Observing Geometrical Torsion

Lucat Stefano & Tomislav Prokopec

Universiteit Utrecht

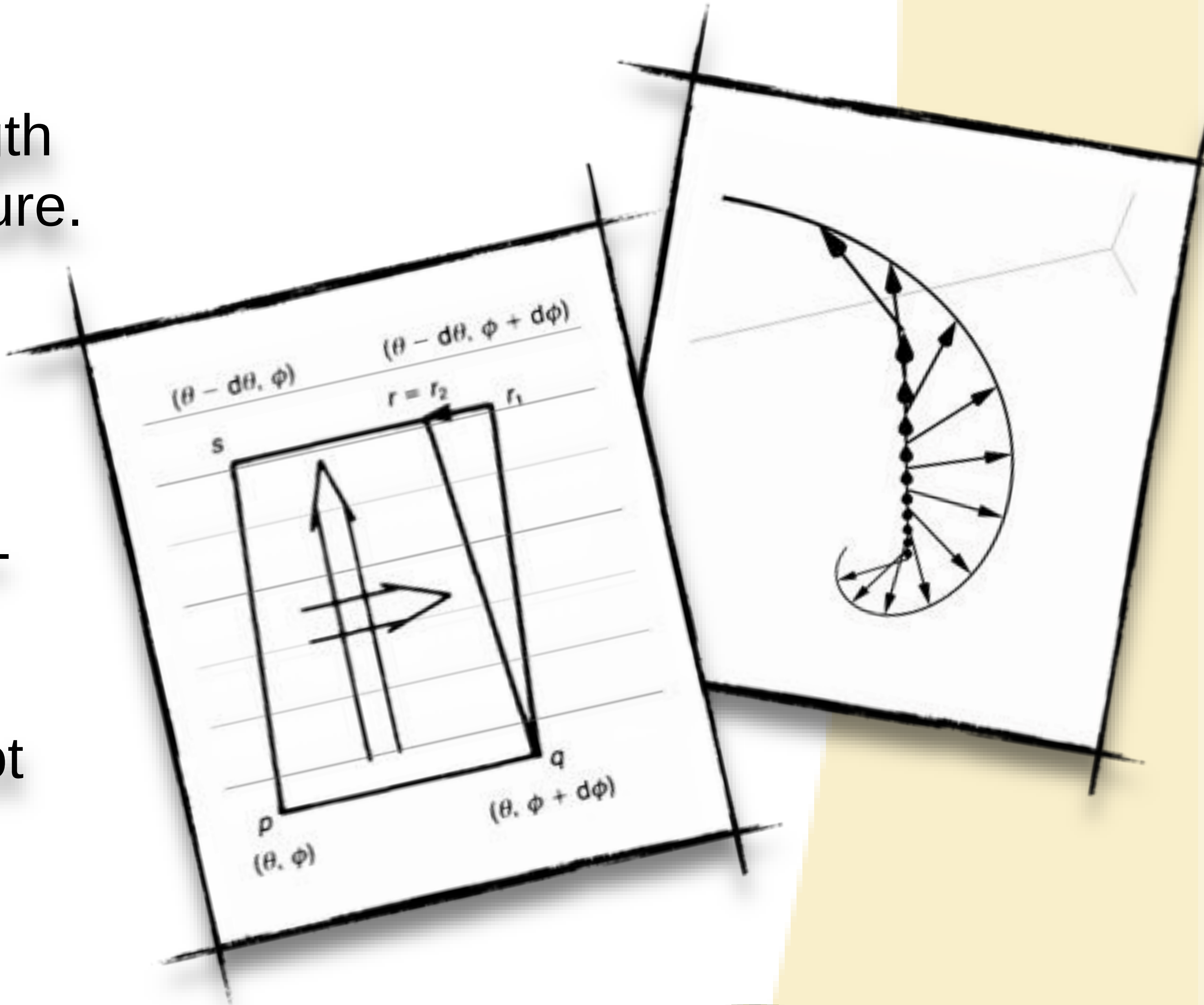
Protvino, July 2017

Content

- Gravity with torsion, “Who, What, When, Where, Why”.
- Weyl invariant gravity + standard model with torsion.
- Possibility of production of torsion waves and detection.
- Perspective on future directions (preliminary results).
- Conclusions

Some geometrical intuition

- Einstein: Gravity is a geometrical force, its strength given by space-time curvature.
- Cartan: adds an additional geometrical structure, separated from curvature, linked to “twisting” of space-time.
- Misconception: torsion is not just an external field. It is a geometrical universal field.



The geometrical field

- As a geometrical field, parallel transport.

$$\nabla_{\dot{\gamma}}\dot{\gamma} = 0 \Rightarrow \frac{\mathrm{d}^2 x^\mu}{\mathrm{d}\tau^2} = -\Gamma^\mu{}_{\alpha\beta}\frac{\mathrm{d}x^\alpha}{\mathrm{d}\tau}\frac{\mathrm{d}x^\beta}{\mathrm{d}\tau}$$

$$\text{if } \nabla_{\dot{\gamma}} g_{\mu\nu} = 0 \implies g_{\mu\nu} = \left\{ \mathcal{P}\left[\exp\left(\int_\gamma \Gamma^\lambda_{\sigma\mu}\dot{\gamma}^\mu\right)\right]\right\}^2$$

- One gains Weyl symmetry. Curvature and geodesics become invariant under rescaling of proper lengths:

$$\mathrm{d}\tau^2 \to \Omega^2(\tau)\mathrm{d}\tau^2 \,, g_{\mu\nu} \to \Omega^2(x) g_{\mu\nu}$$

$$\dot{\gamma} \to \Omega^{-1}\dot{\gamma}$$

→ Space-time dependent rescaling of dimension full quantities

Universiteit Utrecht

Coupling to matter

- Minimal couple prescription, in accordance with GR symmetries

$$\partial_\mu \Phi \to \nabla_\mu \Phi \,,$$

$$\eta^{\mu\nu} \to g^{\mu\nu}$$

$$\nabla_\mu(\Lambda(x)\cdot\Phi) = \Lambda(x)\cdot\nabla_\mu\Phi \,,$$
$$\Lambda(x) \in SO(1,3)$$

$$\nabla_\mu(\Omega(x)\mathbb{1}\cdot\Phi) = \Omega(x)\mathbb{1}\cdot\nabla_\mu\Phi \,,$$
$$\Omega(x) \in \mathbb{R}^+$$

Chiral Symmetry

- Matter couple universally to connection:

$$\mathcal{L}_\psi = \Gamma_{\alpha\beta\lambda}\bar{\psi}\gamma^{[\alpha}\gamma^\beta\gamma^{\lambda]}\psi$$

- Couple microscopically:

$$\mathcal{L}_\Phi = \Phi^\dagger T_\mu \left(2\partial^\mu + T^\mu\right)\Phi$$

Weyl Symmetry

Universiteit Utrecht

The conformal extension of the standard model

- Scalars couple to the torsion trace. At least two scalars, Higgs and an additional dilaton.

$$L = \left(\frac{\Phi^2}{2\alpha^2} R - \frac{1}{2}\bar{\nabla}_\mu \Phi \bar{\nabla}^\mu \Phi - V(\Phi)\right)$$

- Fermions couple to skew symmetric torsion, but not to the trace. The torsion trace contribution cancels in the action.

$$L = \frac{i}{2}\left(\bar{\psi}\gamma^\mu \bar{\nabla}_\mu \psi - \bar{\nabla}_\mu \bar{\psi}\gamma^\mu \psi\right)$$

- Gauge fields are conformal only in four dimensions. For D generic, conformal invariance breaks gauge symmetry.

$$F_{\mu\nu} = \bar{\nabla}_{[\mu} A_{\nu]} \overset{D \to 4}{=} (\mathrm{d}A)_{\mu\nu}$$

$$L = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + L_{int}$$

The link between torsion and Weyl symmetry

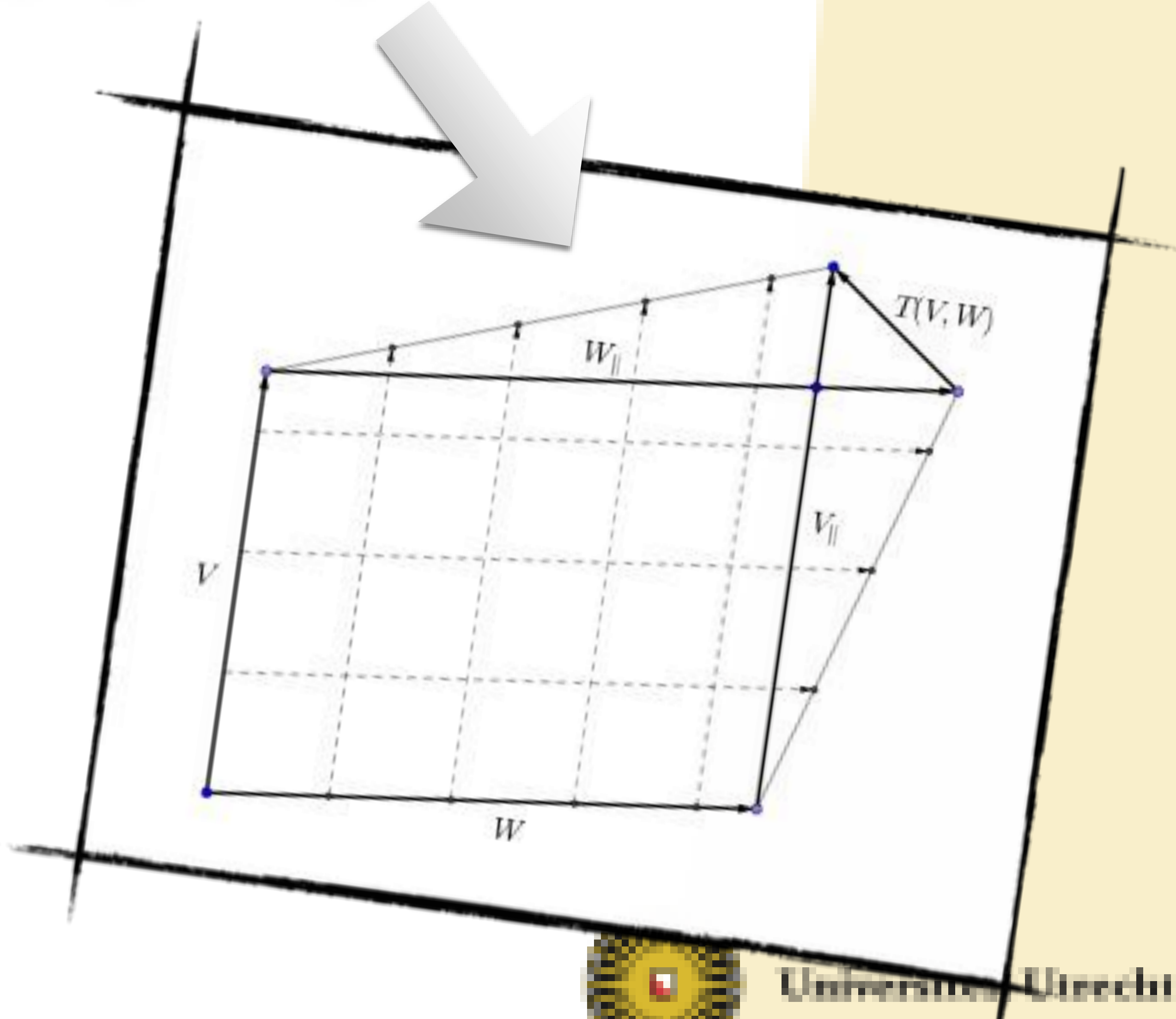
- Why should torsion be linked to Weyl symmetry?

$$e^a_\mu e^b_\nu \eta_{ab} = g_{\mu\nu}$$

$$e^a_\mu \to e^{\theta(x)} e^a_\mu$$

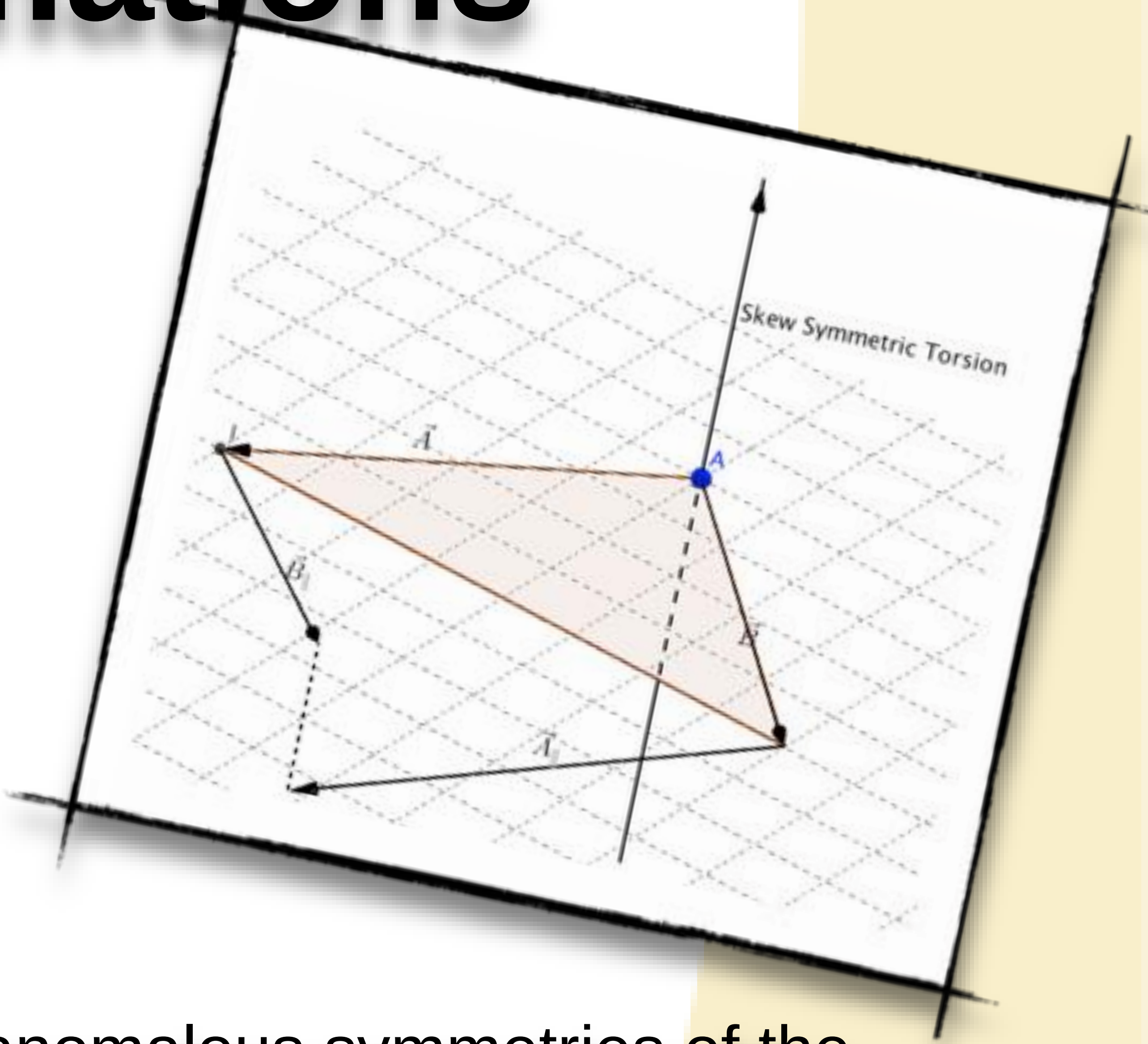
$$\omega^a_b \to \omega^a_b$$

$$T^a \to T^a + e^a \wedge \mathrm{d}\theta$$



- The torsion trace is naturally linked to scale transformations.
- Transforming torsion and vierbein leaves the Cartan connection invariant.

Link to chiral transformations



- Geometrical breaking of the "right hand rule".
- Skew symmetric torsion, couples to chiral fermionic current.

Einstein-Cartan-Kibble-Schiama Theory

- Geometrical fields, linked to two anomalous symmetries of the standard model. Maybe we can learn more about it by studying it carefully.

Universiteit Utrecht

Dynamical Torsion

- If a field couples through covariant derivatives

$$\mathcal{D}_\mu \Phi \subset \mathcal{L}$$

$$\mathcal{F}_{\mu\nu} = [\mathcal{D}_\mu, \mathcal{D}_\nu] \subset \Gamma_{1-loop}$$

- If torsion couples to matter in a universal manner, through Weyl and chiral charge, one loop effect will turn it dynamical.

$$\Gamma_{1\text{-}loop} \subset \left\{ \begin{array}{l} (\partial_\mu T_\nu - \partial_\nu T_\mu)^2 \ ; \\ (\partial_\mu \Sigma^\star_\nu - \partial_\nu \Sigma^\star_\mu)^2 \end{array} \right.$$

$$\Sigma^{\star\mu} = \epsilon^{\mu\alpha\beta\gamma}\Gamma_{[\alpha\beta\gamma]}$$

$+\mathcal{O}(R^2)$ operators

+Kinetic term for third Irreducible component

- Observations: in Colliders, via EFT interactions. Geometrically, via geodesics displacement.

Sources (Torsion trace)

- If our intuition is correct, torsion trace couples to fields dilatation current.

$$T^{\mu}_{\mu} = \partial_{\mu} D^{\mu}$$

$$\Longrightarrow \quad D^{\mu} = x^{\nu} T^{\mu}_{\nu}$$

- If scale symmetry is realised, dilatation current is conserved. Energy momentum is traceless. No torsion production.

$$T_{\mu} = \partial_{\mu}\theta$$

- But we live in the broken phase, effectively there is a scalar mode, satisfying:

$$\Box\theta = \frac{1}{M_P^2} T^{\mu}_{\mu} + \mathcal{O}(\theta^2)$$

Gravitational waves:

$$\Box h_{ij} = \frac{1}{M_P^2} T_{ij} + \mathcal{O}(h^2)$$

Geometrical Detection

- Jacobi fields: give the displacement of nearby parallel geodesics, no skew symmetric torsion

$$\nabla_{\dot\gamma}\nabla_{\dot\gamma}J_\perp + 2\nabla_{\dot\gamma}T(\dot\gamma, J_\perp) = \\ = R(\dot\gamma, J_\perp)\dot\gamma$$

- Linearised equation:

$$\frac{\mathrm{d}^2 J^i}{\mathrm{d}t^2} = \begin{cases} \frac{\ddot{h}^i_j}{2} J^j \\ J^j \partial_j T^i \\ -2\dot{Q}^i{}_{0j} J^j \end{cases} \Rightarrow (\vec{J}\cdot\vec{\nabla})\vec{T} = (\vec{J}\cdot\vec{\nabla})\vec{\nabla}\theta + (\vec{J}\cdot\vec{\nabla})\vec{\nabla}\times\vec{\tau}$$

Perspective: Conformal symmetry breaking

- At the conformal point we can construct a self consistent Weyl invariant theory.

- What is such UV theory? Maybe SO(2,4) local. Symmetry breaking:

$$SO(2,4) \to SO(1,4) \to SO(1,3)$$

$$\mathcal{M} = \Omega^{-1}\mathrm{d}\Omega\,,\ \Omega \subset SO(2,4)/SO(1,3)$$

$$[K^a, P^b] = 2i\eta^{ab}D - 2i\Sigma^{ab}\,,$$
$$P^b\,, \Sigma^{ab}\,, K^a\,, D \in SO(2,4)$$

$$e^a_\mu = e^\theta \delta^a_\mu\,, T^a = e^a \wedge \mathrm{d}\theta$$

Low energy degrees of freedom

Universiteit Utrecht

Perspective: Weyl anomaly

- Conformally invariant theories present quantum anomalies in the Weyl symmetry Ward identities

$$\langle T^\mu_\mu \rangle \neq 0$$

$$\langle T^\mu_\mu \rangle = \langle \nabla_\mu \Pi^\mu \rangle$$

- Local anomaly proportional to topological invariant, Gauss-Bonnet integral

$$\langle T^\mu_\mu \rangle \propto \epsilon^{\alpha\beta\gamma\delta} \epsilon_{\mu\nu\lambda\sigma} R^{\mu\nu}{}_{\alpha\beta} R^{\lambda\sigma}{}_{\gamma\delta}$$

$$\langle \nabla_\mu \Pi^\mu \rangle \propto \epsilon^{\alpha\beta\gamma\delta} \epsilon_{\mu\nu\lambda\sigma} R^{\mu\nu}{}_{\alpha\beta} R^{\lambda\sigma}{}_{\gamma\delta}$$

If trace is added in the geometry

- Solution valid at the conformal fixed point

$$\beta(\{\lambda_i\}) \to 0 \,, \; \mu \to \infty$$

Conclusions

- Study of torsion field yields rich theoretical extensions of gravity and the standard model.
- Extend the symmetry of the theory to include Weyl symmetry (a gravitational gauge symmetry) and chiral transformations (which does not extend to space-time.
- In principle, direct detection is possible, but likely difficult.
- "Well defined" UV completions for these models (know UV theory). Can we use extended symmetries to learn more about quantum gravity?

Thanks for attention

Questions?

Universiteit Utrecht