Lattice QCD
Heavy Ion Collisions
and QCD Phase Structure

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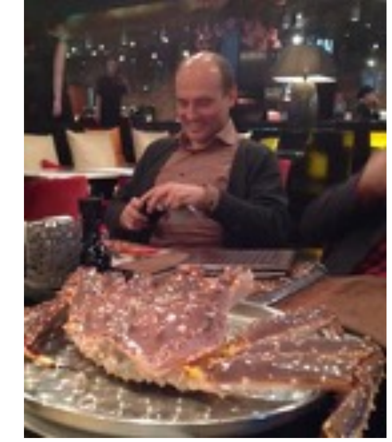
in Collaboration with
FEFU (Far Eastern Federal Univ.) group

CRITICAL POINTS in the MODERN
PARTICLEPHYSICS
Protvino, July 5-7, 2017

FEFU
FAR EASTERN FEDERAL UNIVERSITY

A. Molochkov



V. Bornyakov, D. Boyda,
V. Goy, A. Nikolaev

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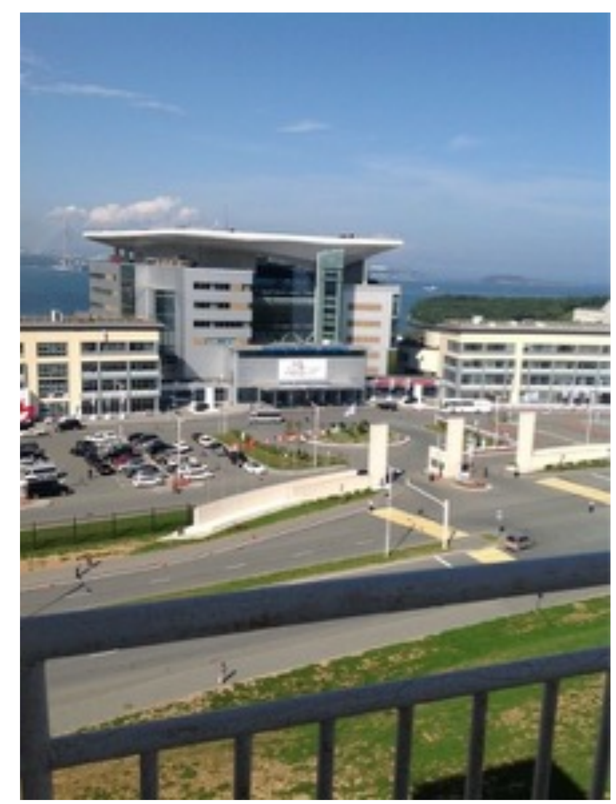


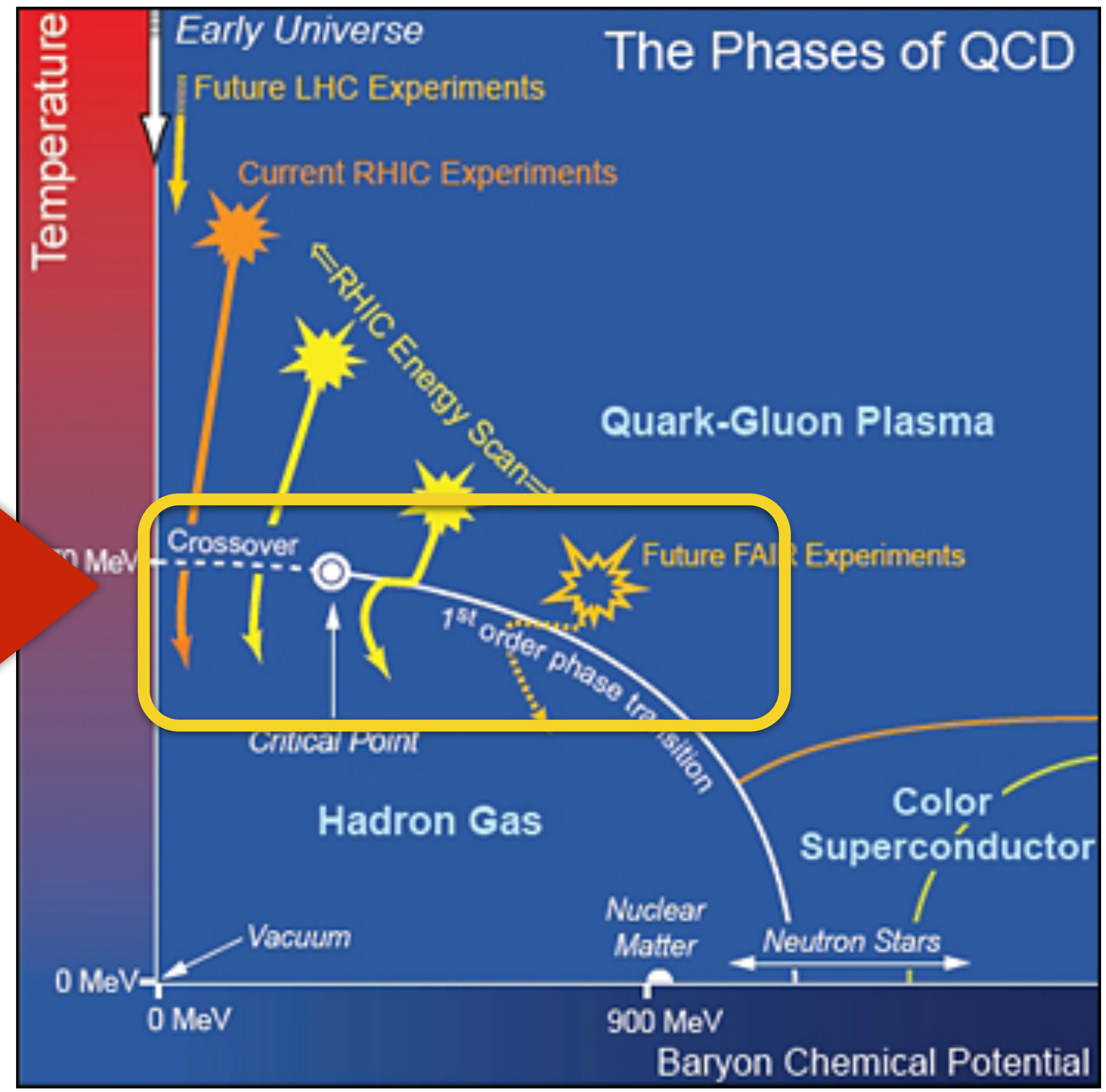
FEFU

A.Nakamura

Objective of Vladivostok Group

Study here with Zn

including LHC(ALICE),RHIC,FAIR, NICA, J-PARC



http://www.bnl.gov/rhic/news2/news.asp?a=1870&t=today

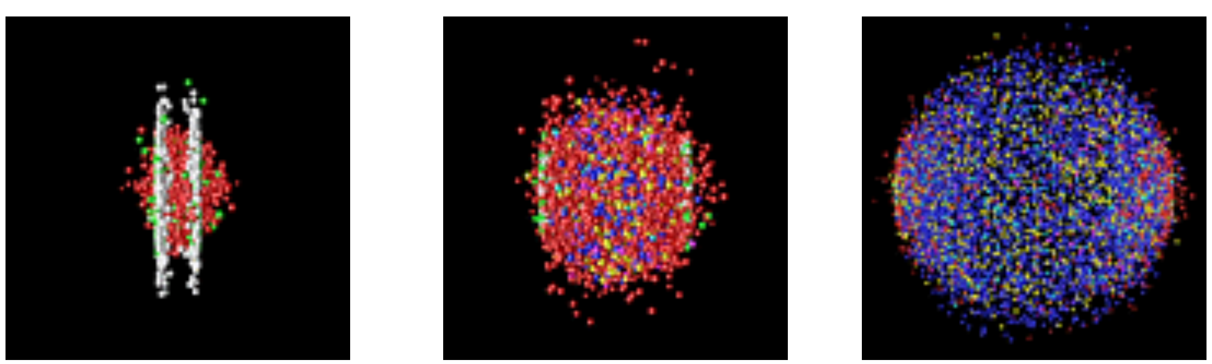
We assume

the Fireballs created in High Energy Nuclear Collisons are described as

a Statistical System.

with μ (chemical Potential)

and T (Temperature)



$$Z(\mu, T)$$

Grand Canonical Partition Function

This Statistical Description is good at least as a first approximation

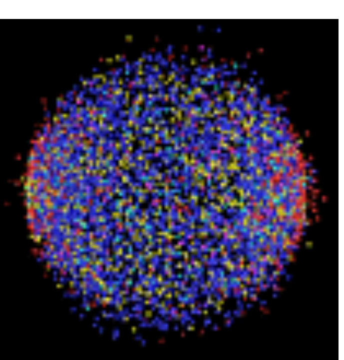
with Two Parameters Chemical Potential, μ

and Temperature, T

$Z_{GC}(\mu, T)$ Grand Canonical Partition Function

Alternative: Number, n and Temperature, T

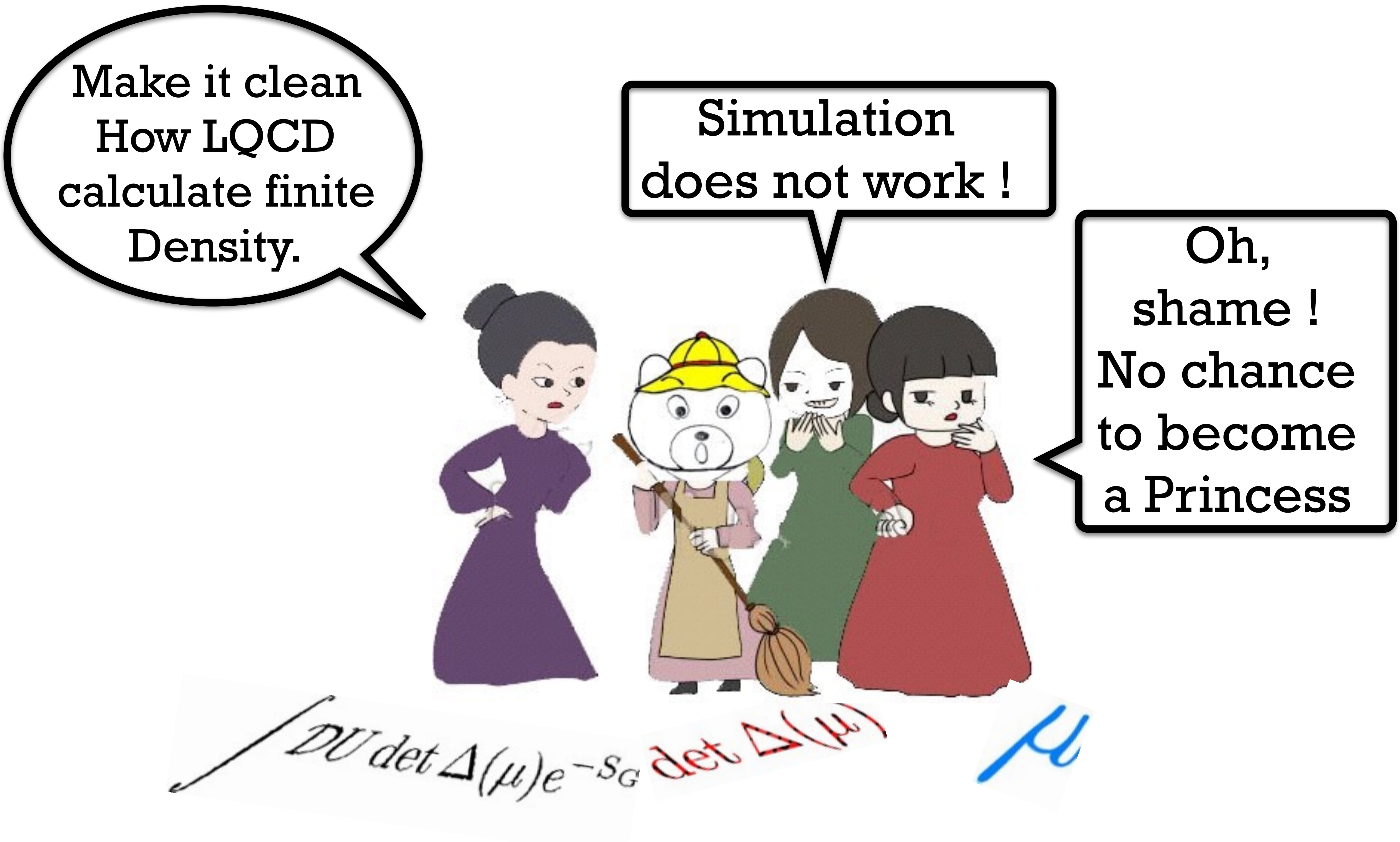
$Z_C(n, T)$ Canonical Partition Function



or Z_N

Lattice QCD simulations provide the fundamental information as a first principle calculation.

However,,,,,



Make it clean How LQCD calculate finite Density.
Simulation does not work !
Oh, shame ! No chance to become a Princess
$\int \mathcal{D}U \det\Delta(\mu) e^{-S_G}$
$\det\Delta(\mu)$
μ

QCD at finite density

μ : Chemical Potential

$$Z = \mathrm{Tr}\, e^{-\beta(H-\mu N)} = \int \mathcal{D}U \mathcal{D}\bar{\psi}\mathcal{D}\psi\, e^{-\beta S_G - \bar{\psi}\Delta\psi}$$

$$= \int \mathcal{D}U \prod_f \det\Delta(\mu)\; e^{-\beta S_G}$$

$$\Delta(\mu) = D_\nu\gamma_\nu + m + \mu\gamma_0$$

$$\Delta(\mu)^\dagger = -D_\nu\gamma_\nu + m + \mu^*\gamma_0 = \gamma_5\Delta(-\mu^*)\gamma_5$$

$$\Rightarrow\quad (\det\Delta(\mu))^* = \det\Delta(\mu)^\dagger = \det\Delta(-\mu^*)$$

$(\det \Delta(\mu))^* = \det \Delta(\mu)^\dagger = \det \Delta(-\mu^*)$

For $\mu = 0$

$$(\det \Delta(0))^* = \det \Delta(0)$$

$\det \Delta$ ⇨ *Real*

For $\mu \neq 0$ (in general)

$\det \Delta$ ⇨ *Complex*

$$Z = \int \mathcal{D}U \prod_f \det \Delta(m_f, \mu_f) e^{-\beta S_G}$$

⇧

Complex ⇨ Sign Problem

$$\langle O \rangle = \frac{1}{Z} \int \mathcal{D}U \, O \, \det \Delta \, e^{-\beta S_G}$$

In Monte Carlo simulation, configurations are generated according to the Probability:

$$\det \Delta \, e^{-\beta S_G} / Z$$

$\det \Delta$: *Complex*

⇒ Monte Carlo Simulations very difficult !

$$\langle O \rangle = \frac{\int DUO \det \Delta e^{-S_G}}{\int DU \det \Delta e^{-S_G}}$$

$$\det \Delta = |\det \Delta| e^{i\theta}$$

$$\langle O \rangle = \frac{\int DUO |\det \Delta| e^{i\theta} e^{-S_G}}{\int DU |\det \Delta| e^{-S_G}} \times \frac{\int DU |\det \Delta| e^{-S_G}}{\int DU |\det \Delta| e^{i\theta} e^{-S_G}}$$

$$= \frac{\langle O e^{i\theta} \rangle_{|\det|}}{\langle e^{i\theta} \rangle_{|\det|}}$$

Our Tool

Canonical Approach

Not so well-known

From Experiments

From Lattice

Z_N

Canonical Partition Functions

Advantage to use Z_n

- We can construct (approximate) Z_n from experimental Baryon number and Charge Distributions.
- We can circumvent the sign problem in Lattice QCD.
- We can construct Grand Partition Function $Z(\mu, T)$ from Z_n .
- New approach, i.e., Challenging !

They are equivalent and related as

$$Z(\xi, T) = \sum_n Z_n(T)\,\xi^n$$

$\xi \equiv e^{\mu/T}$ Fugacity

Quick Proof of Fugacity Expansion

$$Z(\mu, T) = \sum_n Z_n(T)(e^{\mu/T})^n$$

(Left Hand Side)= $\mathrm{Tr}\, e^{-(H-\mu\hat{N})/T}$

If $[H, \hat{N}] = 0$

$$= \sum_n \langle n| e^{-(H-\mu\hat{N})/T} |n\rangle$$

$$= \sum_n \langle n| e^{-H/T} |n\rangle \, e^{\mu n/T}$$

$\langle n| e^{-H/T} |n\rangle \rightarrow Z_n(T)$

This is a very useful relation.

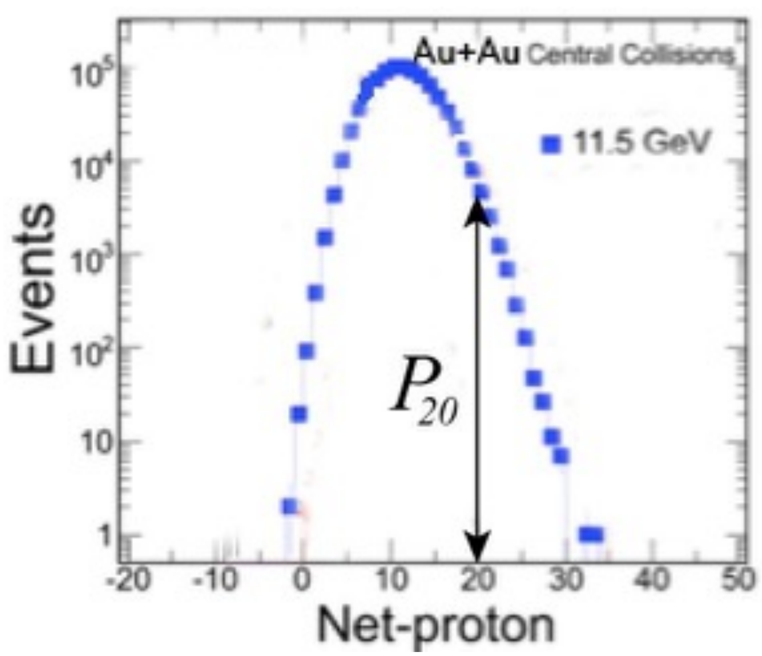
The partition function stands for the Probability

$$Z_{GC}(\mu, T) = \sum_n Z_n(T)\xi^n$$

System with μ and T

Probability to find (net-)baryon number=n

We extract Z_n from experimental multiplicity at RHIC



$$P_n = Z_n \xi^n \qquad \left(\xi \equiv e^{\mu/T}\right)$$

ξ unknown

$$Z_n = P_n / \xi^n$$

Z_n satisfies

$$Z_{+n} = Z_{-n}$$

RHIC tells us Z_n

(Particle-AntiParticle Symmetry)

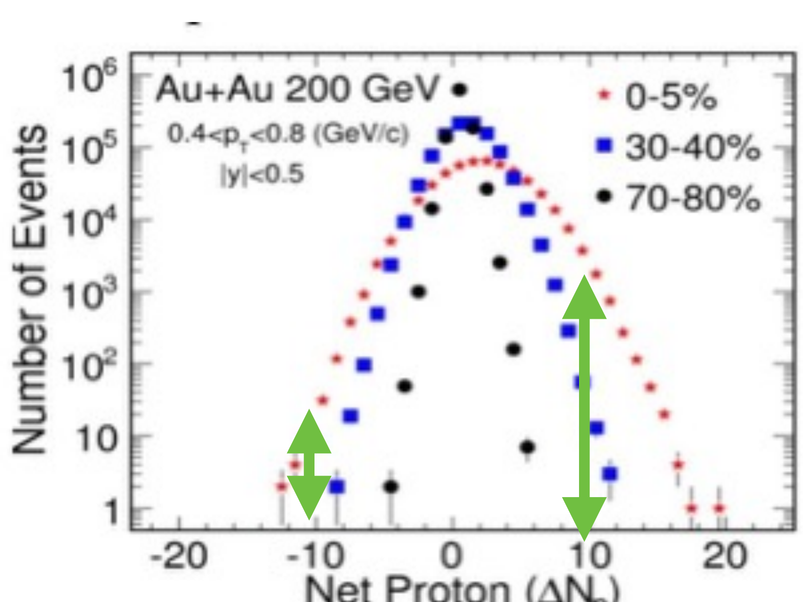
$$P_n = cZ_n\xi^n$$

$$P_{-n} = cZ_{-n}\xi^{-n}$$

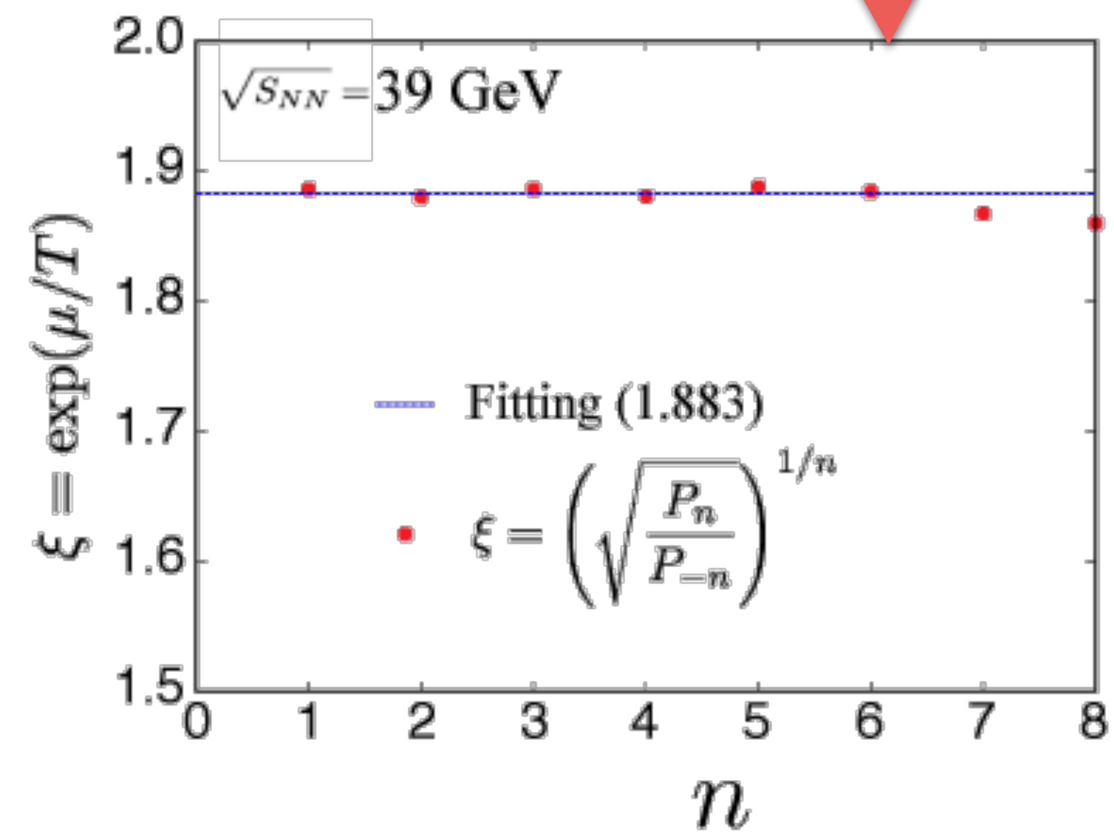
$$Z_{+n} = Z_{-n}$$

$$P_nP_{-n} = c^2 Z_n Z_{-n} = c^2 Z_n^2$$

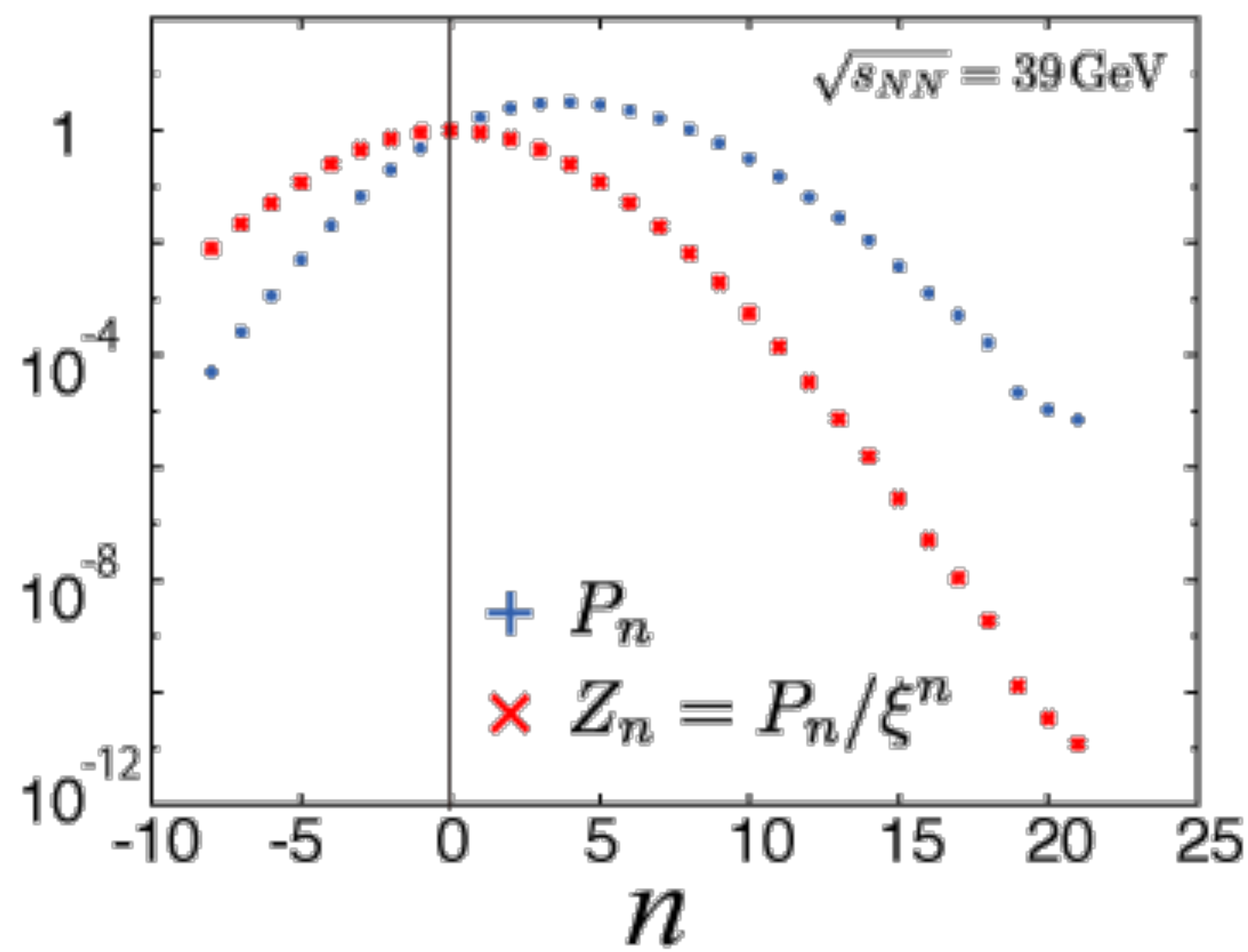
$$\frac{P_n}{\sqrt{P_nP_{-n}}} = \xi^n \quad \xi = \left(\sqrt{\frac{P_n}{P_{-n}}}\right)^{1/n}$$



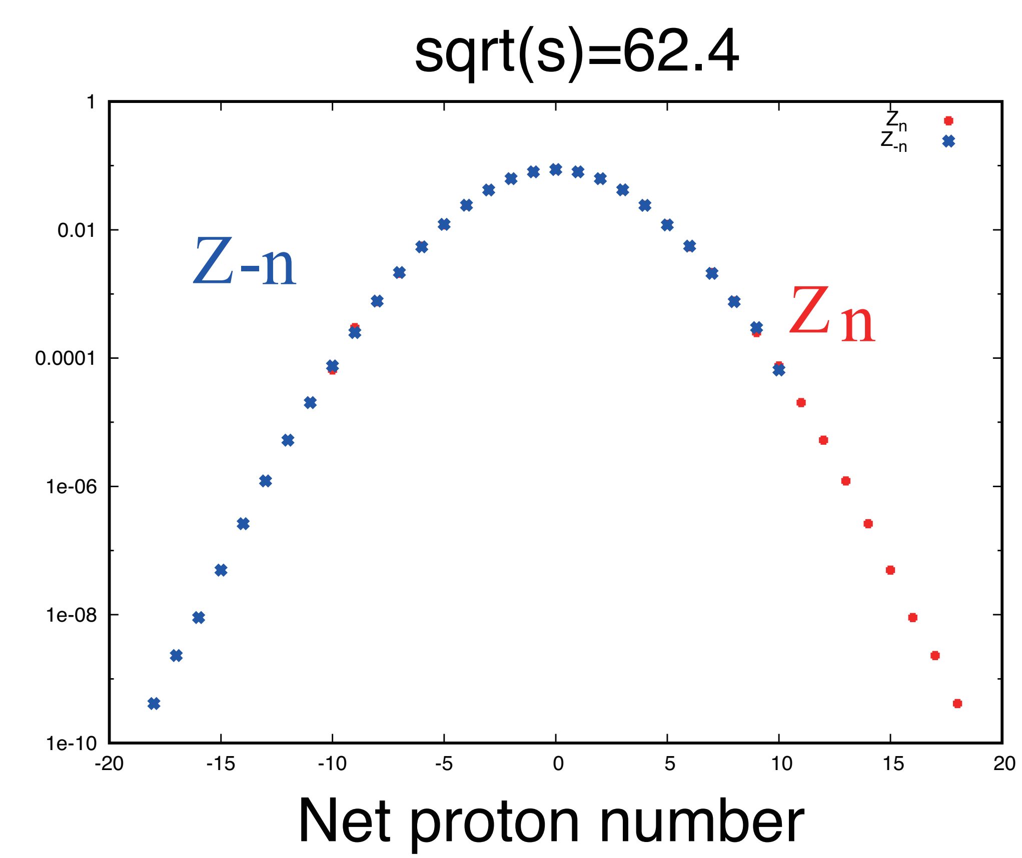
P_{-10} P_{+10}



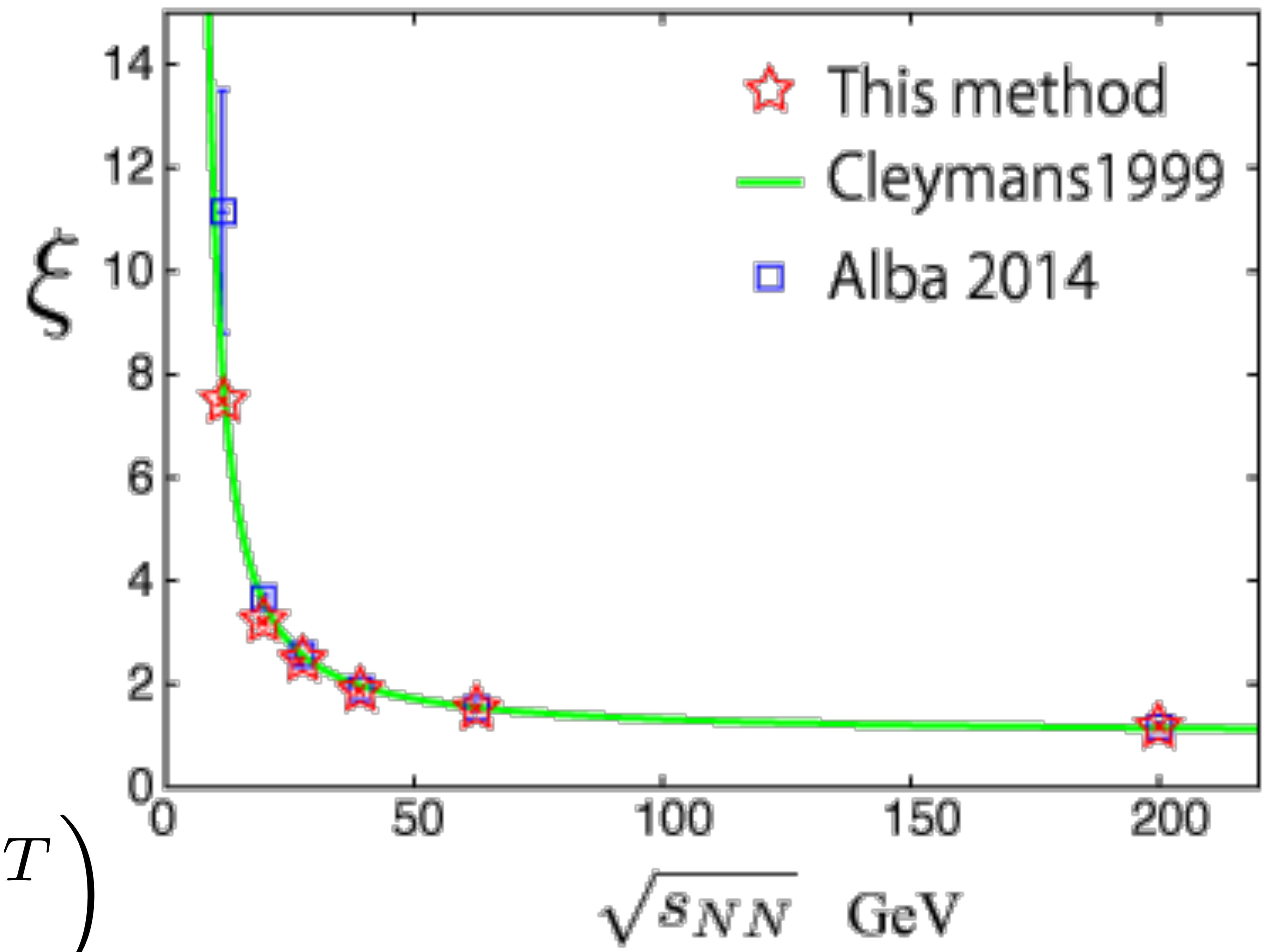
Here we demand $Z_{+n} = Z_{-n}$



$$\xi = 1.88336$$



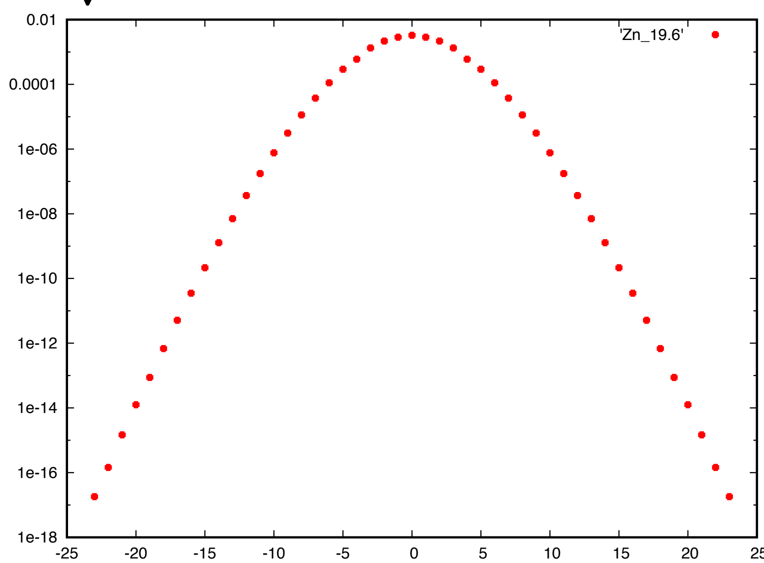
Fitted ξ are very consistent with those by Freeze-out Analysis.



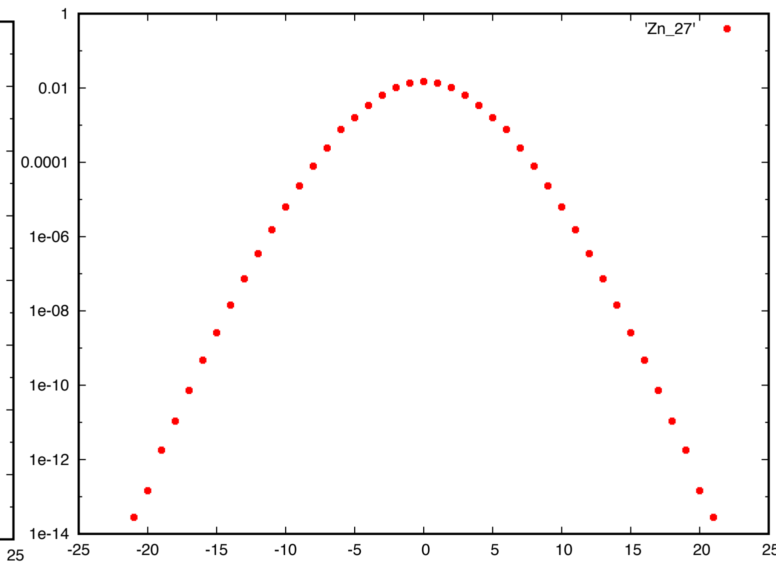
$$\left(\xi \equiv e^{\mu/T}\right)$$

Z_n from RHIC data

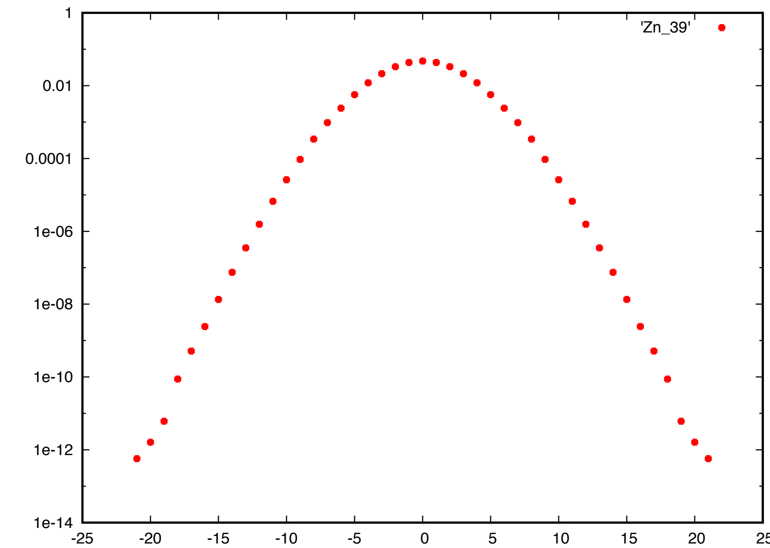
$\sqrt{s} = 19.6\mathrm{GeV}$



$\sqrt{s} = 27\mathrm{GeV}$

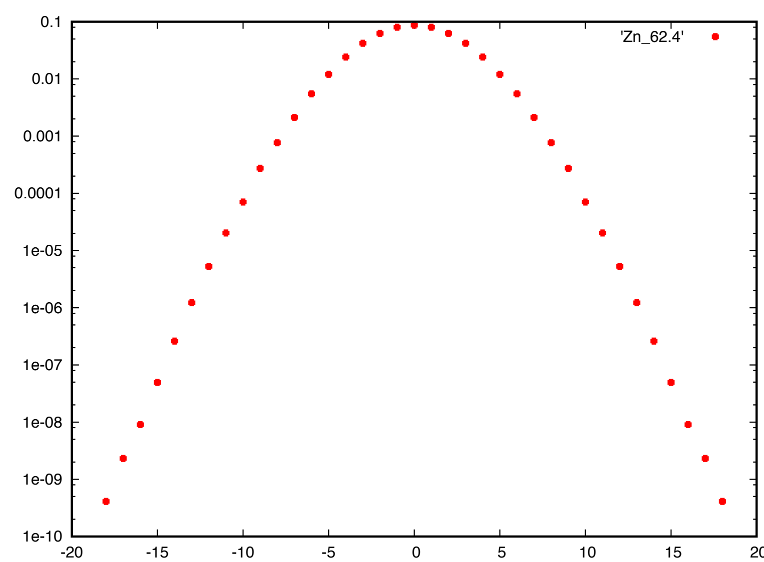


$\sqrt{s} = 39\mathrm{GeV}$

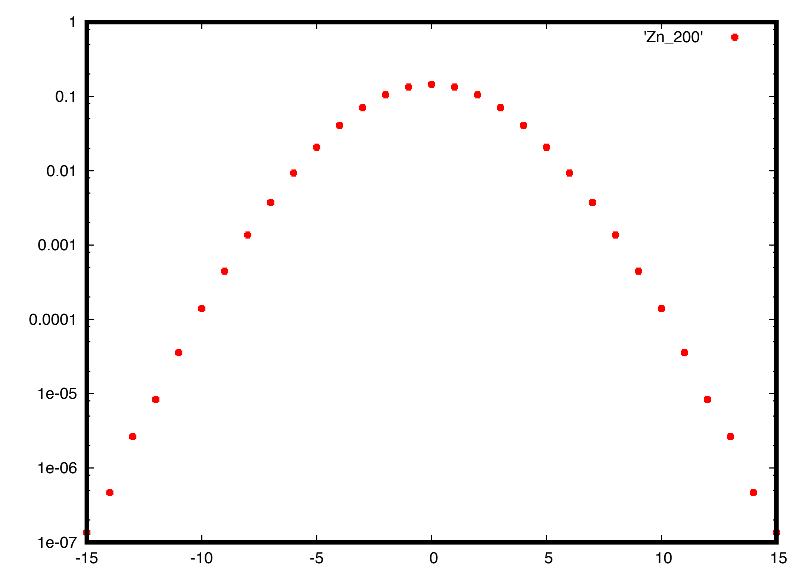


Can I see Difference?

$\sqrt{s} = 62.4\mathrm{GeV}$



$\sqrt{s} = 200\mathrm{GeV}$



Yes, You Can !
We will see it.

Yes, very useful, because

$$Z(\xi, T) = \sum_n Z_n(T)\,\xi^n$$

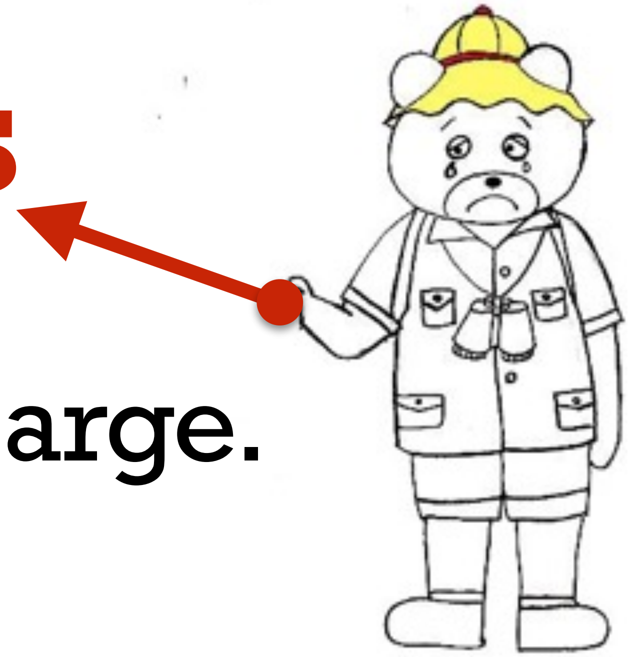
$(\xi \equiv e^{\mu/T}$: Fugacity)

$Z_n(T)$ ← $Z(\xi, T)$ at some ξ and T

$Z_n(T)$ → $Z(\xi, T)$ at ANY ξ

for both Experiments and Lattice

Problems



1) N_{max} is not very large.

$$Z(\xi, T) = \sum_{n=-N_{max}}^{+N_{max}} Z_n(T)\,\xi^n$$

Lower estimation of larger density contribution.

2) Measured multiplicities are not Baryon, but Proton.
Consider results Approximation.

We can calculate Z_n also by Lattice QCD

But Sign Problem on Lattice ?

$$Z_{GC}(\mu, T) = \int \mathcal{D}(\text{Gluon Fields}) \times \det D(\mu) \, e^{-(\text{Gluon Action})}$$

Complex if μ is real.

Our Lattice

- Clover improved Wilson action
- Iwasaki gauge action
- Lattice 4×16^3 (L ≈ 3.2fm, a ≈ 0.2fm)
- $m_\pi / m_\rho = 0.8$ ($m_\pi = 0.7$GeV)
  T/T_c = 0.84, 0.93,0.99, 1.08, 1.20, 1.35
- 20 - 40 points Imμ,
  1800 - 3800 configurations at each point
- Parameters were taken from
  S. Ejiri et. al., PRD 82, 014508 (2010)
- Our cluster: Vostok1 (20 GPU K40)

For Pure Imaginary μ ➡ $\det D$ **real**

A.Hasenfratz and Toussant, 1992

$$Z_C(n,T) = \int \frac{d\theta}{2\pi} e^{i\theta n} Z_{GC}(\theta \equiv \frac{\mathrm{Im}\mu}{T}, T)$$

Great Idea ! But practically it did not work.

We must develop several Engineering Methds.

1) Integration method
2) Multi-Precision Calculations

Integration Method

$$n_B = \frac{1}{3V} T \frac{\partial}{\partial \mu} \log Z_G$$

$$= \frac{N_f}{3N_s^3 N_t} \int \mathcal{D}U e^{-S_G} \mathrm{Tr} \Delta^{-1} \frac{\partial \Delta}{\partial \mu} \det \Delta$$

(For pure imaginary μ, n_B is also imaginary)

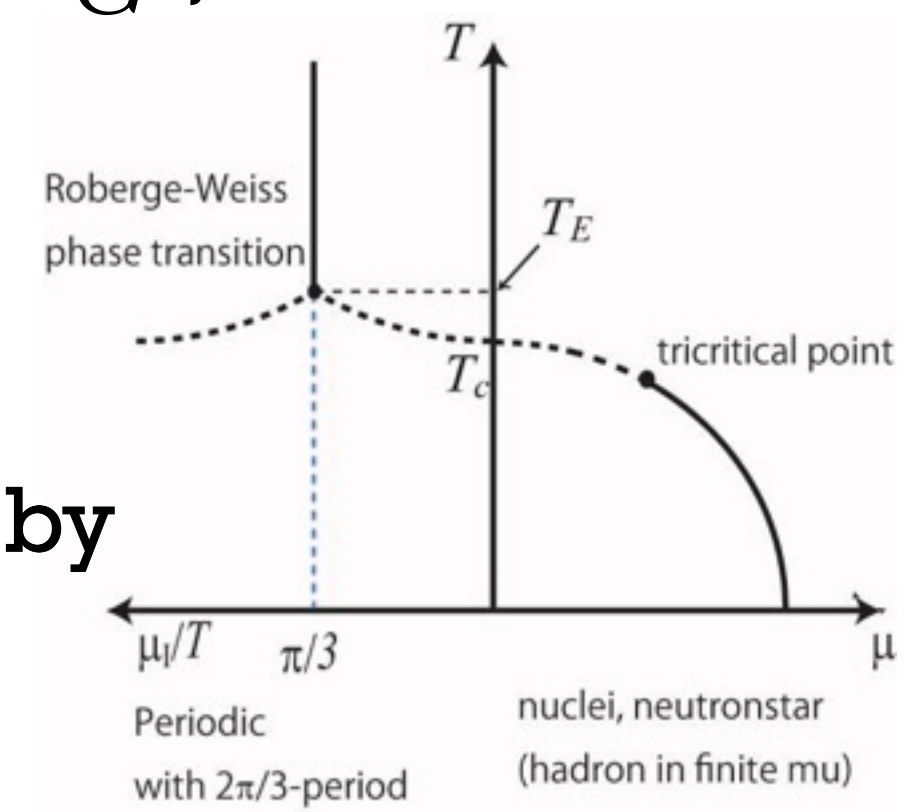
Then, for fixed T

$$Z(\theta \equiv \frac{\mu}{T}) = \exp(V \int_0^\theta n_B d\theta')$$

$$Z_k = \frac{3}{2\pi} \int_{-\pi/3}^{+\pi/3} d\theta \exp\left(i\, k\theta + \int_0^\theta n_B d\theta' \right)$$

We map Information in Pure Imaginary Chemical Potential to Real ones.

- We measure the number density at many pure imaginary chemical potential $n_B(\mu_I)$.
- We construct Grand Partition Function Z_G , by integrating $n_B(\mu_I)$
- By Fourier transformation, we get Z_n
- Then we can calculate Real μ regions by



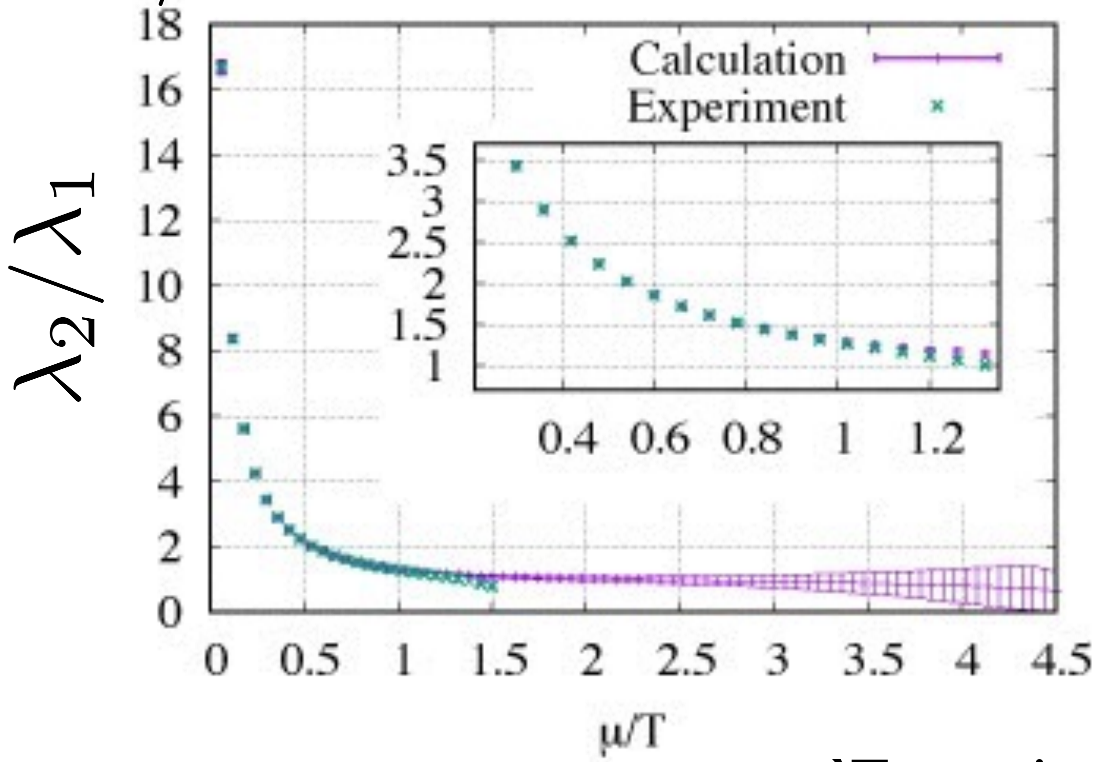
$$Z(\xi, T) = \sum_n Z_n(T)\,\xi^n$$

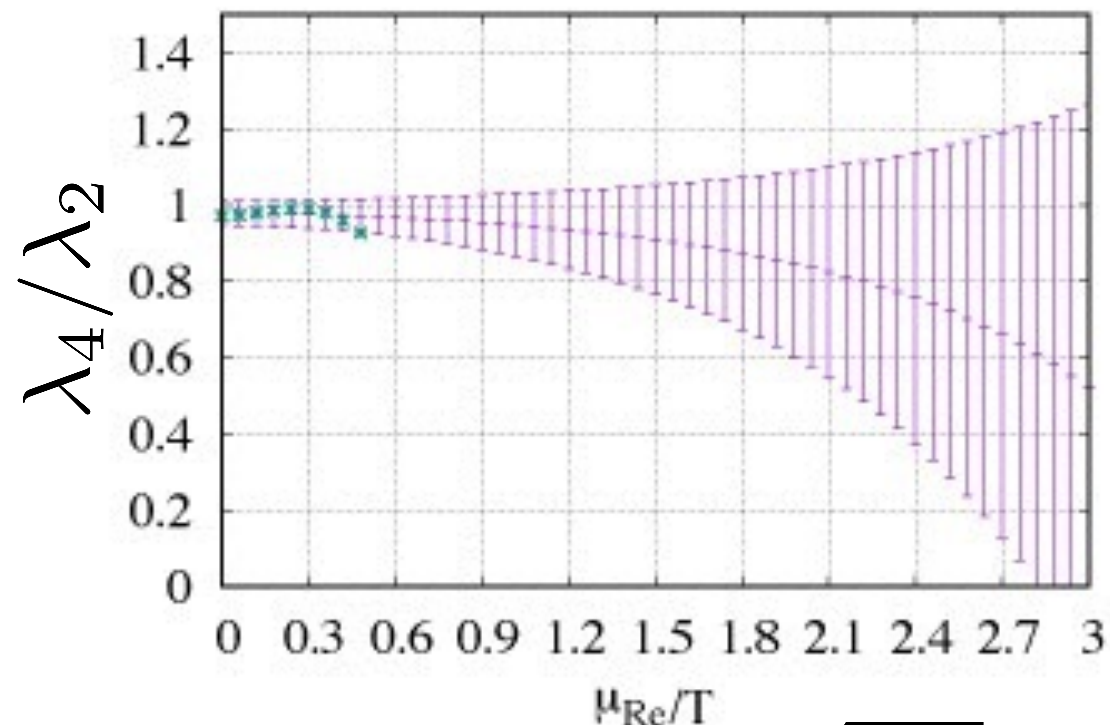
$\xi \equiv e^{\mu/T}$ Fugacity

Moments $\lambda_k = (T\frac{\partial}{\partial\mu})^k \log Z$

D.Boyda

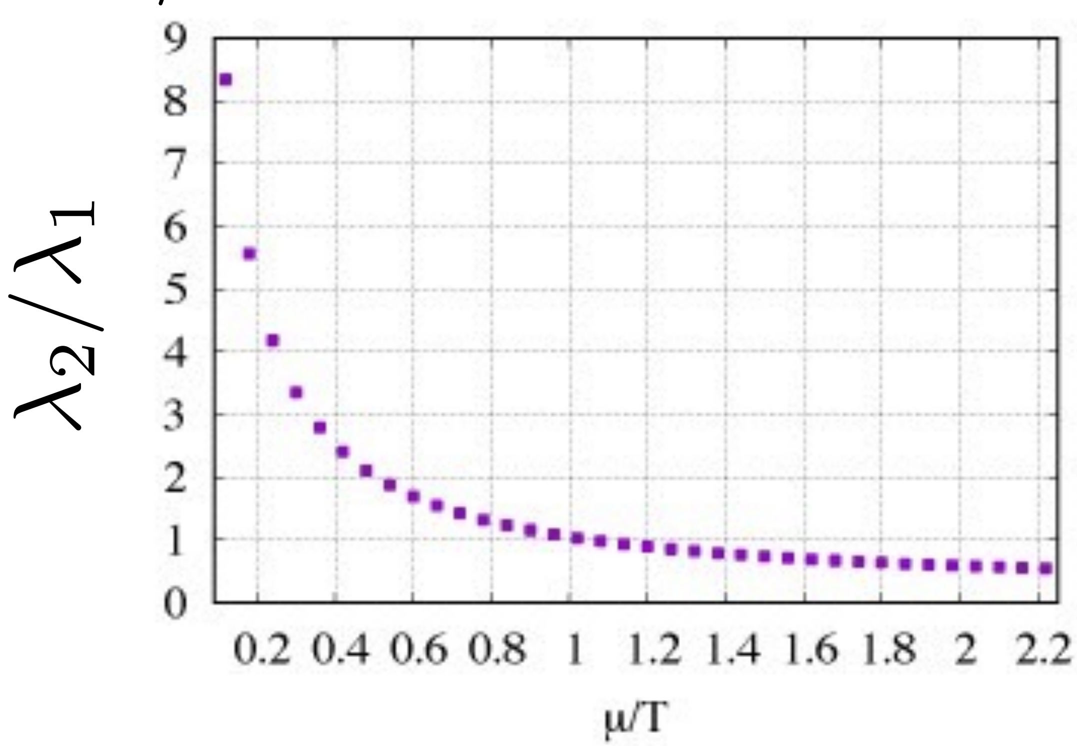
$T/T_c = 0.93$

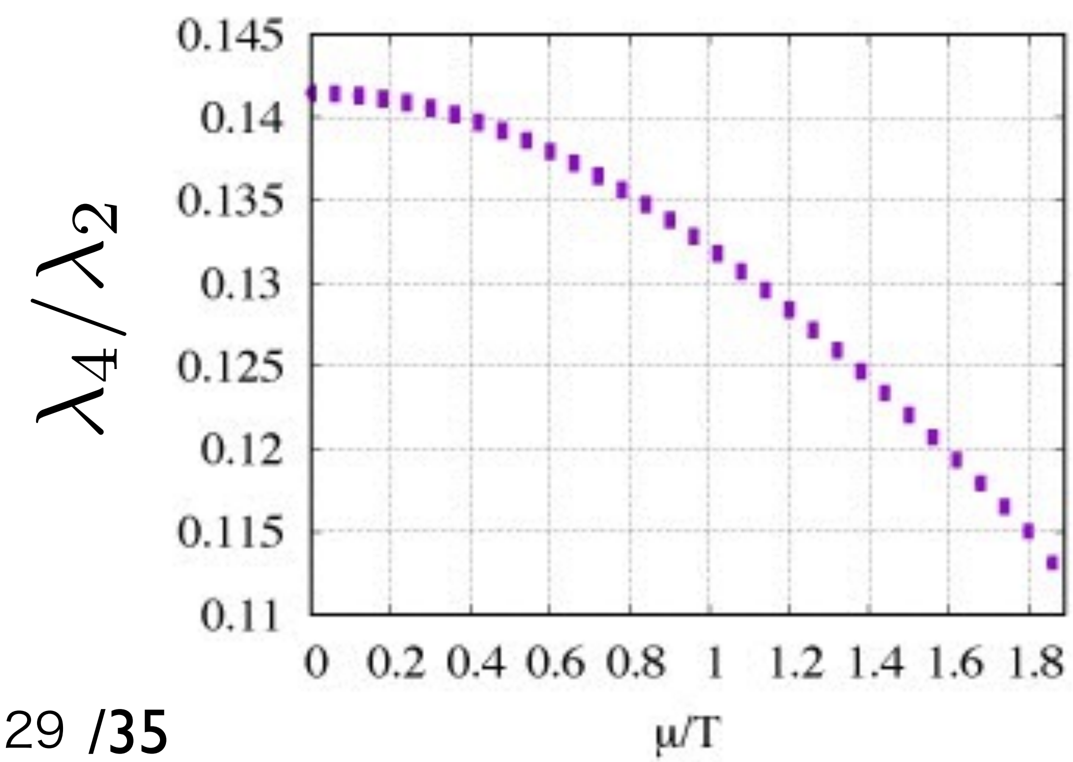




✖ \`Experiment' constructed from RHIC Star $\sqrt{s_{NN}} = 39\,(\mathrm{GeV})$

$T/T_c = 1.35$

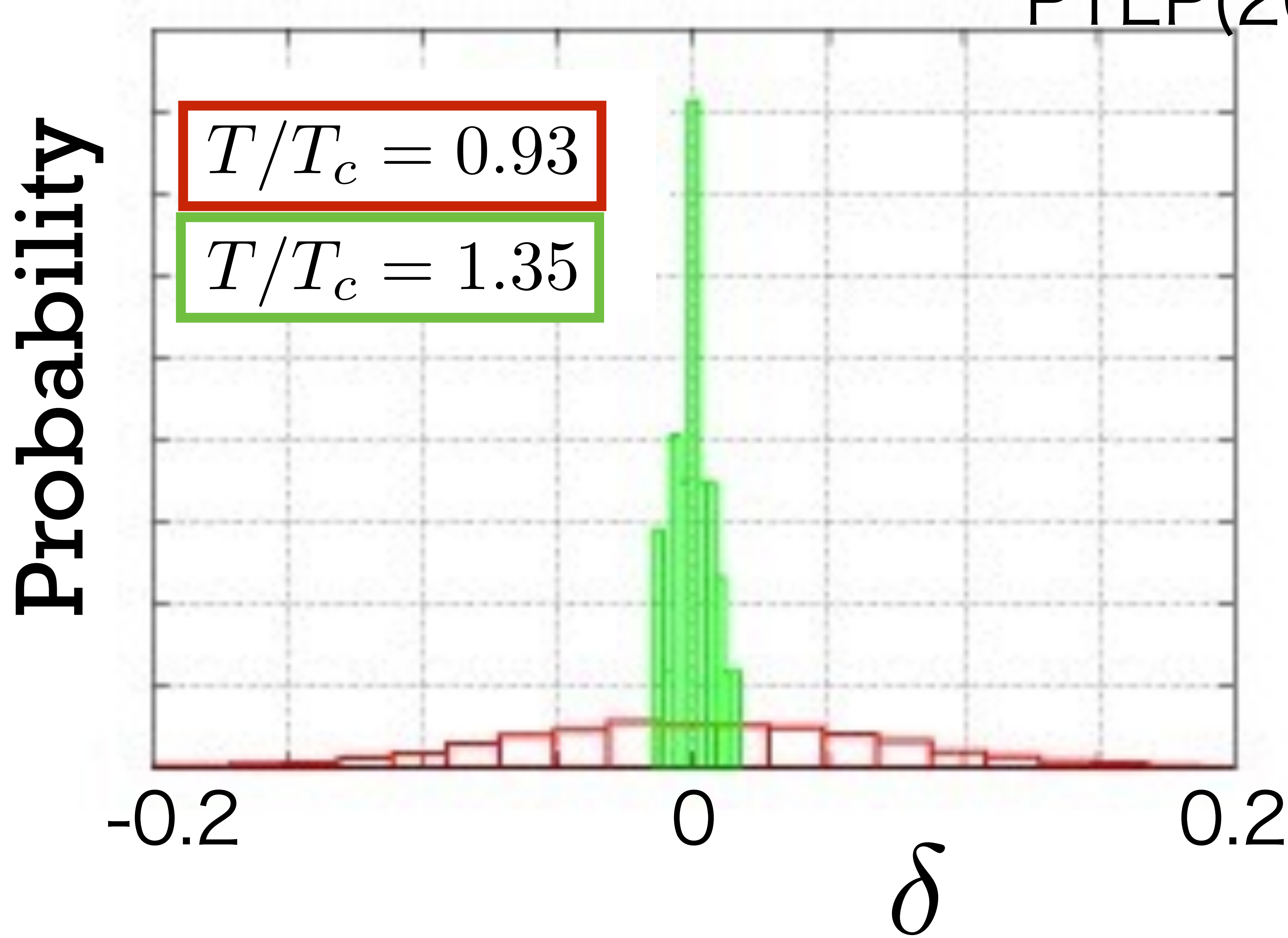




Hidden Sign Problem ?

Z_n **have phase on each configuration !**

V.Goy et al.,
PTEP(2017) 031D01

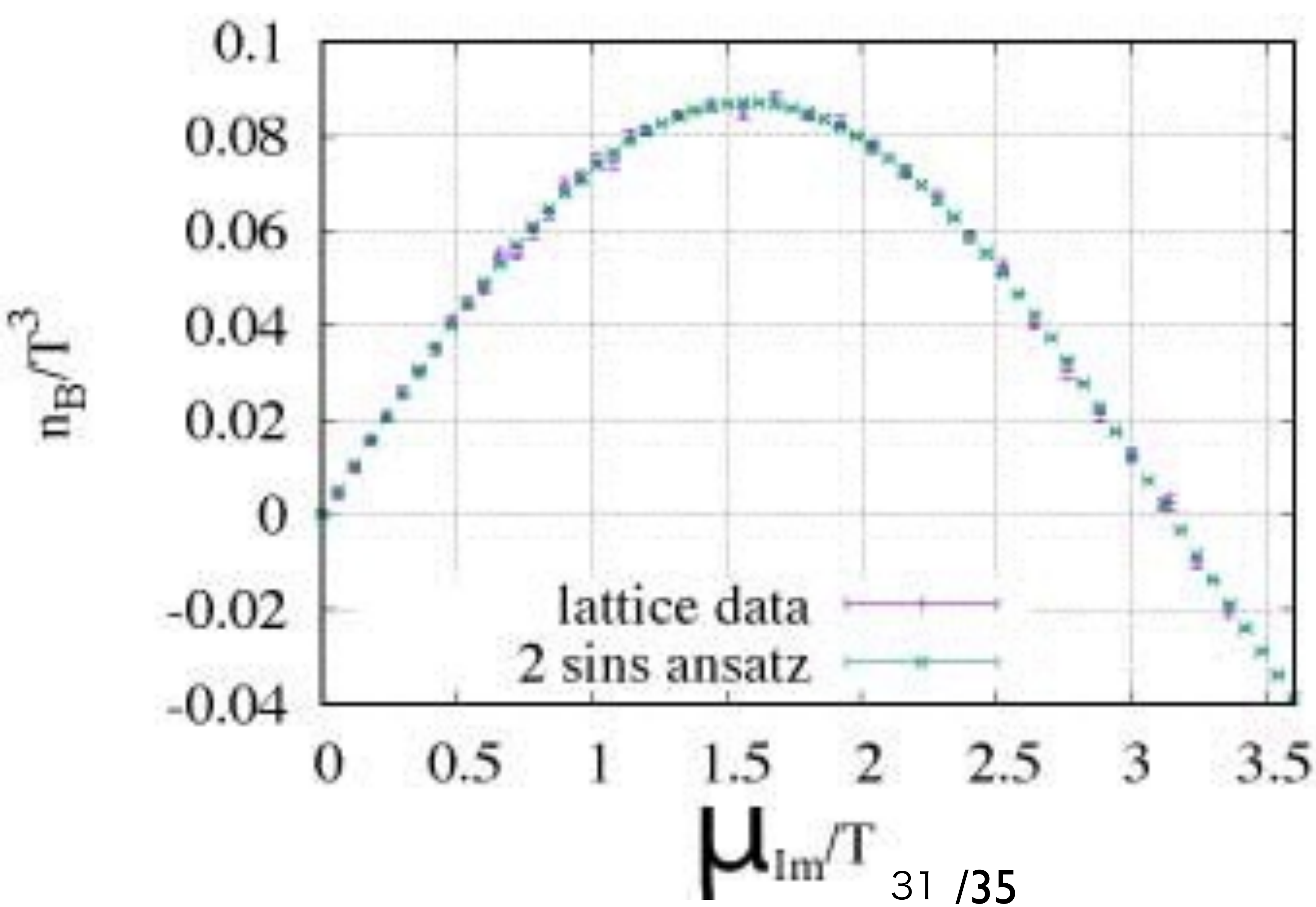


$z_n \simeq |z_n| e^{in\delta}$

$Z_n = \langle z_n \rangle$ are real positive.

A Remark of Function Form of $n_B(\mu_I)$

Preliminary



$n_B(\mu_I)$ is well approximated by sine function at *T*<*Tc*.

Takahashi et al. Phy. Rev. D 91 (1) (2015) 014501.
Bornyakov et al., Phys.Rev. D95, 094506 (2017)

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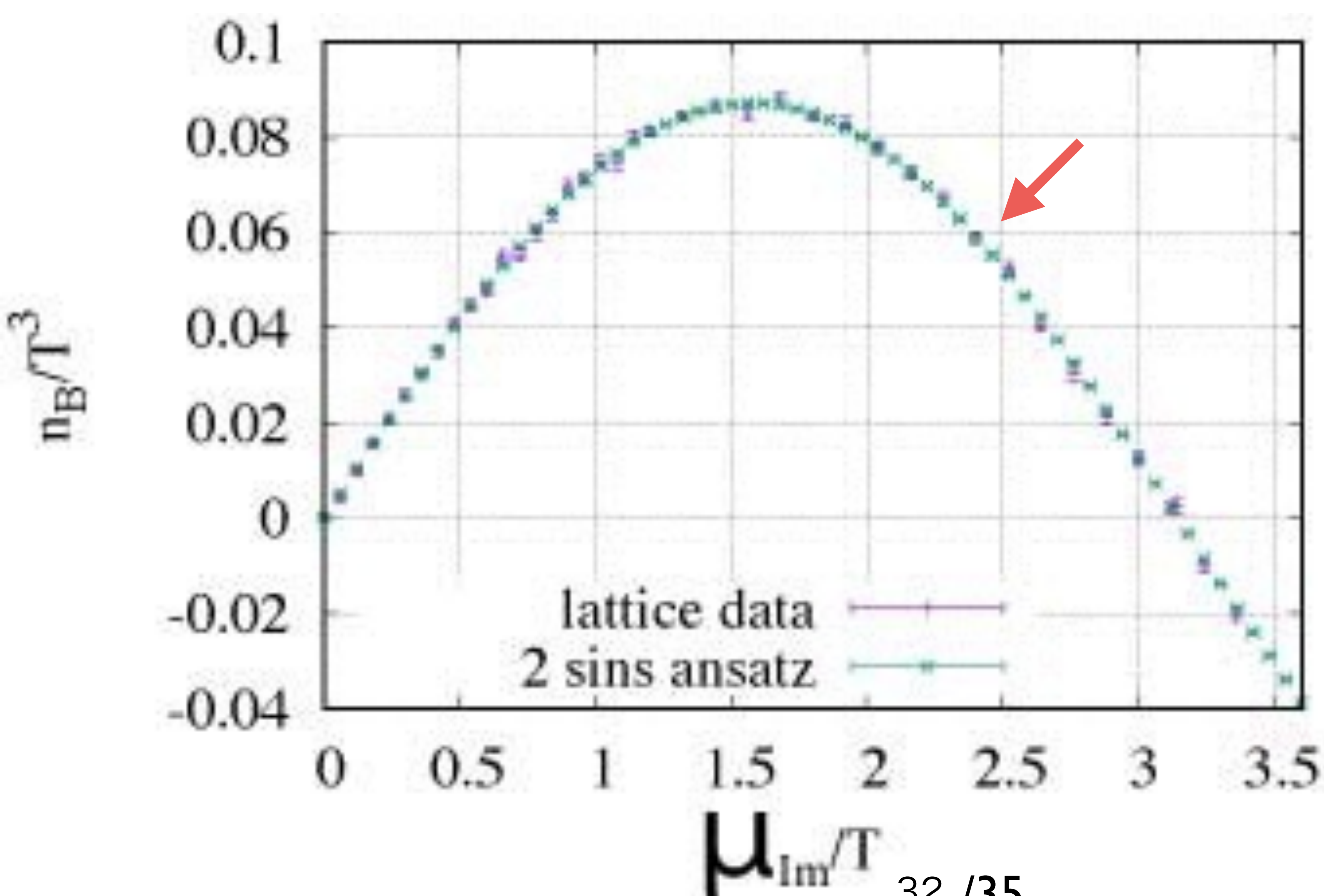

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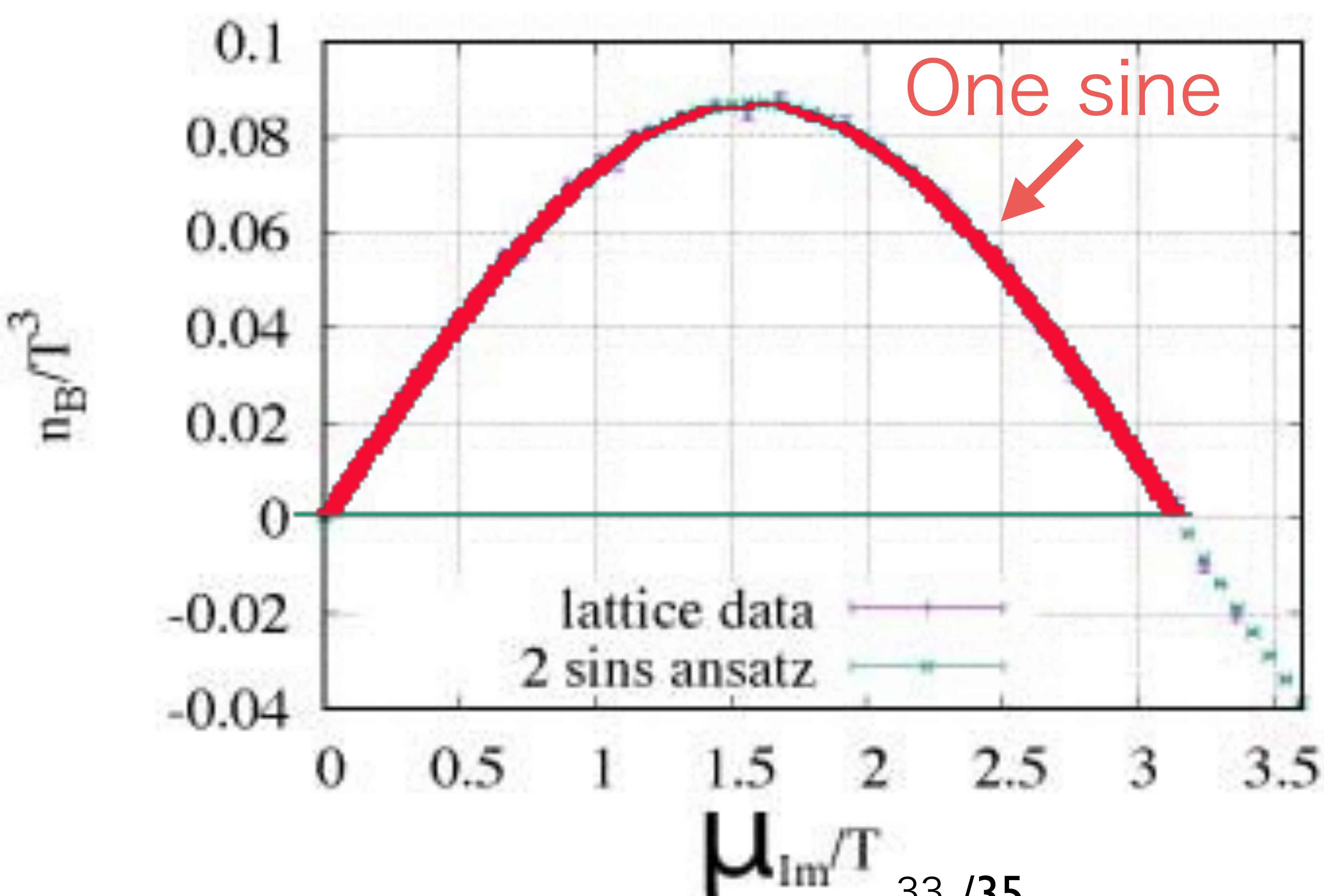

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In general,

$$n_B/T^3 = \sum_k f_{3k} \sin(k\theta_I)$$

$f3 = 0.0871(3)$, $f6 = -0.00032(27)$ ($\chi^2/\text{dof} = 0.93$)

Lowest order,

$$n_B/T^3 \sim f_3 \sin(\theta_I)$$

$$\Rightarrow Z_n \propto I_n(f_3)$$

This is Skellam Model, which is used in Heavy Ion Collisions to describe the gross structure.

(Skellam is the difference of two independent Poisson Distributions.
f6, f9 … include the dynamics.)

Concluding Remarks

- We have developed the Canonical Approach for revealing QCD Phase Structure.
- The canonical partition functions Z_n drop very rapidly as *n* goes large, and we need multi-precision calculations.
- The phase of Z_n fluctuates rapidly as *n* goes large in the confinement phase.
  No such problem in the deconfinement phase.
  For LHC, we can make reliable lattice calculations.
- Quark masses are heavy, because this is a test to see whether the Canonical Approach works for finite baryon density. So now it is time to go towards Physical Parameters.