The Effects Of Majorana Phases in Estimating the Masses Of Neutrinos

Ng. K. Francis

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MOTIVATIONS

To study the Compatibility of Theoretical Calculations with Experimental data:

1. **Absolute neutrino masses (m_1,m_2, m_3).**
2. **$0\nu\beta\beta$ decay mass parameter m_{ee} .**
3. **Cosmological bound / sum $\sum_{i}^{3}(m_i)_{abs}$**
4. **Solar mixing angle θ_{12} for different CP-phases**
5. **Leptons Charged Corrections effects on θ_{13} .**

I BEGIN MY TALK WITH A THOUGHT PROVOKING STATEMENT THAT:

- Neutrino physics is all about its mass
- Besides in others, ***there is a Noble Prize in determination of absolute Mass of neutrinos.***

Noble Prize Amount US $1.24 million, € 0.95 million.

Imagine!!

The amount of money spent on ϑ Experiments world wide, haa ! is all about ϑ mass

TALK IS BASED ON MY WORK

- The Effects of Majorana Phases in Estimating the Masses of Neutrinos.

  arXiv:1705.08288v1 [hep-ph] 20 May 2017

- Validity of quasi-degenerate neutrino mass models and their predictions on baryogenesis

  Nuclear Physics B 863 (2012)19-32

KNOWN PROBLEMS IN NEUTRINOS

parameters	Δm^2_{21} (10^{-5} eV^2)	Δm^2_{23} (10^{-3} eV^2)	$\tan^2 \theta_{12}$	$\sin^2 \theta_{13}$	$\sin^2 \theta_{23}$	$\Sigma\|m_i\|$ eV	$\|m_{ee}\|$ eV
Best-fit	7.67	2.39	0.312	0.016	0.466	≤ 0.23	≤ 0.27
1 σ	7.48-7.83	2.31-2.50	0.294-0.331	0.006-0.026	0.408-0.539	≤ 0.23	≤ 0.27
2 σ	7.31-8.01	2.19-2.66	0.278-0.352	<0.036	0.366-0.602	≤ 0.23	≤ 0.27
3 σ	7.14-8.19	2.06-2.81	0.263-0.375	<0.046	0.331-0.644	≤ 0.23	≤ 0.27

{[21] G.L.Fogli et al, Phys Rev.D 78, 033010 (2008) [arXiv: 0805.2517 [hep-ph]}.

[8] M. C. Gonzalez-Garcia, M. Maltoni and T. Schwetz, JHEP 1411, 052 (2014).

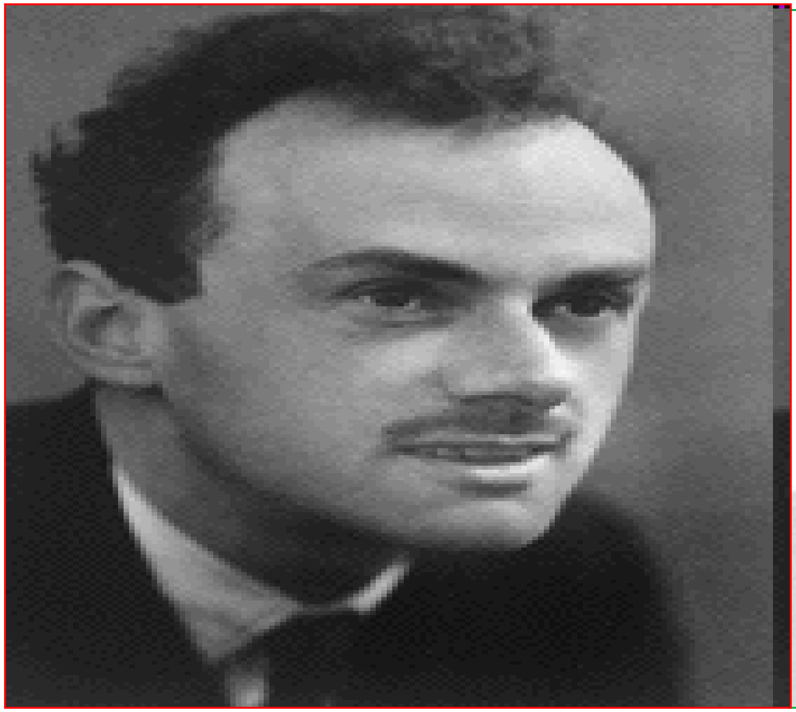
[9] D. V. Forero, M. Tortola and J. W. F. Valle, Phys. Rev. D90, 093006 (2014

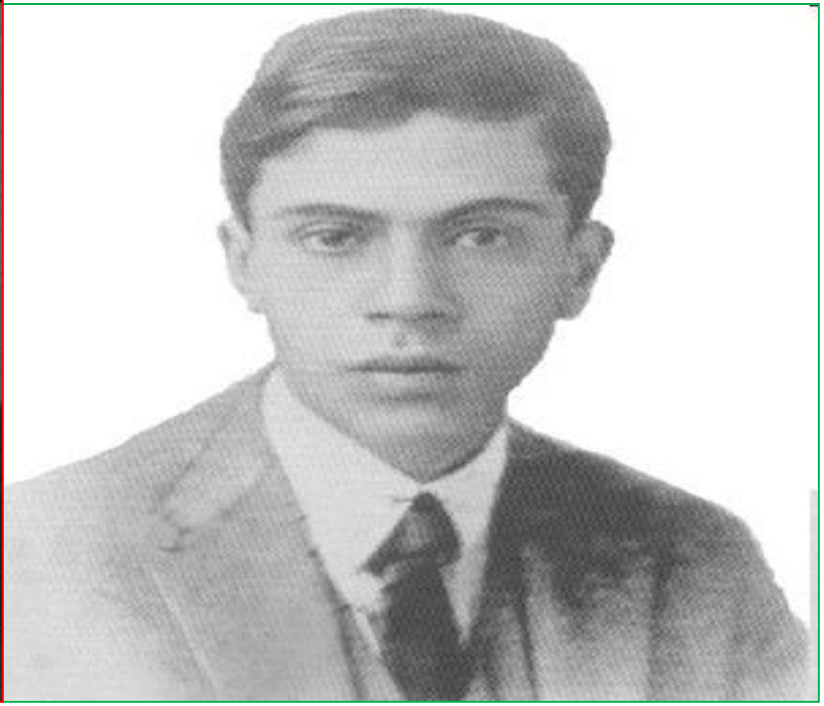
UNKNOWN PROBLEMS IN NEUTRINOS

- Nature of Neutrinos-Dirac or Majorana
- Octant θ_{23} $(= 45^0$?)
- Neutrino Mass Hierarchy (NH / IH / QD ?)
- CP Vioating Phase/s-(Dirac or Majorana)
- Absolute Masses of Neutrinos

TYPE OF NEUTRINOS ???

DIRAC TYPE ?? **MAJORANA TYPE ??**





2015 PHYSICS NOBEL PRIZE FOR NEUTRINO OSCILLATIONS $\Rightarrow \nu$ *has mass*

Takaaki Kajita

Arthur B. McDonald

JOURNEY OF NEUTRINO

- 1930-Pauli (Neutron): 1934-FERMI (Neutrino)

↓ 68 Years Later

- 1998-Kamioka Japan & SNO Canada: Neutrino-Oscillations

implies **Neutrinos are Massive**

↓

- Year ??? of discovery of Absolute Neutrino Mass

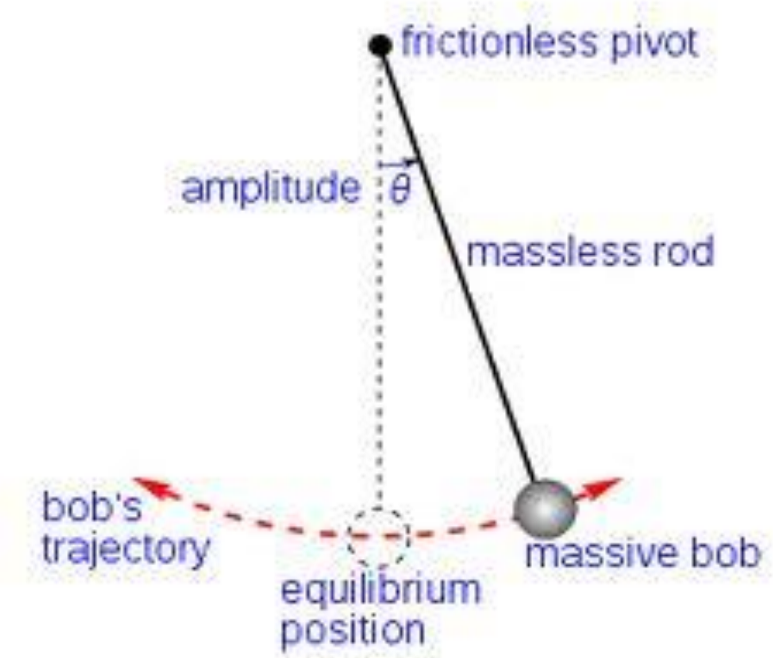
A Nobel Prize in Waiting ☺

Thought Provoking Statement:

Neutrino physics is all about neutrino mass

NEUTRINO OSCILLATION ??

- Simple Pendulum



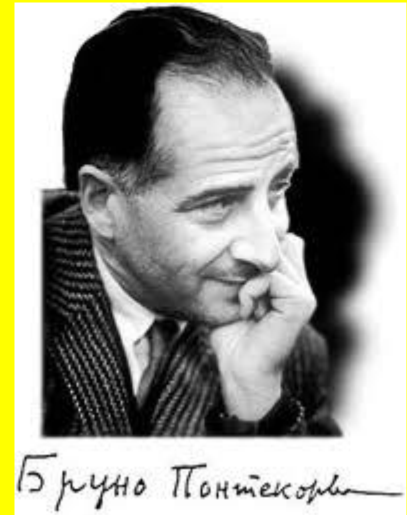
Neutrino Oscillation (1957)

Chameleon

ν-**Oscillation** $\approx$

ν-conversion

- BRUNO PONTECORVO (Italy1913- Russia1993)



- Half of his ashes is now buried in the Protestant Cemetery in Rome, and another half in Dubna, Russia, according to his will.

Absolute Neutrino Masses

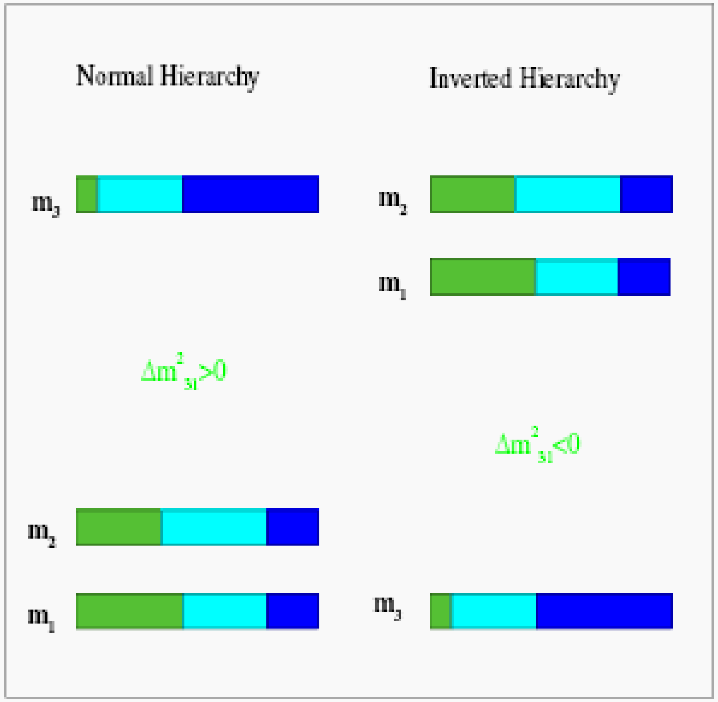
- Tritium β-deay

$$
\begin{aligned}
m_{\nu_e} &= \left(\Sigma |U_{ei}|^2 m_i^2\right)^{\frac{1}{2}} \\
&= [c_{13}^2 c_{12}^2 m_1^2 + c_{13}^2 s_{12}^2 m_2^2 + s_{13}^2 m_3^2
\end{aligned}
$$

- Neutrinoless double beta decay

$$
\begin{aligned}
m_{ee} &= |\ \Sigma U_{ei}^2 m_i\ | \\
&= |\ c_{13}^2 c_{12}^2 m_1 + c_{13}^2 s_{12}^2 m_2 e^{i\phi_2} + s_{13}^2 m
\end{aligned}
$$

Mass Squared Differences and Absolute Masses



- Normal Hierarchy :

$$m_3^2 = m_1^2 + \Delta m_{21}^2 + \Delta m_{32}^2$$
$$m_2^2 = m_1^2 + \Delta m_{21}^2$$

$$m_3^2 \approx \Delta m_{atm}^2 >> m_2^2 \approx \Delta m_{\odot}^2 >> m_1^2$$

- Inverted Hierarchy :

$$m_2^2 = m_3^2 + \Delta m_{23}^2$$
$$m_1^2 = m_3^2 + \Delta m_{23}^2 - \Delta m_{21}^2$$

$$m_2^2 \approx \Delta m_{atm}^2 \approx m_2^2 >> m_3^2$$

- Quasi-Degenerate

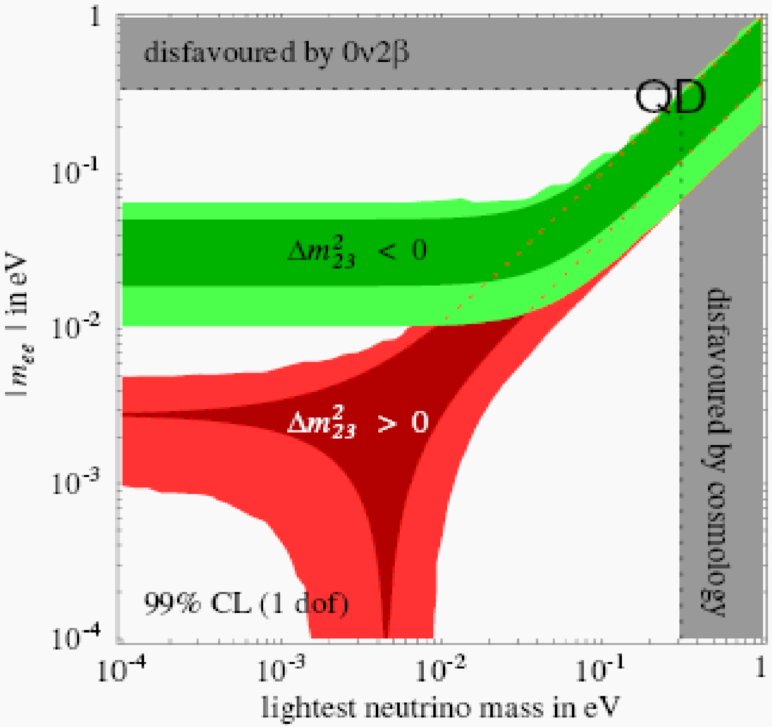
$$m_3 \approx m_2 \approx m_1 >> \sqrt{\Delta m_{atm}^2}$$

Neutrino Less Double Beta Decay

- Can establish Majorana nature of Neutrinos

$$\langle m \rangle = |m_1|U_{e1}|^2 + m_2|U_{e2}|^2 e^{i\phi_1} + m_3|U_{e3}|^2 e^{i\phi_2}|$$

- ν Mass Spectrum
- Absolute ν Mass Scale
- CP phases



Vissani and Strumia. 2005

NH: $m_1 << m_2 << m_3$

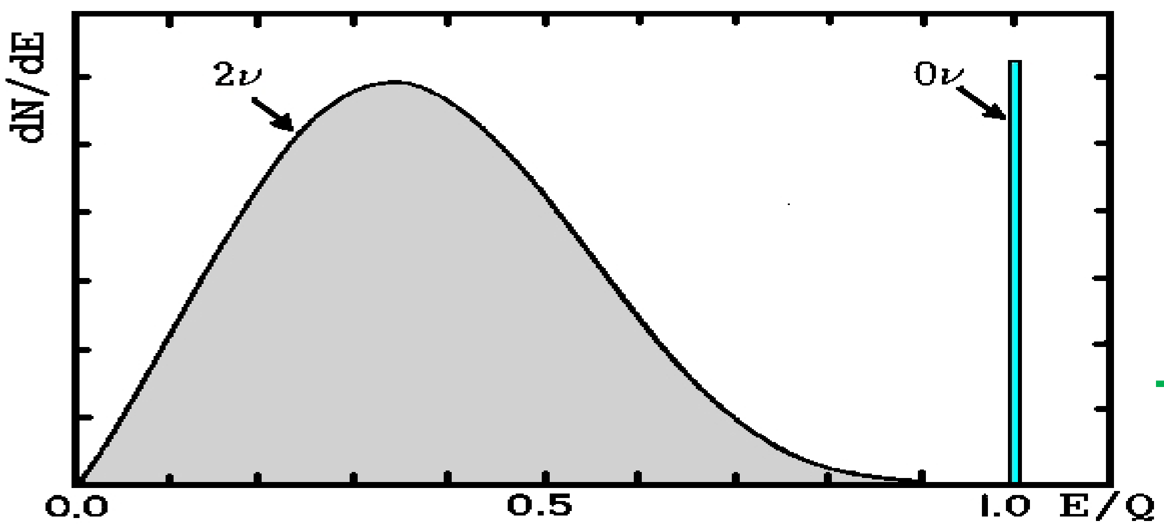
IH: $m_3 << m_1 \approx m_2$

QD: $m_1 \approx m_2 \approx m_3$

- $\langle < m > \rangle$ is in the range of sensitivity of upcoming $0\nu\beta\beta$ experiments for IH and QD
- Uncertainties coming from Nuclear matrix elements

Comparison: $2\nu\beta\beta$ and $0\nu\beta\beta$

- Measuring the energy of both electrons
- $2\nu\beta\beta$: Continuous energy spectrum
- $0\nu\beta\beta$: Sharp peak at Q value of decay
- $Q = E_{mother} - E_{daughter} - 2me$
- Schechter & Valle (1982): Measuring $0\nu\beta\beta \Rightarrow$ Majorana particle



Quasi-Degenerate Neutrinos (QDN)

- QDN is interesting as it can be confirmed or disapproved in the future neutrinoless double beta experiments or in cosmological observations of the cosmic microwave radiation and the large scale structure of the universe.
- Three Types of QDNs
- TBM & μ-τ symmetry matrices
- CPV in the lepton sector
- Neutrinoless double beta decay mass parameter m_{ee}
- Cosmologiccal sum$\sum_i^3 m_i$.

The absolute scale of masses, consequences:

Addresses key issues in particle physics

–hierarchical or degenerate neutrino mass spectrum

–understanding the scale of new physics beyond SM, or a crucial datum for reconstructing PHYSICS Beyond SM

–potential insight into origin of fermion masses

Impacts cosmology and astrophysics

–early universe, relic neutrinos (HDM), structure formation, anisotropies of CMBR

–supernovae, origin of elements

–potential influence on UHE cosmic rays.

[0vbb Phys. Lett. B 586 (2004) 198]

- **Techniques**

–time of flight (SN1987a

–particle decay kinematics

- •beta decay (and electron capture) spectrum shape
- •muon momentum in pion decay
- •invariant mass studies of multi particle semi leptonic decays

- **Advantages**–sensitive to absolute mass scale

–purely kinematical observables

- **Combined ν-oscillation & 0ν*ββ* experiments can:**

–help distinguish 3 or 4 neutrino scenarios

–understand of hierarchy and ordering of masses

–measure CP-violating phases in lepton sector

Four approach absolute ν-mass determination

1. Neutrinoless double beta decay $0\nu\beta\beta$
2. Tritium decay end-point spectrum
3. Cosmic ray spectrum above BZK cutoff (in the Z-burst model)
4. Cosmological measurement of the power spectrum governing the CMB and large scale strcuture.

1 & 2 sensitive to mass eigenstates coupling to the electron neutrino. 3 & 4 sensitive to heavy component of the cosmic neutrino background.

All mass eigenstates related by m^2 inferred from neutrino oscillation data.

Ways to Generate Neutrino Masses

I. Through extension of the scalar section (several ways), but two simplest Methods considered in the Literature are:

- SM+ scalar Higgs triplet (H)
- SM+ singly charged scalar single (h^+).

***II. Through the introduction of right-handed neutrinos (the most straight forward):

SM + SU(2) x U(1) singlet right-handed neutrino.

Seesaw Mechanism-3 Types

- ******Type I***: **SU(3) x SU(2) x U(1)- singlet fermion**
- Type II: SU(2)-triplet scalars;
- Type III: SU(2) –triplet fermions.

NB: They differs from each other by the properties of the exchanged heavy particles.

(Leptogenesis by S.Davidson, E. Nardi and Yosef Nir).

Mass matrices with 2-3 symmetry :

$$m_{LL} = \begin{pmatrix} m_{11} & m_{12} & m_{12} \\ m_{12} & m_{22} & m_{23} \\ m_{12} & m_{23} & m_{22} \end{pmatrix}$$

where the eigenvalues and solar mixings are

$$m_1 = m_{11} - \sqrt{2}\tan\theta_{12} m_{12},$$

$$m_2 = m_{11} + \sqrt{2}\cot\theta_{12} m_{12},$$

$$m_3 = m_{22} - m_{23},$$

$$\tan 2\theta_{12} = \frac{2\sqrt{2} m_{12}}{m_{11} - m_{22} - m_{23}}.$$

$$\theta_{23} = 45^{\circ}$$

$$\theta_{13} = 0$$

$$\theta_{12} = ?$$

MAJORANA PHASES IN MASSES

Majorana Phases:

$$P = \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\alpha} & 0 \\ 0 & 0 & e^{i\beta} \end{pmatrix}$$

Diagonal mass matrix:

$$M^{diag} = \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 e^{2i\alpha} & 0 \\ 0 & 0 & m_3 e^{2i\beta} \end{pmatrix}$$

NEUTRINO OSCILLATIONS AND MNS MIXING MATRIX:

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

$$\nu_f = U\nu_i ; f = e, \mu, \tau ; i = 1,2,3$$

Two neutrino eigenstates: Flavour and mass eigenstates

$$\Delta m^2_{21} = \Delta m^2_{sol} = (m_2^2 - m_1^2) > 0 ;$$
$$\Delta m^2_{31} = \Delta m^2_{atm} = |m_3^2 - m_1^2|$$

Which may be greater or less than zero

(Neutrino mixing matrix is known as MNS mixing matrix due to Z. Maki, M. Nakagawa, S. Sakata)

3.Neutrino mass models

1.Quasi-Degenerate: $\left(m_1 \approx m_2 \approx m_3\right)$

- Quasi-Degenerate Type 1A: $\left(m_1, -m_2, m_3\right)$
- Quasi-Degenerate Type 1B: $\left(m_1, m_2, m_3\right)$
- Quasi-Degenerate Type 1C: $\left(m_1, m_2, -m_3\right)$

2.Inverted Hierarchy $\left(m_1 \approx m_2 > m_3\right)$

- Inverted Hierarchy Tye2A: $\left(m_1, m_2, m_3\right)$
- Inverted Hierarchy Type2B : $\left(-m_1, m_2, m_3\right)$

3. Normal hierarchy $\left(m_1 \approx m_2 < m_3\right)$ $\left(m_1, -m_2, m_3\right)$

Types of Quasi-degenerate mass models

By taking particular choice of phases:

$$A : m_i = (m_1, -m_2, m_3)$$

$$B : m_i = (m_1, m_2, m_3)$$

$$C : m_i = (m_1, m_2, -m_3)$$

We consider 3X3 mass matrices through parameterizations with two parameters in the present analysis.

- No definite information of Dirac Neutrino masses/SO(10) prediction:3 choices.

$$m_{LR} = Diag\left(\lambda^{m}, \quad \lambda^{n}, \quad 1\right)\upsilon$$

$$\text{with } \lambda = 0.3 \ \& \ \upsilon = 174\, GeV$$

$$(m,n) = (6,2) \rightarrow \text{Charged lepton type ,}$$

$$(m,n) = (8,4) \rightarrow \text{Up quark type ,}$$

$$(m,n) = (4,2) \rightarrow \text{Down quark type}$$

- Seesaw model alone cannot solve the problem.

For our calculation:

We consider SO(10)GUT prediction for the choices of Diagonal Dirac neutrino mass matrix:

$$m_{LR} = Diag\left(\lambda^m, \quad \lambda^n, \quad 1\right)\upsilon$$
$$\text{with } \lambda = 0.3 \ \& \ \upsilon = 174\, GeV$$

For

Case(i): $m_{LR} = m_E \equiv Diag\left(\lambda^6, \quad \lambda^2, \quad 1\right)\upsilon$

Case(ii) : $m_{LR} = m_{up} \equiv Diag\left(\lambda^8, \quad \lambda^4, \quad 1\right)\upsilon$

Quasi-degenerate: Type 1A $(m_1, -m_2, m_3)$

$$m^o_{LL} = \begin{pmatrix} 0 & -1/\sqrt{2} & -1/\sqrt{2} \\ -1/\sqrt{2} & 1/2 & -1/2 \\ -1/\sqrt{2} & -1/2 & 1/2 \end{pmatrix} m_0$$

$$m_{LL} = \begin{pmatrix} \epsilon - 2\eta & -c\epsilon & -c\epsilon \\ -c\epsilon & 1/2 - d\eta & -1/2 - \eta \\ -c\epsilon & -1/2 - \eta & 1/2 - d\eta \end{pmatrix} m_0$$

$$\tan 2\theta_{12} = \frac{2\sqrt{2}(-c\epsilon)}{\epsilon - 2\eta - 1/2 + d\eta + 1/2 + \eta} = -\frac{2\sqrt{2}c}{1 + (d-1)\eta/\epsilon}$$

Three-fold quasi-degenerate Types 1B and 1C

quasi- degenerate Type 1B(m_1, m_2, m_3)

$$m_{LL} = \begin{pmatrix} 1-\epsilon-2\eta & -c\epsilon & -c\epsilon \\ -c\epsilon & 1-d\eta & -\eta \\ -c\epsilon & -\eta & 1-d\eta \end{pmatrix} m_0$$

$$m^0_{LL} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} m_0$$

Quasi-degenerate Type 1C$(m_1, m_2, -m_3)$

$$m_{LL} = \begin{pmatrix} 1-\epsilon-2\eta & -c\epsilon & -c\epsilon \\ -c\epsilon & -\eta & 1-d\eta \\ -c\epsilon & 1-d\eta & -\eta \end{pmatrix} m_0$$

$$m^0_{LL} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} m_0$$

$$\tan 2\theta_{12} = \frac{2\sqrt{2}c}{1 + (1-d)\eta/\epsilon}$$

To compute various parameters

- **To calculate m_1 and m_2**
- **NH**: $\alpha = \frac{|\Delta m^2_{23}|}{m^2_0}$, $\beta = \frac{\Delta m^2_{21}}{m^2_0}$,

Where Δm^2_{21} = 7.60 x 10^{-3} eV^2 and $|\Delta m^2_{23}|$=2.40 x 10^{-5} eV^2 [39].

- $m_1 = m_0 \sqrt{1-\alpha(1+\beta)}$; $m_2 = m_0 \sqrt{1-\alpha}$; $m_3 = m_0$.
- **IH**: $\alpha = -\frac{|\Delta m^2_{23}|}{m^2_0}$, $\beta = +\frac{\Delta m^2_{21}}{m^2_0 \alpha}$,

where Δm^2_{21}= 7.60 x 10^{-3} eV^2 and Δm^2_{23}= 2.40 x 10^{-5} eV^2.

- $m_1 = m_0 \sqrt{1+\alpha(\beta-1)}$; m2 $= m_0 \sqrt{1+\alpha}$; $m_3 = m_0$.

{[39] T.Schwetz, M.Tortola.J.W.F.Valle, hep-hp/08082016;
M. C. Gonzales-et al, arXiv: 0812.3161.
G.L.Fogli et al, Phys Rev.D 78, 033010 (2008) }.

TABLE-1

Input	Calculated	QD-NH		QD-IH	
m_3	α	m_1	m_2	m_1	m_2
0.400	0.015	0.3968929	0.39699	0.4028945	0.4029888
0.10	0.24	0.086741	0.08718	0.111035	0.1113552
0.08	0.375	0.0626418	0.06326	0.0934023	0.0938083

Table1. The absolute neutrino masses in eV estimated from oscillation data [8]
(Using calculated value of β = 0.03166).

II. For Calculation of ε and η the following Expressions were used.

For Type-IA (NH or IH):

$m_1 = (2\varepsilon - 2\eta)\, m_0,$

$m_2 = (-\varepsilon - 2\eta)\, m_0,$

$m_3 = m_0.$

For Type – IB/IC (NH or IH):

$m_1 = (1 - 2\varepsilon - 2\eta)\, m_0,$

$m_2 = (1 + \varepsilon - 2\eta)\, m_0,$

$m_3 = m_0.$

Where m_1 and m_2 are directly substituted from Tables A & B corresponding to each case. Calculations is easy so we simply give the final calculations of ε and η in Tables 1 to 4.

Parameters	QD-NH		QD-IH	
	Type-IA	Type-IB	Type-IA	Type-IB
m_1 (eV)	0.08674	0.08672	0.09340	0.09340
m_2 (eV)	-0.0872	0.08717	-0.0938	0.09380
m_3 (eV)	0.10	0.10	0.08	0.08
$\Sigma\|m_i\|$ eV	0.273	0.273	0.267	0.267

Table-2: Predictions for $\tan^2\theta_{12} = 0.50$ & other parameters consistent with observations.

Table-3: Predictions for $\tan^2\theta_{12} = 0.45$ and other parameters consistent with observational data.

Parameters	QD-NH		QD-IH	
	Type-IA	Type-IB	Type-IA	Type-IB
c	0.86	0.945	0.868	0.96
d	1.025	0.998	1.0	1.002
m_3 (eV)	0.10	0.10	0.08	0.08
ε	0.6616	0.00145	0.88762	0.00169
η	0.1655	0.06483	0.22317	-0.0855
m_1 (eV)	0.08754	0.08676	0.09392	0.09341
m_2 (eV)	-0.0879	0.08717	-0.0943	0.09381
m_3 (eV)	0.0996	0.10002	0.08	0.08001
$\Sigma\|m_i\|$ eV	0.275	0.274	0.268	0.267
Δm^2_{21} eV^2	$7.7\text{x}10^{-5}$	$7.3\text{x } 10^{-5}$	$7.6 \text{ x}10^{-5}$	$7.4\text{x } 10^{-5}$
$\|\Delta m^2_{23}\|$ eV^2	$2.2 \text{ x}10^{-3}$	$2.4\text{x } 10^{-3}$	$2.4\text{x } 10^{-3}$	$2.4 \text{ x}10^{-3}$
$\tan^2\theta_{12}$	0.45	0.45	0.45	0.45
$\|m_{ee}\|$ eV	0.0877	0.0869	0.0936	0.0935

LEPTONS CHARGED CORRECTIONS

$$U_{MNS} = U_L^{\tau} \, U_{\vartheta}$$

Where $U_L^{\tau} = \begin{pmatrix} 0.999327 & 0.0366849 & 0.000476075 \\ 0.036688 & -0.999231 & -0.01383 \\ 0.0000316411 & -0.0138381 & 0.999904 \end{pmatrix}$

[Ref [25]-Mohapatra et al Phys Lett **B636** 114 (2006)]

U_{ϑ} is same as in the case of mass matrix diagonalization without Charged Leptons Corrections.

Cal VALUES LEPTONS CHARGED CORRECTIONS

Parameters	Normal Ordering		Inverted Ordering		NH-IA**	NH-IB**
	Type-IA	Type-IB	Type-IA	Type-IB		
c d m_3 (=mo) eV ε η	**0.0863** **1.0** **0.08** **0.88762** **0.22317**	**0.945** **0.998** **0.10** **0.0015** **0.0648**	**0.863** **1.0** **0.08** **0.88762** **0.22317**	**0.96** **1.002** **0.08** **0.0016917** **-0.085456**	**0.863** **1.0** **0.40** **0.6616** **0.1655**	**1.0** **1.0** **0.1** **0.0015** **0.0649**
$\tan^2\theta_{12}$ $\tan^2\theta_{23}$ $\sin\theta_{13}$	0.50 0.94 0.026	0.50 0.94 0.026	0.50 0.94 0.026	0.50 0.94 0.026	0.55** 0.94 0.026	0.55** 0.945 0.0256
m_1 (eV) **m_2 (eV)** **m_3 (eV)** **$\Sigma\|m_i\|$ (eV)**	0.0934533 -0.0938581 0.08 0.07959	0.0867551 0.0871879 0.100013 0.2739	0.0934533 -0.0938581 0.08 0.267	0.0934139 0.093814 0.0800137 0.2672	0.348958 -0.349118 0.40 1.098	0.08672 0.08717 0.10 0.27389
Δm^2_{21} (10^{-5} eV^2) Δm^2_{23} (10^{-3} eV^2)	7.7 2.25	7.3 2.39	7.6 2.39	7.4 2.39	7.6 2.39	7.6 2.39

Comparison

Parameters	Without Lepton Charged Corrections.					
	NH-IA	NH-IB	IH-IA	IH-IB		NH-IA
$\tan^2\theta_{12}$ $\tan^2\theta_{23}$ $\sin\theta_{13}$	**0.45** **1.0** **0.00**	**0.45** **1.0** **0.00**	**0.45** **1.0** **0.00**	**0.45** **1.0** **0.00**		**0.50*** **1.0** **0.00**
	With Lepton Charged Corrections.					
$\tan^2\theta_{12}$ $\tan^2\theta_{23}$ $\sin\theta_{13}$	**0.50** **0.94** **0.026**	**0.50** **0.94** **0.026**	**0.50** **0.94** **0.026**	**0.50** **0.94** **0.026**		**0.55*** **0.945** **0.026**

- **Cosmological Data:**

Wilkinson Microwave Anisotropy Probe (WMAP) result(2010):

$$\frac{n_B}{n_\gamma} = \left(6.1^{+0.3}_{-0.2}\right) \times 10^{-10} \quad \left(1\sigma \text{ level}\right)$$

Future Challenge

1. Neutrino Physics:

To Find a General Model

or

Method without parameterizations.

2. Particle Cosmology: Can apply

CP violation in the leptogenesis/Baryon Asymmetry of the Universe.

In this direction we are working out earnestly.

Final Conclusions

- *The effects of Majorana phases on the predictions of absolute neutrino masses in 3-fold QDN models are consistent with data on the mass-squared differences derived from various oscillation experiments, and from the upper bound on absolute neutrino masses in $0\nu\beta\beta$ decay $m_{ee} \leq 0.27 eV$ and cosmology bound $\sum m_i \leq 0.28 eV$.*
- *But the recent tightest upper cosmology bound of $\sum m_i \leq 0.28 eV$ severe constraint on the scale of absolute neutrino $m_3 \leq 0.10\ eV$.*
- *The solar mixing angle is below TBM, viz, $\tan^2\Theta_{12}=0.45$ which coincides with the fit in the neutrino oscillation data.*
- *QDN Models with $m_i \leq 0.09\ eV$ not yet ruled out.*
- *The result shows the validity of QDN mass models.*

The results presented here are new and have subtle implications in the discrimination of neutrino mass models.

Thank You

Institute for HEP
PROTVINO