QUARTET-METRIC GRAVITY AND DARK COMPONENTS

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XXXI International Workshop on HEP,
5 – 7 July 2017, Protvino, Russia

- Based partly on:
  - Y.F. Pirogov, Eur. Phys. J. C **76**, 215 (2016);
  - Y.F. Pirogov, arXiv:1606.06150 [gr-qc].

Outlook

Why modify GR?

- State of the art: Cosmology Standard Model/ΛCDM:
  - Ingredients:
    - General Relativity (GR) plus Λ-term;
    - Continuous medium: matter and radiation.
  - Universe energy “budget” at present:
    - Radiation negligible;
    - Baryonic Matter $\sim$ 4%;
    - (Cold) Dark Mater (CDM) $\sim$ 23%;
    - Vacuum energy/Cosmological Constant (CC) $\sim$ 73%.

- Problems:
  - CDM nature and properties;
  - CC *(un)naturalness* (though mainly theoretical):
    - Why Λ is small *classically*?
    - Why Λ is not renormalized by *quantum corrections*?
    - Why Λ manifests itself just *lately*?
  - Approaches: nuisance or real problem?
- Beyond ΛCDM:
  - Beyond SM (BSM): new particles as Dark Components (DC's).
  - Beyond GR: Modified Gravity with new gravity fields and interactions as dynamical Dark Energy (DE) and DM;
  - Modified Gravity vs. BSM interpretation ambiguity: division rather conditional.

How modify GR?

- Principles:
  - Effective Field Theory (EFT):
    fields, symmetries and interactions.
  - Minimality principle (Occam's razor):
    "Among competing hypotheses, the one with the fewest assumptions should be preferred.

- Field set:
  - Gravity field: $g_{\mu\nu}(x)$ (not yet metric).
  - A quartet of scalar fields $X^a(x), \quad a = 0, \ldots, 3$, defining some distinguished/"vacuum" coordinates:

$$X^\alpha = \delta^\alpha_a X^a(x)$$

    (patch-vise) reversible: $x^\mu = x^\mu(X)$.
    The most rough characteristics of the vacuum, accounting in essence only for the fact of its existence.
- Quartet-Metric/Quartet-Modified (QM) Gravity.

- Symmetry pattern:
  - General Covariance (GC):
    all (smooth enough) coordinates x^μ are admissible.
  - General diffeomorphism Invariance/general relativity:

$$\mathrm{GDiff}\ :\ x^\mu \to x^\mu + \zeta^\mu(x)$$

  - Global Poincare invariance in $a, b, \ldots$

$$\mathrm{ISO(1,3)}\ :\ X^a \to \Lambda^a{}_b X^b + C^a$$

    Minkowski symbol η_{ab}.
- Local interactions:

$$S = \int \mathcal{L}(g_{\mu\nu}, \partial_\lambda g_{\mu\nu}, \partial_\lambda X^a, \Phi, \partial_\lambda \Phi) d^4 x.$$

- GC building blocks:
  - Auxiliary/"vacuum" metric: $\gamma_{\mu\nu} = \partial_\nu X^a \partial_\nu X^b \eta_{ab}$.
  - Scalar densities: $g = \det(g_{\mu\nu})$ and $\gamma = \det(\gamma_{\mu\nu})$.
    GC scalar field: $\Sigma \equiv e^\sigma = \sqrt{-g}/\sqrt{-\gamma}$.
  - GC connection-like tensor:

$$B^\lambda_{\mu\nu} = \Gamma^\lambda_{\mu\nu}(g_{\mu\nu}) - \gamma^\lambda_{\mu\nu}(\gamma_{\mu\nu}).$$

    In particular, convolution: $B^\lambda_{\mu\lambda} = \partial_\mu \ln \sigma$.
  - "Disformal" metric:

$$\tilde{g}_{\mu\nu} = f(\sigma) g_{\mu\nu} - \varphi(\sigma)\gamma_{\mu\nu}.$$

    Geometry/gravity nonequivalence.
  - GC "Higgs"-like tensor: $H^\mu{}_\nu = g^{\mu\lambda}\gamma_{\lambda\nu}$.

- GC action:

$$S = \int L(B^{\lambda}_{\mu\nu}, \tilde{g}_{\mu\nu}, H^{\mu}{}_{\nu}, \sigma, \Phi)\mathcal{M}(\gamma, \sigma)d^4x.$$

- Lagrangian:

$$L = K - V + L_m$$

.

  - Kinetic term: $K(B^{\lambda}_{\mu\nu}, \tilde{g}_{\mu\nu}, \sigma)$.
  - Potential: $V(H^{\mu}{}_{\nu}, \sigma)$.
  - Matter: $L_m(\Phi, \tilde{g}_{\mu\nu}, \sigma)$.

- Measure:
  - Geometrical: $\mathcal{M} = \sqrt{-\tilde{g}} \equiv \sqrt{-\det(\tilde{g}_{\mu\nu})}$,
  - Gravitational: $\mathcal{M} = \sqrt{-g} \equiv \sqrt{-\det(g_{\mu\nu})}$,
  - Vacuum: $\mathcal{M} = \sqrt{-\gamma} \equiv \sqrt{-\det(\gamma_{\mu\nu})} = \det(\partial_\mu X^a)$.
    - Due to $\sqrt{-g} = e^\sigma \sqrt{-\gamma}$ the last two measures are explicitly equivalent up to Lagrangian redefinition: $L \to e^\sigma L$.
  - Importance of knowing measure:
    - Occam's razor: $\mathcal{M}$ at which L looks most simply (if any) should a priori be preferred.

- Alternatives for QMG: “seed” theories GR vs. UR, with gravitating vs. nongravitating measure.
  - *Classically* equivalent up to matter conservation, but:
    - UR theoretically advantages from CC problem because:
    - Λ_0 from the *classical* Field Equation decouples from the Lagrangian Λ,
    - Λ_0 quantum (*semi-classically*) rnonenormalized.

Quartet-Modified GR

- "Seed" theory: GR.
- QM GR:
  - Geometry/gravity equivalence: $\tilde{g}_{\mu\nu} = g_{\mu\nu}$
  - Gravitational measure: $\mathcal{M} = \sqrt{-g}$.
  - Quadratic gravity Lagrangian:

$$L_2 = K_2(B^\lambda_{\mu\nu}, g_{\mu\nu}, \sigma) - V(H^\mu{}_\nu, \sigma)$$

- Linear approximation:
  - Flat background: $x^\mu \to \xi^\alpha$,

$$X^\alpha = \xi^\alpha + \chi^\alpha,$$

$$g_{\alpha\beta} = \eta_{\alpha\beta} + h_{\alpha\beta},$$

$$\gamma_{\alpha\beta} = \eta_{\alpha\beta} + (\partial_\alpha \chi_\beta + \partial_\beta \chi_\alpha).$$

  - GDiff SSB/BEH mechanism for gravity:

$$h_{\alpha\beta} \to h'_{\alpha\beta} = h_{\alpha\beta} - (\partial_\alpha \chi_\beta + \partial_\beta \chi_\alpha)$$

    Residual Diff's.
  - In particular:
    - Massive tensor graviton $h'_{\alpha\beta}$,
      with $\partial_\beta h'^{\alpha\beta} = 0$ at $K_2 = L_{\rm GR}$, $V = V_{\rm FP}$,
    - Scalar graviton σ,
      with $h' = h^\alpha_\alpha - 2\partial_\alpha \chi^\alpha$ at $K_s \sim 1/2\, g^{\mu\nu} \partial_\mu \sigma \partial_\nu \sigma$.

Quartet-Modified UR

- "Seed" theory: Unimodular Relativity (UR).
- QM UR:
  - Geometry/gravity nonequivalence:
    - *Nongravitating* vacuum measure: $\mathcal{M} = \gamma$.
    - Exceptional conformal metric:

$$\tilde{g}_{\mu\nu} = \hat{g}_{\mu\nu} \equiv e^{-\sigma/2} g_{\mu\nu} = (\sqrt{-\gamma}/\sqrt{-g})^{1/2} g_{\mu\nu},$$

    with $\det(\hat{g}_{\mu\nu}) \equiv \hat{g} = \gamma$.
  - Gravitational local scale/Weyl invariance $g_{\mu\nu} \to s(x) g_{\mu\nu}$:

$$\hat{g}_{\mu\nu} \to \hat{g}_{\mu\nu}, \quad \gamma \to \gamma.$$

    - Elimination of one extra gravity component (residual 6 as in UR and GR).

- UR action :

$$S = \int \Big(-\frac{\kappa_g^2}{2}\Big(R(\hat{g}_{\mu\nu}) - 2\Lambda\Big) + L_m(\Phi, \hat{g}_{\mu\nu})\Big)\sqrt{-\gamma}d^4x$$

(dynamical $\sqrt{-\gamma} = \det(\partial_\mu X^a)$) .

- FE's: transverse (no explicit Λ)

$$\hat{R}_{\mu\nu} - \frac{1}{4}\hat{g}_{\mu\nu}\hat{R} = \frac{1}{\kappa_g^2}\Big(\hat{T}_{m\,\mu\nu} - \frac{1}{4}\hat{g}_{\mu\nu}\hat{T}_m\Big),$$

$+$ longitudinal (explicit Λ).

- Solutions:

$$g_{\mu\nu} = g_{\mu\nu}(\Phi, \gamma, \Lambda_0),$$

$+$ metric-independent $\gamma(x) = \det(\partial_\mu X^a \partial_\nu X^b)$.
Classically equivalent to GR in the gauge $g = \gamma$.

  - Advantage:
    - Transversality: *natural* decoupling of the longitudinal gravitational contribution at the *semi-classical* level. Preservation of the classical Λ_0.
  - Flaws:
    - Ambiguity: yet unrestricted $\gamma(x)$ (though dynamical).
    - Ad hock small *classical* $\Lambda_0 \ll \Lambda$.
  - Naturalness: dynamical setting $\Lambda_0 \ll \Lambda$ starting from an arbitrary Λ needed.

- An artifact of the incompleteness of UR with the exact Weyl invariance.
- A possible way out to eliminate the flaws: an explicit (small) violation of Weyl invariance by means of the Weyl-noninvariant σ-field.

QM and Dark Components

- UG: UR + true dynamical extra gravity component.
  - Minimally: scalar graviton σ as weak-field quasi-particle.
  - Generally: "aquadratic" scalar-graviton Lagrangian:

$$L_s = \upsilon_s^2 \kappa_g^2 \kappa_0^2 F(K_s/\kappa_0^2) - V_s(\sigma),$$

    υ_s a dimensionless parameter,
    κ_0 a scalar-gravity IR scale ($\kappa_0 < \kappa_g$).
  - Kinetic term for σ:

$$K_s \simeq 1/2\, \hat{g}^{\mu\nu} \partial_\mu \sigma \partial_\nu \sigma.$$

    Violation of Weyl invariance: appearance of the extra gravity component/physical scalar graviton σ.

- IR limit:

$$\kappa_0^2 F(K_s/\kappa_0^2) = K_s \text{ at } K_s/\kappa_0^2 \ll 1 \text{ (formally, for all } K_s \text{ at } \kappa_0 \to \infty).$$

- UV suppression, e.g.:

$$F(K_s/\kappa_0^2) = 1 \text{ at } K_s/\kappa_0^2 \gg 1 \text{ (formally, for all } K_s \text{ at } \kappa_0 \to 0).$$

  - Modification of UR at $\kappa_0 \ll \kappa_g$ only for the (very) weak gravity fields (motivated by observations for the galaxy dark halos). Phenomenologically, κ_0 of order H_0 (or, rather, v.v.).
- The physical scalar graviton σ as the unified DM and DE.
- A priori, other contributions also conceivable.

Resume

- QMG presents a bold new type of GR modification built on the BEH mechanism for gravity with SSB of GR. A priori, there is no gravity/geometry equivalence. This is due to existence of the distinguished/"vacuum" coordinates described by a scalar quartet.
- Admixture to metric of the scalar quartet as a gravity counterpart of the Higgs field gives the generic toolkit for a rich spectrum of the emergent gravity phenomena beyond GR. In particular, this may include the massive tensor and scalar gravitons, variable effective CC, DE, DM, etc.
- Further theoretical and phenomenological study of QMG (in particular, QM UR) is needed to verify it as a candidate on the next-to-GR theory of gravity (if any).

THANK YOU