Duality correspondence between chiral symmetry breaking and charged pion condensation in chirally asymmetric dense quark matter: Consideration of an NJL_2 model with spatially inhomogeneous condensates

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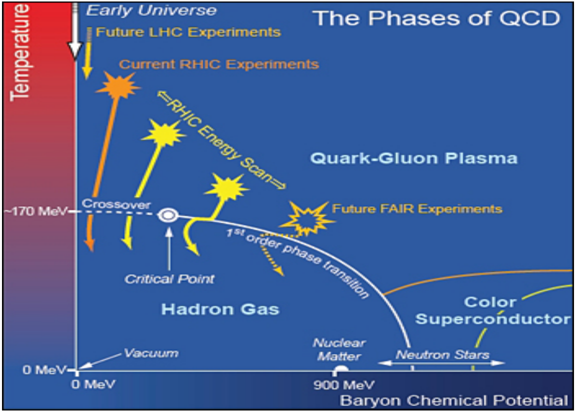
QCD at finite temperature and nonzero chemical potential

QCD at nonzero temperature and baryon chemical potential plays a fundamental role in many different physical systems. QCD at extreme conditions

- neutron stars
- heavy ion collision experiments
- Early Universe

QCD Phase Diagram



Two main phase transition

- confinement-deconfinement
- chiral symmetry breaking phase—chriral symmetric phase

Methods of dealing with QCD

Methods of dealing with QCD

- perturbative QCD, pQCD, high energy
- First principle calcaltion – lattice Monte Carlo simulations, LQCD
- Effective models

  Chiral pertubation theory χPT
  Nambu–Jona-Lasinio model NJL
  Polyakov-loop extended Nambu–Jona-Lasinio model PNJL
  Quark meson model
- $1/N$ expansion (large number of colors) G.t'Hooft.
  the predictions of $\frac{1}{N_c}$ expansions for QCD are mostly of a qualitative nature
- Holographic methods, Gauge/gravity or gauge/string duality AdS/CFT conjecture

(1+1)- dim Gross-Neveu model

(1+1)-dimensional Gross-Neveu (GN) model possess a lot of common features with QCD

- renormalizability
- asymptotic freedom
- sponteneous chiral symmetry breaking in vacuum
- dimensional transmutation
- have the similar $\mu_B - T$ phase diagrams

Also

Relative simplicity, renormalizability and possibility to solve theory in the leading order of $1/N$ expansion

NJL_2 model can be used as a laboratory for the qualitative simulation of specific properties of QCD at arbitrary energies

Gross-Neveu and Nambu–Jona-Lasinio model

Gross-Neveu model

$$\mathcal{L} = \bar{q}\gamma^{\nu}\mathrm{i}\partial_{\nu}q + \frac{G}{N_c}(\bar{q}q)^2$$

$$q \to \gamma_5 q$$

discrete symmetry

$$\widetilde{L} = \bar{q}\left[\gamma^{\rho}\mathrm{i}\partial_{\rho} - \sigma\right]q - \frac{N_c}{4G}\sigma^2$$

Nambu–Jona-Lasinio model

$$\mathcal{L} = \bar{q}\gamma^{\nu}\mathrm{i}\partial_{\nu}q + \frac{G}{N_c}\left[(\bar{q}q)^2 + (\bar{q}\mathrm{i}\gamma^5 q)^2\right]$$

$$q \to e^{i\gamma_5\alpha}q$$

continuous symmetry

$$\widetilde{L} = \bar{q}\left[\gamma^{\rho}\mathrm{i}\partial_{\rho} - \sigma - \mathrm{i}\gamma^5\pi\right]q - \frac{N_c}{4G}\left[\sigma^2 + \pi^2\right.$$

Chiral symmetry breaking

$1/N_c$ expansion, leading order

$$\langle\bar{q}q\rangle \neq 0$$

$$\langle\sigma\rangle \neq 0 \quad \longrightarrow \quad \widetilde{L} = \bar{q}\left[\gamma^{\rho}\mathrm{i}\partial_{\rho} - \langle\sigma\rangle\right]q$$

Isotopic and axial isotopic chemical potentials

Isotopic chemical potential

Dense matter with isotopic imbalance in neutron stars, heavy ion collision experiments

$$n_I = n_u - n_d \quad \longleftrightarrow \quad \mu_I = \mu_u - \mu_d$$

axial chemical potential

Systems with chiral imbalance have attracted some interest in recent years.
Chiral imbalance is a nonzero difference between densities of left- and right-handed fermions,

$$n_5 = n_R - n_L \quad \longleftrightarrow \quad \mu_5 = \mu_R - \mu_L$$

arise from quantum anomaly in the quark-gluon-plasmaand leads to the chiral magnetic effect.

Axial isotopic chemical potentials

axial isotopic chemical potentials

$$\mu_{I5} = \mu_{uR} - \mu_{uL} + \mu_{dL} - \mu_{dR}$$

Model and its Lagrangian

We consider a two-dimensional model which describes dense quark matter with two massless quark flavors (u and d quarks).

$$L = \bar{q}\Big[\gamma^\nu \mathrm{i}\partial_\nu + \frac{\mu_B}{3}\gamma^0 + \frac{\mu_I}{2}\tau_3\gamma^0 + \frac{\mu_{I5}}{2}\tau_3\gamma^0\gamma^5\Big]q +$$

$$+\frac{G}{N_c}\Big[(\bar{q}q)^2 + (\bar{q}\mathrm{i}\gamma^5\vec{\tau}q)^2\Big], \qquad (1)$$

The model (16) is a generalization of the two-dimensional Gross-Neveu model with a single massless quark color N_c-plet to the case of two quark flavors and additional baryon μ_B-, isospin μ_I- and axial isospin μ_{I5} chemical potentials. These parameters are introduced in order to describe in the framework of the model (1) quark matter with nonzero baryon n_B-, isospin n_I- and axial isospin n_{I5} densities, respectively.

Symmetries of Lagrangian

Lagrangian is invariant with respect to the abelian $U_B(1)$, $U_{I_3}(1)$ and $U_{AI_3}(1)$ groups,

$$U_B(1): \;\; q \to \exp(\mathrm{i}\alpha/3)q; \tag{2}$$
$$U_{I_3}(1): \;\; q \to \exp(\mathrm{i}\alpha\tau_3/2)q; \tag{3}$$
$$U_{AI_3}(1): \;\; q \to \exp(\mathrm{i}\alpha\gamma^5\tau_3/2)q. \tag{4}$$

Lagrangian (1) is invariant with respect to the electromagnetic $U_Q(1)$ group,

$$U_Q(1): \;\; q \to \exp(\mathrm{i}Q\alpha)q, \tag{5}$$

where $Q = \mathrm{diag}(2/3, -1/3)$.

Equivalent Lagrangian

To find the thermodynamic potential of the system, we use a semi-bosonized version of the Lagrangian (16), which contains composite bosonic fields $\sigma(x)$ and $\pi_a(x)$ $(a = 1, 2, 3)$

$$\widetilde{L} = \bar{q}\Big[\gamma^\rho \mathrm{i}\partial_\rho + \mu\gamma^0 + \nu\tau_3\gamma^0 + \nu_5\tau_3\gamma^1 - \sigma - \mathrm{i}\gamma^5\pi_a\tau_a\Big]q$$

$$-\frac{N_c}{4G}\Big[\sigma\sigma + \pi_a\pi_a\Big]. \qquad (6)$$

For bosonic fields one has

$$\sigma(x) = -2\frac{G}{N_c}(\bar{q}q); \quad \pi_a(x) = -2\frac{G}{N_c}(\bar{q}\mathrm{i}\gamma^5\tau_a q). \qquad (7)$$

Chiral density wave and pion wave

In vacuum, i.e. $\mu = 0$, $\nu = 0$ and $\nu_5 = 0$, $\langle\sigma(x)\rangle$ and $\langle\pi_a(x)\rangle$ do not depend on space coordinate x. However, in a dense medium, when $\mu \neq 0$, $\nu \neq 0$ and $\nu_5 \neq 0$, the ground state expectation values of bosonic fields might have a nontrivial dependence on x.
We will use the following ansatz:

$$\langle\sigma(x)\rangle = M\cos(2bx),\ \ \langle\pi_3(x)\rangle = M\sin(2bx),$$

$$\langle\pi_1(x)\rangle = \Delta\cos(2b'x),\ \ \langle\pi_2(x)\rangle = \Delta\sin(2b'x),$$
$$\langle\pi_+(x)\rangle = \Delta e^{2b'x},\ \ \langle\pi_-(x)\rangle = \Delta e^{-2b'x},$$

where M, b, b' and Δ are constant dynamical quantities. In fact, they are coordinates of the global minimum point of the thermodynamic potential (TDP) $\Omega(M, b, b', \Delta)$.

In the leading order of the large N_c-expansion it is defined by the following expression:

$$\int d^2x\Omega(M,b,b',\Delta) = -\frac{1}{N_c}\mathcal{S}_{\rm eff}\{\sigma(x),\pi_a(x)\}\big|_{\sigma(x)=\langle\sigma(x)\rangle,\pi_a(x)=\langle\pi_a(x)\rangle}, \tag{8}$$

for the thermodynamic potential one can obtain

$$\Omega(M, b, b', \Delta) = \frac{M^2 + \Delta^2}{4G} + \mathrm{i} \int \frac{d^2 p}{(2\pi)^2} \ln P_4(p_0). \qquad (9)$$

where

$$P_4(p_0) = \eta^4 - 2a\eta^2 - b\eta + c, \eta = p_0 + \mu$$

$$a = M^2 + \Delta^2 + p_1^2 + \tilde{\nu}^2 + \tilde{\nu}_5^2; \quad b = 8p_1\tilde{\nu}\tilde{\nu}_5;$$

$$c = a^2 - 4p_1^2(\tilde{\nu}^2 + \tilde{\nu}_5^2) - 4M^2\tilde{\nu}^2 - 4\Delta^2\tilde{\nu}_5^2 - 4\tilde{\nu}^2\tilde{\nu}_5^2.$$

$$\tilde{\nu} = \nu + b, \qquad \tilde{\nu}_5 = \nu_5 + b'.$$

$$\mu \equiv \mu_B/3, \ \nu = \mu_I/2, \ \nu_5 = \mu_{I5}/2$$

.

Duality

The thermodynamic potential is invariant with respect to the so-called duality transformation

$$\mathcal{D}: \; M \longleftrightarrow \Delta, \quad \nu \longleftrightarrow \nu_5, \quad b \longleftrightarrow b'. \tag{10}$$

If we change axes $\nu \longleftrightarrow \nu_5$ then we should exchange PC $\longleftrightarrow$ CSB.

$$F_1(M) \equiv \Omega^{ren}(M, \Delta = 0)$$

$$F_2(\Delta) \equiv \Omega^{ren}(M = 0, \Delta)$$

$$F_2(\Delta) \quad = \quad F_1(\Delta)\Big|_{\nu \longleftrightarrow \nu_5}. \tag{11}$$

Duality

$$L = \sum_{k=1}^{N} \bar{\psi}_k \left[\gamma^\nu i \partial_\nu + \mu \gamma^0 + \mu_5 \gamma^0 \gamma^5 \right] \psi_k$$

$$+ \frac{G_1}{N} \left[\left(\sum_{k=1}^{N} \bar{\psi}_k \psi_k \right)^2 + \left(\sum_{k=1}^{N} \bar{\psi}_k i \gamma^5 \psi_k \right)^2 \right] +$$

$$+ \frac{G_2}{N} \left(\sum_{k=1}^{N} \psi_k^T \epsilon \psi_k \right) \left(\sum_{j=1}^{N} \bar{\psi}_j \epsilon \bar{\psi}_j^T \right),$$

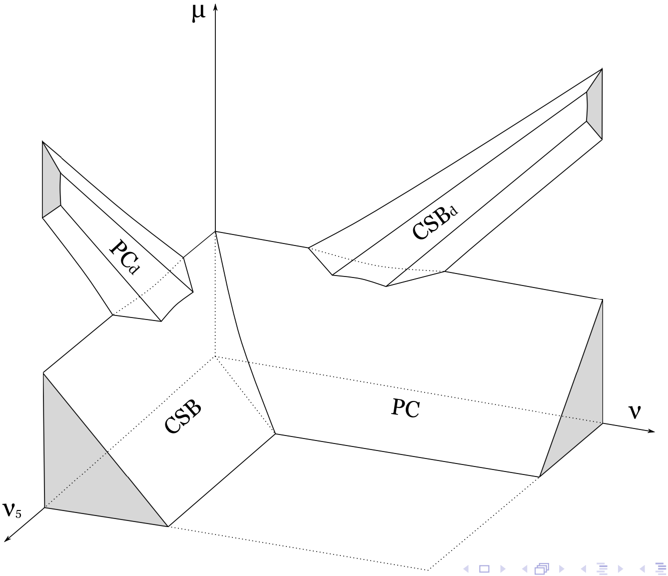
So the TDP (??) is invariant with respect to the following duality transformation D:

$$D: \quad G_1 \longleftrightarrow G_2, \quad \mu \longleftrightarrow \mu_5, \quad M \longleftrightarrow \Delta. \qquad (12)$$

Orbifold equivalences

AdS-CFT duality is strong-weak duality
Orbifold equivalence is strong-strong duality
Orbifold equivalences connect gauge theories with different gauge groups and matter content in the large Nc limit.
There are a class of QCD-like theories which are free from the sign problem.
The whole or the part of the phase diagrams of dual theories should be universal in the large-Nc limit via the orbifold equivalence

Phase portrait (μ, ν, ν_5) in homogeneous case



Phase portrait in homogeneous case

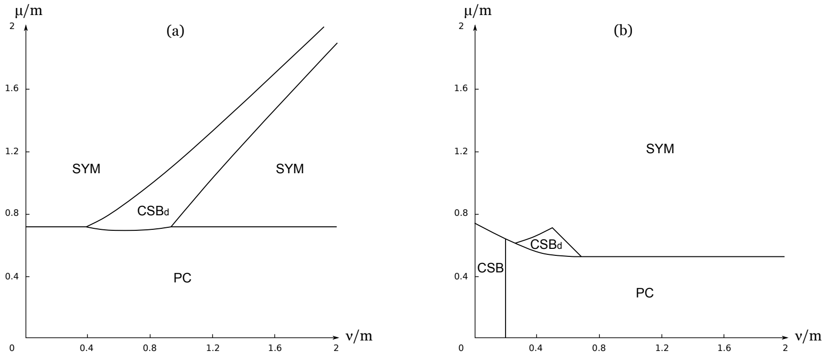


Рис.: The (ν, μ)-phase portrait of the model for different values of the chiral chemical potential ν_5: (a) The case $\nu_5 = 0$. (b) The case $\nu_5 = 0.2m$.

Phase portrait in homogeneous case

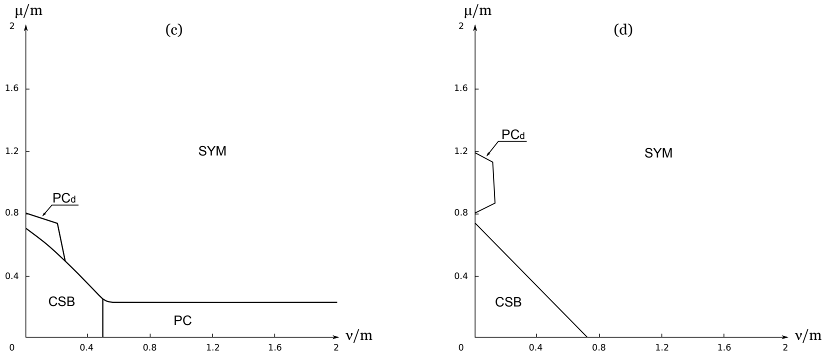


Рис.: The (ν, μ)-phase portrait of the model for different values of the chiral chemical potential ν_5:(a) The case $\nu_5 = 0.5m$. (b) The case $\nu_5 = m$.

Inhomogeneous case

At $M = 0$ and $\Delta = 0$ the expression for thermodynamic potential does depend on b and b'. This is quite unphysical and somehow we need to change the expresssion for thermodynamic potential.

$$\tilde{F}_1(M, b) = \tilde{\Omega}(M, b, b', 0) = F_1(M, b) - F_1(0, b) + F_1(0, 0) \quad (13)$$

$$\tilde{F}_2(\Delta, b') = \tilde{\Omega}(0, b, b', \Delta) = F_2(\Delta, b') - F_2(0, b') + F_2(0, 0) \quad (14)$$

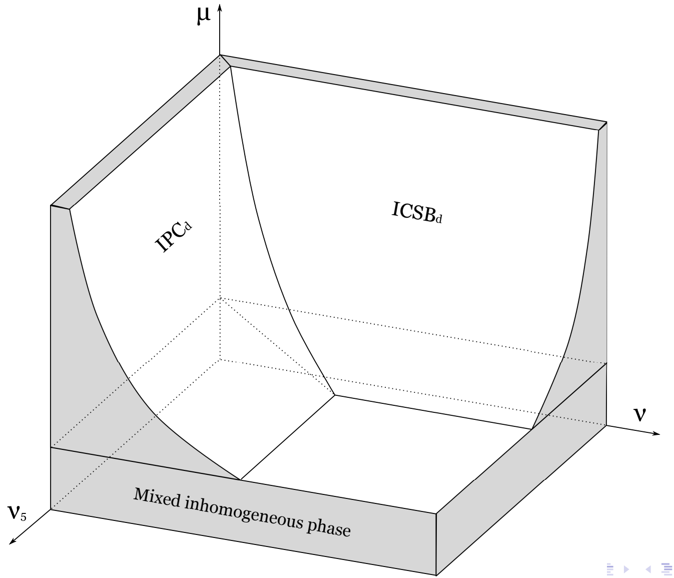
for thermodynamic potential

$$\tilde{\Omega}(M, b, b', \Delta) = \Omega(M, b, b', \Delta) - \Omega(M, b, b', 0) \quad (15)$$

$$+\Omega(M, b, 0, 0) - \Omega(0, b, b', \Delta) + \Omega(0, 0, b', \Delta)$$

$$-\Omega(0, b, 0, 0) - \Omega(0, 0, b', 0) + \Omega(0, b, b', 0) + \Omega(0, 0, 0, 0)$$

Phase portrait (μ, ν, ν_5) in inhomogeneous case



Phase portrait (μ, ν)

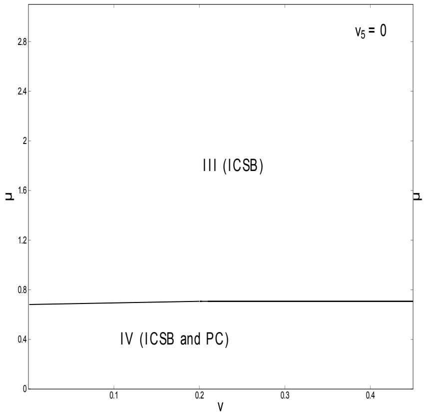


Рис.: (μ, ν) phase diagram at $\nu_5 = 0$

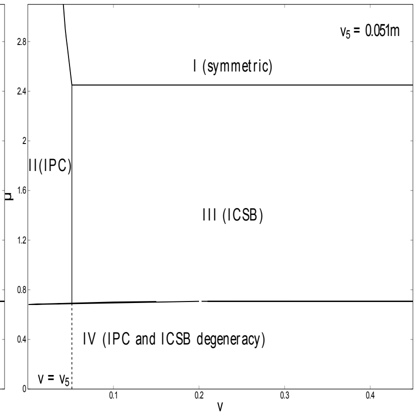


Рис.: (μ, ν) phase diagram at $\nu_5 = 0.051m$

Phase portrait (μ, ν)

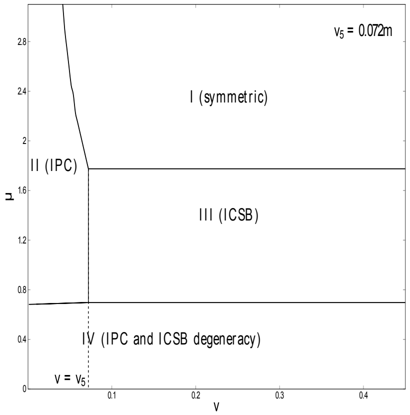


Рис.: (μ, ν) phase diagram at $\nu_5 = 0.072m$

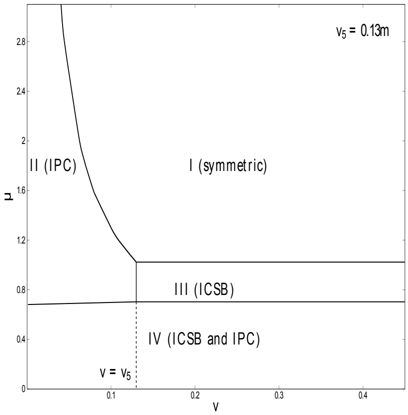


Рис.: (μ, ν) phase diagram at $\nu_5 = 0.13m$

Phase portrait

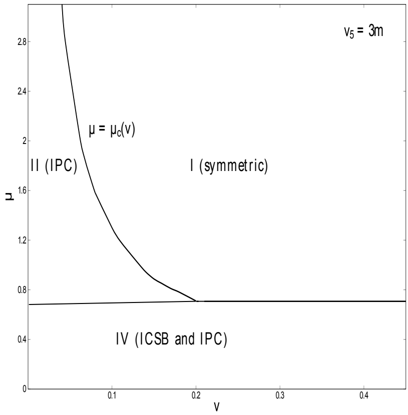


Рис.: (μ, ν) phase diagram at $\nu_5 = 3m$

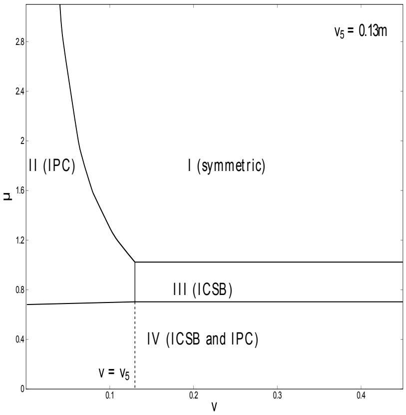


Рис.: (μ, ν_5) phase diagram at $\nu = 0.13m$

Phase portrait

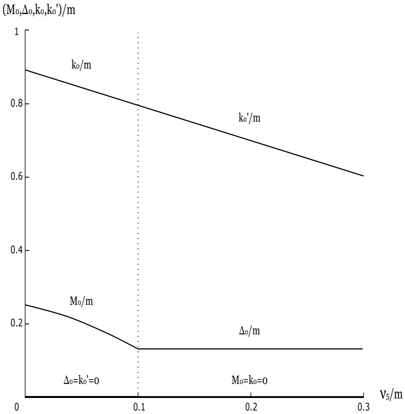


Рис.: The behavior of M_0, k_0, k_0', Δ_0 as functions of ν_5 for fixed $\mu = m$ and $\nu = 0.1m$.

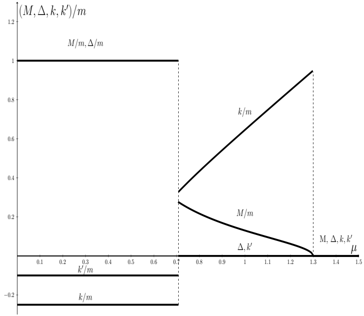


Рис.: The behavior of M_0, k_0, k_0', Δ_0 as functions of μ for fixed $\nu_5 = 0.1m$ and $\nu = 0.25m$.

Phase portrait

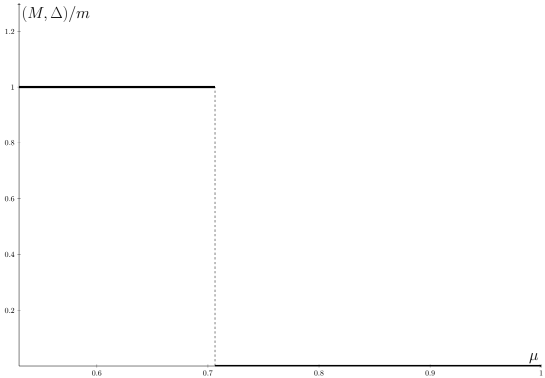


Рис.: The behavior of M_0, k_0, k_0', Δ_0 as functions of μ for fixed $\nu_5 = 0.25m$ and $\nu = 0.251m$.

Phase portrait (ν, ν_5)

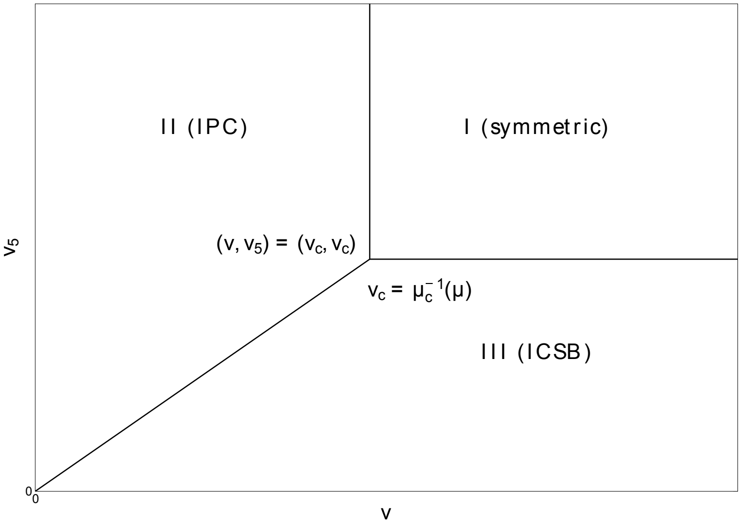


Рис.: (ν, ν_5) phase diagram at $\mu > 0.707$

Conclusions

Our consideration aims at study of the properties of chirally ($\mu_{I5} \neq 0$) and isotopically ($\mu_I \neq 0$) asymmetric dense ($\mu_B \neq 0$) quark matter with inhomogeneous condensates.

- At $\mu_{I5} \neq 0$ even in homogeneous case there is charged PC phase with nonzero baryon density. The main result is that μ_{I5} generates charged pion condensation in dense quark matter.
- Charged PC phase realises at any nonzero $\mu_{I5} \neq 0$ in contrast to homogeneous case where it realises only for rather larger values of μ_{I5}(larger than some value). It means that charged pion condensation happens at even small chiral asymmetry.
- In the leading order of the large-N_c approximation in the framework of the NJL_2 model (1) there is a duality correspondence between CSB and charged PC phenomena.
- Inhomogeneous condensates are quite favoured compared to homogeneous ones.All phases at the phase diagram are inhomogeneous or symmetric ones.