Universal upper bound on the Bose–Einstein condensate and the Hubbard star

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Geneva, 23rd August 2017

Phys. Rev. B 96, 064502 (2017)

Engineering and Physical Sciences Research Council

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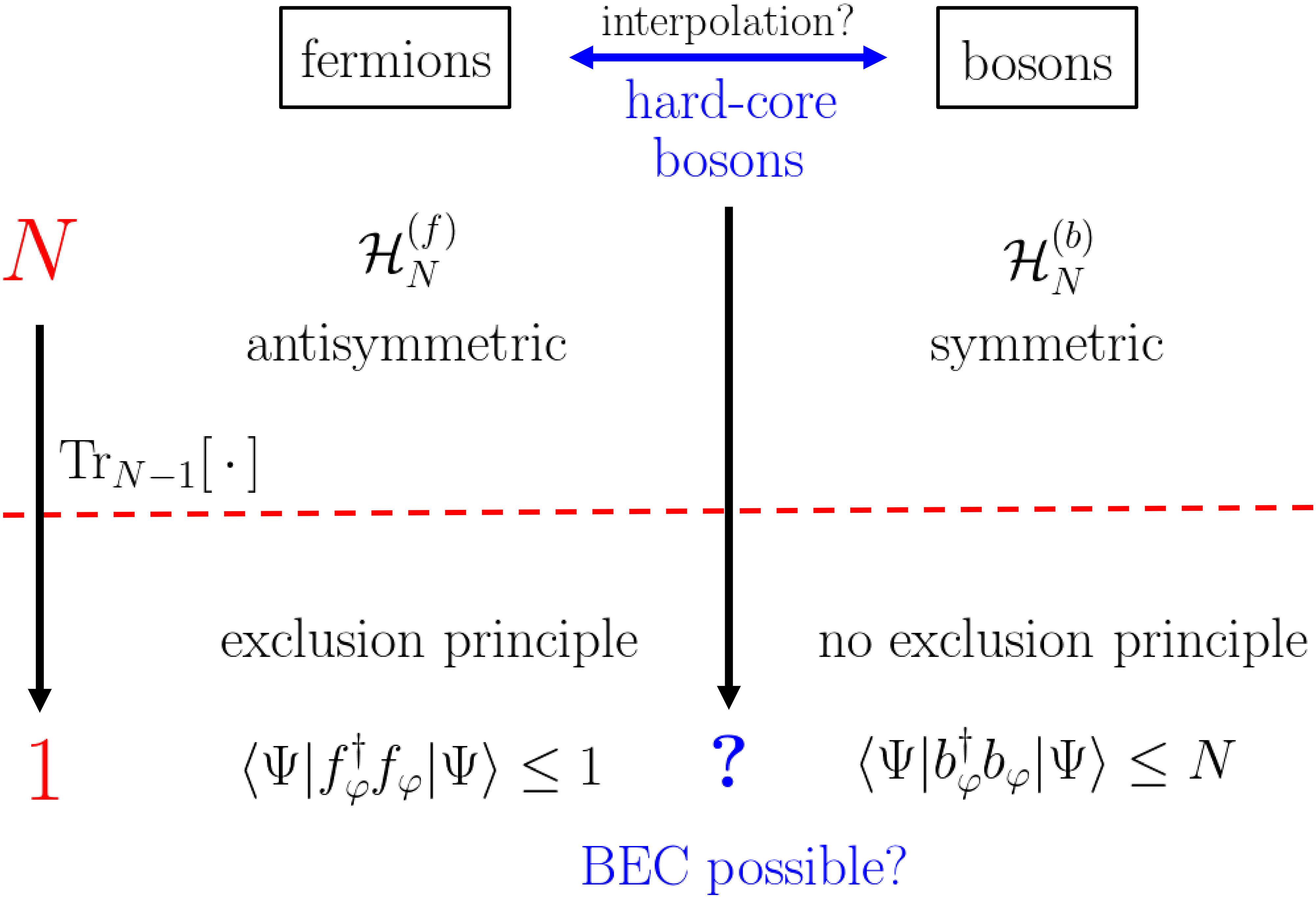
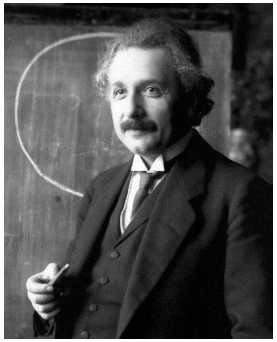

fermions
interpolation?
bosons
hard-core bosons
N
$\mathcal{H}_N^{(f)}$
$\mathcal{H}_N^{(b)}$
antisymmetric
symmetric
$\mathrm{Tr}_{N-1}[\,\cdot\,]$
exclusion principle
no exclusion principle
1
$\langle\Psi|f_\varphi^\dagger f_\varphi|\Psi\rangle \leq 1$
?
$\langle\Psi|b_\varphi^\dagger b_\varphi|\Psi\rangle \leq N$
BEC possible?

BEC & hard-core bosons



- 1924: Prediction for ideal Bose-Gas
- criterion:

$$\boxed{\max_{|\varphi\rangle} \langle \Psi | b_\varphi^\dagger b_\varphi | \Psi \rangle \; \propto \; N}$$

Onsager & Penrose (1956)

- hard-core bosons
  - 1956: Matsubara & Matsuda: liquid Helium
  - 2004: Bloch: experim. realization
  - 1d: $N_{max} \propto \sqrt{N}$

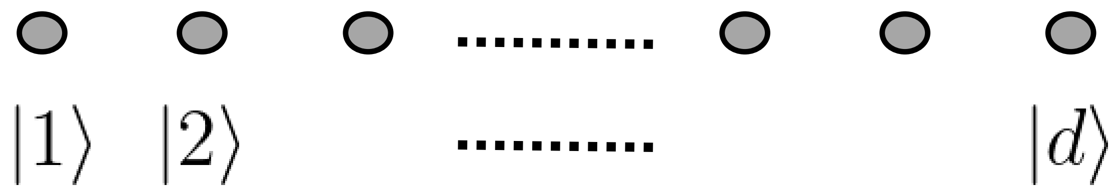
hard-core interaction

$\Downarrow$

universal bound on N_{max}?

independent of $\left\{ \begin{array}{l} \bullet\ \text{dim. \& form of lattice} \\ \bullet\ \text{temperature} \\ \bullet\ \text{microscopic details} \end{array} \right.$

- lattice:



$\Rightarrow$ 1-particle Hilbert space $\mathcal{H}_1^{(d)}$

$\Rightarrow$ basis $\mathcal{B}_1 = \{|j\rangle\}_{j=1}^{d}$ "sites"

- N hard-core bosons: $\mathcal{H}_N^{(hcb)}$

spanned by $|i_1, \ldots, i_N\rangle$, $1 \leq i_1 < i_2 < \ldots < i_N \leq d$

symmetrized

"configuration of N HCB"

- $$|\varphi\rangle = \sum_{j=1}^{d} c_j \, |j\rangle$$ 1 HCB

- $$|\Psi\rangle = \sum_{1\leq i_1<\ldots<i_N\leq d} A_{i_1,\ldots,i_N} \, |i_1,\ldots,i_N\rangle$$ N HCB

- $$N^{(\varphi)}(|\Psi\rangle) \equiv \langle\Psi|h_\varphi^\dagger h_\varphi|\Psi\rangle$$

$$\boxed{N_{max} \equiv \max_{|\varphi\rangle} \max_{|\Psi\rangle} \left[N^{(\varphi)}(|\Psi\rangle) \right]}$$

$$\vec{c} \equiv (c_j)_{j=1}^{d}$$

$$N^{(\varphi)}(|\Psi\rangle) = \sum_{1\leq i_1<\ldots<i_{N-1}\leq d} \left| \langle \vec{A}^{(i_1,\ldots,i_{N-1})}, \vec{c} \rangle \right|^2$$

$$\vec{A}^{(i_1,\ldots,i_{N-1})} \equiv \left(A_{i_1,\ldots,i_{N-1},k}\right)_{k=1}^{d}$$

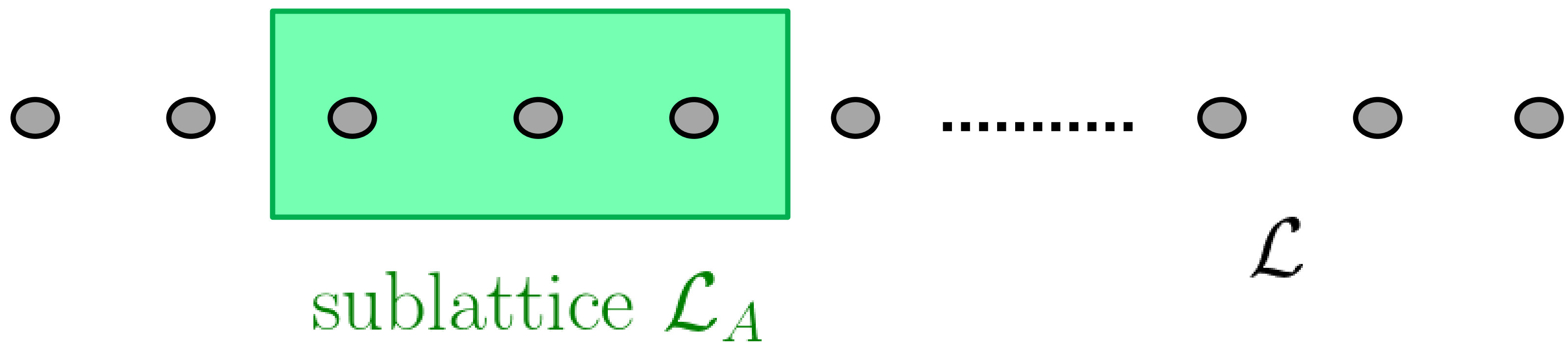
Result I

$$\boxed{N_{max} = \frac{N}{d}\left(d - N + 1\right)}$$

- $|\varphi_{max}\rangle = \frac{1}{\sqrt{d}} \sum_{j=1}^{d} |j\rangle$
- $|\Psi_{max}\rangle = \mathcal{N} \sum_{1 \leq i_1 < \ldots < i_N \leq d} |i_1, \ldots, i_N\rangle$

max. delocalized

- $+$ phases
- reconstruction of $|\Psi\rangle$ from 1-particle information **!**



$$\Rightarrow \boxed{N_{max}^{(A)} = ?}$$

Result II

- Complementary system B as particle resource to maximize $N_{max}^{(N_A,d_A)}$
- $N_{max}^{(N_A,d_A)}$ attained $\Leftrightarrow$ Entanglement A$\leftrightarrow$B minimal

Systems with N_{max}

- lattice $\mathcal{L}$ + infinite-range hopping

[B. Toth, J. Phys. Stat. **61**, 749 (1990)]

- $$\hat{H} = -t \sum_i h_i^\dagger h_{i+1} + h.c. \; + \; V \sum_i \hat{n}_i \hat{n}_{i+1}$$

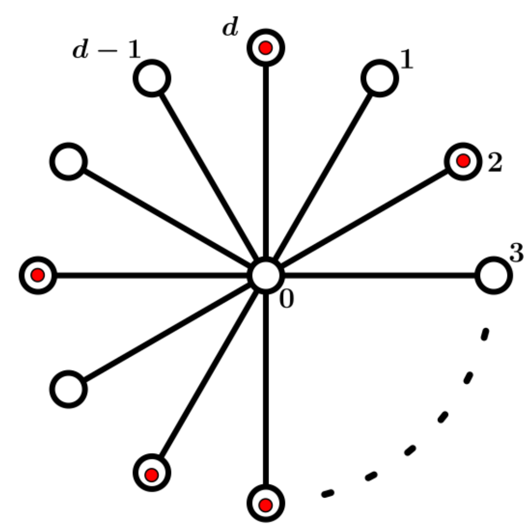
[C. N. Yang, C. P. Yang, Phys. Rev. **151**, 258 (1966)]

$$V = -2t$$

- Hubbard Star

$$\hat{H} \; = \; -t \sum_{j=1}^{d} (h_0^\dagger h_j \; + \; h_j^\dagger h_0)$$

simulates infinite-range hopping



Thank you!

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