# Exploring Non-Local Observables in Shock Wave Collisions

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based on work with
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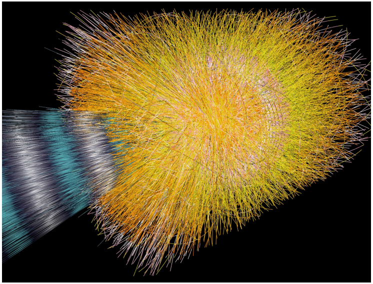
Doktoratskolleg
Particles and Interactions

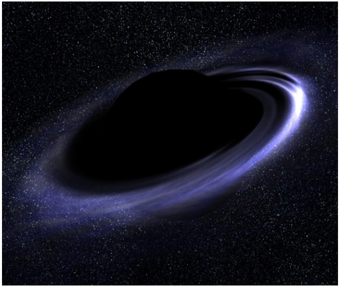
Der Wissenschaftsfonds.

# Contents

# Holographic Shock Wave Collisions

- AdS/CFT Correspondence [Maldacena, 1999]
- In a certain limit: supergravity approximation of full string theory
- $\mathcal{N}=4$ SYM theory instead of QCD





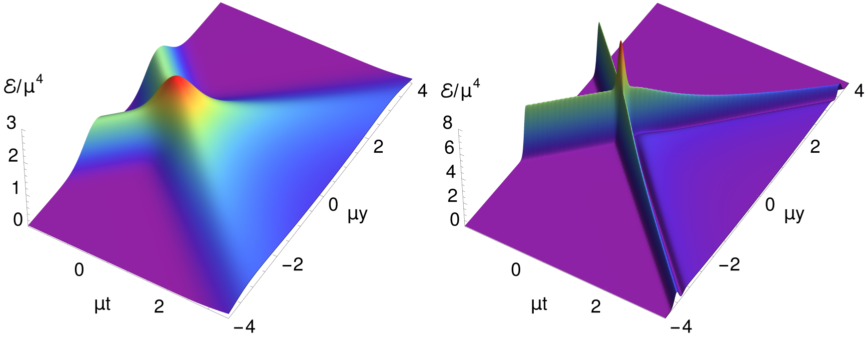
ingoing particles $\iff$ gravitational shock waves

thermalization of SYM plasma $\iff$ formation of a black hole

# Energy-Momentum Tensor

Einstein's equations are solved numerically for different initial conditions:

- Energy-momentum tensor is extracted from the metric



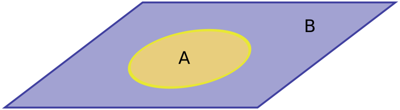

Time evolution of the EMT is qualitatively different for wide and narrow shock waves.

# Entanglement Entropy

EE is a measure of the entanglement between A and B.

In QM the EE is the *von Neumann* entropy of the reduced density matrix.

$S_A = -\mathrm{tr}_A\left(\rho_A \, \log \rho_A\right)$



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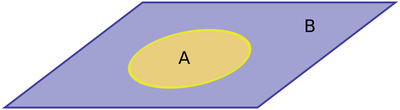
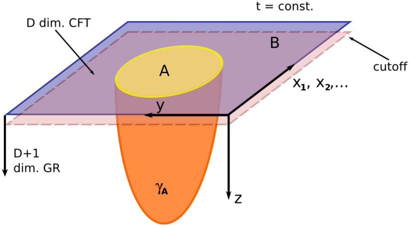



**Area formula** for holographic EE [Ryu and Takayanagi, 2006]

$$S_A = \frac{Area\ of\ \gamma_A}{4\, G_N^{(d+1)}}$$



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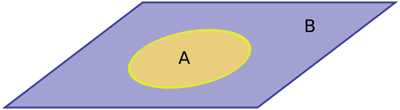
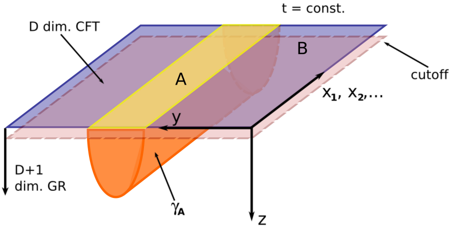
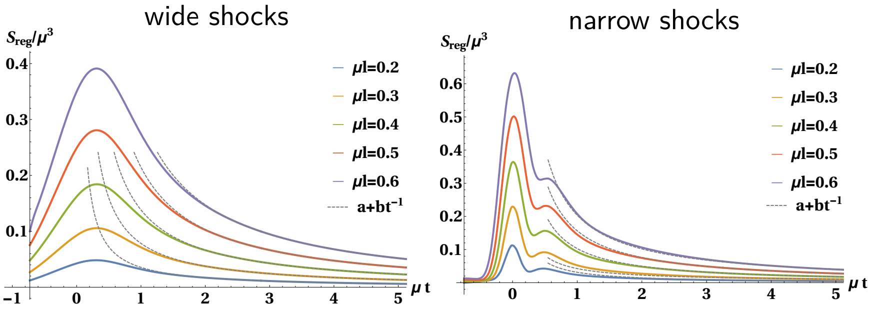



**Area formula** for holographic EE [Ryu and Takayanagi, 2006]

$$S_A = \frac{\mathcal{A}}{4\,G_N} = \frac{L}{4\,G_N} V_0$$



# Time Evolution of EE




- Rapid initial growth
- Linear growth
- (Post-) Collision regime: difference between the two cases
- Late time behavior $\propto t^{-1}$

[C. Ecker, D. Grumiller, PS, S. Stricker, W. van der Schee, JHEP11(2016)054; arxiv/1609.03676]

# (Quantum-) Null Energy Condition (Q)NEC

- Well behaved classical theories satisfy the NEC

$$\langle T_{\mu\nu} k^\mu k^\nu \rangle \geq 0, \;\; k_\mu k^\mu = 0$$

- In quantum theories the NEC can be violated [Epstein, 1965]

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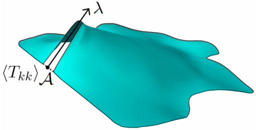
**Recently:**

- The QNEC, a quantum version of the NEC was conjectured [Bousso, 2015]

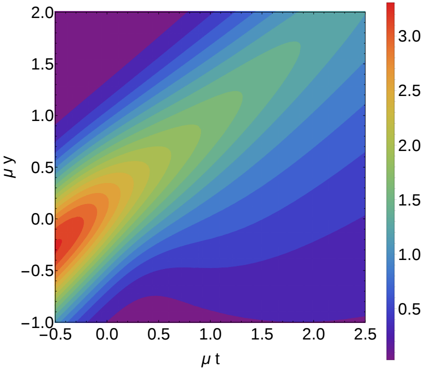
$$\langle T_{\mu\nu} k^\mu k^\nu \rangle \geq \tfrac{1}{2\pi} S''$$

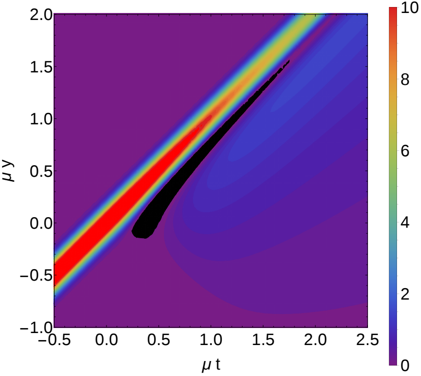



- The QNEC was proven for certain classes of theories [Koeller, 2015; Bousso, 2015] and in general [Balakrishnan et al (1706.09432)]

# Violation of the NEC in Shock Wave Collisions

| wide shocks | narrow shocks |
|---|---|
| $\langle T_{\mu\nu} k^\mu k^\nu \rangle \geq 0$ ✓ | $\langle T_{\mu\nu} k^\mu k^\nu \rangle \geq 0$ ✗ |



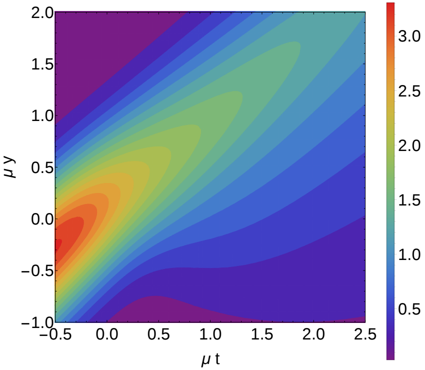




Lightlike projection of the EMT; wide and narrow shock waves.

# Violation of the NEC in Shock Wave Collisions

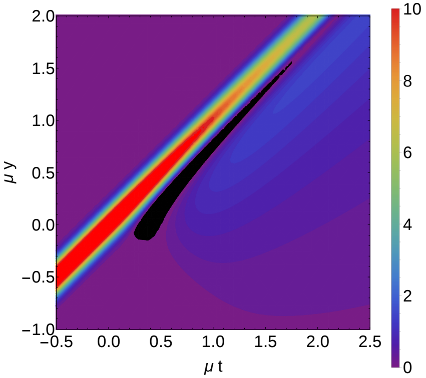
wide shocks

$\langle T_{\mu\nu} k^\mu k^\nu \rangle \geq 0$ ✓



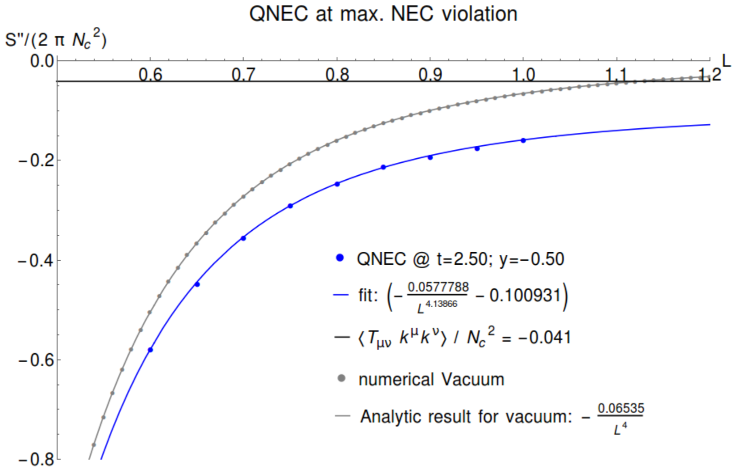

narrow shocks

$\langle T_{\mu\nu} k^\mu k^\nu \rangle \geq S'' \frac{1}{2\pi}$ ?



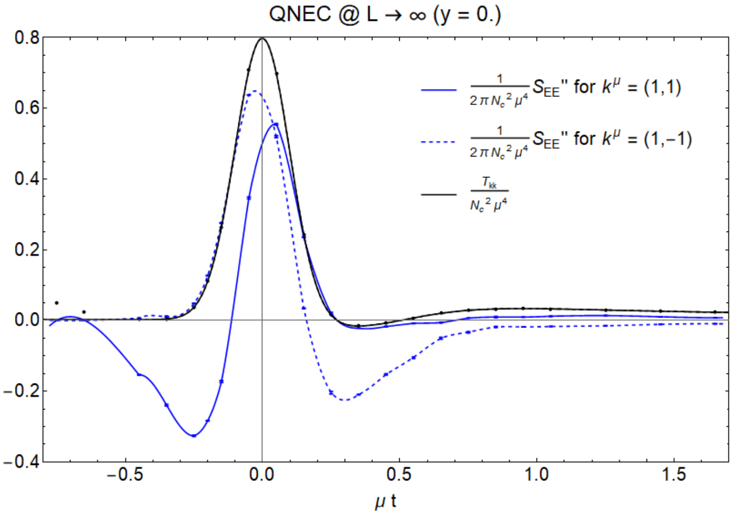

Lightlike projection of the EMT; wide and narrow shock waves.

# QNEC in Shock Wave Collisions




QNEC in the shockwave geometry and vacuum.

# Saturation of QNEC




Different saturation behavior depending on the choice of $k^\mu$.

# Summary

- The entanglement entropy shows features which are characteristic for wide and narrow shocks.
- For narrow shock waves we find a violation of the NEC, while the QNEC is satisfied (or even saturated).

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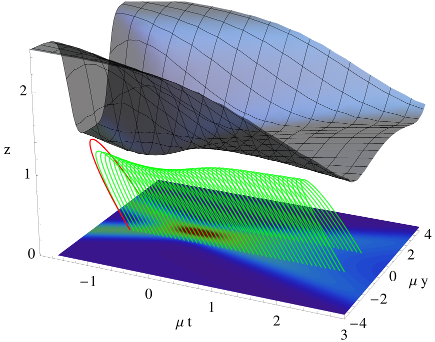
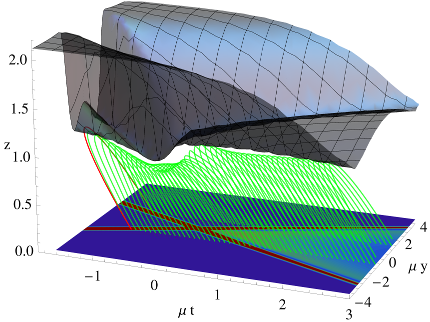
## Outlook

- Trying to get analytic understanding about when QNEC saturates NEC.
- Saturation of QNEC in systems with less symmetry? e.g. non-conformal theories
- Investigate whether geodesics and/or extremal surfaces can reach beyond the horizon.

# Time Evolution of EE

The geodesic equation is solved numerically in the shock wave geometry.

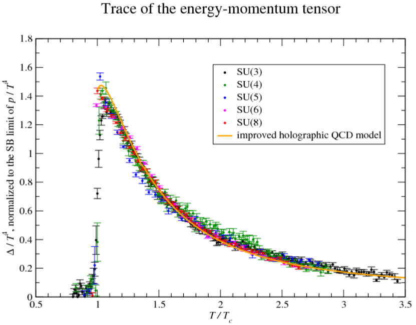



Geodesics in the shock wave geometry for wide (left) and narrow (right) shocks.

# Arguments for $\mathcal{N}=4$ SYM theory

- no confinement (like QCD above $T_C$)
- scale invariance (like QCD above $T_C$)
- QCD is approximately conformally invariant at high energies
- shear viscosity (in large $N_c$ limit) is in the same range as experiments at RHIC suggest




Lattice calculations for the trace anomaly of the QCD plasma at different $N_C$. [Panero, 2009]

# Numerical solution of Einstein's equations

## Shockwave metric

$$\mathrm{d}s^2 = -A\,\mathrm{d}v^2 + 2\,\mathrm{d}r\mathrm{d}v + 2\,F\,\mathrm{d}v\mathrm{d}y + \Sigma^2\,e^{-2B}\mathrm{d}y^2 + \Sigma^2\,e^{B}\mathrm{d}x_1^2 + \Sigma^2\,e^{B}\mathrm{d}x_2^2$$

- Einstein's equations: $R_{MN} - \frac{1}{2}\,g_{MN}\,R + \Lambda\,g_{MN} = 0$
- Using the *characteristic formulation* of general relativity, Einstein's equations decouple to a nested set of ODEs (NOT SHOCKWAVE)

$$0 = \Sigma'' + \frac{1}{2}\,B'^2\,\Sigma, \tag{5.1}$$

$$0 = \Sigma\,(\dot{\Sigma})' + 2\,\Sigma'\,\dot{\Sigma} - 2\,\Sigma^2, \tag{5.2}$$

$$0 = \Sigma\,(\dot{B})' + \frac{3}{2}\,(\Sigma'\,\dot{B} + B'\,\dot{\Sigma}), \tag{5.3}$$

$$0 = A'' + 3\,B'\,\dot{B} - 12\,\frac{\Sigma'\,\dot{\Sigma}}{\Sigma^2} + 4, \tag{5.4}$$

$$0 = \ddot{\Sigma} + \frac{1}{2}\,(\dot{B}^2\,\Sigma - A'\,\dot{\Sigma}). \tag{5.5}$$