Dynamic Mesoscopic Conductors
Single Electron Sources, Full Counting Statistics, Thermal Machines

Patrick P. Hofer
Department of Theoretical Physics, University of Geneva

supervised by

Markus Büttiker
Christian Flindt
Eugene Sukhorukov
Aashish Clerk
Nicolas Brunner
Goal of Thesis
To contribute to the development of technologies that make use of quantum effects e.g. quantum computer, quantum thermal machines,...
Overview & Motivation

Goal of Thesis
To contribute to the development of technologies that make use of quantum effects *e.g.* quantum computer, quantum thermal machines,...

Dynamic Mesoscopic Conductors
- Existing semi-conductor technology
- Quantum transport has long history

PRL 60, 848 (1988)
Overview & Motivation

**Goal of Thesis**
To contribute to the development of technologies that make use of quantum effects *e.g. quantum computer, quantum thermal machines,...*

**Dynamic Mesoscopic Conductors**
- Existing semi-conductor technology
- Quantum transport has long history

**Single Electron Sources**
- Control on single particle level
- Single electrons as carriers of quantum information

---

**Selected Publications**

- *Quantum Spin Hall Insulator*: P. P. Hofer and M. Büttiker, PRB *88*, 241308(R) (2013)
**Overview & Motivation**

**Goal of Thesis**
To contribute to the development of technologies that make use of quantum effects *e.g.* quantum computer, quantum thermal machines,...

**Dynamic Mesoscopic Conductors**
- Existing semi-conductor technology
- Quantum transport has long history

**Single Electron Sources**
- Control on single particle level
- Single electrons as carriers of quantum information

**Full Counting Statistics & Waiting time distributions**
- Probabilistic nature of electron transport
- Quantum fluctuations - Non-classical behaviour

**Selected Publications**
Negative Full Counting Statistics: P. P. Hofer and A. A. Clerk, PRL 116, 013603 (2016)
Overview & Motivation

Goal of Thesis
To contribute to the development of technologies that make use of quantum effects e.g. quantum computer, quantum thermal machines,...

Dynamic Mesoscopic Conductors
- Existing semi-conductor technology
- Quantum transport has long history

Single Electron Sources
- Control on single particle level
- Single electrons as carriers of quantum information

Full Counting Statistics & Waiting time distributions
- Probabilistic nature of electron transport
- Quantum fluctuations - Non-classical behaviour

Thermal Machines
- Energy harvesting, refrigeration, and thermometry
- Ideal test-beds to study quantum thermodynamics

Selected Publications
Quantum Heat Engines & Outline

**Wave-nature of electrons**

PRB 91, 195406 (2015)

P. P. Hofer, B. Sothmann

**Particle-nature of Photons**

PRB 93, 041418(R) (2016)

P. P. Hofer, J.-R. Souquet, A. A. Clerk
Thermoelectricity

- Thermal gradient
- Energy dependent transmission
- Separating electrons from holes

⇒ Thermoelectric Current
Thermoelectricity

- Thermal gradient
- Energy dependent transmission
- Separating electrons from holes

$\Rightarrow$ Thermoelectric Current
Thermoelectricity from interference

- Two paths connecting hot and cold contacts
- Total transmission: \( T(E) = |t_a(E) + t_b(E)|^2 \)
Thermoelectricity from interference

- Two paths connecting hot and cold contacts
- Total transmission: $T(E) = |t_a(E) + t_b(E)|^2$
- Purely interference if: $t_\alpha(E) = A_\alpha e^{i\varphi_\alpha(E)}$

$$T(E) = A_a^2 + A_b^2 + 2A_aA_b \cos(\varphi_a(E) - \varphi_b(E))$$

$\Rightarrow$ Thermoelectricity from wave nature of electrons
Mach-Zehnder Interferometer

- Quantum Hall regime: electrons propagate along edge channels
- Path-length difference: $\tau = (L_a - L_b)/v_D$
- Phase difference: $\varphi_a - \varphi_b = \phi + E\tau$
- Thermopower requires current against voltage: $P = IV$
\[ T = 240 \text{ mK} \quad \Delta T = 60 \text{ mK} \quad \eta_C = 25\% \quad v_D = 0.5 \cdot 10^5 \text{ m/s} \]

- Non-interacting scattering theory
- Linear response

\[ I \approx 44 \text{ pA} \]
\[ P \approx 0.14 \text{ fW} \]
\[ \eta \approx 1.05\% \]
\( T = 240 \text{ mK} \quad \Delta T = 60 \text{ mK} \quad \eta_C = 25\% \quad v_D = 0.5 \cdot 10^5 \text{ m/s} \)

- Non-interacting scattering theory

- Linear response

\[
I \approx 44 \text{ pA} \\
P \approx 0.14 \text{ fW} \\
\eta \approx 1.05\%
\]

\[
I \approx 53 \text{ nA} \\
P \approx 0.36 \text{ fW} \\
\eta \approx 3\%
\]
Superconducting Circuits

Heat current from hot to cold bath drives charge current against the voltage bias
Heat current from hot to cold bath drives charge current against the voltage bias.

- Resonance condition:
  \[2eV = \Omega_h - \Omega_c\]

- Tunneling Cooper pair exchanges hot with cold photon.

- Cooper pairs carry no heat.
Superconducting Circuits

Heat current from hot to cold bath drives charge current against the voltage bias.

- Resonance condition:
  \[ 2eV = \Omega_h - \Omega_c \]

- Tunneling Cooper pair exchanges hot with cold photon.

- Cooper pairs carry no heat.

Heat carried by photons, work provided by Cooper pairs.
Efficiency

**Single Cooper pair**

- Work performed: \( W = 2eV = \Omega_h - \Omega_c \)
- Heat provided by hot bath: \( Q_{in} = \Omega_h \)
Efficiency

Single Cooper pair

- Work performed: \( W = 2eV = \Omega_h - \Omega_c \)
- Heat provided by hot bath: \( Q_{in} = \Omega_h \)

Universal efficiency: \( \eta = W/Q_{in} = 1 - \Omega_c/\Omega_h \)
**Efficiency**

**Single Cooper pair**

- Work performed:  \( W = 2eV = \Omega_h - \Omega_c \)
- Heat provided by hot bath:  \( Q_{in} = \Omega_h \)

**Universal efficiency:**

\[
\eta = \frac{W}{Q_{in}} = 1 - \frac{\Omega_c}{\Omega_h}
\]

- Efficiency bounded by Carnot efficiency as long as work is positive

\[
\eta \leq 1 - \frac{T_c}{T_h}
\]

2\textsuperscript{nd} Law of thermodynamics
More quantitative analysis

\[
\hat{H} = \sum_{\alpha=\text{c},\text{h}} \Omega_{\alpha} \hat{a}_{\alpha}^{\dagger} \hat{a}_{\alpha} - E_J \cos (2eVt + \phi_c + \phi_h)
\]

\[
\hat{I} = -2eE_J \sin (2eVt + \phi_c + \phi_h) \quad \phi_{\alpha} = 2\lambda_{\alpha} (\hat{a}_{\alpha} + \hat{a}_{\alpha}^{\dagger})
\]

Armour et al. PRL 111, 247001 (2013), Gramich et al. PRL 111, 247002 (2013)

Josephson junction driven by voltage and cavity fluxes
More quantitative analysis

\[
\hat{H} = \sum_{\alpha = c, h} \Omega_\alpha \hat{a}^\dagger_\alpha \hat{a}_\alpha - E_J \cos (2eV t + \hat{\varphi}_c + \hat{\varphi}_h)
\]

\[
\hat{I} = -2eE_J \sin (2eV t + \hat{\varphi}_c + \hat{\varphi}_h) \quad \hat{\varphi}_\alpha = 2\lambda_\alpha (\hat{a}_\alpha + \hat{a}^\dagger_\alpha)
\]

Armour et al. PRL 111, 247001 (2013), Gramich et al. PRL 111, 247002 (2013)

Josephson junction driven by voltage and cavity fluxes

- Rotating Wave Approximation (RWA)
- Coupling to baths: Lindblad master equation
- Master equation solved using QuTiP
Universal efficiency

Photons in hot cavity

$$\partial_t \langle \hat{n}_h \rangle = -I/(2e) + J_h/\Omega_h$$
Universal efficiency

Photons in hot cavity

\[ \partial_t \langle \hat{n}_h \rangle = -I/(2e) + J_h/\Omega_h \]

- Steady state: \( I/(2e) = J_h/\Omega_h = -J_c/\Omega_c \)
Universal efficiency

Photons in hot cavity

\[ \partial_t \langle \hat{n}_h \rangle = -I/(2e) + J_h/\Omega_h \]

- Steady state: \( I/(2e) = J_h/\Omega_h = -J_c/\Omega_c \)
- Power: \( P = IV \)
- Efficiency: \( \eta = P/J_h = 1 - \Omega_c/\Omega_h \)
Universal efficiency

Photons in hot cavity

\[ \partial_t \langle \hat{n}_h \rangle = -\frac{I}{2e} + \frac{J_h}{\Omega_h} \]

- Steady state: \( I/(2e) = J_h/\Omega_h = -J_c/\Omega_c \)
- Power: \( P = I V \)
- Efficiency: \( \eta = P/J_h = 1 - \Omega_c/\Omega_h \)
- Entropy: \( \partial_t S_{tot} = -\frac{J_h}{T_h} - \frac{J_c}{T_c} = \frac{I}{2e} \left( \frac{\Omega_c}{T_c} - \frac{\Omega_h}{T_h} \right) \geq 0 \)
Universal efficiency

Photons in hot cavity

\[ \partial_t \langle \hat{n}_h \rangle = -\frac{I}{2e} + J_h / \Omega_h \]

Condition for positive work

\[ \frac{\Omega_c}{T_c} - \frac{\Omega_h}{T_h} \geq 0 \]

Universal efficiency

\[ \eta = 1 - \frac{\Omega_c}{\Omega_h} \leq 1 - \frac{T_c}{T_h} \]

Approximations: RWA and weak coupling to baths
Performance

- $\Omega_h/2\pi = 13.5$ GHz
- $T_h = 960$ mK ($n_B^h = 1$)
- $\eta_C = 93.75\%$
- $E_J = 1.24$ μeV ($0.3 \cdot 2\pi$ GHz)
- $\Omega_c/2\pi = 3$ GHz
- $T_c = 60$ mK ($n_B^c = 0.1$)
- $\kappa/2\pi = 0.06$ GHz
- $2eV = 43.4$ μeV ($10.5 \cdot 2\pi$ GHz)

\[
I = 23\, \text{pA} \quad P = 0.5\, \text{fW} \quad \eta = 77.8\% 
\]

Constraints

- Rotating wave approximation: $E_J \ll \Omega_h, \Omega_c, 2eV$
- Master equation: $\kappa \ll \Omega_c, \Omega_h, 2eV$
Conclusions & Comparison

Wave-nature of electrons

![Diagram of wave-nature of electrons]

\[ \Delta T \simeq 60 \text{ mK} \]
\[ I \simeq 53 \text{ nA} \]
\[ P \simeq 0.36 \text{ fW} \]
\[ \eta \simeq 3\% \]

PRB 91, 195406 (2015)

P. P. Hofer, B. Sothmann

Particle-nature of Photons

![Diagram of particle-nature of Photons]

\[ \Delta T \simeq 900 \text{ mK} \]
\[ I \simeq 23 \text{ nA} \]
\[ P \simeq 0.5 \text{ fW} \]
\[ \eta \simeq 77.8\% \]

PRB 93, 041418(R) (2016)

P. P. Hofer, J.-R. Souquet, A. A. Clerk
**Wave-nature of electrons**
- Optimal quantum interference heat engine
  P. Samuelsson, S. Kheradsoud, B. Sothmann, PRL 118, 256801 (2017)

**Particle-nature of photons**
- Refrigerator
- Thermometer
  P. P. Hofer, J. B. Brask, M. Perarnau-Llobet, N. Brunner, ArXiv:1703.03719 (accepted in PRL)
- Validity of local Lindblad master equation

Courtesy of J.-R. Souquet
**Wave-nature of electrons**

- Optimal quantum interference heat engine
  
P. Samuelsson, S. Kheradsoud, B. Sothmann, PRL 118, 256801 (2017)

**Particle-nature of photons**

- Refrigerator
  

- Thermometer
  
P. P. Hofer, J. B. Brask, M. Perarnau-Llobet, N. Brunner, ArXiv:1703.03719 (accepted in PRL)

- Validity of local Lindblad master equation
  

**Outlook**

- Role of (quantum) fluctuations

- Work extraction from quantum states
  
  Collaboration with N. Loerch & C. Bruder, University of Basel

- Limitations of classical models

Courtesy of J.-R. Souquet