Holographic QCD predictions for glueball decay patterns

Frederic Brünner, Denis Parganlija, Anton Rebhan

Institute for Theoretical Physics
TU Wien, Vienna, Austria

Joint SPS-ÖPG Annual Meeting in Geneva, 2017
Ever elusive: Glueballs

Spectrum of bare glueballs (prior to mixing with $q\bar{q}$ states) more or less known from lattice:

$m_{0^{++}} \sim 1.7$ GeV
$m_{2^{++}} \sim 2.4$ GeV
$m_{0^{+-}} \sim 2.6$ GeV

... 

Morningstar & Peardon hep-lat/9901004
Ever elusive: Glueballs

Spectrum of bare glueballs (prior to mixing with $q\bar{q}$ states) more or less known from lattice:

\[ m_{0^{++}} \sim 1.7 \text{ GeV} \]
\[ m_{2^{++}} \sim 2.4 \text{ GeV} \]
\[ m_{0^{-+}} \sim 2.6 \text{ GeV} \]

... Morningstar & Peardon hep-lat/9901004

Interactions of glueballs still unclear:

- Are glueballs broad or narrow?
- Do they mix with $q\bar{q}$ strongly or weakly?

→ no conclusive identification of any glueball in meson spectrum
Ever elusive: Glueballs

Spectrum of bare glueballs (prior to mixing with $q\bar{q}$ states) more or less known from lattice:

\[ m_{0^{++}} \sim 1.7 \text{ GeV} \]
\[ m_{2^{++}} \sim 2.4 \text{ GeV} \]
\[ m_{0^{--}} \sim 2.6 \text{ GeV} \]

Morningstar & Peardon hep-lat/9901004

Interactions of glueballs still unclear:

- Are glueballs broad or narrow?
- Do they mix with $q\bar{q}$ strongly or weakly?

→ no conclusive identification of any glueball in meson spectrum

most discussed lowest $0^{++}$ candidates:
  - narrow $f_0(1500)$ or $f_0(1710)$ vs. very broad background ("red dragon")

various phenomenological models describe $f_0(1500)$ or $f_0(1710)$
  alternatingly as $\sim 50\% - 70\%$ or $\sim 75\% - 90\%$ glue

$[G$ and two isoscalar $q\bar{q}$ states $u\bar{u} + d\bar{d}$ and $s\bar{s}$ can be shared by $f_0(1370), f_0(1500), f_0(1710)]$
Even more elusive: Pseudoscalar glueball

Pseudoscalar glueball ($\tilde{G}$):

- closely related to $\eta'$ and the $U(1)_A$ problem
- in 1980: first glueball candidate the isoscalar pseudoscalar $\iota(1440)$, now listed as two states $\eta(1405)$ and $\eta(1475)$ in PDG
- together with $\eta(1295) \Rightarrow$ a supernumerary state beyond the first radial excitations of the $\eta$ and $\eta'$ mesons, with $\eta(1405)$ singled out as glueball candidate
- BUT: lattice predicts $m(\tilde{G}) \sim 2.6$ GeV ! $\Rightarrow$ Still to be discovered
  
  indeed: evidence for three $\eta$ states between 1.2 and 1.5 GeV under dispute
  ($\eta(1405)$ and $\eta(1475)$ could after all be one state $\eta(1440)$; also $\eta(1295)$ sometimes questioned)

Seeking help from closest (top-down) holographic model of (large-$N_c$) QCD:

the Witten-Sakai-Sugimoto model

Qualitative + quantitative estimates of glueball decay pattern:

- F. Brüner, AR, PLB770 (2017) 124
Even more elusive: Pseudoscalar glueball

Pseudoscalar glueball ($\tilde{G}$):

- closely related to $\eta'$ and the $U(1)_A$ problem
- in 1980: first glueball candidate the isoscalar pseudoscalar $\iota(1440)$, now listed as two states $\eta(1405)$ and $\eta(1475)$ in PDG
- together with $\eta(1295)$ ⇒ a supernumerary state beyond the first radial excitations of the $\eta$ and $\eta'$ mesons, with $\eta(1405)$ singled out as glueball candidate
- BUT: lattice predicts $m(\tilde{G}) \sim 2.6$ GeV ⇒ Still to be discovered
  indeed: evidence for three $\eta$ states between 1.2 and 1.5 GeV under dispute
  ($\eta(1405)$ and $\eta(1475)$ could after all be one state $\eta(1440)$; also $\eta(1295)$ sometimes questioned)

Seeking help from closest (top-down) holographic model of (large-$N_c$) QCD: the Witten-Sakai-Sugimoto model

Qualitative + quantitative estimates of glueball decay pattern:

- F. Brünner, AR, PLB770 (2017) 124
Witten model: Holographic nonsupersymmetric QCD


Type-IIA string theory with $N_c \to \infty$ $D4$ branes
dual to $4+1$-dimensional super-Yang-Mills theory

supersymmetry completely broken by compactification
on “thermal-like” circle $x_4 \equiv x_4 + 2\pi/M_{KK}$ (Kaluza–Klein)

- antisymmetric b.c. for adjoint fermions: masses $\sim M_{KK}$
- adjoint scalars not protected by gauge symmetry: also masses $\sim M_{KK}$

$\to$ dual to pure-glue YM theory

3+1-dimensional at scales $\ll M_{KK}$

but supergravity approximation needs weak curvature,
cannot take limit $M_{KK} \to \infty$

Glueballs: Constable & Myers 1999; Brower, Mathur & Tan 2000

- scalar and tensor glueballs corresponding to 5D dilaton $\Phi$ and graviton $G_{ij}$
  plus exotic scalar modes (discarded by us)
- pseudoscalar glueball from RR 1-form field $C_1$
Sakai-Sugimoto model: Adding chiral quarks


Add $N_f$ D8- and D8-bar branes, separated in $x_4$, $N_f \ll N_c$ (probe branes)

<table>
<thead>
<tr>
<th></th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>D4</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>D8/\overline{D8}</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>x</td>
<td>x</td>
</tr>
</tbody>
</table>

4-8, 4-\overline{8} strings
→ fundamental, massless chiral fermions

flavor symmetry
$U(N_f)_L \times U(N_f)_R$

spontaneously broken because D8-\overline{D8} have to join in cigar-shaped topology

for now: maximal separation in $x_4$ (antipodal on $x_4$ circle): $L = \pi / M_{KK}$
Quantitative predictions

- Quite good parameter-free prediction of (axial-)vector meson mass pattern!

Other predictions depend on value of 't Hooft coupling $\lambda$ at scale $M_{KK}$:

Matching

1. $m_\rho \approx 776$ MeV fixes $M_{KK} = 949$ MeV ($\Rightarrow T_{decon.f} = 151$ MeV)

2. $f_\pi^2 = \frac{\lambda N_c}{54\pi^4} M_{KK}^2$ gives $\lambda = g_{YM}^2 N_c \approx 16.63$ [Sakai&Sugimoto 2005-7]

(matching instead large-$N_c$ lattice result [Bali et al. 2013] for $m_\rho/\sqrt{\sigma}$ gives $\lambda \approx 12.55$)

yields (for $N_c = 3$ and $\lambda = 16.63 \ldots 12.55$):

- LO decay rate of $\rho$ meson $\sim \lambda^{-1} N_c^{-1}$
  $\Gamma_{\rho \to 2\pi}/m_\rho = 0.1535 \ldots 0.2034$ (exp.: $0.191(1)$)

- decay rate for $\omega \to 3\pi$ (from Chern-Simons part of D8 action) $\sim \lambda^{-4} N_c^{-2}$
  $\Gamma_{\omega \to 3\pi}/m_\omega = 0.0033 \ldots 0.0102$ (exp.: $0.0097(1)$)

**WSS model also predictive regarding glueball decay pattern and rates?**
Glueball decay rates in Sakai-Sugimoto model


Full decay pattern of scalar (Dilatonic) glueball $G_D$

decay $G_D \rightarrow 4\pi$ suppressed (below $2\rho$ threshold): $\Gamma_{G \rightarrow 4\pi}/\Gamma_{G \rightarrow 2\pi} \sim \lambda^{-1} N_c^{-1}$, while $f_0(1500) \rightarrow 4\pi$ dominant:

<table>
<thead>
<tr>
<th>decay</th>
<th>$\Gamma/M$ (PDG)</th>
<th>$\Gamma/M$ [$G_D$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_0(1500)$ (total)</td>
<td>0.072(5)</td>
<td>0.027...0.037</td>
</tr>
<tr>
<td>$f_0(1500) \rightarrow 4\pi$</td>
<td>0.036(3)</td>
<td>0.003...0.005</td>
</tr>
<tr>
<td>$f_0(1500) \rightarrow 2\pi$</td>
<td>0.025(2)</td>
<td>0.009...0.012</td>
</tr>
<tr>
<td>$f_0(1500) \rightarrow 2K$</td>
<td>0.006(1)</td>
<td>0.012...0.016</td>
</tr>
<tr>
<td>$f_0(1500) \rightarrow 2\eta$</td>
<td>0.004(1)</td>
<td>0.003...0.004</td>
</tr>
</tbody>
</table>

$\Rightarrow f_0(1500)$ seemingly disfavored, at least when nearly pure glue
Glueball decay rates in Sakai-Sugimoto model


Full decay pattern of scalar (Dilatonic) glueball $G_D$

decline $G_D \rightarrow 4\pi$ suppressed (below $2\rho$ threshold): $\Gamma_{G\rightarrow 4\pi}/\Gamma_{G\rightarrow 2\pi} \sim \lambda^{-1} N_c^{-1}$, while $f_0(1500) \rightarrow 4\pi$ dominant:

<table>
<thead>
<tr>
<th>decay</th>
<th>$\Gamma/M$ (PDG)</th>
<th>$\Gamma/M[G_D]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_0(1500)$ (total)</td>
<td>0.072(5)</td>
<td>0.027...0.037</td>
</tr>
<tr>
<td>$f_0(1500) \rightarrow 4\pi$</td>
<td>0.036(3)</td>
<td>0.003...0.005</td>
</tr>
<tr>
<td>$f_0(1500) \rightarrow 2\pi$</td>
<td>0.025(2)</td>
<td>0.009...0.012</td>
</tr>
<tr>
<td>$f_0(1500) \rightarrow 2K$</td>
<td>0.006(1)</td>
<td>0.012...0.016</td>
</tr>
<tr>
<td>$f_0(1500) \rightarrow 2\eta$</td>
<td>0.004(1)</td>
<td>0.003...0.004</td>
</tr>
</tbody>
</table>

$\Rightarrow f_0(1500)$ seemingly disfavored, at least when nearly pure glue

$f_0(1710) \rightarrow \pi\pi$ OK: $\Gamma^{(\text{ex})}(f_0(1710) \rightarrow \pi\pi)/(1722\text{MeV}) \sim 0.01$

but $f_0(1710)$ decays predominantly into $K\bar{K}$!

— not reproduced by (chiral=flavor-symmetric) WSS model, but may be due to mechanism of “chiral suppression of scalar glueball decay”

(Chanowitz 2005)
Nonchiral enhancement in mass-deformed WSS!

Holographic realization of mass terms give additional vertices between glueballs and pseudoscalars
Rigorously calculable for Witten-Veneziano mass term of $\eta_0$

$$\mathcal{L}^{\text{chiral}}_{G_D \eta_0 \eta_0} = \frac{3}{2}d_0 m_0^2 \eta_0^2 G_D,$$
$$m_0^2 = \frac{N_f}{27 \pi^2 N_c} \lambda^2 M_{\text{KK}}^2,$$
$$d_0 \approx \frac{17.915}{\lambda^{1/2} N_c M_{\text{KK}}},$$

but not (yet) fixed for current quark masses of the octet.

Parametrize uncertainty by free parameter $x$:

$$\mathcal{L}^{\text{massive}}_{G_D \pi\pi} = \frac{3}{2}d_m G_D \mathcal{L}_m,$$
$$d_m \equiv xd_0$$

Most symmetric choice $x = 1$ (⇔ no $G_D \rightarrow \eta \eta'$)
→ relatively strong enhancement factor for kaons and $\eta$ mesons:

$$\Gamma_{G \rightarrow PP}^{\text{chiral}} \rightarrow \Gamma_{G \rightarrow PP}^{\text{chiral}} \times \left(1 - 4 \frac{m_P^2}{M_G^2}\right)^{1/2} \left(1 + 8.480 \frac{m_P^2}{M_G^2}\right)^2$$
### Comparison with \( f_0(1710) \)

<table>
<thead>
<tr>
<th>decay</th>
<th>( \Gamma/M ) (PDG)</th>
<th>( \Gamma/M[G_D] ) (chiral)</th>
<th>( \Gamma/M[G_D] ) (massive)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f_0(1710) ) (total)</td>
<td>0.081(5)</td>
<td>0.059...0.076</td>
<td>0.083...0.106</td>
</tr>
<tr>
<td>( f_0(1710) \rightarrow 2K )</td>
<td>(*) 0.029(10)</td>
<td>0.012...0.016</td>
<td>0.029...0.038</td>
</tr>
<tr>
<td>( f_0(1710) \rightarrow 2\eta )</td>
<td>0.014(6)</td>
<td>0.003...0.004</td>
<td>0.009...0.011</td>
</tr>
<tr>
<td>( f_0(1710) \rightarrow 2\pi )</td>
<td>0.012((+5) (-6))</td>
<td>0.009...0.012</td>
<td>0.010...0.013</td>
</tr>
<tr>
<td>( f_0(1710) \rightarrow 2\rho, \rho\pi\pi \rightarrow 4\pi )</td>
<td>?</td>
<td>0.024...0.030</td>
<td>0.024...0.030</td>
</tr>
<tr>
<td>( f_0(1710) \rightarrow 2\omega )</td>
<td>0.010((+6) (-7))</td>
<td>0.011...0.014</td>
<td>0.011...0.014</td>
</tr>
<tr>
<td>( f_0(1710) \rightarrow \eta\eta' )</td>
<td>?</td>
<td>0</td>
<td>if 0 : ↑</td>
</tr>
</tbody>
</table>

\[ \Gamma(\pi\pi)/\Gamma(K\bar{K}) \]  
\[ 0.41^{+0.11}_{-0.17} \]  
\[ 3/4 \]  
\[ 0.35 \]

\[ \Gamma(\eta\eta)/\Gamma(K\bar{K}) \]  
\[ 0.48\pm0.15 \]  
\[ 1/4 \]  
\[ 0.28 \]

* PDG ratios for decay rates + \( Br(f_0(1710) \rightarrow KK) = 0.36(12) \) [Albaladejo&Oller 2008]

- decays into 2 pseudoscalars: massive WSS perfectly compatible with PDG data!
- significant decay into 4 pions (after extrapolation to beyond \( 2\rho \) threshold): falsifiable prediction of this model!  
  \( (f_0(1710) \rightarrow 2\rho^0 \) forthcoming from CEP experiments at LHC! \)
Pseudoscalar glueball in Witten-Sakai-Sugimoto model

Pseudoscalar glueballs described by fluctuations of

$$\tilde{F}_2 = dC'_1 + \frac{c}{U^4} \left( \theta + \frac{\sqrt{2N_f}}{f_\pi} \eta_0(x) \right) dU \wedge d\tau$$

(anomaly inflow)

No direct coupling of $C_1$ to flavor D8 branes,

$$\text{vertex } G-\tilde{G}-\eta_0 \propto \sqrt{\frac{N_f}{N_c}} \frac{\sqrt{\lambda}}{N_c}$$

from

$$-\frac{1}{4\pi(2\pi\ell_s)^6} \int d^{10}x \sqrt{-g} |\tilde{F}_2|^2$$

→ very narrow pseudoscalar glueball with dominant decay pattern

$$\tilde{G} \rightarrow G(=f_0(1710)) + \eta('')$$

(resonant)

\[\begin{array}{cccc}
\text{Gamma (GeV)} & \text{Mass (GeV)} \\
0.000 & 2.3 \\
0.005 & 2.4 \\
0.010 & 2.5 \\
0.015 & 2.6 \\
0.020 & 2.7 \\
0.025 & 2.8 \\
0.030 & 2.9 \\
\end{array}\]
Pseudoscalar glueball in Witten-Sakai-Sugimoto model

Pseudoscalar glueballs described by fluctuations of RR field
\[ \tilde{F}_2 = dC_1' + \frac{c}{U^4} \left( \theta + \frac{\sqrt{2N_f}}{f_\pi} \eta_0(x) \right) dU \wedge d\tau \] (anomaly inflow)

No direct coupling of \( C_1 \) to flavor D8 branes,

vertex \( G - \tilde{G} - \eta_0 \propto \sqrt{\frac{N_f}{N_c}} \frac{\sqrt{\lambda}}{N_c} \) from
\[ -\frac{1}{4\pi(2\pi\ell_s)^6} \int d^{10}x \sqrt{-g} |\tilde{F}_2|^2 \]

→ very narrow pseudoscalar glueball with dominant decay pattern
\[ \tilde{G} \rightarrow G (= f_0(1710)) + \eta'(') \rightarrow P P \eta'() \]

\[ \lambda = (12.55 + 16.63)/2 \]
Pseudoscalar glueball production

As with decay, production of $\tilde{G}$ involves $G+\eta(')$ or $G+\text{another }\tilde{G}$

would explain why not yet observed in radiative $J/\psi$ decays; needs excited $\psi$ or $\Upsilon$?

• Another possibility: Central Exclusive Production in high-energy hadron collisions!

Parametric orders of the production amplitudes of pseudoscalar glueballs in double
Pomeron or double Reggeon exchange

$\tilde{G}\tilde{G}$: \hspace{1cm} $\sim \lambda^{-1} N_c^{-2}$

$\eta(')\tilde{G}$: \hspace{1cm} $\sim \lambda^0 N_f^{1/2} N_c^{-5/2}$

$G\tilde{G}$: \hspace{1cm} $\sim \lambda^{-1} N_f^1 N_c^{-3} \ldots$ suppressed

(in the uppermost diagram the full line stands for $G$ or $G_T$)
Production of $\tilde{G}\tilde{G}$ and $\tilde{G}\eta'$ pairs versus $\eta'\eta'$

Production from a virtual scalar glueball
for as functions of the c.o.m. energy of the produced pair (assuming $m(\tilde{G}) = 2.6$ GeV)

The full line gives $N(\tilde{G}\tilde{G})/N(\eta'\eta')$, which is independent of the 't Hooft coupling; upper and lower dashed lines correspond to $N(\tilde{G}\eta')/N(\eta'\eta')$ with 't Hooft coupling 12.55 and 16.63, respectively.

**CEP of $\eta'\eta'$ in Durham [Harland-Lang et al. 2013]:**

$\sigma(\eta'\eta')/\sigma(\pi^0\pi^0) \sim 10^3 \ldots 10^5$ at $\sqrt{s} = 1.96$ TeV
Conclusion

• With just one dimensionless parameter, top-down holographic QCD model of Witten, Sakai and Sugimoto very predictive and surprisingly successful quantitatively:

Meson spectrum and dynamics:
— vector and axial vector mesons masses, $\rho$ and $\omega$ decay rates, anomalous $m'_{\eta}$, …
  with typically 10–30% errors

Glueball spectrum:
— if “exotic mode” discarded, scalar glueball mass close to lattice QCD prediction
tensor and pseudoscalar glueball $\sim 30\%$ too light

WSS model also perhaps good guide for glueball signatures!

Scalar glueball decay pattern consistent with $f_0(1710)$ as nearly pure glueball, if
predictions for $4\pi$ and $\eta\eta'$ decays confirmed

**Golden channel?:** very narrow pseudoscalar glueballs with characteristic decay and
production pattern (would explain why not yet observed in radiative $J/\psi$ decays)
Glueballs in the Witten model

∃ scalar and tensor glueballs corresponding to 5D dilaton \( \Phi \) and graviton \( G_{ij} \)

Csaki, Ooguri, Oz & Terning 1999

Type-IIA supergravity compactified on \( x_4 \)-circle many more modes:
Constable & Myers 1999; Brower, Mathur & Tan 2000

<table>
<thead>
<tr>
<th>Mode</th>
<th>( S_4 )</th>
<th>( T_4 )</th>
<th>( V_4 )</th>
<th>( N_4 )</th>
<th>( M_4 )</th>
<th>( L_4 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( J^{PC} )</td>
<td>( G_{44} )</td>
<td>( \Phi, G_{ij} )</td>
<td>( C_1 )</td>
<td>( B_{ij} )</td>
<td>( C_{ij4} )</td>
<td>( G^\alpha_\alpha )</td>
</tr>
<tr>
<td>( n=0 )</td>
<td>7.30835</td>
<td>22.0966</td>
<td>31.9853</td>
<td>53.3758</td>
<td>83.0449</td>
<td>115.002</td>
</tr>
<tr>
<td>( n=1 )</td>
<td>46.9855</td>
<td>55.5833</td>
<td>72.4793</td>
<td>109.446</td>
<td>143.581</td>
<td>189.632</td>
</tr>
<tr>
<td>( n=2 )</td>
<td>94.4816</td>
<td>102.452</td>
<td>126.144</td>
<td>177.231</td>
<td>217.397</td>
<td>277.283</td>
</tr>
<tr>
<td>( n=3 )</td>
<td>154.963</td>
<td>162.699</td>
<td>193.133</td>
<td>257.959</td>
<td>304.531</td>
<td>378.099</td>
</tr>
<tr>
<td>( n=4 )</td>
<td>228.709</td>
<td>236.328</td>
<td>273.482</td>
<td>351.895</td>
<td>405.011</td>
<td>492.171</td>
</tr>
</tbody>
</table>

Lowest mode not from dilaton, but from “exotic polarization” – in 11D notation:

\[
\delta g_{44} = -r^2 \frac{f}{L^2} H(r)G(x), \quad \delta g_{\mu\nu} = \frac{r^2}{L^2} \left[ \frac{1}{4} H(r) \eta_{\mu\nu} - \left( \frac{1}{4} + \frac{3R^6}{5r^6 - 2R^6} \right) H(r) \frac{\partial_{\mu} \partial_{\nu}}{M^2} \right] G(x)
\]

\[
\delta g_{11,11} = \frac{r^2}{L^2} \frac{1}{4} H(r)G(x), \quad \delta g_{rr} = -\frac{L^2}{r^2} f^{-1} \frac{3R^6 H(r) G(r)}{5r^6 - 2R^6}, \quad \delta g_{r\mu} = \frac{90r^7 R^6 H(r) \partial_\mu G(x)}{M^2 L^2 (5r^6 - 2R^6)^2}
\]
Lattice glueballs vs. supergravity glueballs

Morningstar & Peardon hep-lat/9901004:

Brower, Mathur & Tan 2000:

(mass scales matched on $2^{++}$) $\rightarrow$ seemingly good qualitative agreement!
Quantitative predictions: vector meson spectrum

Parameter-free prediction of (axial-)vector meson mass pattern:

<table>
<thead>
<tr>
<th>Isotriplet Meson</th>
<th>$\lambda_n = m^2/M_{KK}^2$</th>
<th>$m/m_\rho$</th>
<th>$(m/m_\rho)^{\text{exp.}}$</th>
<th>$(m/m_\rho)^{N\rightarrow\infty}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1^{--} (\rho)$</td>
<td>0.669314</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$1^{++} (a_1)$</td>
<td>1.568766</td>
<td>1.531</td>
<td>1.59(5)</td>
<td>1.86(2)</td>
</tr>
<tr>
<td>$1^{--} (\rho^*)$</td>
<td>2.874323</td>
<td>2.072</td>
<td>1.89(3)</td>
<td>2.40(4)</td>
</tr>
<tr>
<td>$1^{++} (a_1^*)$</td>
<td>4.546104</td>
<td>2.606</td>
<td>2.12(3)</td>
<td>2.98(5)</td>
</tr>
</tbody>
</table>

(last column from lattice study by Bali et al. JHEP 06, 071 (2013))

agreement within $\lesssim 20\%$

not bad, given that WSS is not yet large-$N$ QCD (in particular at scales $\gtrsim M_{KK}$)

(near-perfect agreement for $m_{a_1}/m_\rho$ with real QCD certainly fortuitous)
Nonchiral enhancement in mass-deformed WSS?


Current quark masses can be introduced in principle through deformations of the WSS model by either world-sheet instantons [Hashimoto, Hirayama, Liu & Yee 2008] or with bifundamental background scalar $T$ [Bergman, Seki & Sonnenschein 2007] both lead to

$$\int d^4x \int_{u_{KK}}^{\infty} du \ h(u) \ Tr \ (T(u) \ P \ e^{-i \int dz A_z(z,x)} + h.c.),$$

where $h(u)$ includes metric (glueball) fields.

Choosing appropriate boundary conditions for $T$, the quark mass matrix arises through

$$\int_{u_{KK}}^{\infty} du \ h(u) \ T(u) \propto M = \text{diag}(m_u, m_d, m_s),$$

thereby realizing a Gell-Mann-Oakes-Renner relation.
Witten-Veneziano mass term

Already in chiral model:

WSS contains (fully determined) Witten-Veneziano mass term for singlet $\eta_0$ pseudoscalar from $\text{U}(1)_A$ anomaly contributions $\sim 1/N_c$

$$m_0^2 = \frac{N_f}{27\pi^2 N_c} \lambda^2 M_{KK}^2$$

from $S_{C_1} = -\frac{1}{4\pi (2\pi l_s)^6} \int d^{10} x \sqrt{-g} |\tilde{F}_2|^2$ with

$$\tilde{F}_2 = \frac{6\pi u_{KK}^3 M_{KK}^{-1}}{u^4} \left( \theta + \frac{\sqrt{2N_f}}{f_\pi} \eta_0 \right) du \wedge dx^4,$$

where $\theta$ is the QCD theta angle and $\eta_0(x) = \frac{f_\pi}{\sqrt{2N_f}} \int dz \text{Tr} A_z(z,x)$.

With $N_f = N_c = 3$, $M_{KK} = 949$ MeV, $\lambda = 16.63 \ldots 12.55$: $\boxed{m_0 = 967 \ldots 730}$ MeV
Witten-Veneziano mass term

With finite quark masses $\eta_0$ and $\eta_8$ no longer mass eigenstates.

Diagonalizing:

$$N_f = N_c = 3, \quad M_{KK} = 949 \text{ MeV}, \quad \lambda = 16.63 \ldots 12.55: \quad \boxed{m_0 = 967 \ldots 730 \text{ MeV}},$$

(with $\mathcal{M} = \text{diag}(\hat{m}, \hat{m}, m_s)$, fixing $m_\pi = 140 \text{ MeV}$ and $m_K = 497 \text{ MeV})$ →

$$m_\eta = 518 \ldots 476 \text{ MeV}, \quad m_{\eta'} = 1077 \ldots 894 \text{ MeV},$$

$$\theta_P = -14.4^\circ \ldots -24.2^\circ,$$

nice ballpark:

light meson decays values favors [Ambrosini 2009, Pham 2015]: $\theta_P \approx -14^\circ$

radiative charmonium decay [Gerard 2004, 2013]: $\theta_P \approx -21^\circ$

$\Gamma(\eta' \to 2\gamma)/\Gamma(\eta \to 2\gamma)$ leads to [PDG]: $\theta_P = (-18 \pm 2)^\circ$
Constraints on $\eta\eta'$ rates for $f_0(1710)$ as $\approx$ pure glueball

Relaxing $x = 1$: [F. Brünner & AR, PRD92, 1510.07605]

WSS model gives flavor asymmetries consistent with experimental results for $f_0(1710)$ in as long as $\Gamma(G \to \eta\eta')/\Gamma(G \to \pi\pi) \lesssim 0.04$ (upper limit from WA102: < 0.18)
Tensor glueball decay rates in Sakai-Sugimoto model

Tensor glueball in WSS, and extrapolated to higher mass:

<table>
<thead>
<tr>
<th>decay</th>
<th>M</th>
<th>( \Gamma/M[T(M)] )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( T \to 2\pi )</td>
<td>1487</td>
<td>0.013\ldots0.018</td>
</tr>
<tr>
<td>( T \to 2K )</td>
<td>1487</td>
<td>0.004\ldots0.006</td>
</tr>
<tr>
<td>( T \to 2\eta )</td>
<td>1487</td>
<td>0.0005\ldots0.0007</td>
</tr>
<tr>
<td>( T ) (total)</td>
<td>1487</td>
<td>( \approx 0.02\ldots0.03 )</td>
</tr>
<tr>
<td>( T \to 2\rho \to 4\pi )</td>
<td>2000</td>
<td>0.135\ldots0.178</td>
</tr>
<tr>
<td>( T \to 2K^* \to 2(K\pi) )</td>
<td>2000</td>
<td>0.119\ldots0.177</td>
</tr>
<tr>
<td>( T \to 2\omega \to 6\pi )</td>
<td>2000</td>
<td>0.045\ldots0.059</td>
</tr>
<tr>
<td>( T \to 2\pi )</td>
<td>2000</td>
<td>0.014\ldots0.018</td>
</tr>
<tr>
<td>( T \to 2K )</td>
<td>2000</td>
<td>0.010\ldots0.013</td>
</tr>
<tr>
<td>( T \to 2\eta )</td>
<td>2000</td>
<td>0.0018\ldots0.0024</td>
</tr>
<tr>
<td>( T ) (total)</td>
<td>2000</td>
<td>( \approx 0.3\ldots0.45 )</td>
</tr>
<tr>
<td>( T ) (total)</td>
<td>2400</td>
<td>( \approx 0.45\ldots0.6 )</td>
</tr>
</tbody>
</table>

Very broad tensor glueball, if at 2.4 GeV (probably unobservable)

With a mass of 2 GeV, width larger but perhaps comparable with that of the rather broad tensor meson \( f_2(1950) \), which has \( \Gamma/M = 0.24(1) \).

Very narrow (unconfirmed) candidate \( f_J(2220) \) not compatible with WSS