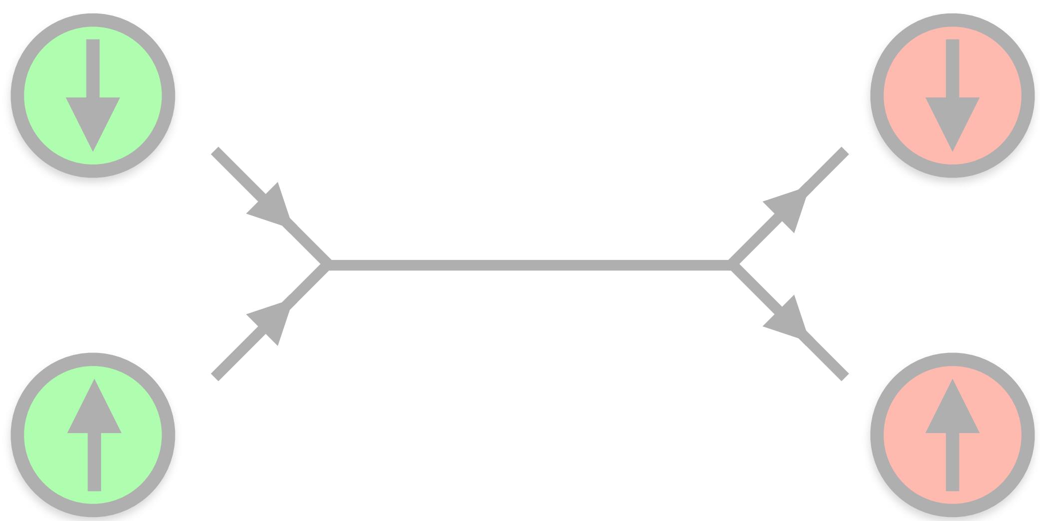
ETH zürich

Symplectic Fermi Liquid and its realization in cold atomic systems

A. Ramires, arXiv:1705.04080

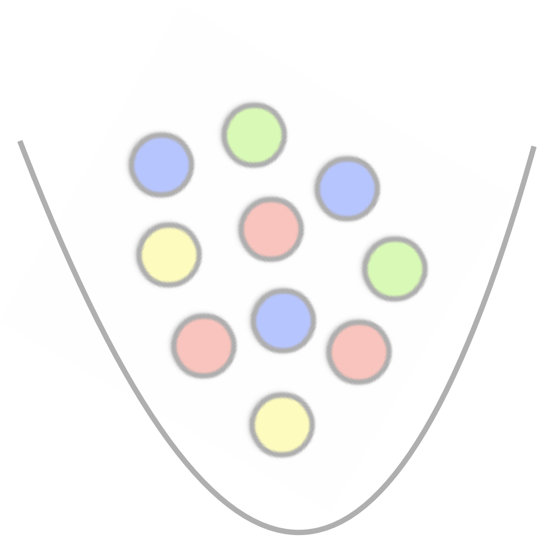


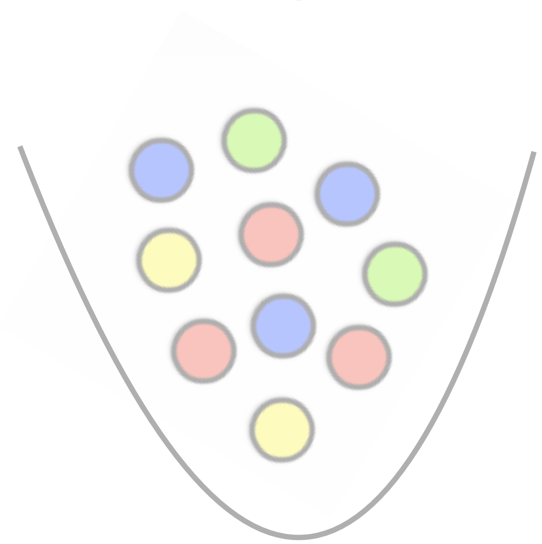
Aline Ramires

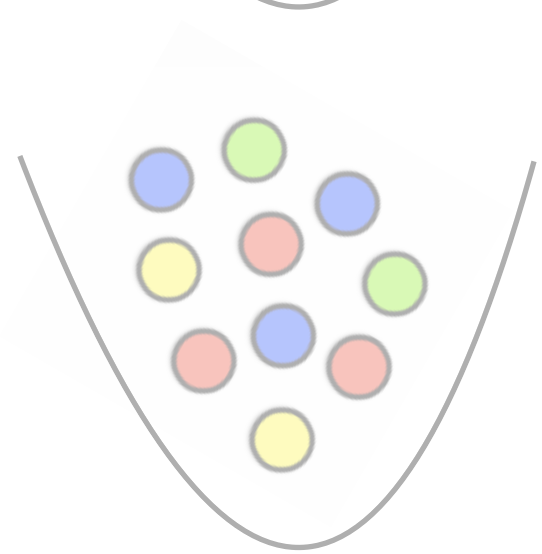
Institute for Theoretical Studies - ETH - Zürich

SPS Meeting - Geneva - August 24th 2017

Outline







- Why Enlarged Symmetries?

  SU(N) and SP(N)

- Why Symplectic Symmetry?

  Time-reversal symmetry

- SU(N) and SP(N) in cold atoms

- SP(N) FL Theory

- Final Remarks

  Experimental realization

  Further connections to theory

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Quantum Chromodynamics

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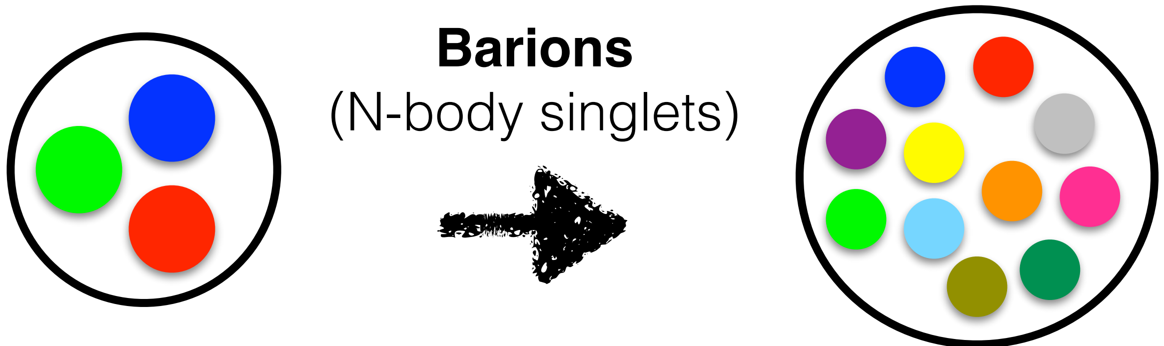

G. t'Hooft, Nucl Phys B 71, 461 (1973)
E. Witten, Nucl Phys B 160, 57 (1979)

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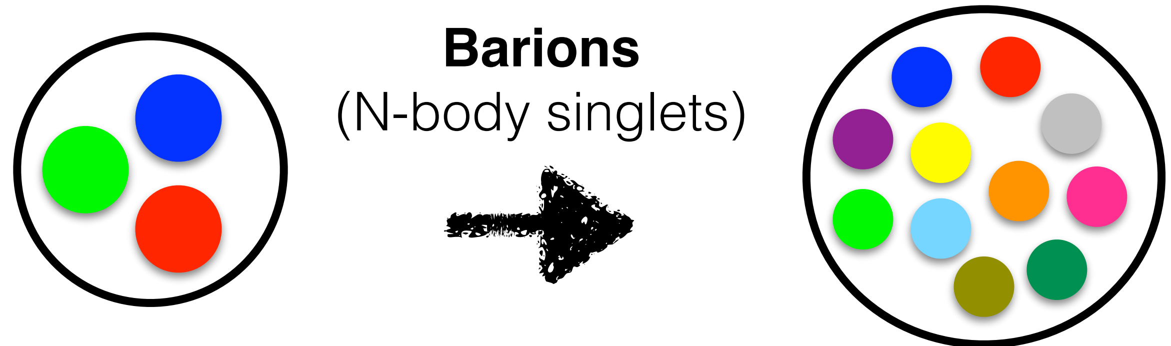

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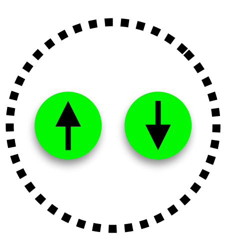
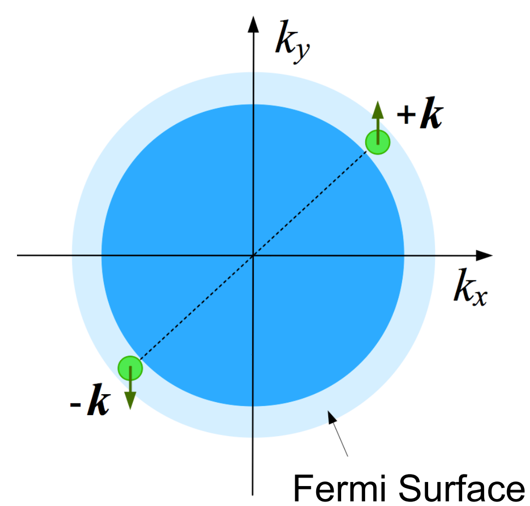
Condensed Matter



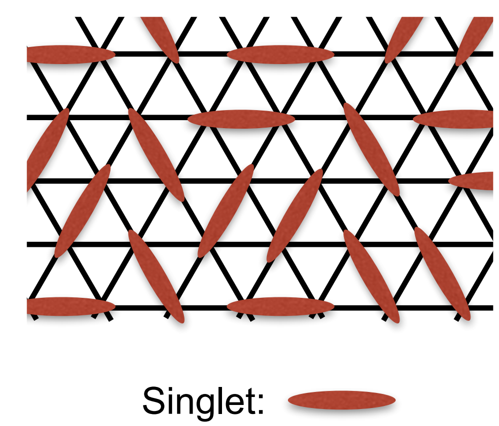
$$= \frac{|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle}{2}$$

$$SU(2) \Rightarrow SU(N)\ ?$$

Cooper Pairs



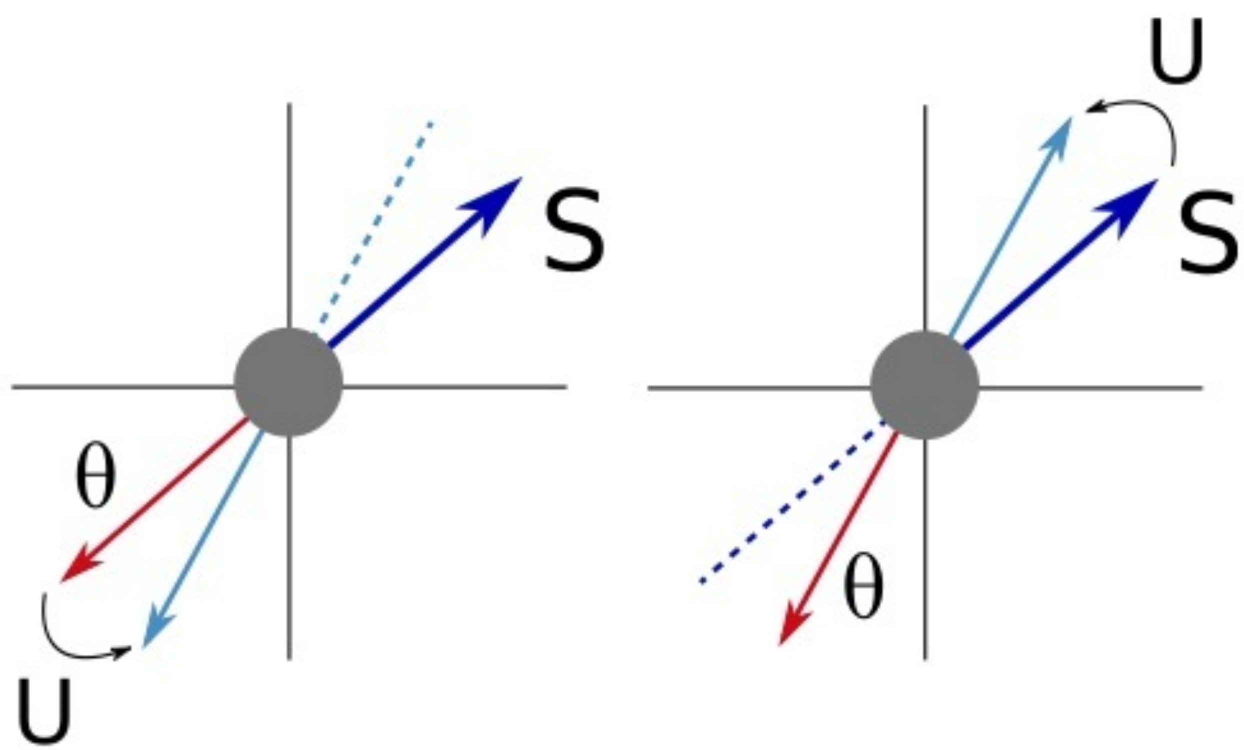
Valence Bonds



Distinct Large-N approaches capture different kinds of correlations and are appropriate to describe different physical phenomena.

Why SP(N) Symmetry?

Consistency requirement:
Enlarged symmetry and time-reversal symmetry (TRS): $$[\theta, U] = 0$$



Writing the time-reversal operator as:

$$\theta = \epsilon K$$

Symplectic condition

$$U \epsilon U^T = \epsilon$$

So the most appropriate generalisation to treat condensed matter systems is to SP(N), not SU(N) symmetry.

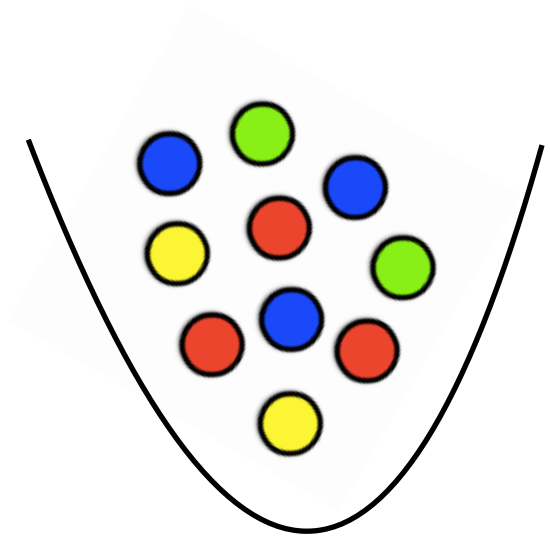
$$SU(2) \Rightarrow SP(N)$$

R. Flint et. al., Nature (2008)

Cold Atomic Systems

Spin-orbit coupling + Hyperfine coupling = Large Hyperfine Spin
Ex.: ^{40}K (f=9/2), ^{87}Sr (f=9/2), ^{41,43}Ca (f=7/2)

$$H = H_0 + H_I,$$



The non-interacting part:

$$H_0 = \int_{\mathbf{r}} \sum_{\alpha=-f}^{f} \Psi_\alpha^\dagger(\mathbf{r}) \left(-\frac{1}{2m}\nabla^2 + V(\mathbf{r}) \right) \Psi_\alpha(\mathbf{r}),$$

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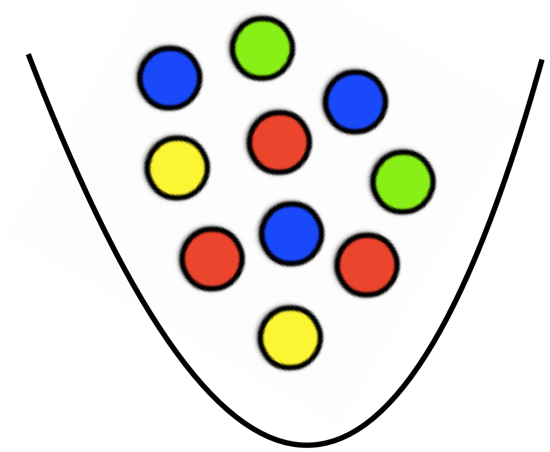

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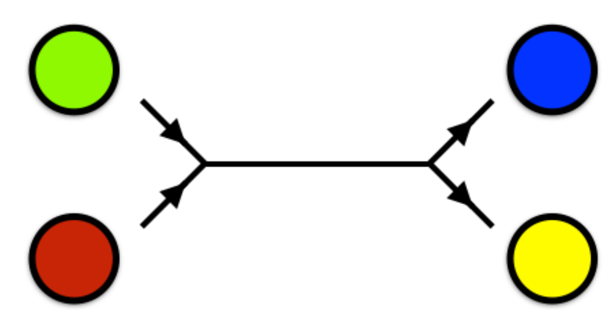
At ultra-low temperatures and in the low density limit, we can model interacting atoms with contact interactions.

$$H_I = \frac{1}{2}\int_{\mathbf{r}} \sum_{\alpha,\beta,\mu,\nu=-f}^{f} \Psi^\dagger_\beta(\mathbf{r})\Psi^\dagger_\alpha(\mathbf{r})\Gamma_{\alpha\beta;\mu\nu}\Psi_\mu(\mathbf{r})\Psi_\nu(\mathbf{r}),$$

$$\Gamma_{\alpha\beta;\mu\nu} = \sum_{F=0}^{2f} g_F \sum_{M=-F}^{F} \langle f\alpha, f\beta | FM\rangle\langle FM | f\mu, f\nu\rangle.$$



Total angular momentum conservation.

f: Hyperfine Spin (Total angular momentum of the atom)
F: Total angular momentum of the PAIR of atoms which are scattering

* Only the even-F channels contribute to scattering

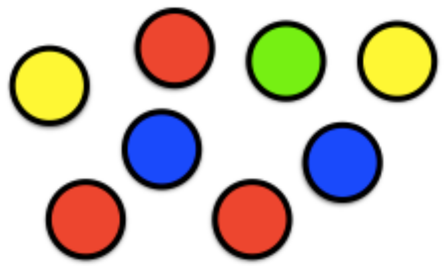
T. D. Lee, K. Huang and C. N. Yang, Phys. Rev. 106, 1135 (1957)

Cold Atoms and enlarged symmetries

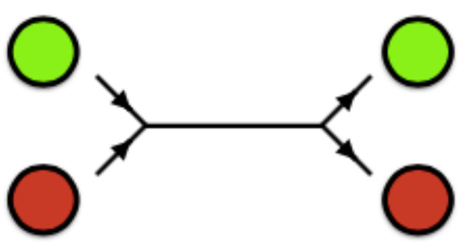
$$\Gamma_{\alpha\beta;\mu\nu} = \sum_{F=0}^{2f} g_F \sum_{M=-F}^{F} \langle f\alpha, f\beta|FM\rangle\langle FM|f\mu, f\nu\rangle.$$

SU(N) Symmetry

- All N colours are equivalent



- Scattering is colour-independent



- Number of particles in each colour is preserved

Realization: Alkaline-Earth atoms

$$L = 0, S = 0 \Rightarrow g_F = g$$

M. A. Cazalilla and A. M. Rey, Rep. Prog. Phys. 77, 124401 (2014)

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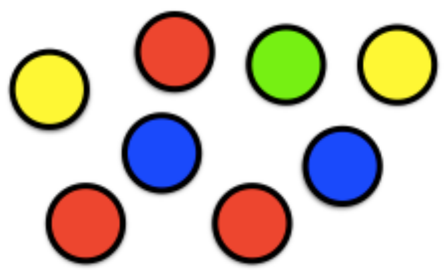

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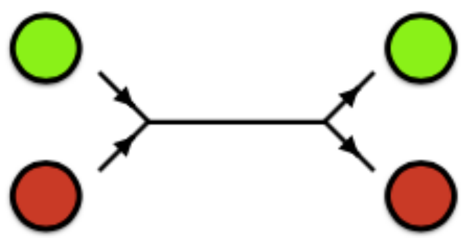

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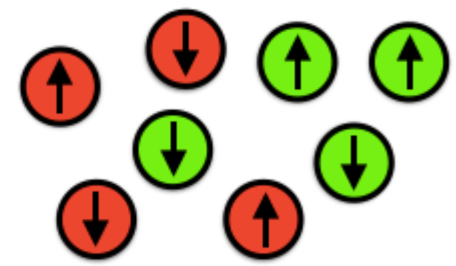
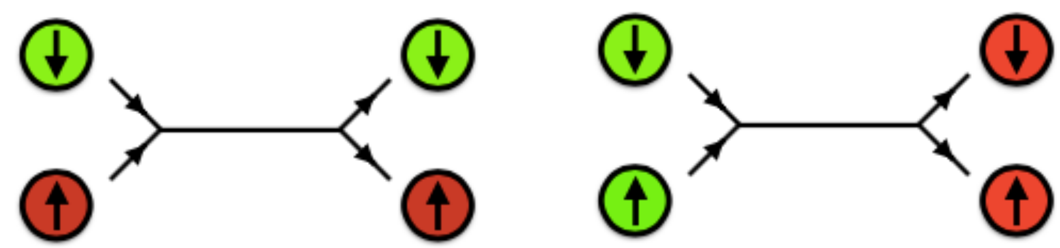
SP(N) Symmetry

- Colours come in pairs: (↑, ↓)

*Only for fermions (N = 2f+1 is even)



- Scattering can allow for colour transmutation in the F=0 channel



- Colour magnetisation is preserved

$$m_\alpha = n_\alpha - n_{-\alpha}$$

Realization under the condition

$$g_0 \neq g_{F>0} = g$$

* SP(4) naturally realized for f=3/2

M. A. Cazalilla and A. M. Rey, Rep. Prog. Phys. 77, 124401 (2014)

A. Ramires, arXiv:1705.04080

SP(N) Fermi Liquid Theory

As for the usual Fermi Liquid theory we can write:

$$\delta\epsilon_{\alpha\beta}(\mathbf{k}) = \sum_{\mathbf{k}'}\sum_{\mu,\nu} f_{\alpha\mu,\beta\nu}(\mathbf{k},\mathbf{k}')\delta n_{\nu\mu}(\mathbf{k}'),$$

Now we parametric the interaction function in terms of three FL parameters:

$$f_{\alpha\mu,\beta\nu}(\mathbf{k},\mathbf{k}') = \underbrace{f_s(\mathbf{k},\mathbf{k}')\delta_{\alpha\beta}\delta_{\mu\nu}}_{\text{Symmetric}} + \underbrace{f_a(\mathbf{k},\mathbf{k}')\sum_A \Gamma^A_{\alpha\beta}\Gamma^A_{\mu\nu}}_{\text{Anti-symmetric}} + \underbrace{f_\epsilon(\mathbf{k},\mathbf{k}')\epsilon_{\alpha\mu}\epsilon_{\beta\nu}}_{\text{Symplectic}}$$

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from which we are able to extract the renormalisation of measurable quantities:

Effective Mass

$$\frac{m^*}{m} = 1 + N\overline{F_s(\theta)\cos\theta},$$

Inverse Compressibility

$$\frac{u^{*2}}{u^2} = \frac{1+N\overline{F_s(\theta)}}{1+N\overline{F_s(\theta)\cos\theta}}.$$

Generalized Susceptibility

$$\chi_G = \frac{2\mu_B^2\rho^*(E_f)}{1+\overline{F_a(\theta)}}$$

where: $F_s(\theta) = \rho^*(E_f)\left(f_s(\theta) + \frac{1}{N}f_\epsilon(\theta)\right),\qquad F_a(\theta) = \rho^*(E_f)\left(f_a(\theta) - f_\epsilon(\theta)\right)$

A. Ramires, arXiv:1705.04080

SP(N) Fermi Liquid Behaviour

Given the interaction term with SP(N) symmetry one can determine the explicit form of the FL parameters and the renormalization of the physical quantities:

$$H_I^{SP(N)} = \frac{g}{2}\sum_{\{\mathbf{k}\}}{}' \sum_{\substack{\alpha,\beta \\ \alpha\neq\beta}} \Psi^\dagger_{\mathbf{k}_1,\alpha}\Psi^\dagger_{\mathbf{k}_2\beta}\Psi_{\mathbf{k}_3\beta}\Psi_{\mathbf{k}_4\alpha} + \frac{\Delta g}{2N}\sum_{\{\mathbf{k}\}}{}' \sum_{\alpha,\beta}(-1)^{\alpha+\beta}\Psi^\dagger_{\mathbf{k}_1\alpha}\Psi^\dagger_{\mathbf{k}_2-\alpha}\Psi_{\mathbf{k}_3\beta}\Psi_{\mathbf{k}_4-\beta},$$

where: $\Delta g = g_0 - g$

S.-K. Yip et al., Phys. Rev. A 89, 043610 (2014)
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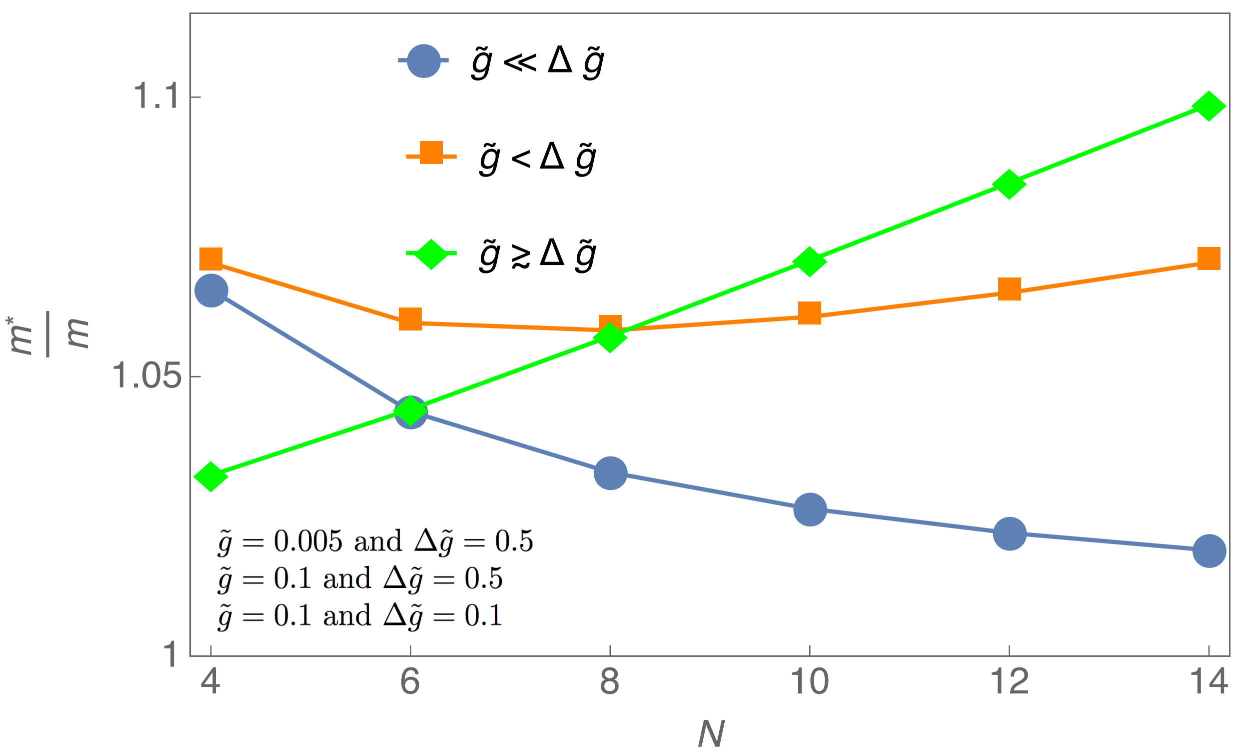

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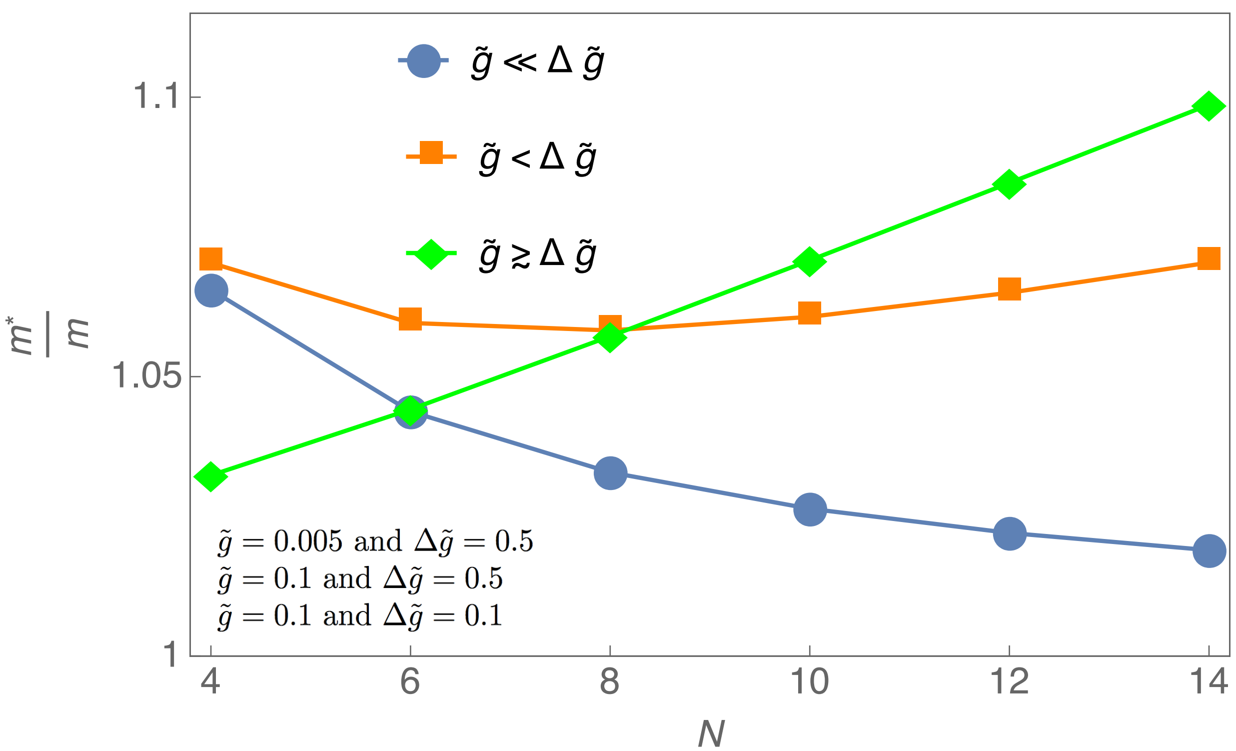
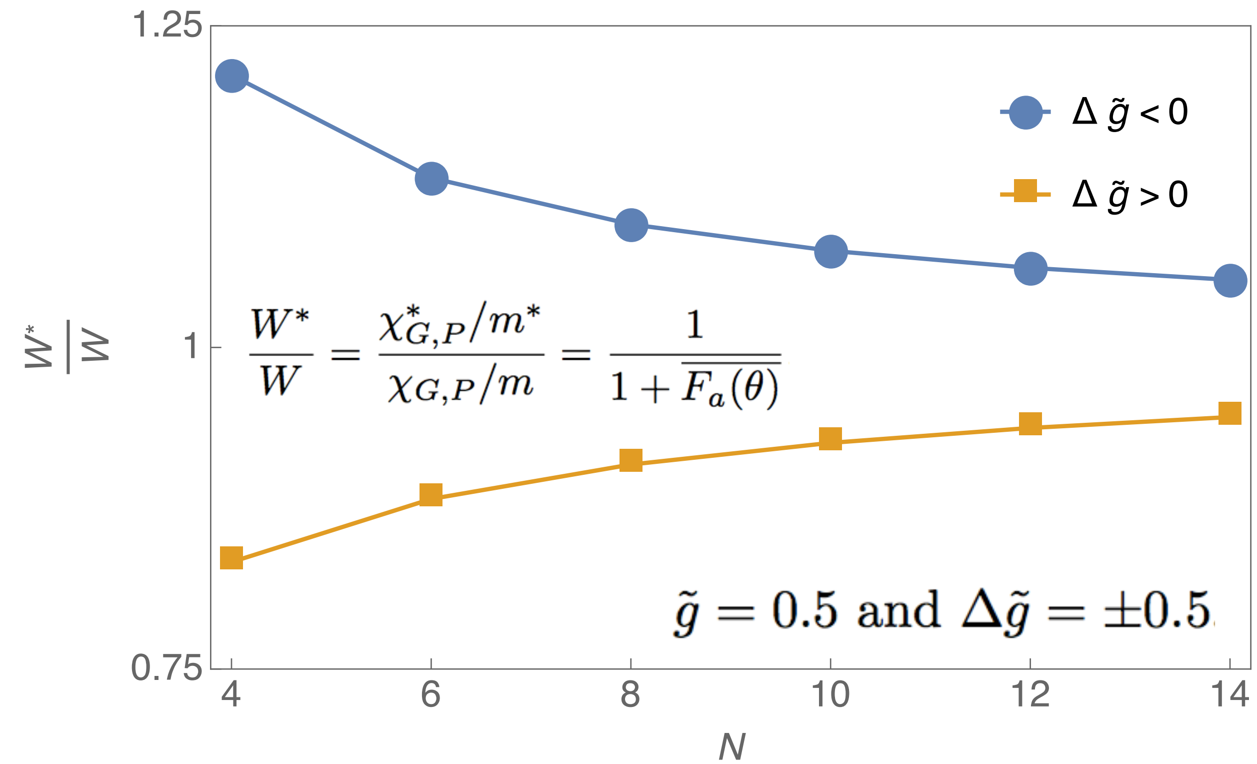


Wilson Ratio



This is in contrast with the results for SU(N), since for N > 2 the second order corrections always take the system away from a magnetic instability.

S.-K. Yip et al., Phys. Rev. A 89, 043610 (2014)
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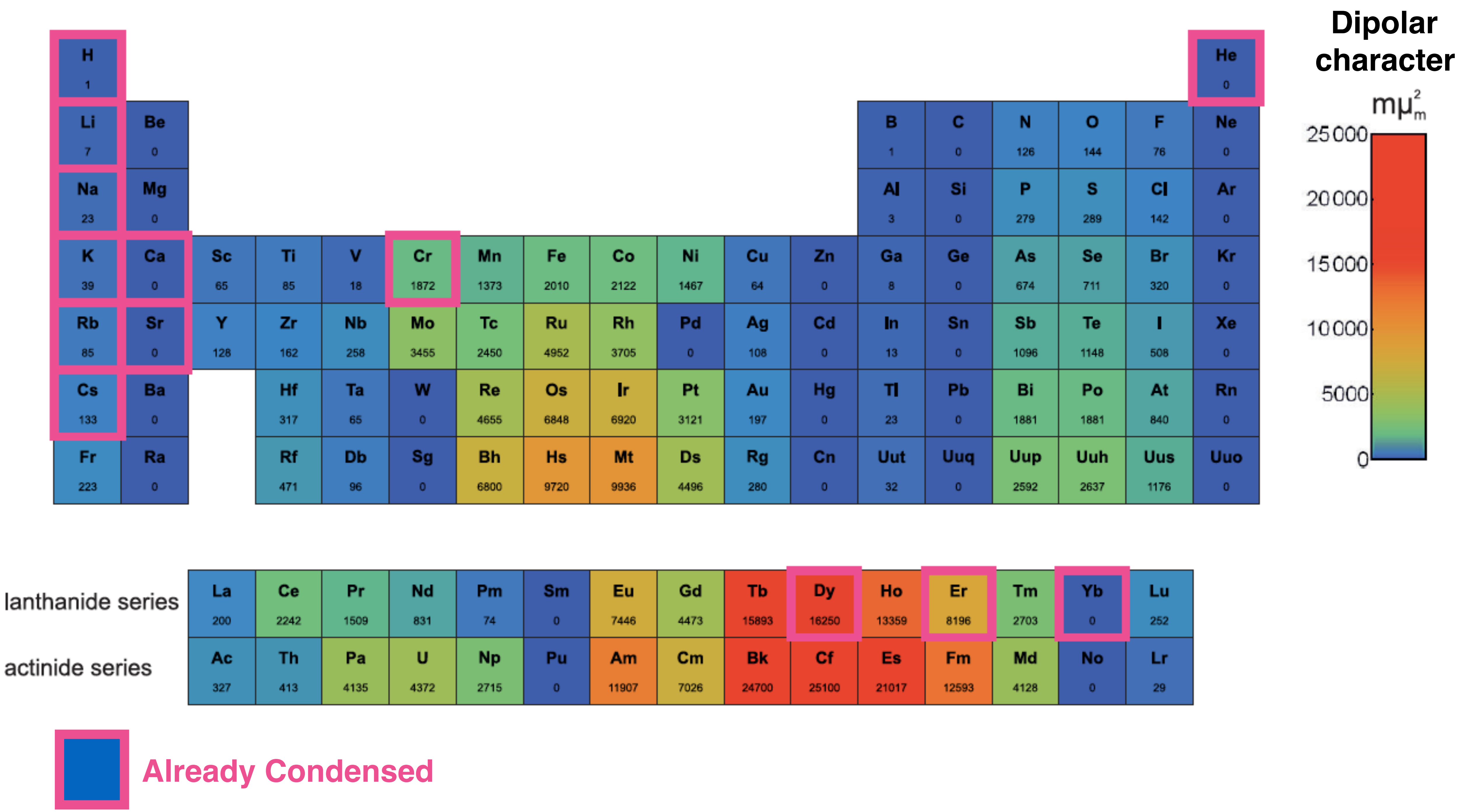
Cold Atoms and enlarged symmetries



T. Maier, PhD Thesis (2015)

Cold Atoms and enlarged symmetries

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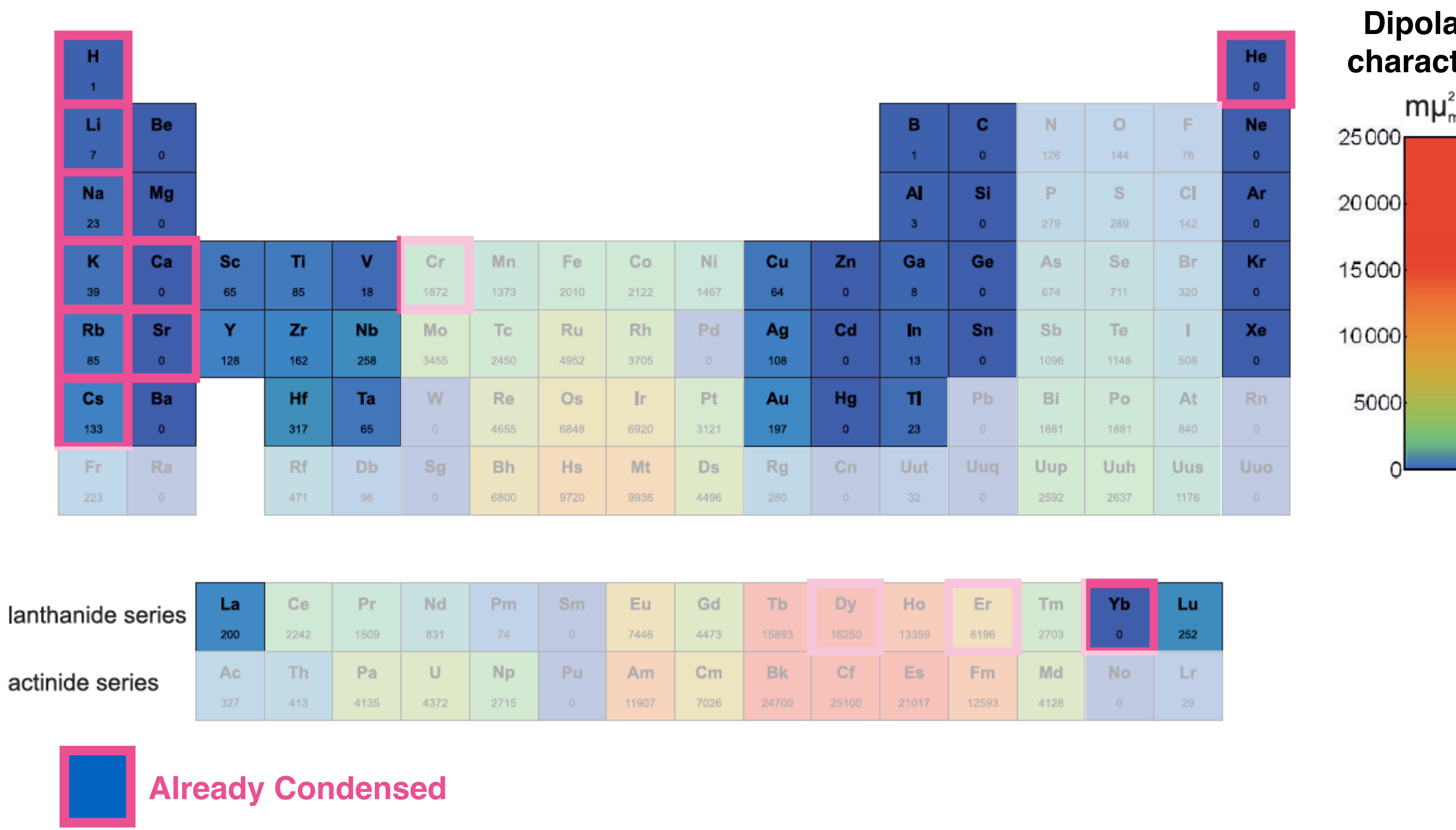
2) Stable Elements



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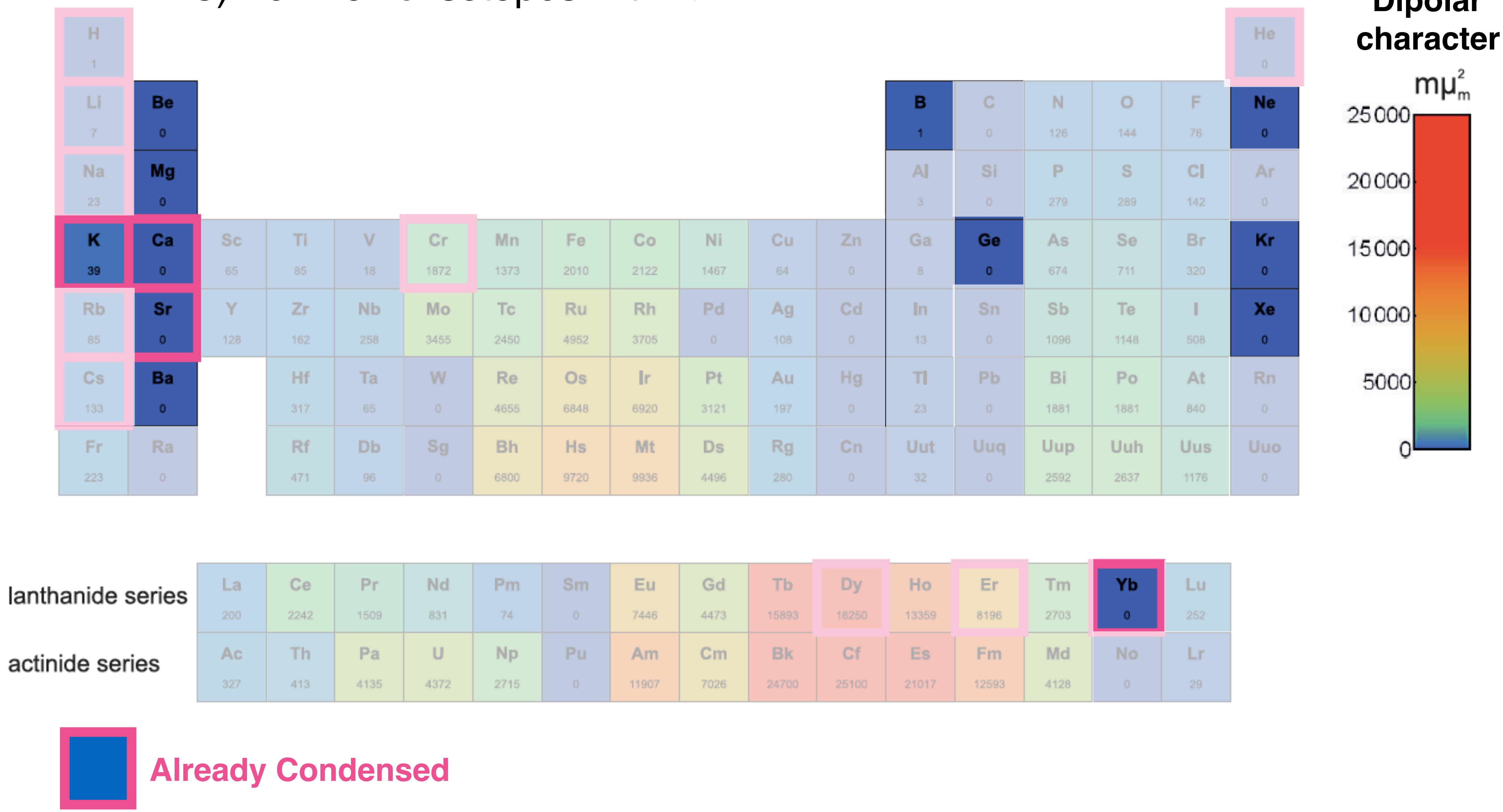

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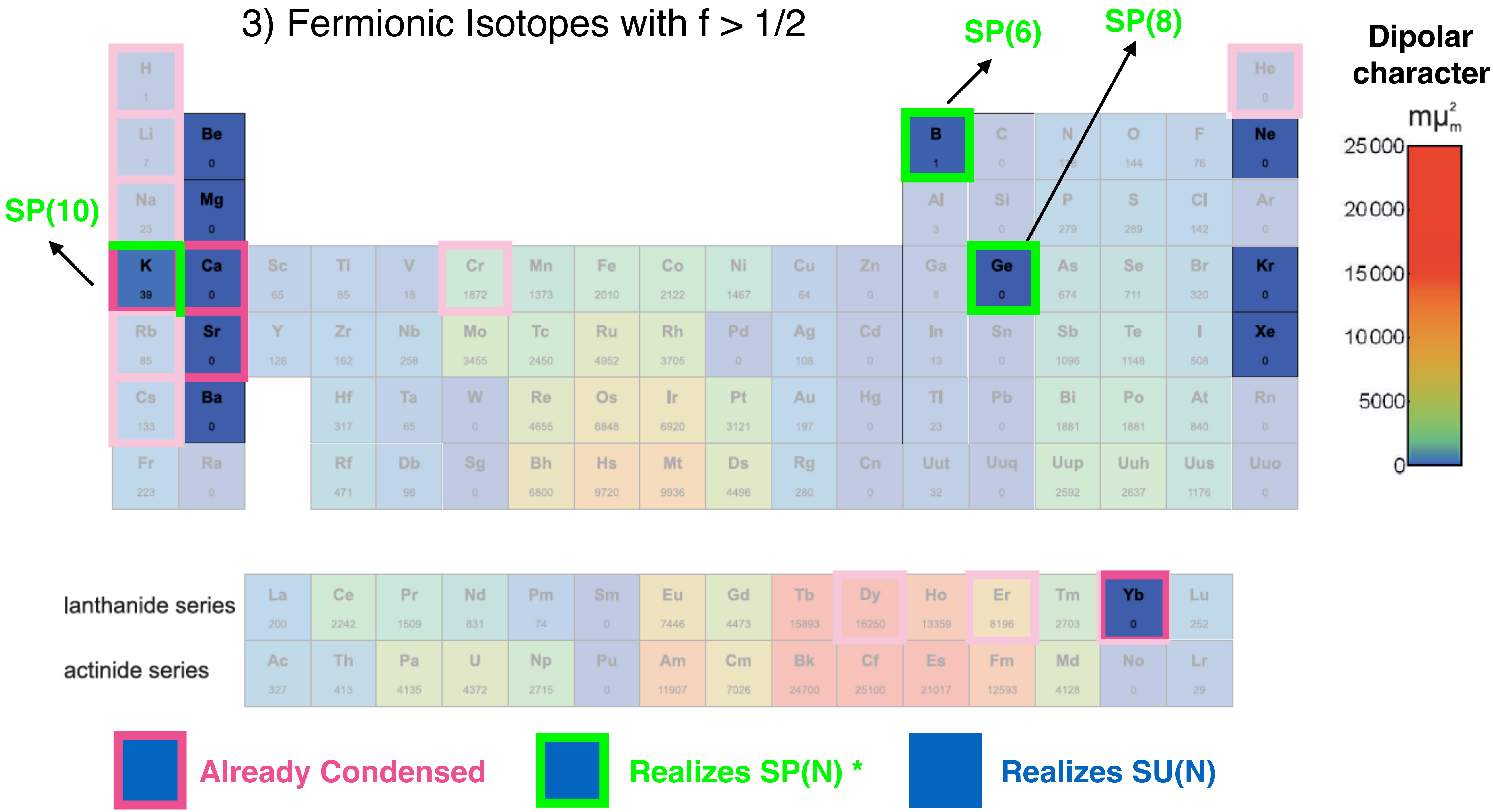

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Final Remarks

- Enlarged symmetries can be realised in cold atomic systems;
- Symplectic symmetry requires a smart experimental setup;
  Candidates ^{40}K (f=9/2), ^{10}B (f=5/2), ^{73}Ge (f=7/2)
- The behaviour of a SP(N)-FL is characterised and can be accessed experimentally (measuring density profiles or fluctuations);
- Order is more easily realised in SP(N) systems
  (enhancement of the Wilson Ratio for $\triangle g<0$);
- Realisation of concepts previously taken only as a theoretical tool:
  - different Large-N schemes (SU(N) and SP(N))
  - physics in higher dimensions (accidental isomorphism: Spin (5) ≈ SP(4))

A. Ramires, arXiv:1705.04080