \pm 0.5_{model}. \tag{3}$$

An independent analysis [21] of the experimental data was carried out by a constrained $\chi^2$ fit. The result [21] agree very well with (3) giving it additional support.

The result [3] is consisted with earlier works investigating the $\gamma p \to \gamma\pi^+ n$ [4] and $\pi^- A \to \gamma\pi^- A'$ [5] reactions, and also with [13], where a global fit to all existing data for the $\gamma\gamma \to \pi^+\pi^-$ reaction was done. On the other hand, the result [3] is in conflict with the prediction of ChPT [22, 23]. This discrepancy can be connected with a different account of the contribution of the $\sigma$-meson and vector mesons in the dispersion relations and ChPT calculations [24].

## III. SCATTERING OF PIONS IN THE COULOMB FIELD OF HEAVY NUCLEI

The first experimental data on the charged pion polarizability was obtained in the work [5]. They studied the scattering of high energy $\pi^-$ mesons off the Coulomb field of heavy nuclei. A connection of the radiative scattering in the Coulomb field of heavy nuclei with the Compton scattering was first predicted in the work [25].

The cross section of the radiative pion scattering $\pi A \to \pi\gamma A'$ via the Primakoff effect can be written as

$$\frac{d\sigma_{\pi A}}{ds dQ^2 d\cos\theta_{\gamma\gamma}^{cm}} = \frac{Z^2\alpha}{\pi(s-\mu^2)} F_{eff}^2(Q^2) \frac{Q^2 - Q_{min}^2}{Q^4} \frac{d\sigma_{\pi\gamma}}{d\cos\theta_{\gamma\gamma}^{cm}}, \tag{4}$$

where $F_{eff} \approx 1$ is the electromagnetic form-factor of nucleus, $\alpha$ is the fine-structure constant, $Z$ is the charge number of the nucleus, and $Q^2$ is the negative 4-momenta transfer squared, $Q^2 = -(p_A - p'_A)^2$. $Q_{min}^2$ is the minimum value of $Q^2$ which is given by the formula

$$Q_{min}^2 = \frac{(s-\mu^2)^2}{4E_{beam}^2}, \tag{5}$$

where $s$ is the square of the total energy of the process $\gamma + \pi^\pm \to \gamma + \pi^\pm$, $E_{beam}$ is the pion beam energy.

This cross section has a Coulomb peak at $Q^2 = 2Q_{min}^2$ with a width equal to $\simeq 6.8Q_{min}^2$.

The experiment [5] was carried at a beam energy equal to 40 GeV. In this case if the energy of the incident photon in the pion rest frame $\omega_1 = 600$ MeV, then $Q_{min}^2$ is equal to $4.4 \times 10^{-6}$ (GeV/c)$^2$. It was shown that the Coulomb amplitude dominates in this case for $Q^2 \le 2 \times 10^{-4}$(GeV/c)$^2$. The experiment [5] was carried out at $Q_{cut}^2 < 6 \times 10^{-4}$(GeV/c)$^2$. Events in the region of $Q^2$ of $(2-8) \times 10^{-3}$(GeV/c)$^2$ were used to estimate the strong interaction background. This background was assumed to behave either as $\sim Q^2$ in the region $Q^2 \le 6 \times 10^{-4}$(GeV/c)$^2$ or as a constant. The polarizability was determined from the ratio (assuming $(\alpha_1 + \beta_1)_{\pi^\pm} = 0$)

$$R_\pi = (\frac{d\sigma_{\gamma\pi}}{d\Omega})/(\frac{d\sigma_{\gamma\pi}^0}{d\Omega}) = 1 - \frac{3}{2}\frac{\mu^3}{\alpha}\frac{x_\gamma^2}{1-x_\gamma}\alpha_\pi, \tag{6}$$

where $\frac{d\sigma_{\gamma\pi}}{d\Omega}$ refers to the measured cross section and $\frac{d\sigma_{\gamma\pi}^0}{d\Omega}$ to simulated cross section expected for $\alpha_\pi = 0$, $x_\gamma = E_\gamma/E_{beam}$ in the laboratory system of the process $\pi A \to \pi\gamma A'$. As a result they have obtained

$$(\alpha_1 - \beta_1)_{\pi^\pm} = 13.6 \pm 2.8 \pm 2.4. \tag{7}$$

The new result of the COMPASS collaboration [6] for the charged pion electric polarizability $\alpha_\pi = 2.0 \pm 0.6_{stat.} \pm 0.7_{syst.}$ has been found also by studying the $\pi^-$-meson scattering off the Coulomb field of heavy nuclei. The result was obtained assuming that $\alpha_1 = -\beta_1$. This value is at variance with the result obtained in a very similar experiment in Serpukhov [5], but also with [3].

The COMPASS experiment [6] was performed with $E_{beam} = 190$ GeV. For such values of $E_{beam}$ the quantity of $Q_{min}^2(COMPASS)$ must be smaller by 22.5 times than $Q_{min}^2$(Serpukhov). However, the authors of the experiment [6] considered $Q_{cut}^2 \lesssim 0.0015$ (GeV/c)$^2$, which are greater than $Q_{cut}^2$ in work [5].

As shown in [26] the basic ratio $R_\pi$ is applicable for the Coulomb peak only. On the other hand, in Ref. [27] it is elaborated that the Coulomb amplitude interference with the coherent nuclear amplitude is important for $0.0005 \leqslant Q^2 \leqslant 0.0015\,$(GeV/c)$^2$. This means that the Serpukhov analysis could safely apply the ratio $R_\pi$ in (6), whereas COMPASS has to consider the interference of the Coulomb and strong amplitude. The phase determined with the simple considerations in Ref. [28] for the Serpukhov experiment [5] is close to $\pi/2$ meaning that the subtraction of a nuclear background assumed to be incoherent is justified.

Moreover, the simulation of the distribution in Fig. 3(c) in the work ([6]) does not reproduce the diffraction bumps at $Q > 0.04\,$(GeV/c).

In Refs. [26, 27], it is suggested that the correct description of the strong amplitude is by the Glauber model (elastic multiple scattering of hadrons in nuclei). The conditions and limitations of the Glauber approximation are discussed in the classical article about diffraction by U. Amaldi, M. Jacob, and G. Matthiae [29]. Göran Fäldt and Ulla Tengblad [26, 27] assume that the hadron-nucleon potential of the nucleons in the nucleus is local and also real then the phases between the incoming hadron and the nucleons add up linearly. However, at high energies - and the COMPASS energy of the incoming $\pi$ with 180 (GeV/c)$^2$ is high - the strong phases become complex and the summed amplitude acquires an additional energy dependent phase. The associated profile function must take into account multiple scattering and will be complex, i.e. an unknown phase appears.

In the COMPASS analysis the contributions to the fit are not shown in Fig. 3(c) of Ref. [6]. With a more realistic "absorbing disc" for the profile function [29] all bumps in Fig. 3(c) could be reproduced well and again a phase would be close to $\pi/2$ [30]. Without a real fit to the data it is impossible to estimate the effect of the model dependence of the diffractive background, but that it will have an influence is clear from Ref. [27].

Comparison of data with different targets provides the possibility to check the $Z^2$ dependence for the Primakoff cross section and estimate a possible contribution of the nuclear background. Such an investigation was performed by the Serpukhov collaboration and they have obtained $Z^2$ dependence with good enough accuracy. The COMPASS collaboration really have gotten their main result using only $Ni$ target but they wrote that they also considered other targets on small statistic and obtained approximate $\sim Z^2$ dependence.

It should be noted that in order to get an information about the pion polarizabilities, the authors considered the cross section of the process $\gamma\pi^- \to \gamma\pi^-$ equal to the Born cross section and the interference of the Born amplitude with the pion polarizabilities only. The COMPASS collaboration analyzed this process up to the total energy $W = 490$ MeV in the angular range $0.15 > \cos\theta_{\gamma\pi}^{cm} > -1$. However, the contribution of the $\sigma$-meson to the cross section of the Compton scattering on the pion could be very substantial in this region of the energy and angles. Therefore, we consider this contribution.

## IV. $\sigma$-MESON CONTRIBUTION

The cross section of the elastic $\gamma\pi$ scattering can be written as [20]:

$$\frac{d\sigma_{\gamma\pi\to\gamma\pi}}{d\Omega} = \frac{1}{256\pi^2}\frac{(s-\mu^2)^4}{s^3}\left[(1-z)^2|M_{++}|^2 + s^2(1+z)^2|M_{+-}|^2\right], \tag{8}$$

where $z = \cos\theta_{\gamma\pi}^{cm}$. The amplitudes $M_{++}$ and $M_{+-}$ have no kinematical singularities and zeros [31].

The dispersion relation (DR) for the amplitude $M_{++}$ at fixed $t$ with one subtraction was obtained in [20]:

$$\begin{aligned} ReM_{++}(s,t) = \quad & Re\overline{M}_{++}(s=\mu^2,t) + B_{++} \\ +\frac{(s-\mu^2)}{\pi}P\int\limits_{4\mu^2}^{\infty} ds'\; ImM_{++}(s',t) \quad & \left[\frac{1}{(s'-s)(s'-\mu^2)} - \frac{1}{(s'-u)(s'-\mu^2+t)}\right], \end{aligned} \tag{9}$$

where $B_{++}$ is the Born term equal to

$$B_{++} = \frac{2e^2\mu^2}{(s-\mu^2)(u-\mu^2)}. \tag{10}$$

Via the cross symmetry this DR is identical to a DR with two subtraction. The subtraction function $Re\overline{M}_{++}(s = \mu^2,t)$ was determined with help of the DR at fixed $s = \mu^2$ with one subtraction where the subtraction constant was expressed through the difference $(\alpha_1-\beta_1)_{\pi^\pm}$:

$$Re\overline{M}_{++}(s=\mu^2,t) = 2\pi\mu(\alpha_1-\beta_1)_{\pi^\pm} + \frac{t}{\pi}\left\{P\int\limits_{4\mu^2}^{\infty}\frac{ImM{+}{+}(t',s=\mu^2)\;dt'}{t'(t'-t)} - P\int\limits_{4\mu^2}^{\infty}\frac{ImM_{++}(s',u=\mu^2)\;ds'}{(s'-\mu^2)(s'-\mu^2+t)}\right\}. \tag{11}$$

The DRs for the amplitude $M_{+-}(s,t)$ have the same expressions (9) and (11) with substitutions: $ImM_{++} \to ImM_{+-}$, $B_{++} \to B_{+-} = B_{++}/\mu^2$ and $2\pi\mu(\alpha_1-\beta_1)_{\pi^\pm} \to 2\pi/\mu(\alpha_1+\beta_1)_{\pi^\pm}$.

The amplitudes $M_{++}$ and $M_{+-}$ were calculated with help of these DRs taking into account contribution of the following mesons: $\rho(770)$, $b_1(1235)$, $a_1(1260)$, and $a_2(1320)$ mesons in the $s$-channel and $\sigma(600)$, $f_0(980)$, $f_0'(1370)$, $f_2(1270)$ mesons in the $t$-channel. It has been shown that the contribution of all these mesons, with exception of the $\sigma$-meson, is very small in the region of the energy and the angles of the COMPASS experiment.

According DRs (11) the contribution of the $\sigma$-meson can be determined as

$$ReM_{++}^{\sigma} = \frac{t}{\pi}\int\limits_{4\mu^2}^{\infty}\frac{ImM_{++}^{\sigma}(t',s=\mu^2)\;dt'}{t'(t'-t)}. \tag{12}$$

The imaginary amplitude $ImM_{++}^{\sigma}(t,s=\mu^2)$ has to be evaluated taking into account that the $\sigma$-meson is a pole on the second Riemann sheet. The relation between amplitudes on the first and the second sheets can be written [32] as

$$F_0^{II}(t+i\epsilon) = F_0^{I}(t+i\epsilon)(1+2i\rho T_0^{II}(t+i\epsilon)), \tag{13}$$

where

$$T_0^{II} = -\frac{g_{\sigma\pi\pi}^2}{t_\sigma - t}, \quad F_0^{II} = \sqrt{2}\,\frac{g_{\sigma\gamma\gamma}g_{\sigma\pi\pi}}{t_\sigma - t}, \tag{14}$$

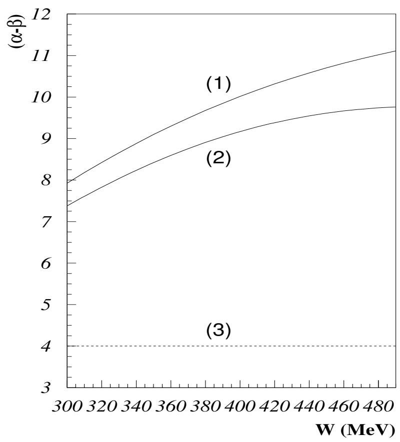


FIG. 1: Dependence of $(\alpha_1 - \beta_1)_{\pi^\pm}$ on $W$. Lines (1) and (2) correspond to the calculation of Eqs. (19) and (20), respectively. Line (3) is the COMPASS result [6].

$$t_\sigma = (M_\sigma - i\Gamma_{\sigma 0}/2)^2, \quad \rho = \frac{\sqrt{1-4\mu^2/t}}{16\pi}, \quad \Gamma_{\sigma 0} = \Gamma_\sigma \left(\frac{t-4\mu^2}{M_\sigma^2 - 4\mu^2}\right)^{1/2}. \tag{15}$$

Using the relation (13) we have

$$ImM^\sigma_{++}(t, s=\mu^2) = \frac{1}{t}\sqrt{\frac{2}{3}}\, \frac{g_{\sigma\gamma\gamma} g_{\sigma\pi\pi} R}{D^2 + R^2}, \tag{16}$$

where

$$D = (M_\sigma^2 - t - \frac{1}{4}\Gamma^2_{\sigma 0}), \quad R = M_\sigma \Gamma_{\sigma 0} + 2\rho g^2_{\sigma\pi\pi}. \tag{17}$$

We can get influence of the $\sigma$-meson on the extracted value of $(\alpha_1 - \beta_1)_{\pi^\pm}$ by equating the cross section without $\sigma$-meson contribution to the cross section when $\sigma$-meson is taking into account [33]:

$$d\sigma_{\gamma\pi\to\gamma\pi}(B, (\alpha_1 - \beta_1)^0_{\pi^\pm})/d\Omega = d\sigma_{\gamma\pi\to\gamma\pi}(B, M^\sigma_{++} (\alpha_1 - \beta_1)_{\pi^\pm})/d\Omega, \tag{18}$$

where $(\alpha_1 - \beta_1)^0_{\pi^\pm}$ is the value of $(\alpha_1 - \beta_1)_{\pi^\pm}$ without of the $\sigma$ contribution obtained in [6]) and $B$ is the Born term.

For backward scattering ($z = -1$), we have the following expression:

$$(\alpha_1 - \beta_1)_{\pi^\pm} = \frac{1}{4\pi\mu}\left\{-(B + ReM^\sigma_{++}) + \frac{B^2 + 4\pi\mu B(\alpha_1 - \beta_1)^0_{\pi^\pm}}{B + ReM^\sigma_{++}}\right\}. \tag{19}$$

In the case of integration over the region $-1 \le z \le 0.15$ we have

$$(\alpha_1 - \beta_1)_{\pi^\pm} = F_0/F_1, \tag{20}$$

where

$$F_0 = \frac{1}{4\pi\mu}\left\{\int_{-1}^{0.15} \left[-ReM^\sigma_{++}(ReM^\sigma_{++} + 2B) + 4\pi\mu B(\alpha_1 - \beta_1)^0_{\pi^\pm}\right](1-z)^2\, dz\right\}, \tag{21}$$

$$F_1 = \left\{\int_{-1}^{0.15} (B + ReM^\sigma_{++})(1-z)^2\, dz\right\}. \tag{22}$$

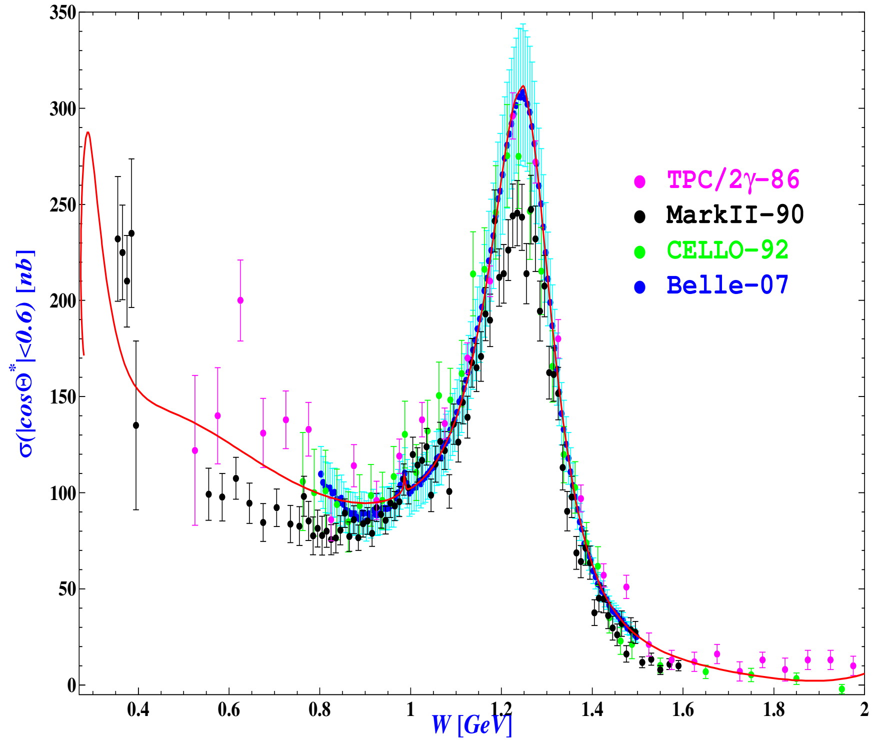


FIG. 2: The cross section of the process $\gamma\gamma \to \pi^+\pi^-$ (with $|\cos\theta_{cm}| < 0.6$). Experimental data of TPC/2$\gamma$ [14], Mark II [10], and CELLO [15] Collaborations are shown with statistical uncertainties only. Statistical uncertainties for the most of the Belle Collaboration data [18] are smaller then corresponding blue circles. Vertical light-blue error bars are systematic uncertainties for these data.

In the calculation we used the parameters of the $\sigma$-meson from Ref. [32]: $M_\sigma = 441$MeV, $\Gamma_\sigma = 544$MeV, $\Gamma_{\sigma\gamma\gamma} = 1.98$keV, $g_{\sigma\pi\pi} = 3.31$GeV, $g^2_{\sigma\gamma\gamma} = 16\pi\Gamma_{\sigma\gamma\gamma}M_\sigma$. The results of the calculations using Eq. (19) (line (1) ) and Eq. (20) (line (2)) are shown in Fig. 1. Line (3) is the result of Ref. [6]. As a result we have obtain $(\alpha_1 - \beta_1)_{\pi^\pm} \sim 10$. However the magnitude of $(\alpha_1 - \beta_1)_{\pi^\pm}$ is very sensitive to parameters of the $\sigma$-meson and can reach a value of $\sim 11$ for the parameters from [34].

So, the contribution of the $\sigma$-meson can essentially change the COMPASS result. It should be noted that the contribution of the $\sigma$-meson was not considered in Serpukhov as well. However, in this case, the contribution of the $\sigma$-meson for the Serpukhov kinematics is $\Delta(\alpha - \beta)_\sigma \gtrsim 2.7$ within the experimental error of the Serpukhov result.

## V. TWO-PHOTON PRODUCTION OF PION PAIRS

The information about pion polarizabilities could be obtained also by studying the cross section of the reaction $\gamma\gamma \to \pi^+\pi^-$.

Investigation of this process at low and middle energies was carried out in the frameworks of different theoretical models and, in particular, within dispersion relations [7, 11, 12, 19, 35, 36].

Authors of most dispersion approaches used DRs for partial waves taking into account the contribution of $S$ and $D$ wave only. Moreover, they often used additional assumptions, for example, to determine subtraction constants. The dipole polarizabilities of charged pions in a number of the works were obtained from the analysis of the experimental data in the region of the low energy ($W < 700$ MeV) mainly (where $W$ is the total energy in $\gamma\gamma$ c.m. system). The most of results for the charged pion polarizabilities obtained in these works are close to the ChPT prediction [22, 23]. On the other hand, the values of the experimental cross section of the process $\gamma\gamma \to \pi^+\pi^-$ in this region are very ambiguous. Therefore, as it has been shown in Ref. [11, 13], even changes of these values by more than 100% are still compatible with the present error bars in the energy region considered. Therefore at present, more realistic values of the polarizabilities could be obtained analyzing the experimental data on $\gamma\gamma \to \pi^+\pi^-$ in a wider energy region.

The processes $\gamma\gamma \to \pi^0\pi^0$ and $\gamma\gamma \to \pi^+\pi^-$ were analyzed in Ref. [2, 13, 20] using DRs with subtraction for the invariant amplitudes $M_{++}$ and $M_{+-}$ without an expansion over partial waves. The subtraction constants are uniquely determined in these works through the pion polarizabilities. The values of polarizabilities have been found from the fit to the experimental data of the processes $\gamma\gamma \to \pi^+\pi^-$ and $\gamma\gamma \to \pi^0\pi^0$ up to 2500 MeV and 2250 MeV, correspondently. As a result the following values of $(\alpha_1-\beta_1)_{\pi^\pm} = 13.0^{+2.6}_{-1.9}$ and $(\alpha_1-\beta_1)_{\pi^0} = -1.6\pm 2.2$ have been found in these works. In addition, for the first time there were obtained quadrupole polarizabilities for both charged and neutral pions.

The new fit to the total cross section of the process $\gamma\gamma \to \pi^+\pi^-$ at $|\cos\theta^{cm}_{\gamma\gamma}| < 0.6$ in the frame of the DRs [13] has been performed with the $\sigma$-meson considered as a pole on the second Riemann sheet.

The DSRs for the charged pions were saturated by the contributions of the $\rho(770)$, $b_1(1235)$, $a_1(1270)$, and $a_2(1320)$ mesons in the $s$ channel and $\sigma$, $f_0(980)$, $f_0'(1500)$, $f_0(1710)$, $f_0(2020)$, $f_2(1270)$, and $f_2(1565$ in the $t$ channel.

As the two $K$ mesons give a big contribution to the decay width of the $f_0(980)$ meson and the threshold of the reaction $\gamma\gamma \to K\overline{K}$ is very close to the mass of the $f_0(980)$ meson, the Flatté's expression [37] for $f_0(980)$ meson contribution to the process $\gamma\gamma \to \pi\pi$ was used.

Besides we took into account a nonresonance contribution of the $S$ waves with the isospin $I = 0$ and 2 using $\pi^+\pi^-$ loop diagrams.

The result using Eq. (16) for $ImM^{\sigma}_{++}(t, s=\mu^2)$ with the following parameters of the $\sigma$-meson: $M_\sigma = 441$ MeV, $\Gamma_\sigma = 544$ MeV, $\Gamma_{\sigma\gamma\gamma} = 1.298$ keV, $g_{\sigma\pi\pi} = 3.31$ GeV, is shown in Fig. 2. As a result of the fit we have obtained: $(\alpha_1-\beta_1)_{\pi^\pm} = 10$, $(\alpha_1+\beta_1)_{\pi^\pm} = 0.107$, $(\alpha_2-\beta_2)_{\pi^\pm} = 30$, and $(\alpha_2+\beta_2)_{\pi^\pm} = 0.151$, and good agreement with the data of the Belle Collaboration [18].

However, the region $W \lesssim 800$ MeV is most sensitive to the contribution of $(\alpha_1-\beta_1)_{\pi^\pm}$. Therefore, in order to obtain real values of the polarizabilities of the charged pions, it is necessary to have new more accurate data for the process $\gamma\gamma \to \pi^+\pi^-$ in the energy region $W < 800$ MeV. It should be noted that even in the region of $W \lesssim 500$ MeV the main contribution is given by the Born term, the dipole and quadrupole polarizabilities, and the $\sigma$-meson.

## VI. DSR AND CHPT

Dispersion sum rules (DSRs) for the difference and sum of electric and magnetic pion polarizabilities have been constructed using dispersion relations for the helicity amplitudes $M_{++}$ and $M_{+-}$, respectively.

In Refs [2, 13, 20] DSRs have been constructed for $(\alpha_1-\beta_1)_\pi$ using DRs for the amplitude $M_{++}$ at fixed $u=\mu^2$ (where $u = 2\mu^2 - s - t$). In this case, the Regge-pole model allows the use of DRs without subtractions [31]. Such a DSR is

$$(\alpha_1-\beta_1)_{\pi^\pm} = \frac{1}{2\pi^2\mu^2}\left(\int\limits_{4\mu^2}^{\infty}\frac{ImM_{++}(t', u=\mu^2)\; dt'}{t'} + \int\limits_{4\mu^2}^{\infty}\frac{ImM_{++}(s', u=\mu^2)\; ds'}{s'-\mu^2}\right). \tag{23}$$

The imaginary parts of the amplitudes in these DSRs can be approximated by the contributions of meson resonances using Breit-Wigner expressions. For example, the contributions of the vector mesons are determined as

$$ImM^{(V)}_{++}(s,t) = -4g_V^2 s\frac{\Gamma_0}{(M_V^2-s)^2+\Gamma_0^2} \tag{24}$$

for $s > 4\mu^2$ and $ImM^{(V)}_{++}(s,t) = 0$ for $s < 4\mu^2$, where

$$g_V^2 = 6\pi\sqrt{\frac{M_V^2}{s}}\left(\frac{M_V}{M_V^2-\mu^2}\right)^3\Gamma_{V\to\gamma\pi}, \qquad \Gamma_0 = \left(\frac{s-4\mu^2}{M_V^2-4\mu^2}\right)^{\frac{3}{2}} M_V\Gamma_V. \tag{25}$$

Let us consider possible reasons of the discrepancy between the predictions of DSRs and ChPT for $(\alpha_1-\beta_1)_{\pi^\pm}$ [24]. The main contribution to the DSRs for $(\alpha_1-\beta_1)_{\pi^\pm}$ is given by the $\sigma$-meson. However, this meson is taken into account only partially in the present ChPT calculations.

Consider the methods of the calculation of the vector meson contribution in the frameworks of DSRs and ChPT. In the narrow width approximation we have from Eq. (24)

$$ImM^{(V)}_{++}(s,t) = -\frac{4}{\pi}g_V^2 s\delta(s-M_V^2).$$

Then the DSRs calculation gives

$$ReM_{++}(s=\mu^2, t=0) = \frac{-4g_V^2M_V^2}{(M_V^2-\mu^2)}. \tag{26}$$

In the case of ChPT the authors of Ref. [22] used

$$ReM_{++}(s=\mu^2, t=0) = \frac{-4g_V^2\mu^2}{(M_V^2-\mu^2)}. \tag{27}$$

The absolute value of the amplitude (27) is smaller than (26) by a factor $M_V^2/\mu^2$.

In the case of the difference of the dipole polarizabilities of the $\pi^0$-meson, the big contribution of the $\sigma$-meson to DSRs is cancelled by the big contribution of the $\omega$-meson. On the other hand, $\sigma$-meson is only partially included in the ChPT calculations and, according Eq. (27), the $\omega$-meson gives a very small contribution to this difference. As a result, the DSRs and ChPT predictions for $(\alpha_1-\beta_1)_{\pi^0}$ are rather close.

## VII. CONCLUSION

We have considered the main experimental works, where the values of $(\alpha_1-\beta_1)_{\pi^\pm}$ were determined.

The values of $(\alpha_1-\beta_1)_{\pi^\pm}$ obtained in the Serpukhov [5], Mainz [3], and LPI [4] experiments are at variance with the ChPT predictions [22].

The result of the COMPASS Collaboration [6] is in agreement with the ChPT calculations. However, this result is very model dependent. It is necessary to correctly investigate the interference between Coulomb and nuclear amplitudes and take into account the contribution of the $\sigma$-meson.

In order to check and improve the results obtained in the Mainz experiment [3], it is necessary to use a better neutron detector and in additional consider other theoretical models to fit to the cross section of the process under consideration.

It should be noted that the most model independent result was obtained in the Serpukhov experiment [5].

New, more accurate experimental data on the process $\gamma\gamma\to\pi^+\pi^-$ in the region $W\le 800$ MeV are needed to obtain reliable values of $(\alpha_1-\beta_1)_{\pi^\pm}$ in this experiment.

In conclusion, further experimental and theoretical investigations are needed to determine the true value of the pion polarizabilities.

The authors thank Th. Walcher and A.I. L'vov for useful discussions.

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