F-theory and AdS_3/CFT_2

Dario Martelli

King's College London

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C. Couzens, C. Lawrie, S. Schäfer-Nameki, and J.-M. Wong

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Plan

- Background and motivations
- Frameworks for holography
- Holography vs F-theory
- Examples of $\mathrm{AdS}_3/\mathrm{CFT}_2$ [overlap with Bobev's talk]
- AdS_3 holography in F-theory
- Outlook

Plan

- Background and motivations
- Frameworks for holography
- Holography vs F-theory
- Examples of AdS_3/CFT_2

$\Big\}$ light & entertaining

- AdS_3 holography in F-theory

$\}$ new & technical

- Outlook

Background and motivations

- Since the formulation of the AdS/CFT correspondence, searching for $\mathrm{AdS}_{\mathbf{d+1}}$ solutions has been used as an approach to explore $\mathrm{CFT}_{\mathbf{d}}$'s
- Although in principle AdS/CFT should be valid also without supersymmetry, this is often useful for obtaining exact (analytic) results, and I will assume it
- Since a lot more was known about CFTs in $\mathbf{d = 2}$ than about CFTs in $\mathbf{d > 2}$, the attention has been devoted to $\mathbf{d = 4}$, then $\mathbf{d = 3}$
- More recently also the more elusive $\mathbf{d = 5}$ and $\mathbf{d = 6}$ have been explored
- $\mathbf{d = 1}$ and $\mathbf{d = 2}$ have been considered, but so far less systematically

Background and motivations

- AdS_2 and AdS_3 arise quite generally in the near-horizon limit of various black holes/black string in $\mathbf{D=4}$ and $\mathbf{D=5}$ dimensions
- Counting the microstates of the dual CFT_1 or CFT_2 provides a window into the microscopic origin of the entropy of these black holes
- E.g. in favourable circumstances one can hope to calculate exactly the degeneracies of these states computing appropriate indices, perhaps employing localization methods
- AdS_3 holography is interesting *per se*
    - Connection to integrability, higher spin, quantum gravity, condensed matter, ...
    - **2d** SCFT's richer and more interesting than expected: e.g. "exotic" theories obtained from wrapped branes, **c**-extremisation...

Frameworks for top-down holography

- If we are interested in studying holography in the context of string or M-theory, we need to find solutions of type II or 11d supergravities
- This is referred to as top-down approach
- This has motivated a research program of systematically scanning through supersymmetric solutions with an $\mathrm{AdS}_{\mathbf{d+1}}$ factor

Classification of supersymmetric $\mathrm{AdS_p}$ solutions

Some representative (though partial) references:

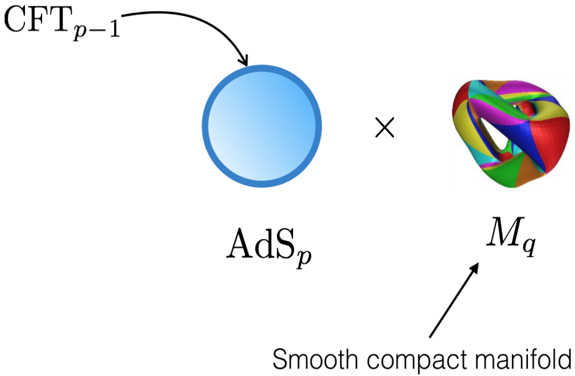
- $\mathrm{AdS_5}$ solutions in 11d sugra [Gauntlett,DM,Sparks,Waldram] (2004)
- $\mathrm{AdS_5}$ solutions in type IIB [Gauntlett,DM,Sparks,Waldram] (2005)
- $\mathbf{(0,2)}$ $\mathrm{AdS_3}$ solutions in type IIB, with D3 branes only [Kim] (2005)
- $\mathcal{N} = 2$ $\mathrm{AdS_4}$ solutions in 11d [Gabella,DM,Passias,Sparks] (2012)
- $\mathrm{AdS_7}$ solutions in type IIA/B [Apruzzi,Fazzi,Rosa,Tomasiello] (2013)
- $\mathrm{AdS_6}$ solutions of type IIA/B [Apruzzi,Fazzi,Passias,Rosa,Tomasiello] (2014)
- $\mathrm{AdS_5}$ solutions in massive type IIA [Apruzzi,Fazzi,Passias,Tomasiello] (2015)

AdS with 7-branes?

- Type IIB supergravity comprises: $\mathbf{F_5}$ (D3 branes), $\mathbf{F_3}$ (D1 and D5 branes), $\mathbf{H}$ (NS5 branes and F1 strings), $\boldsymbol{\tau} = \mathbf{C_0} + \mathbf{i}\mathbf{e}^{-\Phi}$ (D7 branes)
- Although $\boldsymbol{\tau}$ is included in the analysis of type IIB solutions in the references above, there are no examples of $\mathrm{AdS}_\mathbf{p}$ solutions with $\boldsymbol{\tau}$ varying non-trivially, i.e. including non-trivial $\mathbf{SL(2, Z)}$ monodromy ($\mathbf{(p, q)}$ 7-branes)
- The fact that 7-branes lead to singularities is not unexpected: 7-branes in 1+9d are like strings in 1+3d, log-behaviour and monodromies going around them fully expected [Greene,Shapere,Vafa,Yau]
- On the other hand, type IIB + non-trivial $\boldsymbol{\tau}$ = F-theory [Vafa]

Can we incorporate in some meaningful way holography in F-theory?

Holography



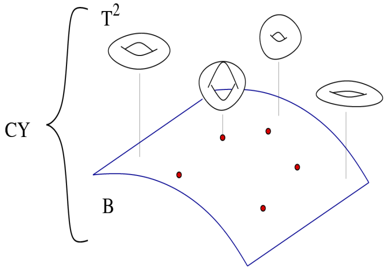
$$\mathbf{ds^2_{10d/11d} = e^{\Delta} ds^2(AdS_p) + ds^2(M_q)}$$ with fluxes on $\mathbf{M_q}$ and τ=constant

F-theory

$$\mathbf{ds^2_{10d} = ds^2(M) + ds^2(B)} \text{ and non-trivial } \tau$$

$$\Downarrow$$

$$\mathbf{ds^2_{12d} = ds^2(M) + \underbrace{ds^2(B) + \frac{1}{\tau_2}\left((dx + \tau_1 dy)^2 + \tau_2^2 dy^2\right)}_{\text{elliptic Calabi-Yau}}}$$



$\mathbf{x, y}$ are coordinates on an auxiliary $\mathbf{T^2}$, with complex structure $\boldsymbol{\tau = \tau_1 + i\tau_2}$

Holography meets F-theory

Holographer: likes smooth spaces, prefers explicit metrics, enjoys checking Killing spinors....

F-theorist: lives in a world with singularities, does not need explicit metrics, relies on algebraic geometry....

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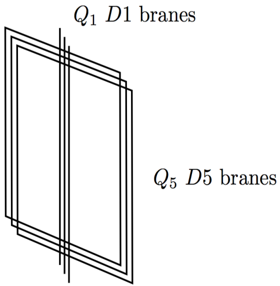
AdS$_3$ from D1D5 systems - 1

AdS$_3$ $\times$ $\mathbf{S^3}$ $\times$ $\mathbf{T^4}$ background in type IIB

[Maldacena]: Multiple D1D5 intersection in type IIB $\rightarrow$ AdS$_3$ $\times$ $\mathbf{S^3}$ $\times$ $\mathbf{T^4}$ is dual to "1+1 dimensional $\mathbf{(4,4)}$ SCFT describing the Higgs branch of the D1+D5"

- $\mathbf{Q_1}$ D1 strings inside $\mathbf{Q_5}$ D5 branes: brane system solution straightforward to write
- In the near horizon AdS$_3$ $\times$ $\mathbf{S^3}$
- Preserves $\mathbf{(4,4)}$ supersymmetry
- $\mathbf{SU(2)_R} \subset \mathbf{SO(4)}$ isometry of $\mathbf{S^3}$ $\rightarrow$ *small* superconformal algebra



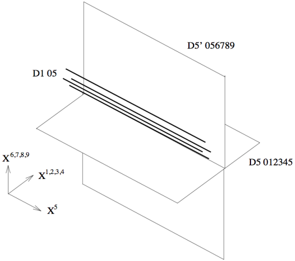
Holographic central charge computed using Brown-Henneaux $\mathbf{c_{sugra}} = \dfrac{\mathbf{3R_{AdS_3}}}{\mathbf{2G_N^{(3)}}}$,

where $\dfrac{\mathbf{1}}{\mathbf{G_N^{(3)}}} \propto \mathrm{vol}(\mathbf{S^3} \times \mathbf{T^4}) \quad \longrightarrow \quad \mathbf{c = 6Q_1Q_5}$

AdS_3 from D1D5 systems - 2

$AdS_3 \times \mathbf{S^3} \times \mathbf{S^3} \times \mathbf{S^1}$ background in type IIB [Boonstra,Peeters,Skenderis]

- $\mathbf{Q_1}$ D1 strings along the intersection of $\mathbf{Q_5}$ D5 branes and $\mathbf{Q'_5}$ D5 branes
- Preserves again $\mathbf{(4,4)}$ supersymmetry
- $\mathbf{SU(2)_R \times SU(2)'_R \subset SO(4) \times SO(4)'}$ isometry of $\mathbf{S^3 \times S^3} \rightarrow$ *large* superconformal algebra



Dual 2d SCFT proposed by [Tong]: reproduces microscopically the holographic central charge $\mathbf{c = 2Q_1 \frac{Q_5 Q'_5}{Q_5 + Q'_5}}$

AdS_3 from wrapped D3 branes

- Consider a space-time $\mathbb{R}^{1,1} \times \mathbf{M_8}$ (with $\mathbf{M_8}$ non-compact). Wrapping $\mathbf{N}$ D3 branes on a two-cycle $\Sigma \subset \mathbf{M_8}$ gives rise to strings transverse to $\mathbf{M_8}$
- Choosing $\mathbf{M_8}$ and Σ appropriately, different fractions of supersymmetry may be preserved
- There is a 2d supersymmetric field theory on the strings, which may be thought of as the compactification on Σ of the 4d field theory living on the D3 branes ($\mathcal{N} = 4$ SYM in the simplest case)
- Question: is this 2d field theory a 2d SCFT (in the IR)?
- If there exists a supersymmetric background comprising AdS_3 which can be thought of as the near horizon limit of this brane configuration, then according to the AdS/CFT correspondence the answer is "yes"
- One can then turn attention directly to supersymmetric AdS_3 solutions, performing a general analysis ("classification" of solutions) [Kim]

$\mathrm{AdS_3/CFT_2}$ from wrapped D3 branes

Input

Metric: $\mathbf{ds^2 = e^{2\Delta}ds^2(\mathrm{AdS_3}) + ds^2(M_7)}$

Fluxes: $\mathbf{F_5 = (1+*)\mathrm{Vol}(\mathrm{AdS_3}) \wedge F^{(2)}}$ and all other vanishing

Susy: $\mathbf{\nabla_M \epsilon + \frac{i}{192}\Gamma^{P_1 \ldots P_4} F_{MP_1 \ldots P_4}\epsilon = 0}$ (requirement of at least $\mathbf{(0,2)}$)

Output

Metric: $\mathbf{ds^2(M_7) = e^{2\Delta}ds^2(d\psi + \rho)^2 + e^{-2\Delta}ds^2(M_6)}$ with $\mathbf{M_6}$ a Kähler space, with Kähler two-form $\mathbf{J}$ and Ricci form $\mathbf{\mathcal{R} = \frac{1}{2}d\rho}$

$\mathbf{U(1) \to M_7 \to M_6}$: $\mathbf{U(1)}$ corresponds to R-symmetry of dual $\mathbf{(0,2)}$ SCFT

Flux: $\mathbf{F^{(2)} = \frac{1}{2}J - \frac{1}{4}d(e^{2\Delta}(d\psi + \rho))}$

AdS_3/CFT_2 from wrapped D3 branes

Input

Metric: $ds^2 = e^{2\Delta} ds^2(\mathrm{AdS}_3) + ds^2(M_7)$

Fluxes: $F_5 = (1+*)\mathrm{Vol}(\mathrm{AdS}_3) \wedge F^{(2)}$ and all other vanishing

Susy: $\nabla_M \epsilon + \frac{i}{192} \Gamma^{P_1 \dots P_4} F_{MP_1 \dots P_4} \epsilon = 0$ (requirement of at least $(0,2)$)

Output

Metric: $ds^2(M_7) = e^{2\Delta} ds^2 (d\psi + \rho)^2 + e^{-2\Delta} ds^2(M_6)$ with M_6 a Kähler space, with Kähler two-form J and Ricci form $\mathcal{R} = \frac{1}{2} d\rho$

$U(1) \to M_7 \to M_6$: $U(1)$ corresponds to R-symmetry of dual $(0,2)$ SCFT

Flux: $F^{(2)} = \frac{1}{2} J - \frac{1}{4} d(e^{2\Delta}(d\psi + \rho))$

Moreover, there is a non-trivial constraint on the Kähler metric on M_6!

$$\Box_6 R_6 - \frac{1}{2} R_6^2 + R_{6\mu\nu} R_6^{\mu\nu} = 0$$

AdS_3/CFT_2 from wrapped D3 branes: examples

- 2d $(0,2)$ theories arising from twisted compactification on Σ_g of $\mathcal{N}=4$ SYM, where Σ_g is a genus g Riemann surface inside a CY four-fold $\mathcal{L}_1 \oplus \mathcal{L}_2 \oplus \mathcal{L}_3 \to M_8 \to \Sigma_g$ [Benini,Bobev]

- Twisting $D_\mu = \partial_\mu + is\tilde{\omega}_\mu - A_\mu$ with $A_\mu = a_1 A^1_\mu + a_2 A^2_\mu + a_1 A^2_\mu$ where A^I_μ are background fields for $U(1)_1 \times U(1)_2 \times U(1)_3 \subset SO(6)$

- Central charges c_L, c_R of $(0,2)$ SCFTs in a nutshell

$$\boxed{\begin{array}{c} \nabla^\mu J^R_\mu = \frac{k_R}{8\pi} F^R_{\mu\nu}\epsilon^{\mu\nu} \qquad \nabla_\mu T^{\mu\nu} = \frac{c_R - c_L}{96\pi} g^{\nu\alpha}\epsilon^{\mu\rho}\partial_\mu\partial_\beta\Gamma^\beta_{\alpha\rho} \\ c_R = 3k_R \quad U(1)_R \ \text{fixed by} \ \mathrm{Tr}U(1)_R U(1)_I = 0 \ \leftrightarrow \ c\text{-extremization} \end{array}}$$

- Trial R-symmetry $U(1)_R = \epsilon_1 U(1)_1 + \epsilon_2 U(1)_2 + \epsilon_3 U(1)_3$

- c-extremization determines $\epsilon_I = \epsilon_I(a_J) \ \Rightarrow \ c_R = c_L \propto N^2 a_1 a_2 a_3$

AdS_3/CFT_2 from wrapped D3 branes: examples

- AdS$_3$ solutions: the internal seven-dimensional manifold is topologically $S^5 \rightarrow M_7 \rightarrow \Sigma_g$. Metric and $F^{(2)}$ depend explicitly on a_1, a_2, a_3

- Holographic central charge $c_{grav} = c_L = c_R \propto N^2 a_1 a_2 a_3$ ✓

- Generalisation by [Benini,Bobev,Crichigno]: 2d $(0,2)$ theories arising from twisted compactification on Σ_g of 4d $\mathcal{N} = 1$ quiver gauge theories arising from N D3 branes transverse to the Calabi-Yau singularities $Y^{p,q}$

- Supergravity solutions again contained in the [Kim] equations: the internal seven-dimensional manifold is topologically $Y^{p,q} \rightarrow M_7 \rightarrow \Sigma_g$

- The holographic central charge c_{grav} can be computed explicitly in terms of twisting parameters, p, q and it matches exactly the field theory central charge, computed using c-extremisation ✓

- Further AdS$_3$ solutions constructed in a series of papers by [Gauntlett et al]

AdS$_3$ in F-theory

[Couzens,Lawrie,DM,Schäfer-Nameki,Wong]

1. Characterise supersymmetric AdS$_3$ solutions of F-theory (generalising [Kim])
2. Find solutions and dual SCFTs (generalising [Benini,Bobev,(Crichigno)])

Input

Metric: $\mathbf{ds^2 = e^{2\Delta}ds^2(\mathrm{AdS}_3) + ds^2(M_7)}$

Fluxes: $\mathbf{F_5 = (1+*)\mathrm{Vol}(\mathrm{AdS}_3) \wedge F^{(2)}}$, arbitrary τ (three-form flux $\mathbf{G = 0}$)

Output

Metric: $\mathbf{ds^2(M_7) = e^{2\Delta}ds^2(d\psi + \rho)^2 + e^{-2\Delta}ds^2(M_6)}$ with $\mathbf{M_6}$ a Kähler space

"F-theory metric" $\quad \mathbf{ds^2(M_8) = \frac{1}{\tau_2}\left((dx + \tau_1 dy)^2 + \tau_2^2 dy^2\right) + ds^2(M_6)}$

$\mathbf{M_8}$ is an auxiliary Kähler elliptic fibration (not CY!) obeying the constraint

$$\mathbf{\Box_8 R_8 - \frac{1}{2}R_8^2 + R_{8ij}R_8^{ij} = 0}$$

All $(0,4)$ AdS$_3$ solutions of F-theory

- Requiring more susy gives further constraints on the geometry: e.g. $(2,2)$ and $(0,4)$ $\leftarrow$ focus on this

- It turns out that $(0,4)$ is very constrained: $\Delta =$constant, $\mathbf{M_8} = \mathbf{S^2} \times \mathbf{Y}$, where $\mathbf{Y}$ is an elliptic Calabi-Yau three-fold

- The "physical" type IIB metric is (locally)

$$\begin{array}{c} \mathbb{E}_\tau \\ \downarrow \\ \mathbf{AdS_3} \times \mathbf{S^3} \times \mathbf{B_4} \end{array}$$

  with $\mathbf{B_4}$ Kähler and $\mathbb{E}_\tau \to \mathbf{Y} \to \mathbf{B_4}$ is an elliptic Calabi-Yau three-fold

- An explicit analysis of the Killing spinors shows that these transform as $\mathbf{2}$ of $\mathbf{SU(2)_r} \subset \mathbf{SO(4)} = \mathbf{SU(2)_r} \times \mathbf{SU(2)_\ell}$ isometry of $\mathbf{S^3}$

  $\rightarrow$ $\mathbf{SU(2)_r} = \mathbf{R}$-symmetry of a *small* $(0,4)$ superconformal algebra

  $\rightarrow$ A quotient by $\Gamma \subset \mathbf{SU(2)_\ell}$ preserves supersymmetry

- Most general $(0,4)$ solution is AdS$_3 \times \mathbf{S^3}/\Gamma \times \mathbf{B_4}$, with flux $\mathbf{F^{(2)}} = \mathbf{J_4}$

Central charges of dual 2d $(0, 4)$ SCFTs

- We can compute[1] the holographic central charge of the dual 2d $(0, 4)$ SCFTs using the Brown-Henneaux formula $c_{\text{sugra}} = \frac{3R_{\text{AdS}_3}}{2G_N^{(3)}}$

$$c_{\text{sugra}}^{\text{IIB}} = N^2 \frac{3\,\mathrm{vol}(S^3/\mathbb{Z}_M)\,\mathrm{vol}(B_4) 32\pi^2}{\mathrm{vol}(S^3/\mathbb{Z}_M)^2} = 6N^2 M\,\mathrm{vol}(B_4)$$

- N is an integer that arises from quantization of F_5 flux through five-cycles $S^3/\mathbb{Z}_M \times \Sigma_2 \subset S^3/\mathbb{Z}_M \times B_4$, interpreted as number of D3 branes
- $c_{\text{sugra}}^{\text{IIB}}$ gives the leading order in N to c_L^{IIB}, c_R^{IIB} which are a priori different. Sub-leading corrections can be computed by considering anomalous couplings of 7-branes
- Delicate point: the Kähler metric on B_4 solving the supersymmetry equations is singular! [Greene,Shapere,Vafa,Yau]
- What do we mean by $\mathrm{vol}(B_4)$?

[1]We now take $\Gamma = \mathbb{Z}_M$.

Deal?

Making sense of the volume

- Quite trivially $\mathrm{vol}(\mathbf{B_4}) = \frac{1}{2}\int_{\mathbf{B_4}} \mathbf{J_4} \wedge \mathbf{J_4}$

- Recall that from an F-theory point of view $\mathbf{B_4}$ is the base of an elliptically fibered Calabi-Yau three-fold: $\mathbb{E}_\tau \to \mathbf{Y} \to \mathbf{B_4}$

- For a wide class of $\mathbf{B_4}$, this defines algebraically a smooth and compact variety with $\mathbf{c_1(Y) = 0}$: there exists a non-singular Ricci-flat metric and a non-singular Kähler form $\mathbf{J_Y} = \mathbf{J_4^{smooth}} + \mathbf{k_0}\omega_0$:

$\mathbf{J_4^{smooth}} = \mathbf{k_a}\omega_\mathbf{a}$, $\omega_\mathbf{a} \in \mathbf{H^2}(\mathbf{B_4}, \mathbb{Z})$ and ω_0 is Poincaré dual to the section σ of the elliptic fibration $\mathbf{B_4}$, with $\sigma \cdot (\sigma + \mathbf{c_1}(\mathbf{B_4})) = 0$

$$\mathrm{vol}(\mathbf{B_4}) = \frac{1}{2}\int_{\mathbf{Y}} \omega_0 \wedge \mathbf{J_4^{smooth}} \wedge \mathbf{J_4^{smooth}}$$

$$= \frac{1}{2}\int_{\mathbf{B_4}} \mathbf{J_4^{smooth}} \wedge \mathbf{J_4^{smooth}} = \frac{1}{2}\int_{\mathbf{C}} \mathbf{J_4^{smooth}} = \frac{1}{2}\mathbf{C}\cdot\mathbf{C}$$

The central charge at leading order in $\mathbf{N}$

- $\mathbf{C}$ is a curve in $\mathbf{B_4}$, Poincaré dual to $\mathbf{J_4^{smooth}}$: $\mathbf{N}$ D3 branes wrapped on $\mathbf{C}$
- In the singular limit $\mathbf{J_4^{smooth}} \to \mathbf{J_4}$ but the volume above is a topological invariant and so it is robust in this limit

$$\mathbf{c^{IIB}_{sugra} = 6N^2M\,\mathrm{vol}(B_4) = 6N^2M\tfrac{1}{2}C \cdot C = 3N^2MC \cdot C}$$

- We would like to compare this result (and the sub-leading corrections, that we can compute) to some microscopic computation of the central charges

1. We can think about the field theory living on the brane set up in type IIB/F-theory
2. We can perform the F/M duality and then think about the brane set up in M-theory

Physical interpretation of $\mathbf{M}$

- $\mathbf{N}$ is the number of D3 branes wrapped on $\mathbf{C} \subset \mathbf{B_4}$ ($\mathbf{N} = \mathbf{M} = \mathbf{1}$ discussed in [Dabholkar,Gomes,Murthy,Sen] and [Haghighat,Murthy,Vafa,Vandoren] from the point of view of effective six dimensional supergravity)
- $\mathbf{S^3}/\mathbb{Z}_\mathbf{M}$ can be thought of as the "near horizon" limit of a Taub-NUT metric
- Indeed we can construct an explicit (smeared) brane solution of the type $\mathbb{R}^{\mathbf{1,1}} \times \mathbf{TN_M} \times \mathbf{B_4}$, with $\mathbf{N}$ D3 branes with world-volume $\mathbb{R}^{\mathbf{1,1}} \times \mathbf{C}$, with near horizon $\mathrm{AdS}_3 \times \mathbf{S^3}/\mathbb{Z}_\mathbf{M} \times \mathbf{B_4}$
- There are $\mathbf{M}$ Kaluza-Klein monopoles with world-volume $\mathbb{R}^{\mathbf{1,1}} \times \mathbf{B_4}$: 2d UV field theory rather complicated to figure out... $\longrightarrow$ for generic $\mathbf{M}$ we will make progress by studying the M-theory dual realization in terms of **M5** branes, and then use anomaly inflow on these [Harvey,Minasian,Moore]
- However, $\mathbf{M} = \mathbf{1}$ is a special case: solution is also (clearly!) the near horizon of $\mathbf{N}$ D3 branes wrapped $\mathbf{C}$, with transverse $\mathbb{R}^\mathbf{4}$ (again, we can construct the explicit pre-near horizon solution)

Central charges from "D3 strings" anomaly inflow

- For $\mathbf{M = 1}$ we can consider the (UV) 2d field theory obtained from wrapping $\mathbf{N}$ D3 branes on a curve $\mathbf{C} \subset \mathbf{B_4}$

- Anomalies of these theories computed from anomaly polynomial of so-called self-dual strings [Shimizu,Tachikawa] (6d point of view)

  Global symmetry: $\underbrace{\mathbf{SO(4)_T = SU(2)_R \times SU(2)_L}}_{\text{transverse rotation to the strings in 6d}} \times \underbrace{\mathbf{SU(2)_I}}_{\text{R-symmetry of 6d (1,0) theory}}$

$$\mathbf{I_4 = k_R c_2(R) + k_L c_2(L) + k_I c_2(I) - \tfrac{1}{24}(c_L - c_R) p_1(T)}$$

$$\mathbf{k_R = \tfrac{1}{2} N^2 C \cdot C + \tfrac{1}{2} N c_1(B_4) \cdot C}$$

$$\mathbf{k_L = -\tfrac{1}{2} N^2 C \cdot C + \tfrac{1}{2} N c_1(B_4) \cdot C}$$

- Identifying $\mathbf{SU(2)_R = SU(2)_r}$ as the IR $\mathbf{R}$-symmetry we obtain $\mathbf{c_R = 6k_R = 3N^2 C \cdot C} + \mathcal{O}(\mathbf{N})$, in agreement with the holographic $\mathbf{c}^{\mathrm{IIB}}_{\mathrm{sugra}}$

- We can match also the sub-leading terms in $\mathbf{N}$, including agreement for $\mathbf{N = 1}$ with the results of [Lawrie,Schäfer-Nameki,Weigand]

M-theory duals

- The statement that F-theory on an elliptically fibered CY is dual to M-theory is familiar..
- However, if one insists in following through this duality explicitly at the level of supergravity solutions, making sure that supersymmetry is preserved at every step, one has to be careful
- In order to perform T-duality from IIB to IIA, there must be a $\mathbf{U(1)}$ symmetry of the solution, under which the Killing spinors are neutral (a non-$\mathbf{R}$ symmetry): in our case $\mathbf{U(1)} \subset \mathbf{SU(2)}_\ell \subset \mathbf{SO(4)}$ rotating $\mathbf{S}^3$
- After lifting to 11d: the metric is $\mathrm{AdS}_3 \times \mathbf{S}^2 \times \mathbf{Y}$ with four-form flux $\mathbf{G} = \mathrm{vol}(\mathbf{S}^2) \wedge \mathbf{J}_\mathbf{Y}$
- After the duality, we can de-singularise the elliptic Calabi-Yau $\mathbf{Y}$, replacing the metric and Kähler form with their non-singular versions!
- Quantization of the $\mathbf{G_4}$ flux reveals that $\mathbf{M}$ is the flux through $\mathbf{S}^2 \times \mathbb{E}_\tau$

M-theory duals

- $\mathbf{J} \propto \mathbf{N}\mathbf{J}_4^{\text{smooth}} + \mathbf{M}\omega_0 \longrightarrow$ **N** M5 branes wrapped on a divisor $\hat{\mathbf{C}}$ (dual to $\mathbf{J}_4^{\text{smooth}}$) plus **M** M5 branes wrapped on the base $\mathbf{B_4}$ (dual to ω_0)

- Equivalently, can think of **N** M5 branes wrapped on the divisor $\mathbf{P} = \hat{\mathbf{C}} + \frac{\mathbf{M}}{\mathbf{N}}\mathbf{B_4}$. This is an ample divisor

- Recalling the type IIB brane picture, for any $\mathbf{M} > \mathbf{1}$ this configuration is the expected configuration dual to **N** D3 branes and **M** KK monopoles (see also [Bena,Diaconescu,Florea])

- However, for $\mathbf{M} = \mathbf{1}$ this is a bit surprising, if we think about the AdS_3 solution as the near horizon limit of **N** D3 branes and *no KK monopoles*

- The expected dual M5 brane configuration involves **N** M5 branes wrapped on $\hat{\mathbf{C}}$ only – thus formally, $\mathbf{M} = \mathbf{0}$ in M-theory

- The issue is that in this case the divisor $\mathbf{P} = \hat{\mathbf{C}}$ is not ample and there is not an AdS_3 solution in 11 dimensions!

Central charges from "M5 string" anomaly inflow

Anomaly polynomial of the world-volume theory on a stack of **N** M5-branes [Harvey]

$$\mathbf{I_8[N] = N I_8^{free}[1] + \frac{1}{24}(N^3 - N) p_2(\mathcal{N})}$$

where

$$\mathbf{I_8^{free}[1] = \frac{1}{48}\left[p_2(\mathcal{N}) - p_2(TW) + \frac{1}{4}(p_1(TW) - p_1(\mathcal{N}))^2\right]}$$

TW is 6d tangent bundle of the worldvolume of the M5 brane

$\mathcal{N}$ is the normal (**SO(5)** R-symmetry) bundle associated to the transverse directions to the M5-brane worldvolume in 11 dimensions

Anomaly polynomial $\mathbf{I_4}$ for the world-volume theory of strings from **N** M5-brane wrapping a surface **P** inside a CY three-fold **Y** determined by integrating $\mathbf{I_8}$ over **P**

$$\mathbf{TW = TP \oplus TW_2} \qquad\qquad \mathcal{N} = \mathcal{N}_{\mathbf{P/M}} \oplus \mathcal{N}_{\mathbf{3}}$$

$\mathbf{W_2}$ is the world-volume of the strings, $\mathcal{N}_{\mathbf{P/M}}$ is the normal bundle of **P** inside **Y** and $\mathcal{N}_{\mathbf{3}}$ is the bundle associated to the $\mathbf{SO(3)_T}$ global symmetry from the rotations of the 3 transverse directions to the string in $\mathbf{5d = 11d - dim(Y)}$

Central charges from "M5 string" anomaly inflow

The result is [Harvey,Minasian,Moore]

$$\mathbf{I_4^{int}[N] = NI_4^{free}[1] + \frac{1}{24}(N^3 - N)P^3 p_1(\mathcal{N}_3)}$$

$$\mathbf{I_4^{free}[1] = \frac{1}{48}\left[2P^3 p_1(\mathcal{N}_3) + c_2(Y)\cdot P(p_1(W_2) + p_1(\mathcal{N}_3))\right]}$$

$$\mathbf{k_3 = \frac{1}{6}N^3P^3 + \frac{1}{12}Nc_2(Y)\cdot P}$$

This is valid for any four-cycle $\mathbf{P}$, therefore $\mathbf{P}$ can be an ample divisor or not!

If $\mathbf{P}$ is ample there exist an AdS_3 11d supegravity solution [Maldacena,Strominger,Witten] $\Rightarrow \mathbf{SO(3)_T = SO(3)_{isometry} = SU(2)_r}$, namely the IR $\mathbf{R}$-symmetry $\Rightarrow \mathbf{c_R = 6k_r = 6k_3}$

Denoting $\mathbf{h = h^{1,1}(B_4)}$, evaluating this for our ample divisor $\mathbf{P}$ gives

$$\mathbf{c_R = 3N^2MC\cdot C + 3N(2 - M^2)c_1(B_4)\cdot C + M^3(10 - h) + M(h - 4)}$$

For any $\mathbf{M \geq 1}$ (so that $\mathbf{P}$ is ample) we get also agreement with the holographic M-theory central charge $\mathbf{c_R^{11}}$ [Kraus,Larsen]

$\mathbf{M = 0}$ case: $\mathbf{P}$ not ample

- When $\mathbf{P} = \hat{\mathbf{C}}$ is not ample there is no AdS_3 11d supegravity solution: $\mathbf{SO(3)_T = SO(3)_{\text{isometry}} \neq SU(2)_r \Rightarrow c_R = 6k_r \neq 6k_3}$
- In the IR $\mathbf{SO(3)_T \to SU(2)_r \times SU(2)_F}$ [Witten], [Putrov,Song,Yan]

$$\mathbf{k_3 = k_r + k_F}$$

- We identify $\mathbf{SU(2)_F = SU(2)_L^{D3}} \Rightarrow$

$$\mathbf{k_r(M=0) = k_3(M=0) - k_L^{D3} = \tfrac{1}{2}N^2 C \cdot C + \frac{1}{2} N c_1(B_4) \cdot C = k_R^{D3}}$$

- Reproduce the F-theory results also from the M-theory UV microscopic point of view, although in this case there is no AdS_3 11d supergravity solution

Outlook

- Started to explore systematically holographic constructions in the context of F-theory, focusing on $\mathrm{AdS}_3/\mathrm{CFT}_2$
- Characterised the general local solutions preserving $\mathbf{(0,4)}$, $\mathbf{(2,2)}$, $\mathbf{(0,2)}$ (to appear)
- For a complete classification of AdS_3 solutions in type IIB/F-theory, one could in principle include complex (RR+NS) three-form $\mathbf{G}$ (in progress)
- For $\mathbf{(0,4)}$ we have obtained a "microscopic" understanding of the holographic constructions: this highlighted subtleties and challenges that need to be addressed in order to realise holography in F-theory
- We have constructed explicit $\mathbf{(0,2)}$ AdS_3 geometries and we are attempting to obtain a "microscopic" description of the dual SCFT's... (in progress)