$SU(2)$ Gluon Propagators and the Asymmetry $\langle A^2 \rangle$ in the Postconfinement Domain

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[1] IHEP, Protvino; [2] JINR, Dubna

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- Motivation
  - Phase and crossover transitions in QCD-like theories at $\mu_B = 0$
  - The concept of semi-QGP and postconfinement domain
  - Old effective theories and the Polyakov loop
  - Why the asymmetry $\langle \Delta_{A^2} \rangle$ and the propagators are of interest?
- Definitions and details of simulation
- Gribov-copy and finite-volume effects
- Temperature dependence of the asymmetry
- Propagators and transition from electric to magnetic dominance
- Conclusions

QCD at $\mu_B = 0$ predicts two crossover transitions:

- the chiral transition ($T_c = 151(3)(3)$ MeV [1],
  $T_c = 192(7)(4)$ MeV [2]);
  the width $\Delta T_c(\chi_{\bar{\psi}\psi}) = 28(5)(1)$ MeV [1].
- the deconfinement transition ($T_c = 176(3)(4)$ MeV [1],
  $T_c = 192(7)(4)$ MeV [2]);
  the width $\Delta T_c(\mathcal{P}) = 38(5)(1)$ MeV [1].

[1] Aoki et al., hep-lat/0609068
[2] M. Cheng et al., hep-lat/0608013

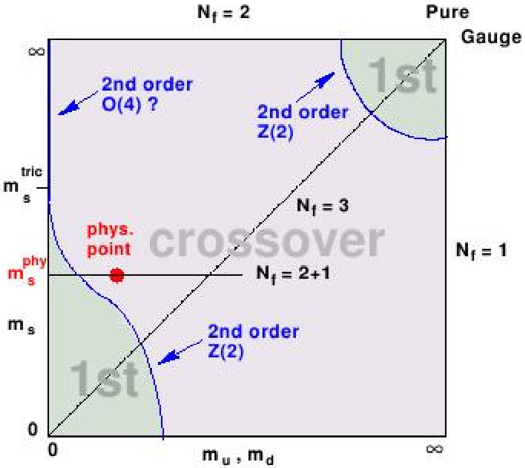
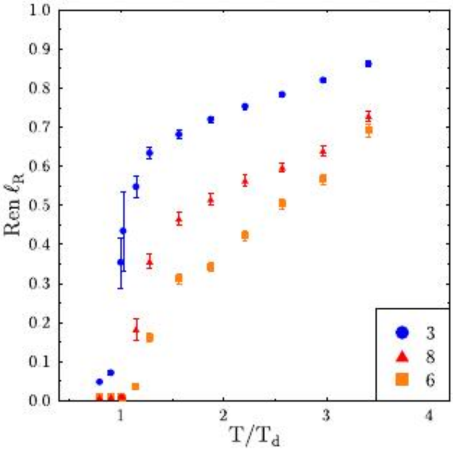


Figure 1: Sketch of the QCD phase diagram in the $m_{ud} - m_s$ plane.



Dumitu et al., 2004; SU(3) Polyakov loop vs.T

$$\mathcal{P} = \frac{1}{N_c}\, Tr \left\langle \exp\left(ig \int_0^{\frac{1}{T}} d\tau A_0(\tau, \vec{x}) \right) \right\rangle = \frac{c}{a} + \mathcal{P}_R + \underline{O}(a)$$

- $SU(3)$ gluodynamics - first order PT
  - $\mathcal{P}_R$ jumps from 0.0 to 0.4 at $T_c \approx 270$ MeV
  - increases from 0.4 to ~ 1.0 at $T_c < T < 4T_c$

- $SU(2)$ gluodynamics - second order PT
  - $\mathcal{P}_R \simeq (T - T_c)^\beta$, $3D$ Ising model universality class ($\beta \approx 0.326$).

The renormalized Polyakov loop
gradually increases
over some range above T_c

$\mathcal{P}_R$ gradually increases
at $T_c < T < T_p$

What does it mean?

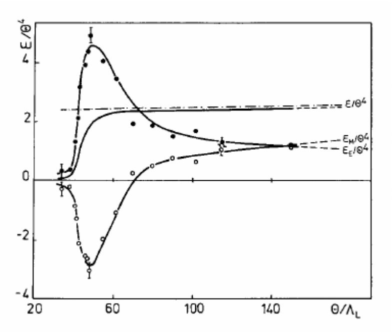
$\mathcal{P}_R$ is related to the static quark propagator:

$$\mathcal{P}_R \sim C\langle \bar{q}(0,\vec{x}) q(1/T,\vec{x})\rangle$$

- $\mathcal{P}_R \to 0$ — (heavy static)
  quarks are completely screened (confinement)
- $\mathcal{P}_R \to 1$ — quarks are unscreened (QGP)
- 0< $\mathcal{P}_R < 1$ — quarks are partially screened (semi-QGP)

(confinement)	(postconfinement)	(deconfinement)
$T < T_c$	$T_c < T < T_p$	$T > T_p$

Postconfinement domain



[Mitrjushkin, Zadorozhny, Zinoviev 1988]

- Polaykov loop behavior
- Failure of PT and old effective theories to evaluate pressure at $T_c < T < 2 \div 3 T_c$.
- Monopole density (condensate-liquid-gas)

Old effective 3D theories

$$L = \frac{1}{2} Tr\ G_{ij}^2 + Tr|D_i A_0|^2 + m_D^2 Tr\ A_0^2 + \lambda_1 \left(Tr\ A_0^2 \right)^2 + \lambda_2 Tr\ A_0^4 \qquad (1)$$

A_0 plays the role of the Higgs field

Valid for $A_0 \to 0$ ($T \to \infty$)

Fail to reproduce pressure at $T_c < T < 3T_c$ and the confinement phase transition

NO CENTER SYMMETRY (Z_N)

Effective models based on the Wilson line

Z_N symmetric

$$L = \frac{1}{2} Tr\ G_{ij}^2 + \frac{T^2}{g^2} Tr|W^\dagger D_i W|^2 + F(W) \tag{2}$$

$$W(A_0) = \exp\left(\int_0^{1/T} d\tau A_0(\tau, \vec{x})\right) \tag{3}$$

$F(W)$ can be fitted to the lattice data on the pressure, then the model predicts 't Hooft loop etc.
However! The the simplest model yields
very narrow postconfinement domain
in terms of the Polyakov loop ($T_p \sim 1.2 T_c$).

The A_0 background field can be considered as an imaginary chemical potential for gluons

Models based on the Wilson line

can be used for the computation of

- energy density, pressure
- shear viscosity
- elliptic flow
- dilepton and photon emission rate

Pisarski, Hidaka, Skokov etc.

The emergence of large spatially constant A_0 background field is associated with the zero-mode dynamics and chromoelectric degrees of freedom

Zero-mode dynamics

- Effective models for the Wilson line
- The chromoelectric-chromomagnetic asymmetry $\Delta_{A^2} = \langle A_0^2 - \frac{1}{3}\sum_{i=1}^{3} A_i^2 \rangle$
- Zero-momentum gluon propagators

Propagators in the gauge theory at $T \neq 0$

Fields dependent on the temperature parameter

$$\hat{A}(\tau, \vec{x}) = \exp(\tau H)\hat{A}(0, \vec{x}) \exp(-\tau H)$$

We compute the "gluon" propagator in the $SU(2)$ theory at the temperature $T = \frac{1}{\beta}$:

$$D^{bc}_{\mu\nu}(\tau, \vec{x}) = \frac{1}{\mathcal{Z}} \int_{A_\mu(0,\vec{x}) = A_\mu(\beta,\vec{x})} DA^a_\mu(x) A^b_\nu(\tau, \vec{x})\, A^c_\mu(0,0) e^{-S_E[A]} |\det\ M_{FP}(A)| \qquad (4)$$

$$S_E[A] = \int_0^\beta d\tau \int_V d^3\vec{x}\ \left(\frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} + \frac{1}{2\alpha} (\partial_\mu A^a_\mu)^2 \right) \qquad (5)$$

$$D_{\mu\nu}(p) = D_L(p)P^L_{\mu\nu} + D_T(p)P^T_{\mu\nu} + \alpha\frac{p_\mu p_\nu}{p^4}$$

$$D_L(p) = \frac{1}{p^2 + F(p)}, \qquad D_T(p) = \frac{1}{p^2 + G(p)}$$

$$\Pi_{\mu\nu} = F(p)P^L_{\mu\nu} + G(p)P^T_{\mu\nu}$$

The quantities under study:

$$D_{ii}(|\vec{p}|^2) = 2D_T(0, \vec{p}), \qquad D_{44}(|\vec{p}|^2) = D_L(0, \vec{p}),$$

We consider $\alpha = 0$ (the Landau gauge)

Linear response theory:

An external perturbation E^{cl} results in a small change of the electric field:

$$\langle\!\langle \delta E(t,\vec{x}) \rangle\!\rangle = \int_{-\infty}^{\infty} dt' \, d\vec{x}' G_R(t-t',\vec{x}-\vec{x}') E^{cl}(t',\vec{x}') \; + \bar{o}\big(E^{cl}\big).$$

The retarded Green's function G_R is related to the temperature Green's function $\tilde{D}_n$ as follows:

$$\tilde{G}_R\left(-i\,\frac{2\pi n}{T} \; - \; i\epsilon, \vec{x}\right) = - \, \tilde{D}_n(\vec{x}) \; ,$$

$$\tilde{D}_n(\vec{x}) = \int_0^{1/T} d\tau \exp\left(- \; \frac{2i\pi\tau n}{T}\right) \langle\!\langle \hat{E}(\tau,\vec{x})\hat{E}(0,\vec{0}) \rangle\!\rangle$$

Screening mass in QED

We consider two charges in QED plasma,

$$\vec{E}_1^{cl} = -i\frac{\vec{p}}{|\vec{p}|^2}Q_1 e^{-i\vec{p}\vec{x}_1} \qquad \vec{E}_2^{cl} = -i\frac{\vec{p}}{|\vec{p}|^2}Q_2 e^{-i\vec{p}\vec{x}_2}$$

Each of them can be considered as a small perturbation in the linear response theory:

$$h = \int d\vec{x}\ \vec{E}^{cl}(\vec{x})\vec{E}(\vec{x}) \ ,$$

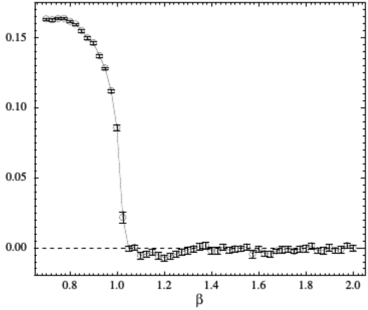
the resulting field has the form

$$E_i^{tot}(\vec{p}) = E_i^{cl} + \langle\langle \delta E_i \rangle\rangle = \frac{p_i p_j E_j^{cl}(\vec{p})}{|\vec{p}|^2 + F(0,\vec{p})} \ .$$

$$
\begin{aligned}
V &\simeq \frac{1}{2}\int d\vec{x}\ \left(\langle\!\langle \vec{E}_1^{tot}\rangle\!\rangle \vec{E}_2^{cl} + \langle\!\langle \vec{E}_2^{tot}\rangle\!\rangle \vec{E}_1^{cl}\right) \\
&= Q_1 Q_2 \int \frac{d\vec{k}}{(2\pi)^3}\ \frac{e^{i\vec{k}(\vec{x}_1-\vec{x}_2)}}{|\vec{k}|^2 + F(0,\vec{k})} \\
&\simeq \frac{Q_1 Q_2}{4\pi}\ \frac{e^{-\,m_e|\vec{x}_1-\vec{x}_2|}}{|\vec{x}_1 - \vec{x}_2|}
\end{aligned}
$$

$$
m_e = \frac{eT}{\sqrt{3}} = \frac{1}{\sqrt{D_L(0)}}
$$

Interest in $A_\mu^a A_\mu^a$ aroused in 2001



F.V.Gubarev, L.Stodolsky, V.I.Zakharov, Phys.Rev.Lett.(2001)

Rapid change of

$$\langle A^2 \rangle_{noncompact} - \langle A^2 \rangle_{compact}$$

is correlated with the confinement-deconfinement transition in the compact $U(1)$ theory.

$U(1)$ - critical coupling

$SU(2)$ - critical temperature

At nonzero temperatures there are two condensates,

$$\langle A_E^2 \rangle = g^2 \langle A_4^a(x) A_4^a(x) \rangle, \qquad \langle A_M^2 \rangle = g^2 \langle \sum_{i=1}^{3} A_i^a(x) A_i^a(x) \rangle.$$

The A^2 asymmetry is defined by the formula

$$\langle \Delta_{A^2} \rangle \equiv \langle A_E^2 \rangle - \frac{1}{3} \langle A_M^2 \rangle \qquad \mathcal{A} = \frac{\langle \Delta_{A^2} \rangle}{T^2}.$$

The asymmetry in terms of the propagators:

$$\mathcal{A} = \frac{4 N_t}{\beta a^2 N_s^3} \left[3(D_L(0) - D_T(0)) + \sum_{p \neq 0} \left(\frac{3|\vec{p}|^2 - p_4^2}{p^2} D_L(p) - 2 D_T(p) \right) \right]$$

Lattice settings

$$S = \frac{4}{g^2} \sum_{P=x,\mu,\nu} \left(1 - \frac{1}{2} \mathrm{Tr} \; U_P \right)$$

where

$$U_P = U_{x,\mu} U_{x+\hat{\mu},\nu} U^{\dagger}_{x+\hat{\nu},\mu} U^{\dagger}_{x,\nu}$$

$$U_{x,\mu} \; = \; u_0 \; + \; i \sum_{a=1}^{3} u_a \sigma_a, \qquad (6)$$

$$A^a_\mu = - \frac{2Z \; u^a_\mu}{ga}, \qquad (7)$$

$$\Lambda : \quad U_{x,\mu} \to \Lambda^\dagger_x U_{x,\mu} \Lambda_{x+\hat{\mu}},$$

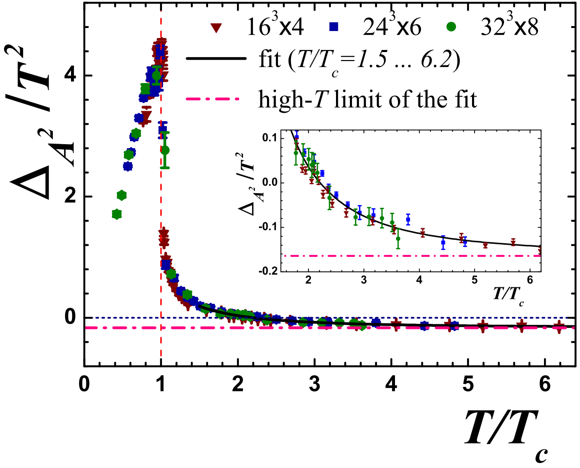
We fix the absolute Landau gauge by finding the global maximum of the functional

$$\mathcal{F}[\,\mathcal{U}\,] = \frac{1}{2} \sum_{x,\mu} \mathrm{Tr}\, U_{x,\mu}, \qquad (8)$$

Stationarity condition:

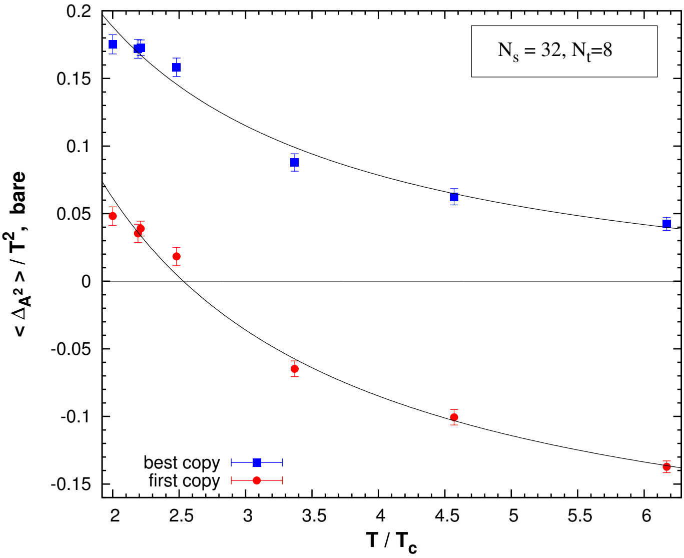
$$\partial_\nu A^a_\nu = 0.$$

We use the simulated annealing algorithm with subsequent over-relaxation



M.N. Chernodub and E.-M. Ilgenfritz, Phys.Rev.D (2008)

main result



Lattice size decreases from 1.3 fm to 0.4 fm

our result

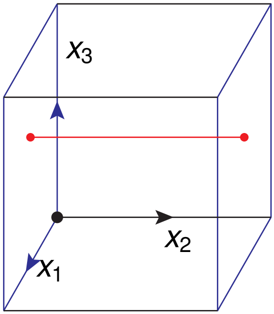
$$A_\mu \to A_\mu^\Lambda = (\Lambda Z)^\dagger A_\mu (\Lambda Z) + \frac{i}{g} (\Lambda Z)^\dagger \partial_\mu (\Lambda Z).$$

$$A_\mu \to A_\mu^\Lambda = \Lambda^\dagger A_\mu \Lambda + \frac{i}{g} \Lambda^\dagger \partial_\mu \Lambda.$$

For $SU(3)$, as an example:

$$Z \in \left\{ \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} e^{\frac{2i\pi}{3}} & 0 & 0 \\ 0 & e^{\frac{2i\pi}{3}} & 0 \\ 0 & 0 & e^{\frac{2i\pi}{3}} \end{pmatrix}, \begin{pmatrix} e^{\frac{4i\pi}{3}} & 0 & 0 \\ 0 & e^{\frac{4i\pi}{3}} & 0 \\ 0 & 0 & e^{\frac{4i\pi}{3}} \end{pmatrix} \right\}$$

Gauge transformation is the same on both sides!



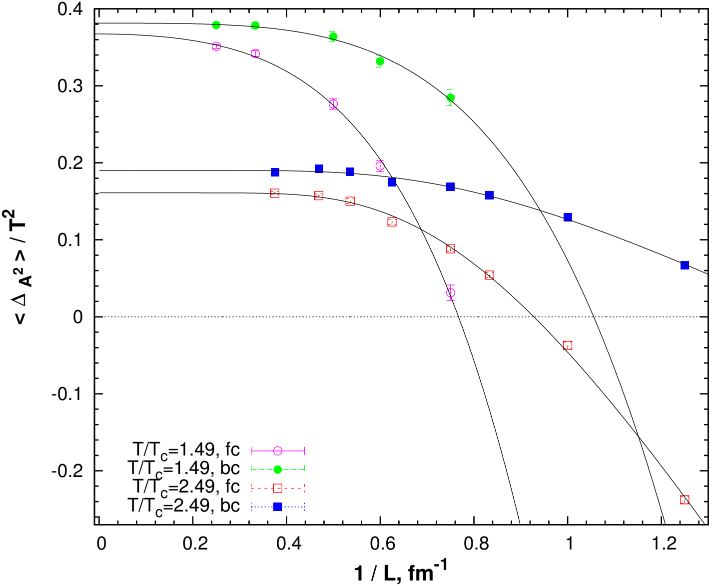
We extend the gauge group by nonperiodic gauge transformations:

$$\Lambda(x_1, b, x_3) = Z\Lambda(x_1, 0, x_3) \text{ etc.}$$

$$P\exp\left(ig\int_0^b A_2(x_1, z, x_3)dz\right) =$$

$$= L(x_1, x_3) \longrightarrow L(x_1, x_3)Z$$

Thus the Hilbert space is broken into 8 superselection sectors



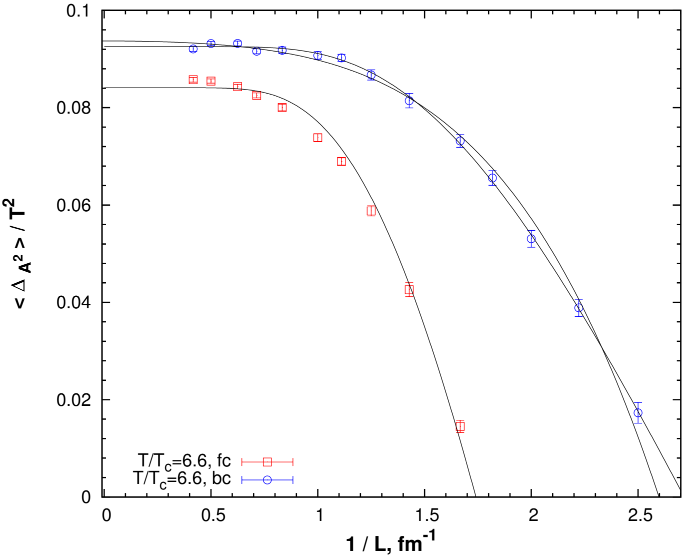
Finite-volume effects

$$\overline{\mathcal{A}}(L) = \overline{\mathcal{A}}_\infty^{pol} - \frac{c_2}{L^2} - \frac{c_4}{L^4} \,,$$

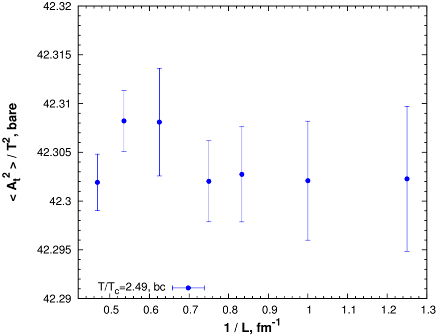
$$\overline{\mathcal{A}}(L) \simeq \overline{\mathcal{A}}_\infty^{exp} - c\exp\left(-\ L/L_0 \right)$$

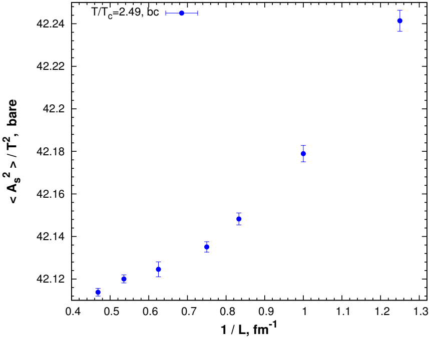
$\frac{T}{T_c}$	Gauge fixing algorithm	$\overline{\mathcal{A}}_\infty^{exp}$	c	L_0 (fm)	$\frac{\chi^2}{N_{dof}}$
1.49	*bc*	0.380(2)	1.7(1.0)	0.41(5)	0.34
1.49	*fc*	0.352(1)	4.7(1.0)	0.47(8)	0.06
2.49	*bc*	0.190(2)	1.7(5)	0.31(3)	1.71
2.49	*fc*	0.161(2)	5.6(5)	0.31(1)	2.60
6.60	*bc*	0.09254(21)	1.06(11)	0.151(5)	0.89

Table: Parameters are given for the exponential fit. The quadratic fit function works worse: at $T/T_c = 6.6$ quality of the exponential fit $Q = 0.55$, polynomial - $Q = 0.00072$.



For the best copy both exponential and polynomial fit functions are shown ($N_t = 4$).

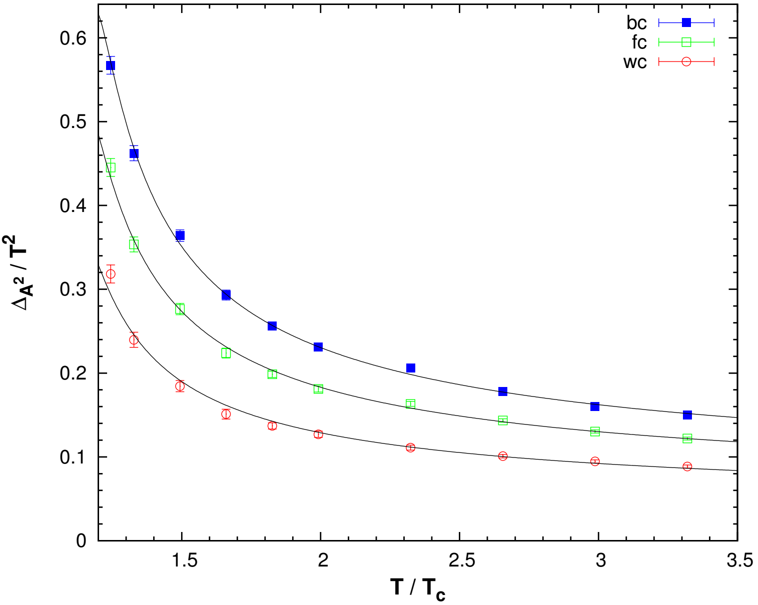




"Chromoelectric" condensate

"Chromomagnetic" condensate

$$\langle \Delta_{A^2} \rangle \equiv \langle A_t^2 \rangle - \langle A_s^2 \rangle \, .$$



bc
fc
wc
0.6
0.5
0.4
0.3
0.2
0.1
0
1.5
2
2.5
3
3.5
Δ_A^2 / T^2
T / T_c

Fitting high-temperature behavior

$$\bar{\mathcal{A}} \simeq b_0 + b_2 \left(\frac{T_c}{T} \right)^2$$

Gauge fixing algorithm	b_0	b_2	$\frac{\chi^2}{N_{dof}}$
bc	0.1036(27)	0.517(16)	1.40
fc	0.0893(22)	0.372(13)	0.92
wc	0.0682(5)	0.231(3)	0.05

Table: $1.65 < T/T_c < 3.32$, fixed lattice size $L = 2$fm.

$b_0 > 0$ in all cases in agreement with perturbation theory

$$\bar{\mathcal{A}} \simeq \frac{zg^2(T)}{4}\left(1 - \frac{g(T)}{3\pi}\sqrt{\frac{2}{3}}\right) ,$$

where the running coupling is taken in the two-loop approximation,

$$\frac{1}{g^2(T)} = \frac{1}{4\pi^2}\left(\frac{11}{6}\ln\left(\frac{T^2}{\Lambda^2}\right) + \frac{17}{11}\ln\ln\left(\frac{T^2}{\Lambda^2}\right)\right) ,$$

z and Λ are the fit parameters, $1.24 < \frac{T}{T_c} < 3.32$.

$$z = 0.1284(14), \quad \Lambda/T_c = 0.845(7), \quad \frac{\chi^2}{N_{dof}} = 1.50$$

One-loop estimate at high temperatures [Vercauteren *et al.*, 2010]

$$\langle \Delta_{A^2} \rangle \simeq c\, g^2 T^2 \left(1 - \frac{g}{3\pi} \sqrt{\frac{2}{3}} \right) \tag{9}$$

- Perturbation theory (2010): c>0
- Lattice simulations (2008): c<0

We consider propagators only for <u>*soft modes*</u> $p_4 = 0$, where

$$P^T_{\mu\nu} = \begin{pmatrix} 0 & 0 \\ 0 & \delta_{ij} - \frac{p_i p_j}{|\vec{p}|^2} \end{pmatrix} \qquad P^L_{\mu\nu} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

$D_L(0)$ — chromoelectric forces
$D_T(0)$ — chromomagnetic forces

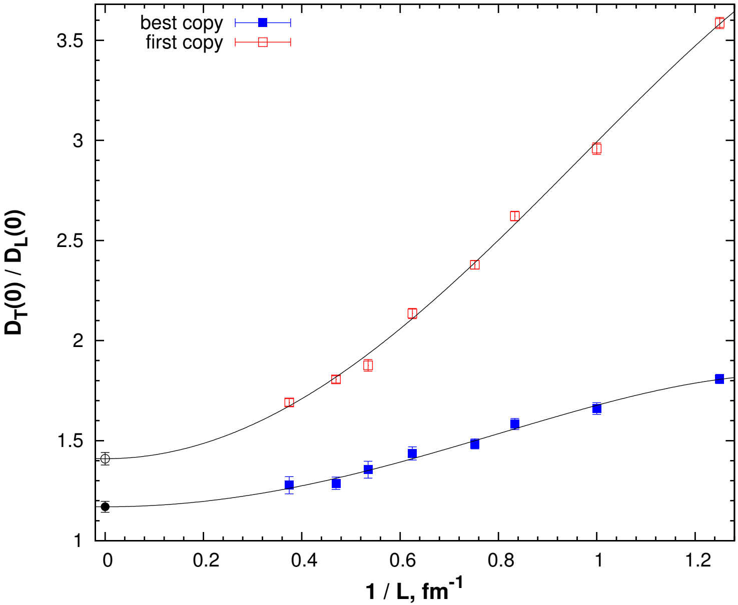
Another definition of screening masses [Heller, Karsch, Rank 97]:

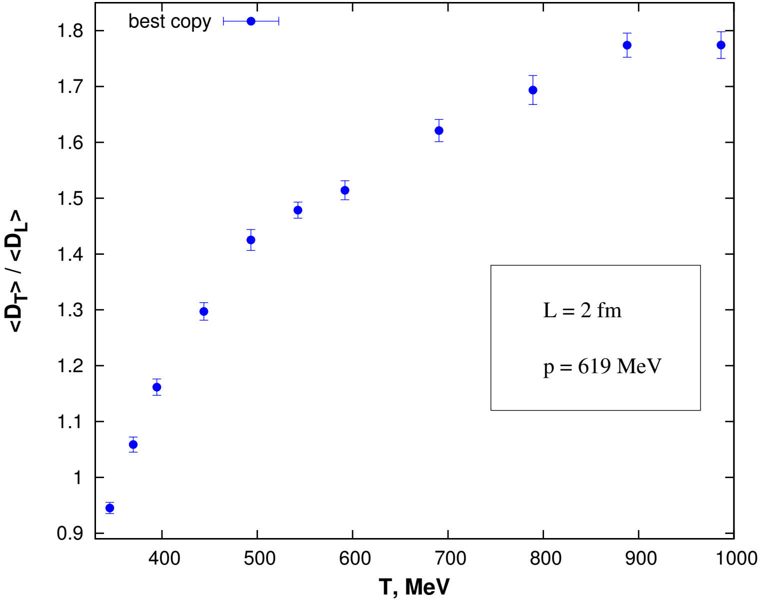
$$\begin{aligned}\tilde{D}_L(p_\perp = 0, x_3) &\sim \exp(-m_e|x_3|),\\ \tilde{D}_T(p_\perp = 0, x_3) &\sim \exp(-m_m|x_3|),\ |x_3| \to \infty\end{aligned}$$

Approximations $m_e = \sqrt{\frac{2}{3}}g(T)T + ...$ and $m_m \sim g^2(T)T$ suggest the fit function ($T > 2T_c$)

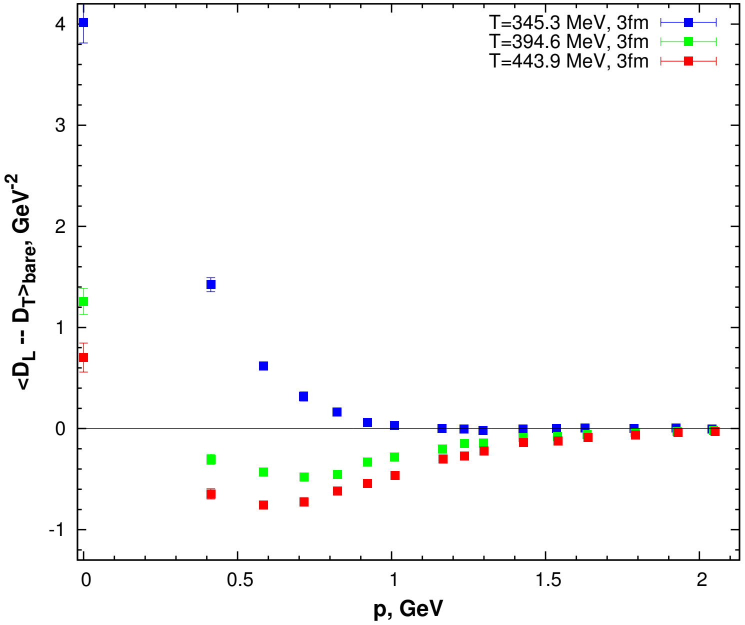
$$\frac{m_e^2(T)}{m_m^2(T)} = \frac{C}{g^2(T)} = 1 \quad \text{at} \quad \frac{T}{T_c} = 0.9(1)$$

we consider the ratio $r(T) = \frac{D_T(0)}{D_L(0)}$ instead of $\frac{m_e^2}{m_m^2}$


best copy
first copy
3.5
3
2.5
2
1.5
1
0
0.2
0.4
0.6
0.8
1
1.2
$D_T(0) / D_L(0)$
$1 / L, fm^{-1}$



Ratio of the “magnetic” to the “electric” propagator at $p = p_{min}$



T=345.3 MeV, 3fm
T=394.6 MeV, 3fm
T=443.9 MeV, 3fm
$<D_L - D_T>_{bare}$, GeV^{-2}
p, GeV
4
3
2
1
0
-1
0
0.5
1
1.5
2

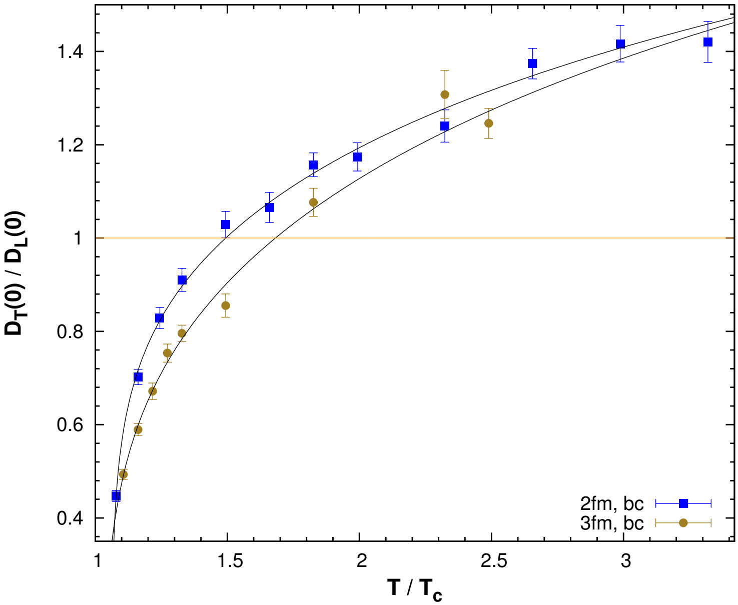
$$r(T) \simeq r_0 + \frac{r_1}{g^2(T)}$$

where

$$\frac{1}{g^2(T)} = \frac{1}{4\pi^2} \left(\frac{11}{6} \ln \left(\frac{T^2}{\Lambda^2} \right) + \frac{17}{11} \ln \ln \left(\frac{T^2}{\Lambda^2} \right) \right) ,$$

Lattice size	r_0	r_1	Λ / T_c	T_p / T_c	$\frac{\chi^2}{N_{dof}}$
2 fm	0.94(1)	3.78(12)	1.060(3)	1.494(30)	0.64
3 fm	0.79(3)	4.59(37)	1.02(2)	1.68(12)	1.42

Table: Fit parameters for the best-copy values of $r(T)$.

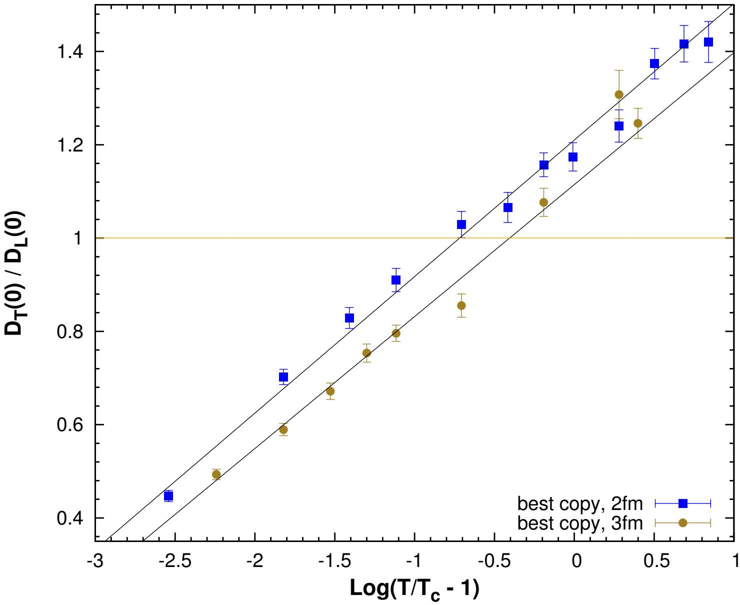


$D_T(0) / D_L(0)$
T / T_c
2fm, bc
3fm, bc
1.4
1.2
1
0.8
0.6
0.4
1
1.5
2
2.5
3

$$r(T) \simeq R_0 + R_1 \ln\left(\frac{T}{T_c} - 1\right) = R_1 \ln\left(\frac{T - T_c}{Q}\right) \ .$$

Lattice size	R_0	R_1	T_p/T_c	$\frac{\chi^2}{N_{dof}}$
2 fm	1.21(1)	0.293(6)	1.488(13)	1.35
3 fm	1.115(15)	0.283(9)	1.667(27)	1.92

Table: Fit parameters for the best-copy values of $r(T)$.



$D_T(0)$ / $D_L(0)$
1.4
1.2
1
0.8
0.6
0.4
-3
-2.5
-2
-1.5
-1
-0.5
0
0.5
1
Log(T/T_c - 1)
best copy, 2fm
best copy, 3fm

Conclusions

- The flip-sector effect is substantial at $L \simeq 2$ fm and crucial at $L < 1$ fm. In the latter case, it dramatically changes the behavior of the asymmetry.
- Finite-volume effects for $\overline{\mathcal{A}}$ and r are significant at lattice sizes < 2 fm .
- The data can be fitted to the function motivated by perturbation theory down to temperatures as low as $1.25T_c$
- Contrary to the conclusions by Chernodub and Ilgenritz (2008), $\overline{\mathcal{A}} > 0$ at all temperatures under consideration
- Boundary of the postconfinement domain T_p is indicated by the condition $D_T(0)/D_L(0) = 1$ rather than by criteria based on $\overline{\mathcal{A}}$. At $L = 3fm \quad T_p = 1.68(12)T_c$.