Cosmology of Bigravity

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  — distance from home to work typical for Protvino.

We are between the two abysses (Blaise Pascal, 1623 – 1662)

"For after all what is man in nature? A nothing in regard to the infinite, a whole in regard to nothing, a mean between nothing and the whole; infinitely removed from understanding either extreme. The end of things and their beginnings are invincibly hidden from him in impenetrable secrecy..."

Minimal length available

Gravitational versus electromagnetic

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- Binary pulsars orbiting: $10^{-3} - 10^{-4}$ Hz (registered through pulsar timing, PSR B1913+16)

Alexander Fridman (1879 – 1925)

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The Creator Differential Equation

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The Creator Differential Equation

$$\boxed{\frac{3\dot{R}^2}{R^2} + \frac{3c^2}{R^2} - \lambda = \frac{\kappa}{2}c^2\rho}$$

Fridman's Cosmology as Newtonian Mechanics

$$\frac{m\dot{R}^2}{2} + U(R) = E,$$

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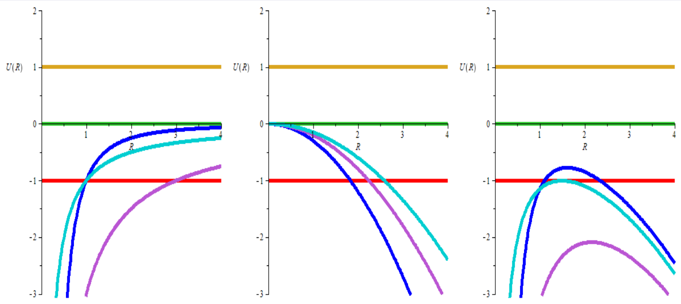
$$U = U_{Newton} + U_{anti-Hooke}.$$

Potential Energy in pictures

These potential energies are drawn below with the three cases of the space curvature marked by color: negative, zero or positive.



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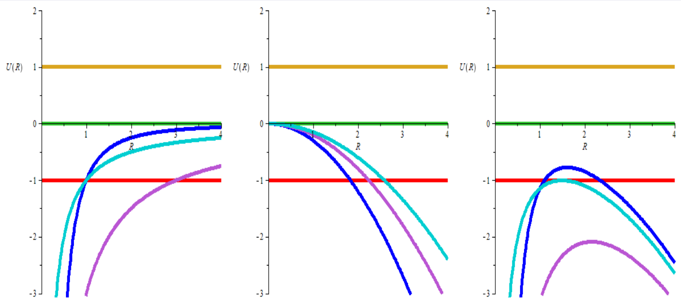
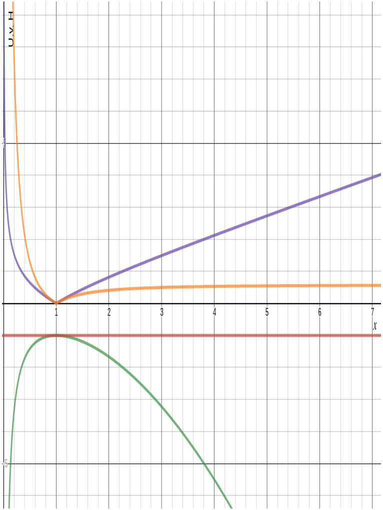


Where we are now? What are the initial conditions?

$U(R)$, E; $U(R)$, E, $v = \dot{R}$, $H = \dot{R}/R$



Einstein Case

Experience

- Dark energy to dust matter including cold dark matter ratio is 70/30,

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- it follows $x = R/R_E \approx 1.7$, where $R_E = \sqrt[3]{\frac{3GM}{\lambda}}$.

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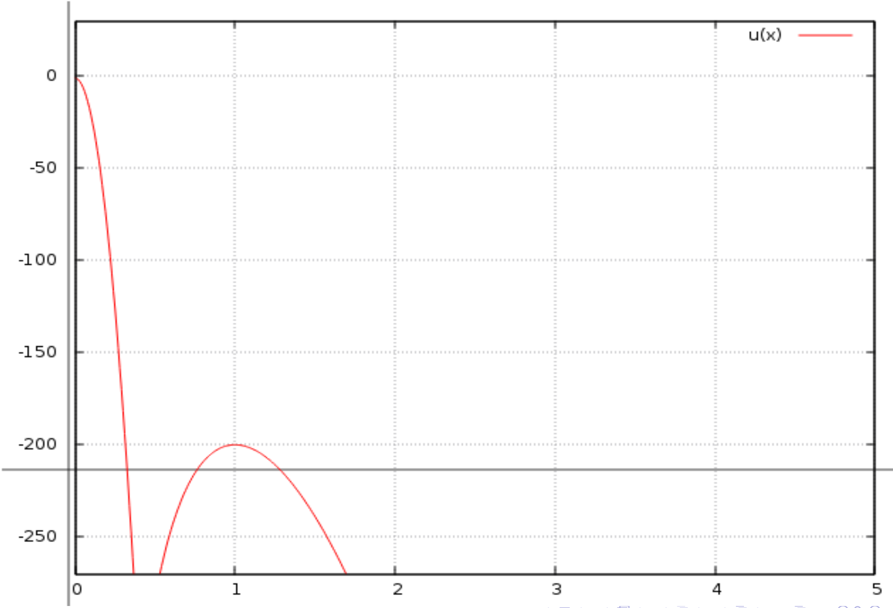
- it follows $x = R/R_E \approx 1.7$, where $R_E = \sqrt[3]{\frac{3GM}{\lambda}}$.
- The Hubble constant is now about $H = 2.3 \times 10^{-18}$ sec^{-1},
- and it follows that the parameter β in the dimensionless potential energy

$$\bar{U}(x) = -\frac{\beta}{3}\left(\frac{2}{x} + x^2\right).$$

is large enough $\beta > 190$, where $\beta \propto \sqrt[3]{\lambda (GM)^2}$.

Inflation scenario

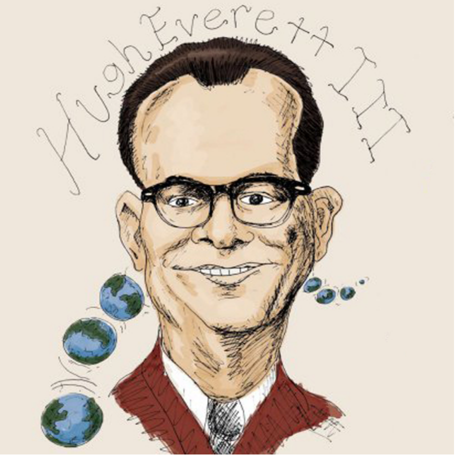


Georg Wilhelm Friedrich Hegel (1770 – 1831)

"Was vernünftig ist, das ist Wirklich; und was wirklich ist, das ist vernünftig." = "What is rational is real; And what is real is rational."

Everything happened!

Hugh Everett (1930 – 1982)



Does $10^{10^{50}}$ Universes provide the Spurious Infinite? [Die schlechte Unendlichkeit]

"Es hat Astronomen gegeben, die sich auf das Erhabene ihrer Wissenschaft gern darum viel zu Gute thaten, weil sie mit einer unermeßlichen Menge von Sternen, mit so unermeßlichen Räumen und Zeiten zu thun habe, in denen Entfernungen und Perioden, die für sich schon groß sind, zu Einheiten dienen, welche noch so vielmal genommen, sich wieder zur Unbedeutenheit verkürzen. Das schaale Erstaunen, dem sie sich dabei überlassen, die abgeschmackten Hoffnungen, erst noch in jenem Leben von einem Sterne zum anderen zu reisen und ins Unermeßliche fort dergleichen neue Kenntnisse zu erwerben, gaben sie für ein Hauptmoment der Vortreflichkeit ihrer Wissenschaft aus, –welche bewundernswürdig ist, nicht um solcher quantitativen Unendlichkeit willen, sondern im Gegentheil um der Maaßverhältnisse und der Gesetze willen, welche die Vernunft in diesen Gegenständen erkennt, und die das vernünftige Unendliche gegen jene unvernünftige Unendlichkeit sind."

Does $10^{10^{50}}$ Universes provide the Spurious Infinite? – [Дурная бесконечность]

“Среди астрономов были такие, которые очень охотно похвалялись возвышенностью их науки, усматривая эту возвышенность в том, что астрономия имеет дело с таким неизмеримым множеством звезд, с такими неизмеримыми пространствами и временами, в которых расстояния и периоды, уже и сами по себе столь огромные, служат единицами и которые, сколь бы многократно их ни брали, все же снова оказываются малыми до незначительности. Пустое удивление, которому они при этом предаются, плоские надежды, что в загробной жизни они будут перекочевывать с одной звезды на другую и, странствуя так по неизмеримому пространству, будут приобретать все новые и новые сведения того же сорта, — эти свои пустое удивление и плоские надежды они выдавали за основную черту превосходства их науки.”

Bimetric gravity I

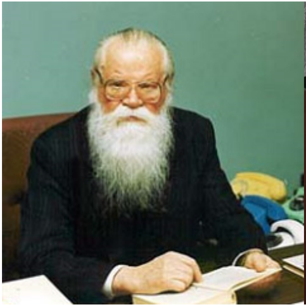
Nathan Rosen (1909 – 1995)

Period of his work in the USSR: 1936 – 1938

Bimetric gravity II

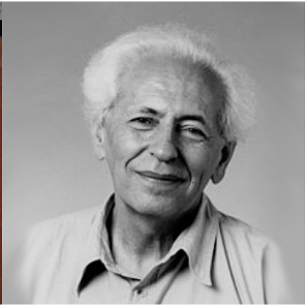
- N. Rosen (started in 1940)
- R. Feynman and some others (started in 1950's)
- C.J. Isham, A. Salam and J. Strathdee (1970)
- J. Wess and B. Zumino (1970)
- A.A. Logunov and his collegues (started in 1977)
- T. Damour and J. Kogan (2002)
- F. Hassan and R. Rosen (2011)

RTG (Relativistic Theory of Gravitation)



A.A. Logunov

M.A. Mestvirishvili



S.S. Gershtein

Massive gravity (RTG model)

- The potential added to the Hilbert-Einstein lagrangian:

$$\frac{m^2}{16\pi G}\left[\sqrt{-g}\left(\frac{1}{2}g^{\mu\nu}f_{\mu\nu}-1\right)-\sqrt{-f}\right].$$

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- Cosmological ansatz: no spatial derivatives

$$\sqrt{-g}=NuR^3, \qquad \sqrt{-f}=NR_0^3, \qquad g^{\mu\nu}f_{\mu\nu}=\frac{1}{u^2}+3\frac{R_0^2}{R^2}.$$

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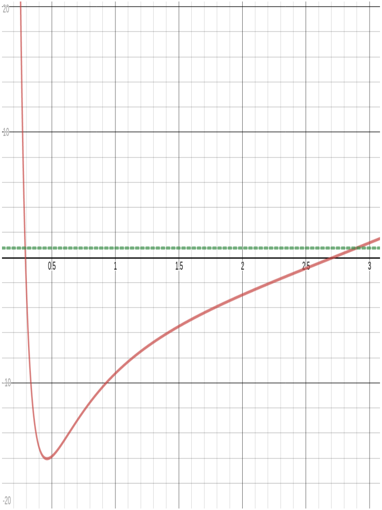
- The gravitational potential energy density of the Universe

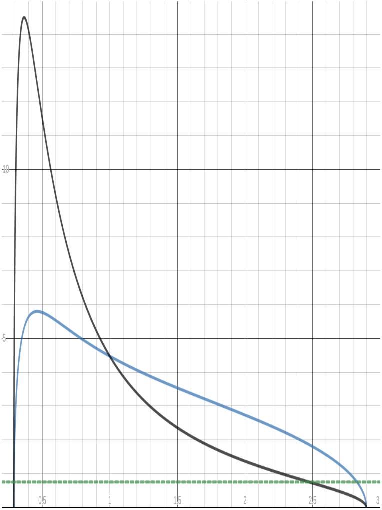
$$U(R)=-\frac{GM}{R}+\frac{m^2}{2}\left(R^2+\frac{1}{2}\frac{R_0^6}{R^4}\right).$$

RTG in graphics

$U(R)$, E;



E, $v = \dot{R}$, $H = \dot{R}/R$



de Rham, Gabadadze, Tolley (dRGT)

dRGT potential (I)

The potential is formed by means of the symmetric polynomials of matrix $X^\mu_\nu = \left(\sqrt{g^{-1}f}\right)^\mu_\nu$ given through eigenvalues of X:

$$\begin{aligned}
e_0 &= 1, \\
e_1 &= \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4, \\
e_2 &= \lambda_1\lambda_2 + \lambda_2\lambda_3 + \lambda_3\lambda_4 + \lambda_4\lambda_1 + \lambda_1\lambda_3 + \lambda_2\lambda_4, \\
e_3 &= \lambda_1\lambda_2\lambda_3 + \lambda_2\lambda_3\lambda_4 + \lambda_1\lambda_3\lambda_4 + \lambda_1\lambda_2\lambda_4, \\
e_4 &= \lambda_1\lambda_2\lambda_3\lambda_4,
\end{aligned}$$

$$\boxed{U = \frac{2m^2}{\kappa}\sqrt{-g}\sum_{n=0}^{4}\beta_n e_n(X) = \frac{m^2}{8\pi G}\left(\beta_0\sqrt{-g} + \ldots + \beta_4\sqrt{-f}\right)},$$

Bigravity with dRGT potential

- Let both space-time metrics f and g be dynamical with Lagrangian

$$\mathcal{L} = \mathcal{L}_f + \mathcal{L}_g + \mathcal{L}_M - \sqrt{-g}\,U(f_{\mu\nu}, g_{\mu\nu}),$$

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- where

$$U(f_{\mu\nu}, g_{\mu\nu}) = \frac{2m^2}{\kappa}\sqrt{-g}\sum_{n=0}^{4} \beta_n e_n(X)$$

Let us take the cosmologilcal ansatz for both metrics

$$f_{\mu\nu} = (-N^2(t), {R_f}^2(t)\delta_{ij}), \qquad g_{\mu\nu} = (-\bar{N}^2(t), R^2(t)\delta_{ij}),$$

then new variables appear

$$\boxed{u = \frac{\bar{N}}{N}}, \qquad \boxed{r = \frac{R_f}{R}}.$$

$$Y^{\mu}_{\nu} = \left(g^{-1}f\right)^{\mu}_{\nu} = g^{\mu\alpha}f_{\alpha\nu} = \mathrm{diag}\left(u^{-2}, r^2\delta_{ij}\right),$$

The positive square root of this diagonal matrix is here

$$X = \sqrt{Y} = \mathrm{diag}\left(+\sqrt{u^{-2}}, +\sqrt{r^2}\delta_{ij}\right) \equiv \mathrm{diag}\left(u^{-1}, r\delta_{ij}\right),$$

λ_i and e_i are as follows

$$\begin{aligned}
\lambda_1 &= u^{-1}, \qquad \lambda_2 = \lambda_3 = \lambda_4 = r,\\
e_0 &= 1,\\
e_1 &= \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 = u^{-1} + 3r,\\
e_2 &= \lambda_1\lambda_2 + \lambda_1\lambda_3 + \lambda_1\lambda_4 + \lambda_2\lambda_3 + \lambda_2\lambda_4 + \lambda_3\lambda_4 = 3ru^{-1} + 3r^2,\\
e_3 &= \lambda_1\lambda_2\lambda_3 + \lambda_2\lambda_3\lambda_4 + \lambda_1\lambda_3\lambda_4 + \lambda_1\lambda_2\lambda_4 = 3r^2u^{-1} + r^3,\\
e_4 &= \lambda_1\lambda_2\lambda_3\lambda_4 = r^3u^{-1}.
\end{aligned}$$

Bi-dRGT potential in cosmology

$$\boxed{U = \frac{2m^2}{\kappa} N\,(uV + W)}.$$

The potential is linear in u and

$$V = \frac{1}{N}\frac{\partial U}{\partial u} = R^3 B_0(r),$$
$$W = \frac{1}{N}\left(U - u\frac{\partial U}{\partial u}\right) = R^3 B_1(r) \equiv R_f^3 \frac{B_1(r)}{r^3},$$

deformed formulae for $(1+r)^3$ arise above

$$B_i(r) = \beta_i + 3\beta_{i+1} r + 3\beta_{i+2} r^2 + \beta_{i+3} r^3.$$

Matter couplings

1. One matter minimally couples to $g_{\mu\nu}$ (no BD ghost at all).
2. g-matter and f-matter minimally couple to $g_{\mu\nu}$ and $f_{\mu\nu}$ (no BD ghost at all).
3. One matter minimally couples to both $g_{\mu\nu}$ and $f_{\mu\nu}$ (BD ghost is present).
4. One matter minimally couples to “the effective metric” (no BD ghost below the cut-off).

Coupling to effective metric (I)

$$\mathcal{G}_{\mu\nu} = g_{\mu\nu} + 2\beta g_{\mu\alpha}\sqrt{g^{-1}f}^{\alpha}_{\nu} + \beta^2 f_{\mu\nu} = \left(E^A_\mu + \beta F^A_\mu\right)\left(E_{A\nu} + \beta F_{A\nu}\right),$$

$$\mathcal{L}_\phi = \sqrt{-\mathcal{G}}\left(\frac{1}{2}\mathcal{G}^{\mu\nu}\phi_{,\mu}\phi_{,\nu} - U(\phi)\right),$$

Cosmological ansatz is as follows

$$\mathcal{G}_{00} = -\mathcal{N}^2, \qquad \mathcal{G}_{ij} = a^2\delta_{ij},$$

$$\mathcal{N} = N(u+\beta), \qquad a = R + \beta R_f.$$

$$\sqrt{-\mathcal{G}} = \mathcal{N}a^3,$$

$$\mathcal{L}_\phi = \mathcal{N}a^3\left(\frac{1}{2}\left(\frac{\dot\phi}{\mathcal{N}}\right)^2 - U(\phi)\right), \qquad \pi_\phi = \frac{a^3}{\mathcal{N}}\dot\phi.$$

Coupling to effective metric (II)

The primary constraints ($\mu = G_f/G_g$)

$$
\begin{aligned}
\mathcal{S} &= \frac{3R^3}{8\pi G_g}\left[-{H_g}^2 + (1+\beta r)^3\frac{8\pi G_g\rho}{3} + \frac{m^2}{3}B_0(r)\right] = 0,\\
\mathcal{R}' &= \frac{3R^3}{8\pi G_g}\left[-\frac{r^3{H_f}^2}{\mu} + \beta(1+\beta r)^3\frac{8\pi G_g\rho}{3} + \frac{m^2}{3}B_1(r)\right] = 0
\end{aligned}
$$

The secondary constraint

$$
\begin{aligned}
\Omega &= \frac{3R}{8\pi G_g}\Omega_1\Omega_2 = 0,\\
\Omega_1 &= rH_f - H_g,\\
\Omega_2 &= \beta_1 + 2\beta_2 r + \beta_3 r^2 - \beta\,(1+\beta r)^2\,8\pi G_g p = 0.
\end{aligned}
$$

First branch

The Friedmann equation for the observable Hubble constant

$$H = \frac{\dot{a}}{\mathcal{N} a}$$

is as follows

$$\boxed{H^2 = \frac{8\pi \tilde{G} \rho}{3} + \frac{\Lambda(r)}{3}}, \qquad \tilde{G} = (1 + \beta r) G.$$

The cosmological term and matter density are functions of r:

$$\begin{aligned} \Lambda(r) &= m^2 \frac{B_0(r)}{(1+\beta r)^2}, \\ \rho &= \frac{m^2}{8\pi G} \frac{\frac{\mu B_1(r)}{r} - B_0(r)}{(1+\beta r)^3 \left(1 - \frac{\mu\beta}{r}\right)}. \end{aligned} \quad (1)$$

Cosmology as evolution of "hidden variable" r

The study of cosmological dynamics transforms into a study of dynamics for r (we suppose equation of state $p = w\rho$)

$$\dot{r} = \frac{3NHa(1+w)(1+\beta r)\left(\frac{\mu B_1}{r} - B_0\right)}{B_0 - (B_{-1})' + \frac{\mu B_0'}{r} + \left(\frac{\mu B_1}{r} - B_0\right)\left(\frac{1}{1-\frac{\mu\beta}{r}} + \frac{3w}{1+\beta r}\right)}.$$

Critical points may be

$$r = -\frac{1}{\beta},$$

$$r = \mu\beta,$$

and the roots of quartic equation

$$\frac{\mu B_1(r)}{r} - B_0(r) = 0.$$

Conclusion: bigravity is cosmologically viable

1. dRGT bigravity has background solutions reproducing the standard cosmological model for the Early Universe.
2. Cosmological constant problem may be solved by postulating the graviton mass value $m \approx \sqrt{\Lambda}$.
3. There is a hope to address the dark matter problem also (Luc Blanchet and Lavinia Heisenberg, 1504.00870).
4. There is a hope to overcome instabilities of local perturbations by a succesful choice of parameters μ, β_i, β (Akrami et al, 1503.07521).