Effective mass signatures in multiphoton pair production

Christian Kohlfürst

Helmholtz-Institut Jena & Friedrich-Schiller-Universität Jena

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CK, H. Gies and R. Alkofer, Phys. Rev. Lett. 112, 050402 (2014)
arxiv.org/abs/1310.7836;

CK, PhD thesis, arXiv:1512.06082;

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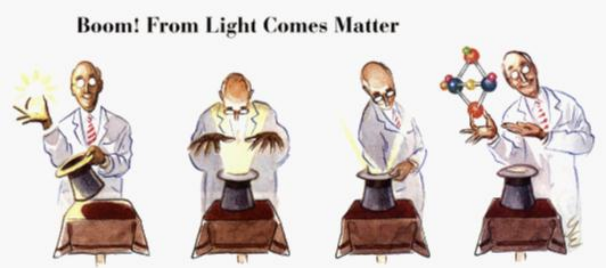
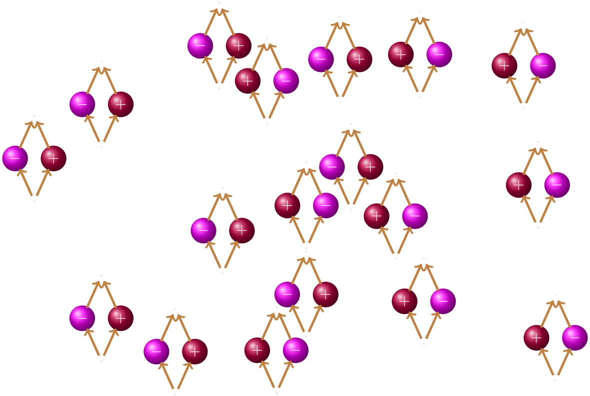
Why Pair Production?

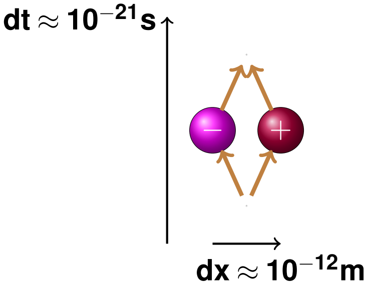


Cite: www.slac.stanford.edu/exp/e144

QED Vacuum



Electron-Positron Pair



- Vacuum fluctuations
- QED scale

F. Sauter: Z. Phys. 69(742), 1931

W. Heisenberg et al.: Z. Phys. 98(714), 1936

J. S. Schwinger: Phys. Rev. 82(664), 1951

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Multiphoton Pair Production

- Electric field E (photons γ) $\rightarrow$ charge separation
- Particles $\rightarrow$ measurable
- Critical field strength $E_{cr} \approx 10^{16} V/cm$
- Critical energy $\mathscr{E}_{e^+e^-} \approx 1 MeV$

E. Brezin and C. Itzykson: Phys. Rev. D 2, 1191 (1970)

V. S. Popov: Sov. Phys. JETP 35, 659 (1972)

Pair Production and Atom Physics

Electric Field: $E(t) = \varepsilon \, E_{cr} \cos(\omega t)$

	Atom Physics	Pair Production
Keldysh-Parameter	$\gamma_K = \frac{\omega\sqrt{2E_{IP}}}{eE}$	$\gamma_K = \frac{\omega}{eE}$
Absorption($\gamma_K \gg 1$)	$P \sim \left(\frac{eE}{2\omega\sqrt{2E_{IP}}}\right)^{2(E_{IP}/\omega)}$	$P \sim (\frac{eE}{2\omega})^{4/\omega}$
Tunneling($\gamma_K \ll 1$)	$P \sim \exp(-\frac{2}{3}\frac{2(E_{IP})^{3/2}}{eE})$	$P \sim \exp(-\pi\frac{E_{cr}}{eE})$

L. V. Keldysh: Sov. Phys. JETP 20(1307), 1965

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Motivation

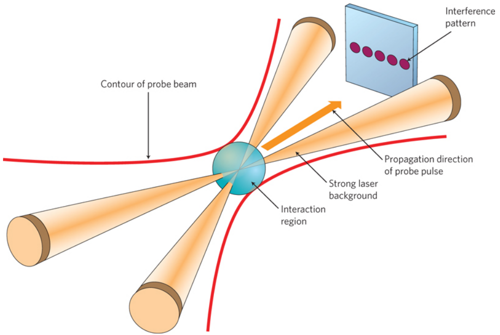
Features

- Non-equilibrium physics
- Non-perturbative effect
- Probing strong-field QED

Experiment

- SLAC
- Extreme Light Infrastructure (ELI)
- FAIR, HiBEF, XFEL

Sketch of an Experimental Setup



- Two colliding laser fields
- Electric field in an antinode of a standing-wave

M. Marklund: Nature Photonics 4, 72-74 2010

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Considerations

Goal

Describe $e^- e^+$ pair production

Requirement

- Electric background field
- Dynamics - field is rapidly oscillating
- Particle statistics
- Observables

Quantum Kinetic Theory

Equation of motion

$$\overline{D}_t \, \mathbb{w} = \overline{M} \, \mathbb{w} \tag{1}$$

Wigner components $\mathbb{w}$
Source and interaction matrix $\overline{M}$
Pseudo-differential operator D_t

Mean-Field approximation

$F_{\mu\nu} \approx \langle \hat{F}_{\mu\nu} \rangle \to$ classical background field

Observables

Combinations of $\mathbb{w} \to$ particle density $F(\mathbf{q})$

I. Bialynicki-Birula: Phys. Rev. D 44(1825), 1991

Quantum Kinetic Theory

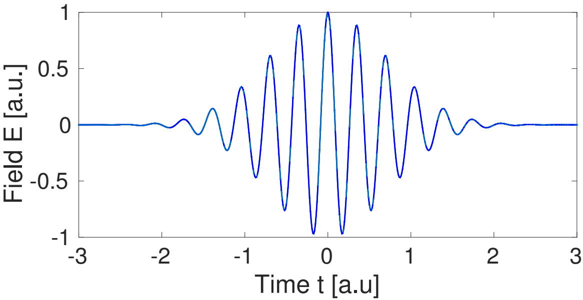
Positive Aspects

- Electric fields as input
- Time evolution
- Phase-space approach $\rightarrow$ particle spectrum

Challenges

- Beyond mean-field approximation
- Back-reaction and particle collisions
- Numerics

Model for the Field



- **Electric field**: $E(t) = \varepsilon \exp\left(-t^2/(2\tau^2)\right)\cos(\omega t)$
- Photon energy ω; Field strength ε; Pulse duration τ

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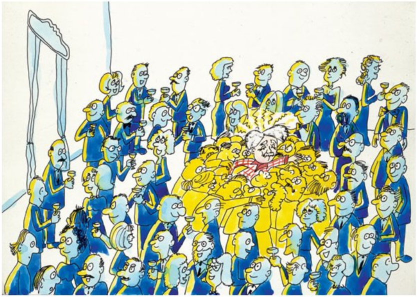
Effective Mass Model



- $e^- e^+$ interact with electric background field
- Particles behave as if they had a higher mass

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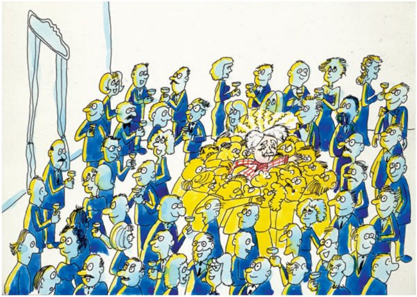
inspired by Prof. Miller, University College London

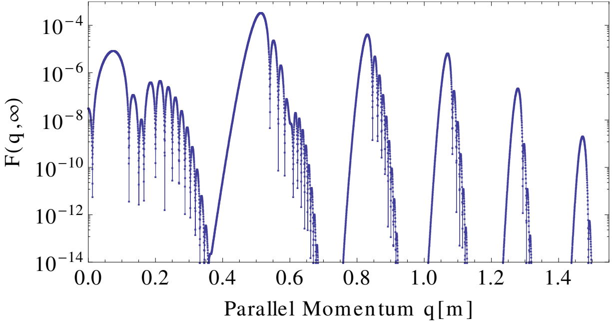
Effective Mass Model



- “Ionization” energy depends on laser field $E_{\text{Kin}} = n\omega - m_*$
- Effective mass $m_* = m\sqrt{1 + \varepsilon^2/(2\omega^2)}$
- Above-Threshold peak position $\left(\frac{n\omega}{2}\right)^2 = m_*^2 + q_n^2$

inspired by Prof. Miller, University College London

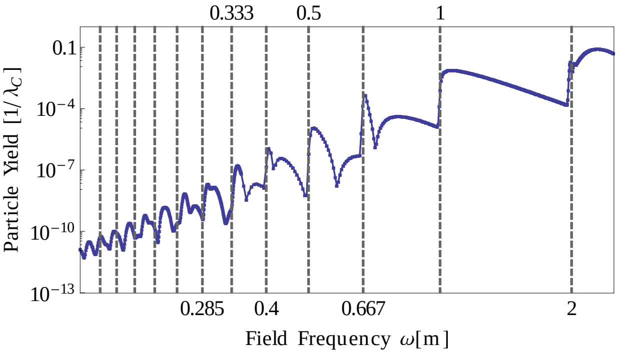
Particle Distribution



Parameters: $\tau = 300[1/m],\ \varepsilon = 0.2\,E_{cr},\ \omega = 0.3[m]$

- Above-Threshold peaks
- Peak position predictable via effective mass concept

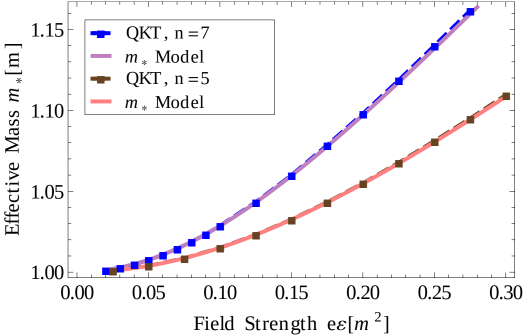
Particle Yield



Parameters: $\tau = 100[1/m]$, $\varepsilon = 0.1\, E_{cr}$

- Resonant at n-photon frequencies: $\omega_n = 2m_*/n$

Effective Mass

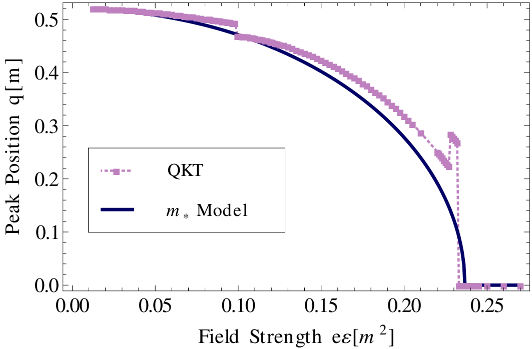


Parameters: $\tau = 100[1/m], n = 7$

Parameters: $\tau = 100[1/m], n = 5$

- Comparison: numerical simulation - m_* model

Channel Closing



Parameters: $\tau = 300[1/m]$ $\omega = 0.322[m]$

- Above-Threshold peak position varies with ε
- Resonance: Peak at threshold ($q = 0$)

Takeaways

Summary

- Phase-space formalism $\rightarrow$ pair production processes in the non-perturbative threshold domain
- Effective mass
- Channel closing

Outlook

- Back-reaction
- Beyond mean-field
- $3D$-simulation

Thank you!

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Quasi-probability Distribution

Wigner operator

$$\mathbb{W}(x,p) = \frac{1}{2}\int d^4y\ \mathrm{e}^{\mathrm{i}p\cdot y}\ U(A_\mu,x,y)\ \left[\bar{\psi}(x-\frac{y}{2}),\psi(x+\frac{y}{2})\right] \qquad (2)$$

- A_μ in mean field approach, $\mathbb{W}(x,p)$ is gauge invariant

Equal-time Approach

$$W(\mathbf{x},\mathbf{p},t) = \int \frac{dp_0}{2\pi}\mathbb{W}(x,p)$$

D. Vasak et al.: Annals of Physics 173(462-492), 1987

I. Bialynicki-Birula et al.: Phys. Rev. D 44(6), 1991

Quasi-probability Distribution

Wigner operator

$$\mathbb{W}(x,p) = \frac{1}{2} \int d^4y \; \mathrm{e}^{\mathrm{i}p \cdot y} \; U(A_\mu, x, y) \; \left[\bar{\psi}(x - \frac{y}{2}), \psi(x + \frac{y}{2}) \right] \qquad (2)$$

- A_μ in mean field approach, $\mathbb{W}(x,p)$ is gauge invariant

Equal-time Approach

$$W(\mathbf{x}, \mathbf{p}, t) = \int \frac{dp_0}{2\pi} \mathbb{W}(x,p) = \frac{1}{4} \left(\mathbb{s} + \mathrm{i}\gamma_5 \mathbb{p} + \gamma^\mu \mathbb{v}_\mu + \gamma^\mu \gamma_5 \mathbb{a}_\mu + \sigma^{\mu\nu} \mathbb{t}_{\mu\nu} \right)$$

D. Vasak et al.: Annals of Physics 173(462-492), 1987

I. Bialynicki-Birula et al.: Phys. Rev. D 44(6), 1991

Dirac-Heisenberg-Wigner Formalism

Equation of motion

$$\left(D_t \mathbb{1} + D_1 \overline{A}_1 + D_2 \overline{A}_2 + D_3 \overline{A}_3 + \Pi_1 \overline{B}_1 + \Pi_2 \overline{B}_2 + \Pi_3 \overline{B}_3\right) \mathbb{w} = \overline{M} \mathbb{w} \tag{3}$$

Wigner vector $\mathbb{w}$; Matrices $\mathbb{1}$, $\overline{A}_i$, $\overline{B}_i$ and $\overline{M}$

Pseudo-differential operators

$$D_t = \partial_t + e \int d\xi \, \mathbf{E}\left(\mathbf{x} + \mathrm{i}\xi \nabla_p, t\right) \cdot \nabla_p$$

$$\mathbf{D} = \nabla_x + e \int d\xi \, \mathbf{B}\left(\mathbf{x} + \mathrm{i}\xi \nabla_p, t\right) \times \nabla_p$$

$$\mathbf{\Pi} = \mathbf{p} - \mathrm{i}e \int d\xi \, \xi \, \mathbf{B}\left(\mathbf{x} + \mathrm{i}\xi \nabla_p, t\right) \times \nabla_p.$$

F. Hebenstreit: Dissertation, 2011

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Dirac-Heisenberg-Wigner Formalism

$$
\begin{aligned}
&D_t \mathbb{s} && -2\boldsymbol{\Pi}\cdot\mathbb{t}_1 &&= 0\\
&D_t \mathbb{p} && +2\boldsymbol{\Pi}\cdot\mathbb{t}_2 &&= -2\mathbb{a}_0\\
&D_t \mathbb{v}_0 + \mathbf{D}\cdot\mathbb{v} && &&= 0\\
&D_t \mathbb{a}_0 + \mathbf{D}\cdot\mathbb{a} && &&= 2\mathbb{p}\\
&D_t \mathbb{v} \;\; + \mathbf{D}\cdot\mathbb{v}_0 && +2\boldsymbol{\Pi}\times\mathbb{a} &&= -2\mathbb{t}_1\\
&D_t \mathbb{a} \;\; + \mathbf{D}\cdot\mathbb{a}_0 && +2\boldsymbol{\Pi}\times\mathbb{v} &&= 0\\
&D_t \mathbb{t}_1 + \mathbf{D}\times\mathbb{t}_2 && +2\boldsymbol{\Pi}\cdot\mathbb{s} &&= 2\mathbb{v}\\
&D_t \mathbb{t}_2 - \mathbf{D}\times\mathbb{t}_1 && -2\boldsymbol{\Pi}\cdot\mathbb{p} &&= 0
\end{aligned}
$$

F. Hebenstreit: Dissertation, 2011

Observables

Particle Density

$$N(t \to \infty) = \int n(\mathbf{p}, t \to \infty) \; d^3p, \tag{4}$$

$$n(\mathbf{p}, t) = \int d^3x \; \frac{\mathbb{s}\,(\mathbf{x}, \mathbf{p}, t) + \mathbf{p} \cdot \mathbb{v}\,(\mathbf{x}, \mathbf{p}, t)}{\omega(\mathbf{p})} \tag{5}$$

with one-particle energy $\omega(\mathbf{p}) = \sqrt{1 + \mathbf{p}^2}$

Charge Density

$$Q(t) = \int d^3x \; d^3p \; \mathbb{v}_0\,(\mathbf{x}, \mathbf{p}, t) \tag{6}$$

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Quantum Kinetic Theory

Ordinary differential equation

$$\begin{pmatrix} \dot{F} \\ \dot{G} \\ \dot{H} \end{pmatrix} = \begin{pmatrix} 0 & W & 0 \\ -W & 0 & -2\omega \\ 0 & 2\omega & 0 \end{pmatrix} \begin{pmatrix} F \\ G \\ H \end{pmatrix} + \begin{pmatrix} 0 \\ W \\ 0 \end{pmatrix} \tag{7}$$

$$W(\mathbf{q}, t) = \frac{eE(t)\,\varepsilon_\perp(\mathbf{q}, t)}{\omega^2(\mathbf{q}, t)}, \qquad \varepsilon_\perp^2(\mathbf{q}, t) = m^2 + \mathbf{q}{\perp}^2$$

$$\mathbf{p} = \mathbf{q} - e\mathbf{A}(t), \qquad \omega^2(\mathbf{q}, t) = \varepsilon_\perp^2(\mathbf{q}, t) + (q_z - eA(t))^2$$

- Homogeneous background field, $\mathbf{E} = E(t)\ \mathbf{e}_z$
- Particle density $F(t)$

S. A. Smolyansky et al. hep-ph/9712377 GSI-97-72, 1997

S. Schmidt et al.: Int.J.Mod.Phys. E7 709-722, 1998

J. C. R. Bloch et al.: Phys. Rev. D 60(116011), 1999

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Problem Statement

Quantum electrodynamic

Only QED-Lagrangian and Dirac equation is known

$$\mathscr{L} = \bar{\psi}(i\gamma^\mu D_\mu - m)\psi - \frac{1}{4}\mathscr{F}_{\mu\nu}\mathscr{F}^{\mu\nu} \quad (8)$$

$$i\gamma^\mu \partial_\mu \psi - m\psi = e\gamma_\mu A^\mu \psi \quad (9)$$

Covariant derivative $D_\mu = \partial_\mu + ieA_\mu$
Field strength tensor $\mathscr{F}_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$

Open question

How to obtain statistical quantity $F(\mathbf{q}, t)$ from particle description?

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Quantization

Vector Potential

Spatial-independent vector potential in one direction

$$A_\mu = (0, A(t)\mathbf{e_3}) \tag{10}$$

- Very strong fields $\rightarrow$ regarded as classical
- Mean field approximation(background field)

S. Schmidt et al.: Int. J. Mod. Phys. E 7(709), 1998

F. Hebenstreit: Dissertation, 2011

Quantization

Dirac Field

Fully quantized

$$\Psi \sim \chi^{+} a(q) + \chi^{-} b^{\dagger}(q) \tag{11}$$

Equation of motion

$$\left(\partial_t^2 + m^2 + (q_3 - eA(t))^2 + ieE(t)\right)\chi^{\pm} = 0 \tag{12}$$

Creation/Annihilation Operators

- Operators $a(q)$, $b^{\dagger}(q)$ hold information about particle statistics
- Fermions $\rightarrow$ anti-commutation relations

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Operator Transformation

Hamiltonian

- Non-vanishing off-diagonal elements
- Diagonalization by Bogoliubov transformation
- Switching to quasi-particle picture

$$A(\mathbf{q},t) = \alpha(\mathbf{q},t)\, a(\mathbf{q}) - \beta^*(\mathbf{q},t)\, b^\dagger(-\mathbf{q}) \tag{13a}$$

$$B^\dagger(-\mathbf{q},t) = \beta(\mathbf{q},t)\, a(\mathbf{q}) + \alpha^*(\mathbf{q},t)\, b^\dagger(-\mathbf{q}) \tag{13b}$$

Bogoliubov coefficients have to fulfill $|\alpha(\mathbf{q},t)|^2 + |\beta(\mathbf{q},t)|^2 = 1$

Particle Distribution

One-particle distribution function

$$F(\mathbf{q}, t) = \langle A^{\dagger}(\mathbf{q}, t) A(\mathbf{q}, t) \rangle \tag{14}$$

Fulfills equation of motion

$$\partial_t F(\mathbf{q}, t) = S(\mathbf{q}, t) \tag{15}$$

- Gives distribution in momentum space
- Time-dependent quantity
- Interpretation as electron/positron distribution for $t \to \pm\infty$ only