# Dynamic Mesoscopic Conductors

## Single Electron Sources, Full Counting Statistics, Thermal Machines


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supervised by

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Christian Flindt

Eugene Sukhorukov

Aashish Clerk

Nicolas Brunner

# Overview & Motivation

**Goal of Thesis**

To contribute to the development of technologies that make use of quantum effects *e.g. quantum computer, quantum thermal machines,...*

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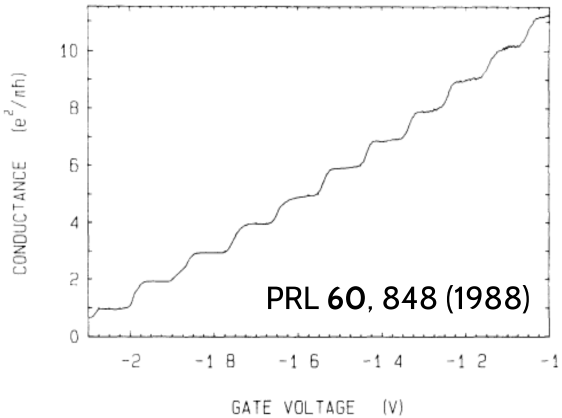
**Dynamic Mesoscopic Conductors**

- Existing semi-conductor technology
- Quantum transport has long history




PRL **60**, 848 (1988)

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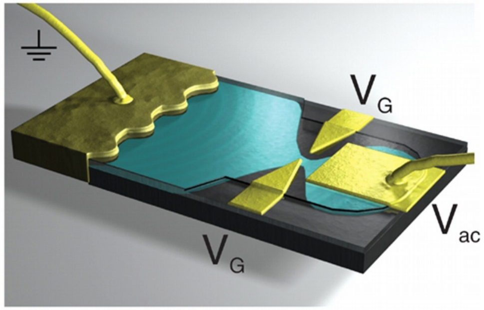
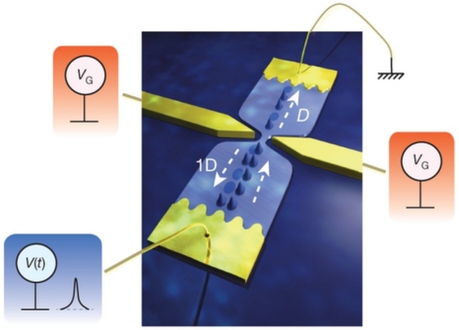
- Single electrons as carriers of quantum information




Science **313**, 499 (2006)




Nature **502**, 659 (2013)

**Selected Publications**

Quantum Spin Hall Insulator: P. P. Hofer and M. Büttiker, PRB **88**, 241308(R) (2013)

Single-Electron Entanglement: D. Dasenbrook, J. Bowles, J. B. Brask, P. P. Hofer, C. Flindt, N. Brunner, NJP **18**, 043036 (2016)

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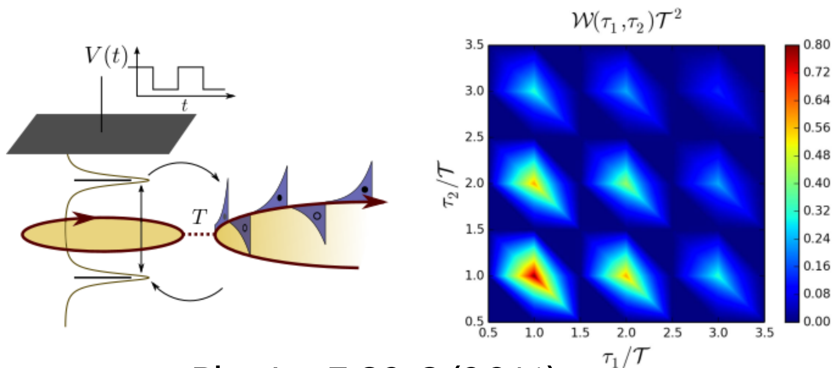
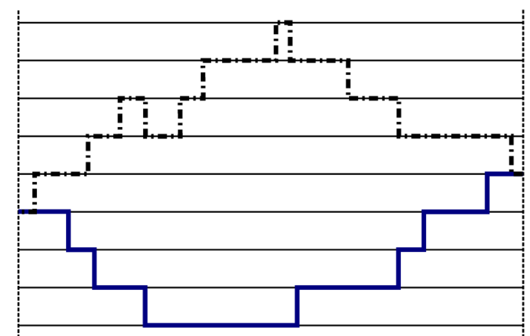
### Full Counting Statistics & Waiting time distributions

- Probabilistic nature of electron transport
- Quantum fluctuations - Non-classical behaviour




Physica E **82**, 3 (2016)



PRL **116**, 013603 (2016)

## Selected Publications

Waiting Time Distributions: D. Dasenbrook, P. P. Hofer, C. Flindt, PRB **91**, 195420 (2015)

Negative Full Counting Statistics: P. P. Hofer and A. A. Clerk, PRL **116**, 013603 (2016)

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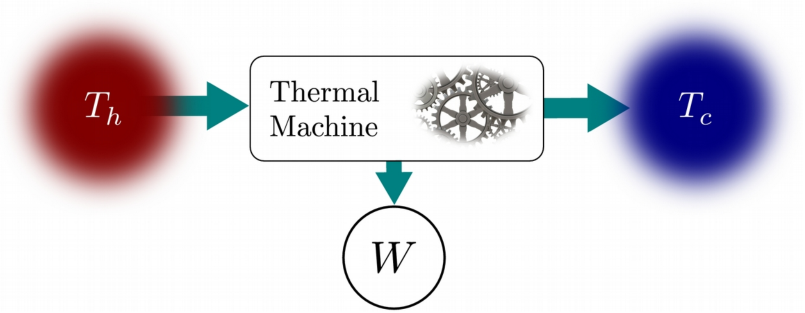
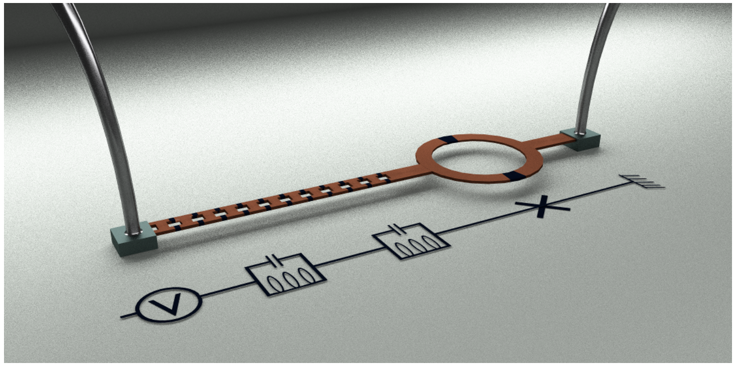
**Thermal Machines**

- Energy harvesting, refrigeration, and thermometry
- Ideal test-beds to study quantum thermodynamics



Courtesy of J.-R. Souquet

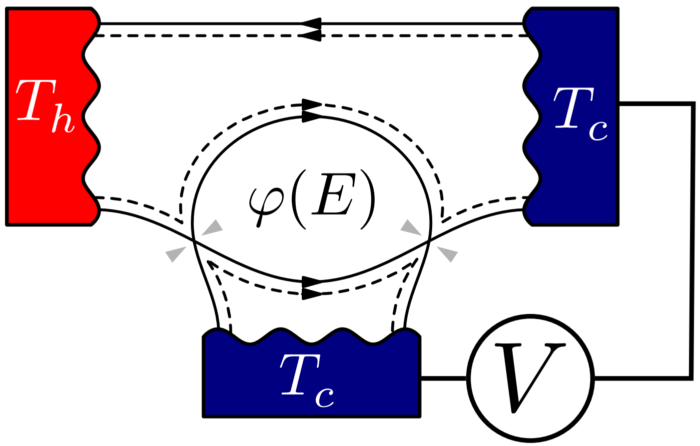
**Selected Publications**

Heat Engines: P. P. Hofer, B. Sothmann, PRB **91**, 195406 (2015), P. P. Hofer, J.-R. Souquet, A. A. Clerk, PRB **93**, 041418(R) (2016)

Refrigerator: P. P. Hofer, M. Perarnau-Llobet, J. B. Brask, R. Silva, M. Huber, N. Brunner, PRB **94**, 235420 (2016)

# Quantum Heat Engines & Outline

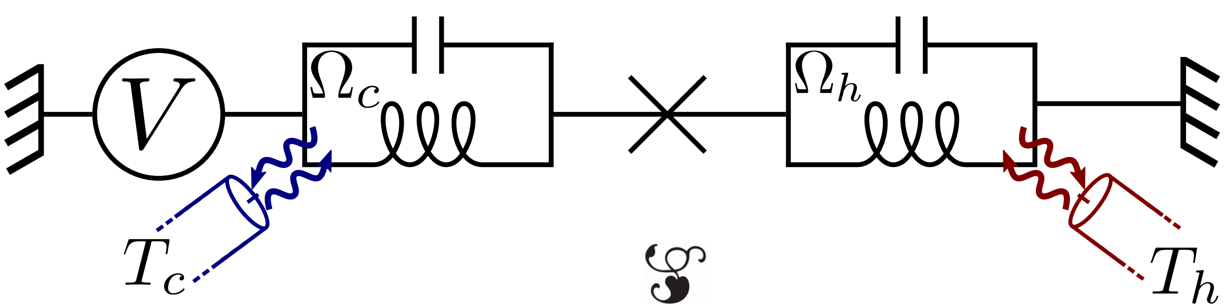
**Wave-nature of electrons**




PRB **91**, 195406 (2015)

P. P. Hofer, B. Sothmann

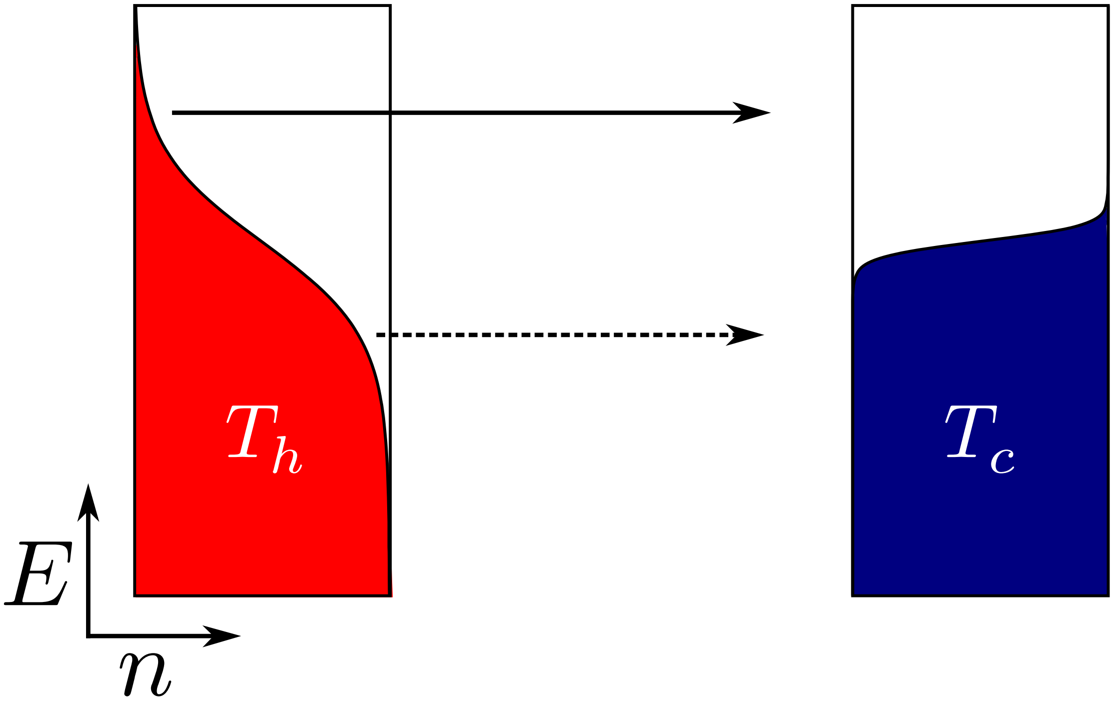
**Particle-nature of Photons**




PRB **93**, 041418(R) (2016)

P. P. Hofer, J.-R. Souquet, A. A. Clerk

# Thermoelectricity




- Thermal gradient
- Energy dependent transmission
- Separating electrons from holes

$\Rightarrow$ **Thermoelectric Current**

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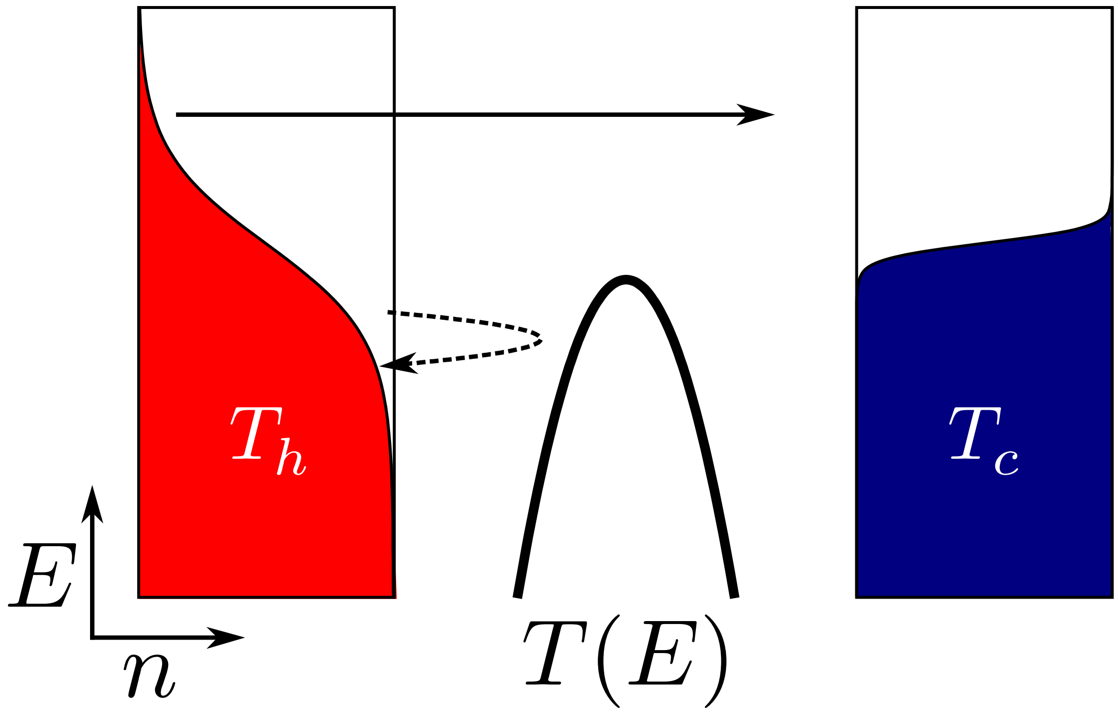


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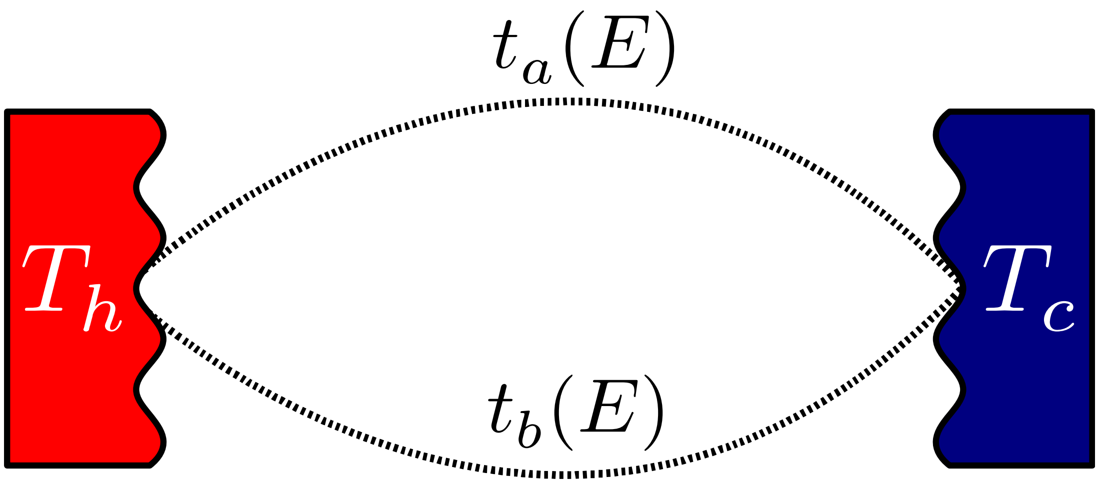
# Thermoelectricity from interference




- Two paths connecting hot and cold contacts
- Total transmission: $T(E) = |t_a(E) + t_b(E)|^2$

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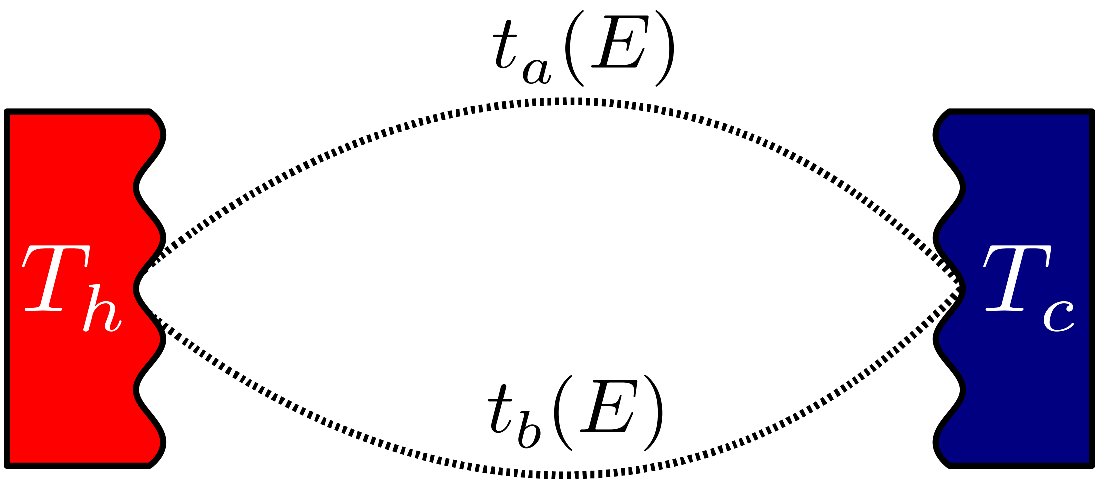
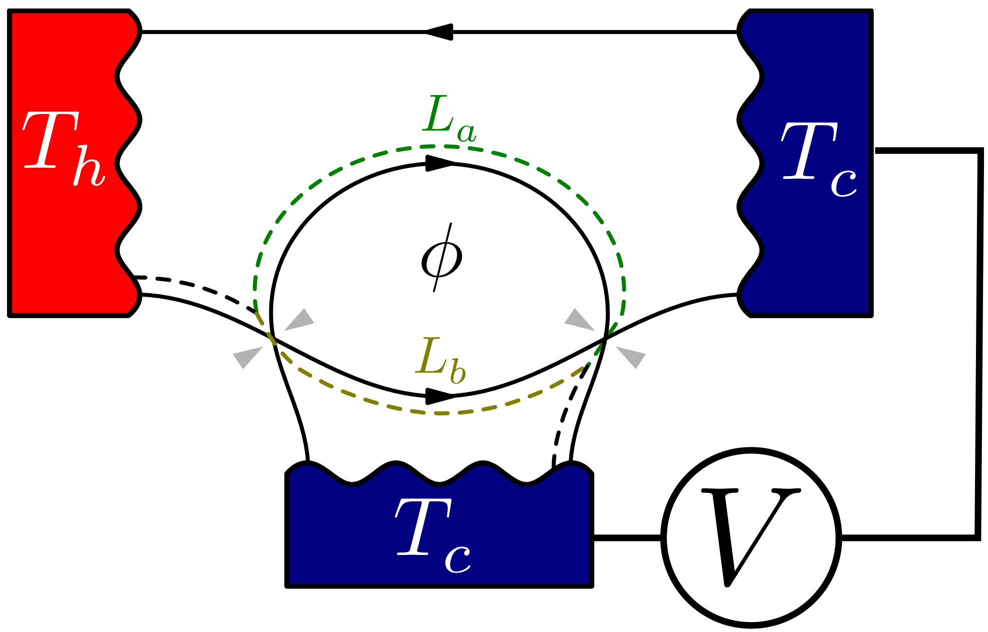



- Two paths connecting hot and cold contacts
- Total transmission: $T(E) = |t_a(E) + t_b(E)|^2$
- Purely interference if: $t_\alpha(E) = A_\alpha e^{i\varphi_\alpha(E)}$

$$T(E) = A_a^2 + A_b^2 + 2A_aA_b\cos(\varphi_a(E) - \varphi_b(E))$$

$\Rightarrow$ **Thermoelectricity from wave nature of electrons**

# Mach-Zehnder Interferometer

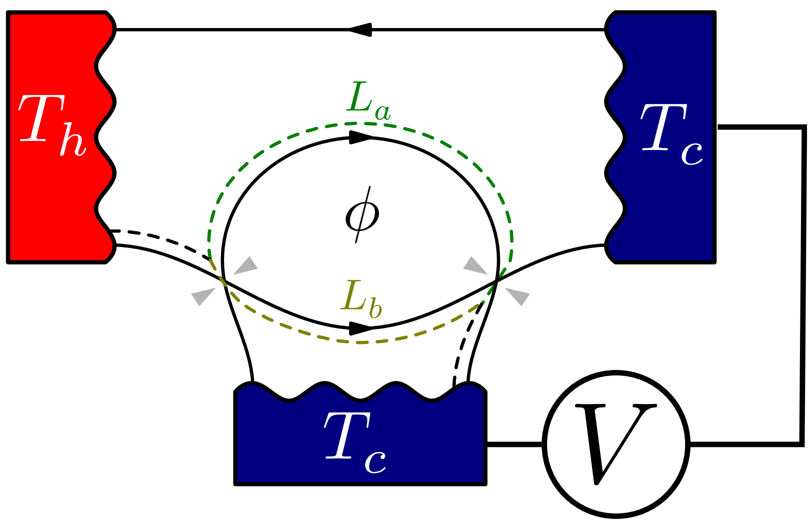



- Quantum Hall regime: electrons propagate along edge channels
- Path-length difference: $\tau = (L_a - L_b)/v_D$
- Phase difference: $\varphi_a - \varphi_b = \phi + E\tau$
- Thermopower requires current against voltage: $P = IV$

# Performance

$$T = 240\,\mathrm{mK} \quad \Delta T = 60\,\mathrm{mK} \quad \eta_C = 25\,\% \quad v_D = 0.5 \cdot 10^5\,\mathrm{m/s}$$

- Non-interacting scattering theory
- Linear response




$$I \simeq 44\,\mathrm{pA}$$
$$P \simeq 0.14\,\mathrm{fW}$$
$$\eta \simeq 1.05\,\%$$

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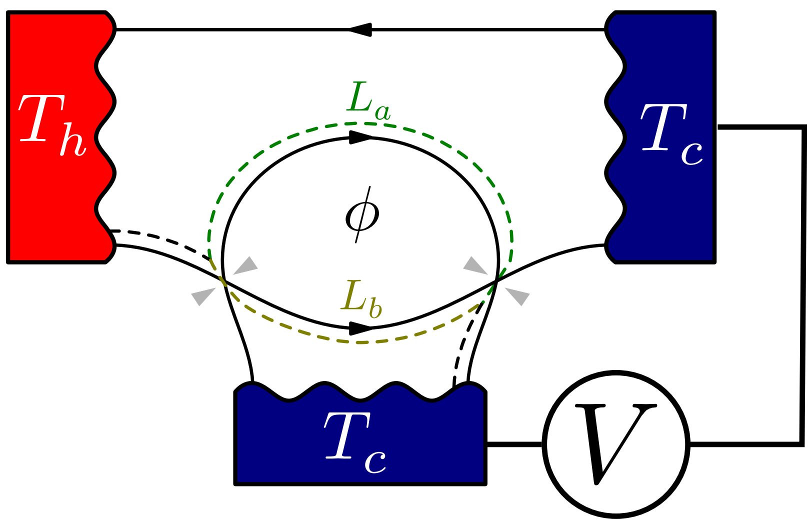


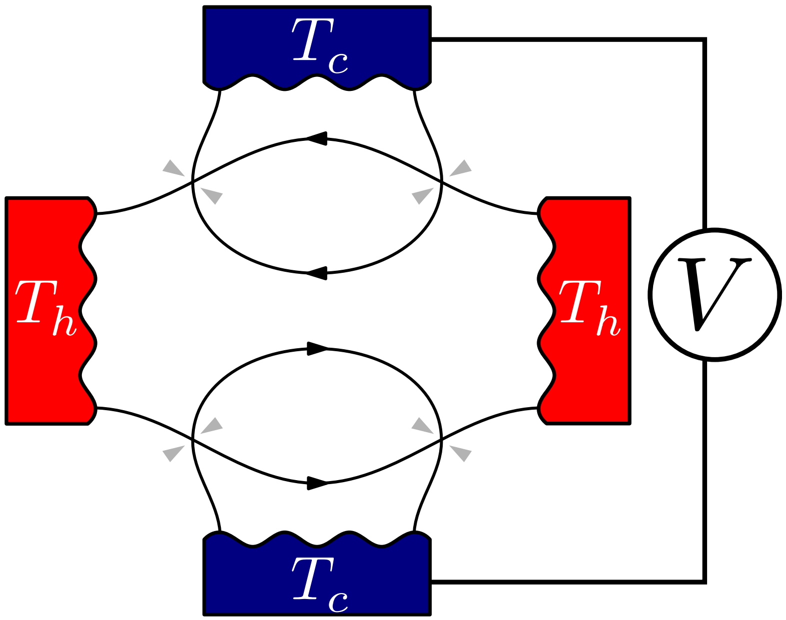


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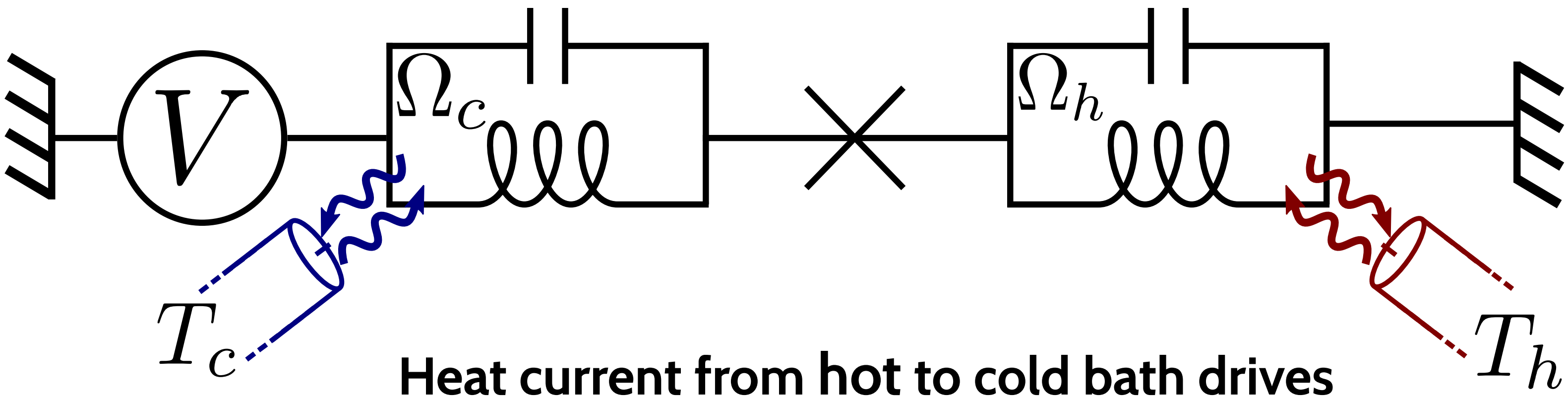
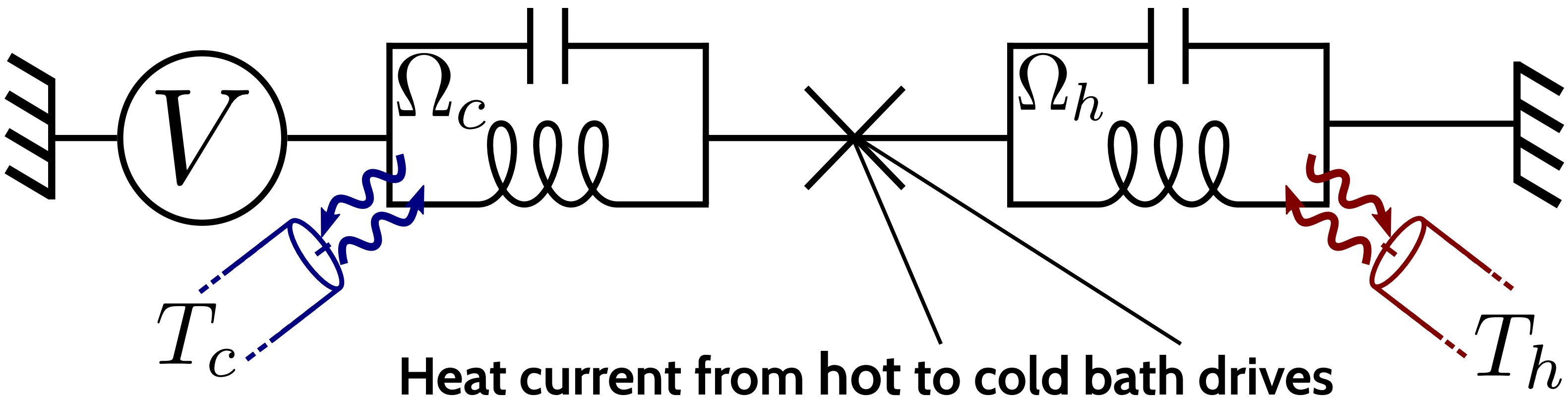
# Superconducting Circuits




**Heat current from hot to cold bath drives charge current against the voltage bias**

# Superconducting Circuits

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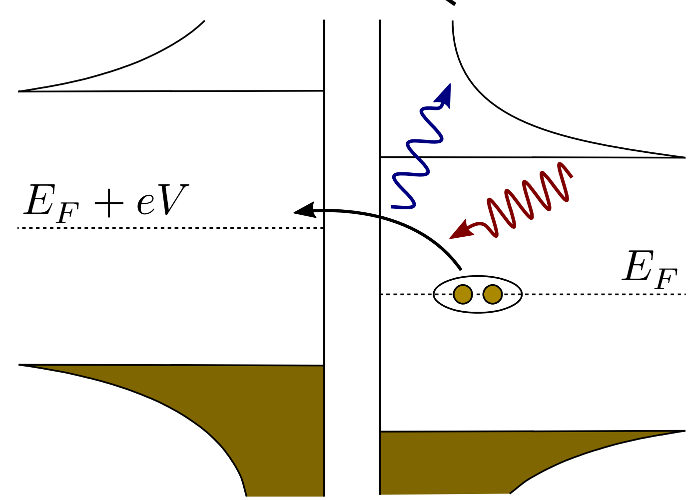
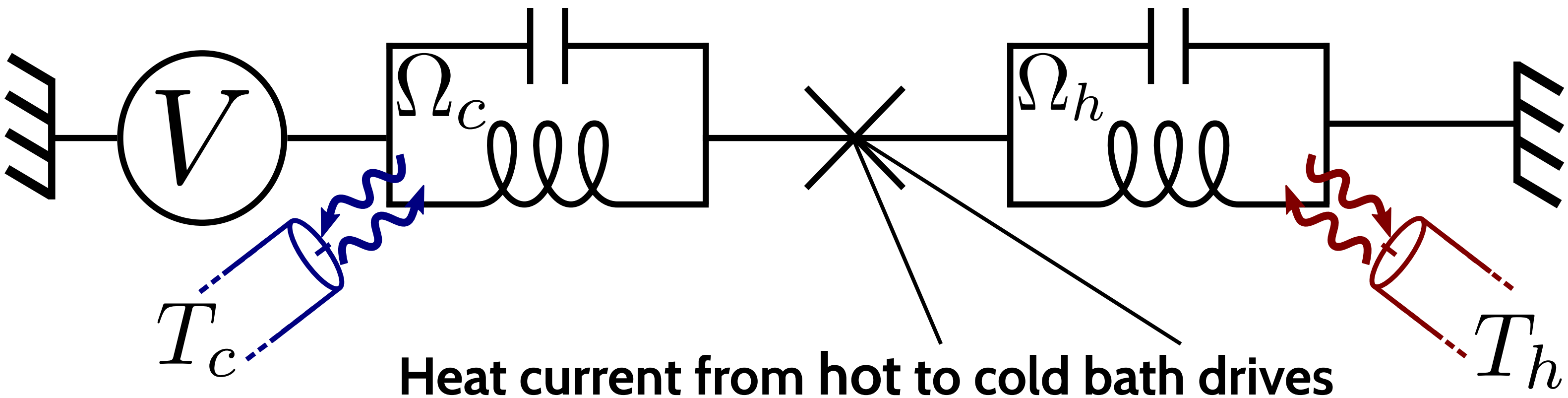



- Resonance condition:

$$2eV = \Omega_h - \Omega_c$$

- Tunneling Cooper pair exchanges hot with cold photon
- Cooper pairs carry no heat

# Superconducting Circuits

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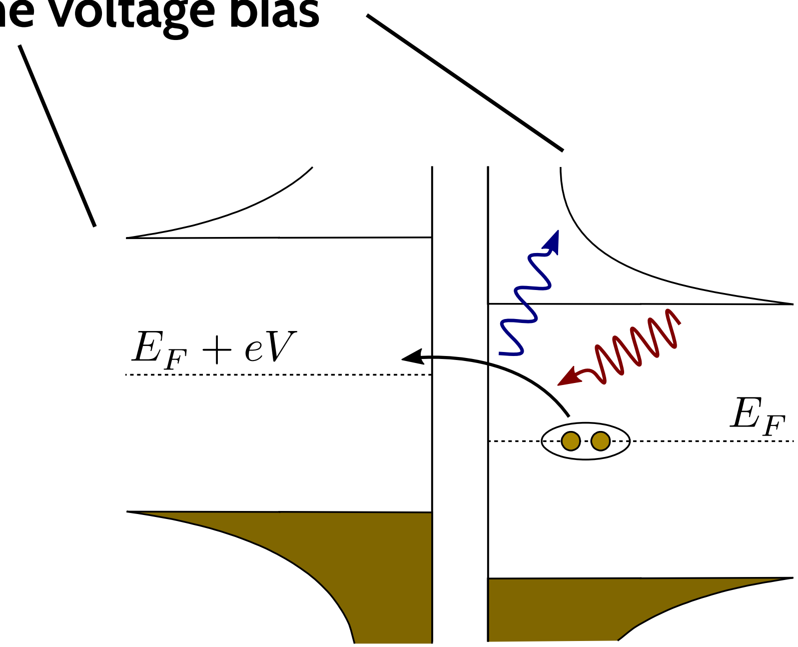


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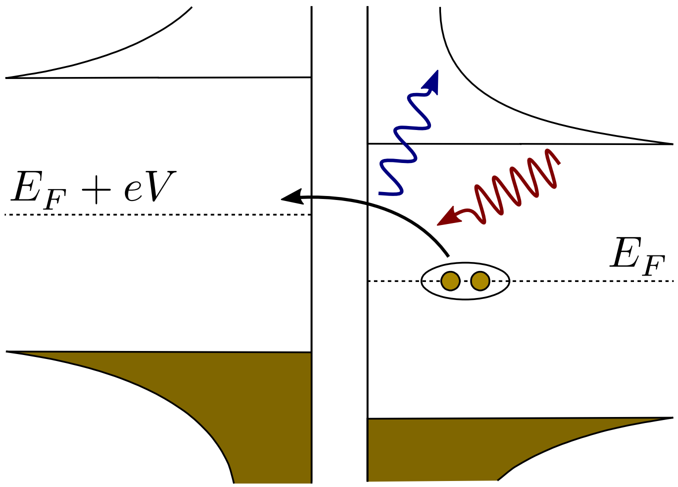
**Heat carried by photons, work provided by Cooper pairs**

# Efficiency

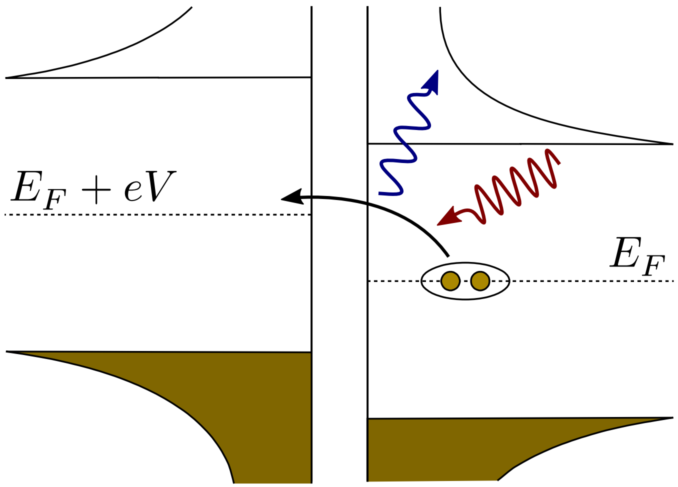



## Single Cooper pair

- Work performed: $W = 2eV = \Omega_h - \Omega_c$
- Heat provided by hot bath: $Q_{in} = \Omega_h$

# Efficiency

<u>Single Cooper pair</u>

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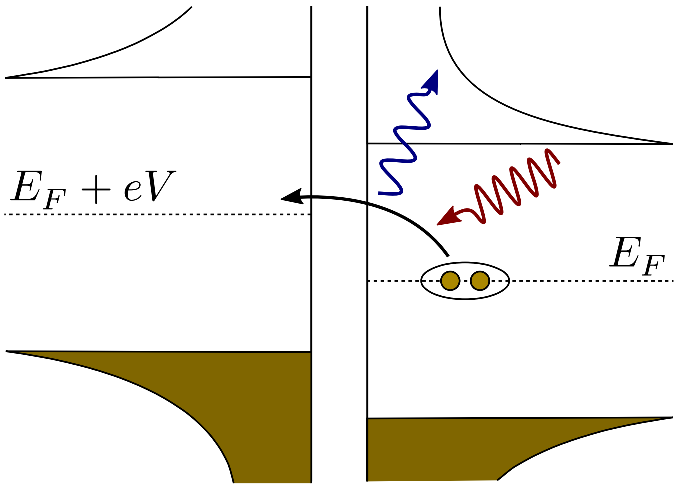


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- Efficiency bounded by Carnot efficiency as long as work is positive

2nd Law of thermodynamics

$$\eta \leq 1 - T_c/T_h$$

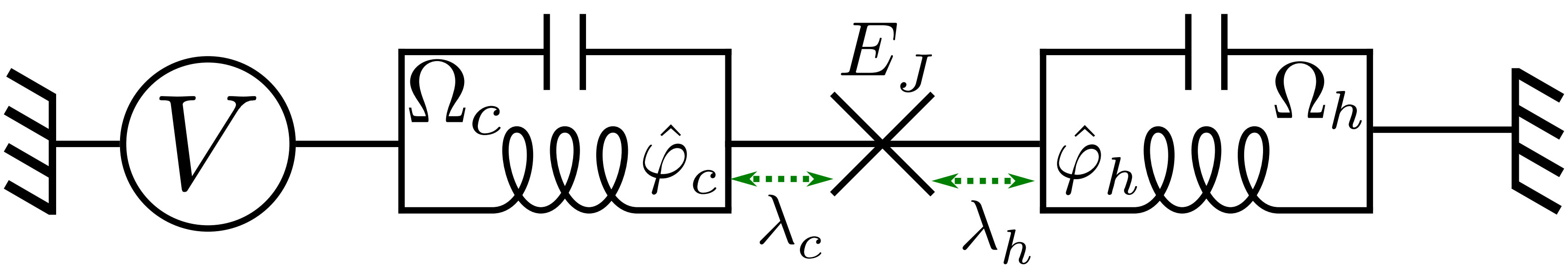
# More quantitative analysis

$$\hat{H} = \sum_{\alpha=c,h} \Omega_\alpha \hat{a}_\alpha^\dagger \hat{a}_\alpha - E_J \cos\left(2eVt + \hat{\varphi}_c + \hat{\varphi}_h\right)$$

$$\hat{I} = -2eE_J \sin\left(2eVt + \hat{\varphi}_c + \hat{\varphi}_h\right) \qquad \hat{\varphi}_\alpha = 2\lambda_\alpha \left(\hat{a}_\alpha + \hat{a}_\alpha^\dagger\right)$$

Armour *et al.* PRL **111**, 247001 (2013), Gramich *et al.* PRL **111**, 247002 (2013)

**Josephson junction driven by voltage and cavity fluxes**



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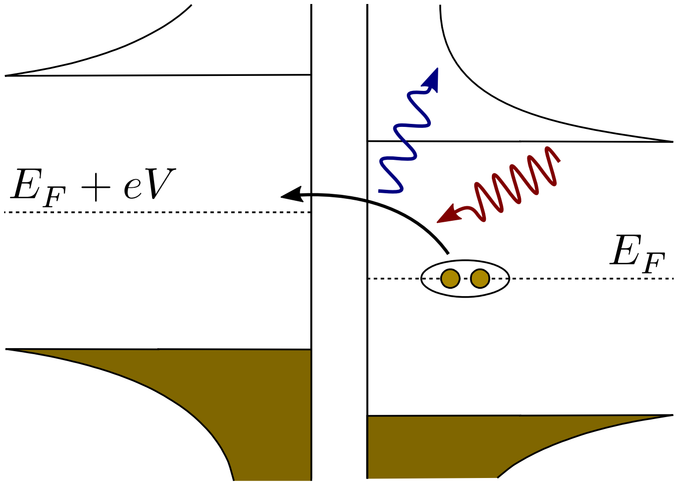
**Josephson junction driven by voltage and cavity fluxes**

- Rotating Wave Approximation (RWA)
- Coupling to baths: Lindblad master equation
- Master equation solved using QuTiP

# Universal efficiency

**Photons in hot cavity**

$$\partial_t \langle \hat{n}_h \rangle = -I/(2e) + J_h/\Omega_h$$



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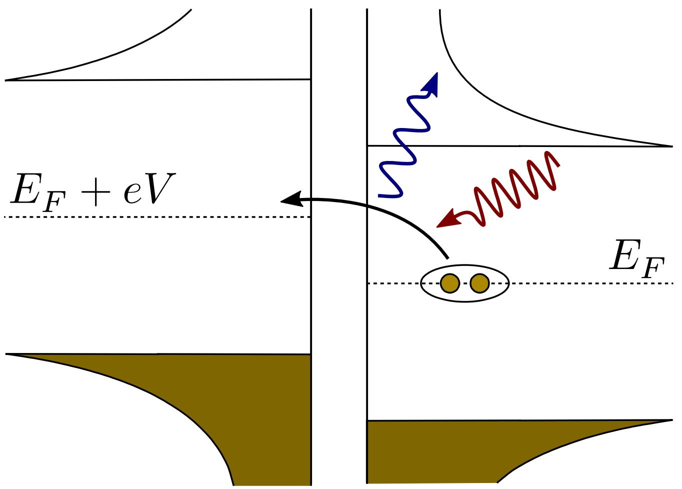

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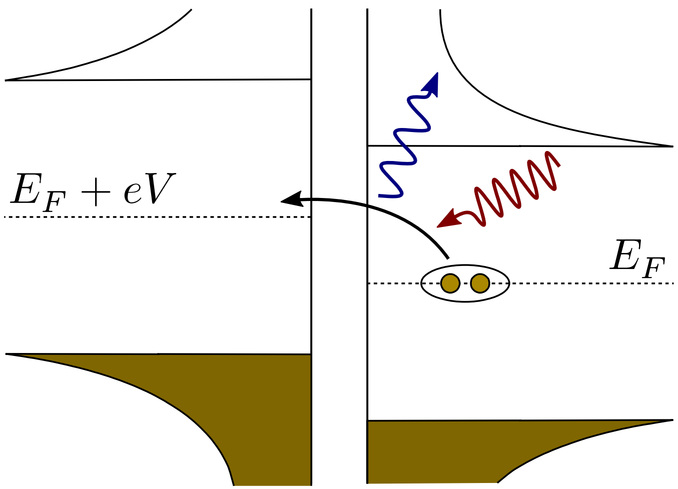

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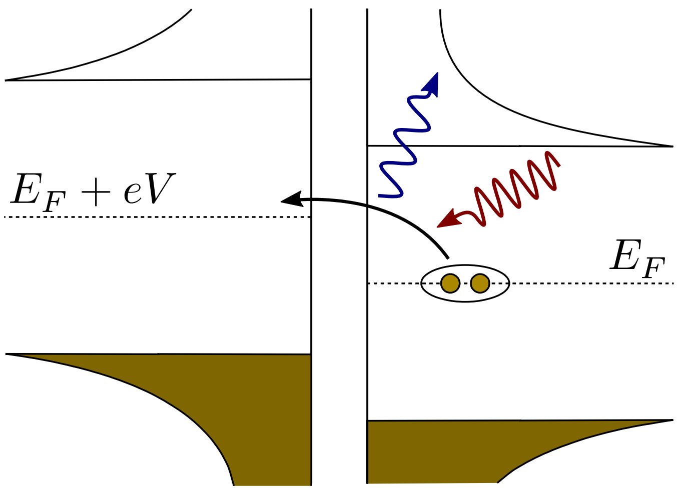



- Power: $P = IV$
- Efficiency: $\eta = P/J_h = 1 - \Omega_c/\Omega_h$

- Entropy: $\partial_t S_{\text{tot}} = -\frac{J_h}{T_h} - \frac{J_c}{T_c} = \frac{I}{2e}\left(\frac{\Omega_c}{T_c} - \frac{\Omega_h}{T_h}\right) \geq 0$

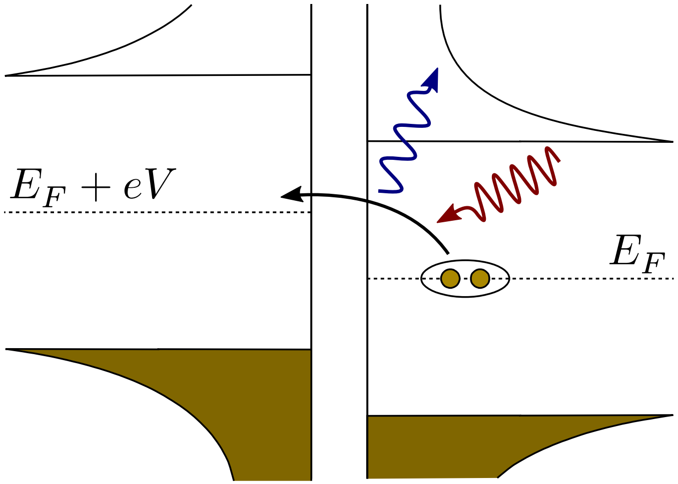
# Universal efficiency

**Photons in hot cavity**

$$\partial_t \langle \hat{n}_h \rangle = -I/(2e) + J_h/\Omega_h$$

**Condition for positive work**

$$\boxed{\frac{\Omega_c}{T_c} - \frac{\Omega_h}{T_h} \geq 0}$$




**Universal efficiency**

$$\boxed{\eta = 1 - \frac{\Omega_c}{\Omega_h} \leq 1 - \frac{T_c}{T_h}}$$

**Approximations: RWA and weak coupling to baths**

# Performance

- $\Omega_h/2\pi = 13.5\,\text{GHz}$
- $T_h = 960\,\text{mK}\ (n_B^h = 1)$
- $\eta_C = 93.75\,\%$
- $E_J = 1.24\,\mu\text{eV}\ (0.3 \cdot 2\pi\,\text{GHz})$
- $\Omega_c/2\pi = 3\,\text{GHz}$
- $T_c = 60\,\text{mK}\ (n_B^c = 0.1)$
- $\kappa/2\pi = 0.06\,\text{GHz}$
- $2eV = 43.4\,\mu\text{eV}\ (10.5 \cdot 2\pi\,\text{GHz})$

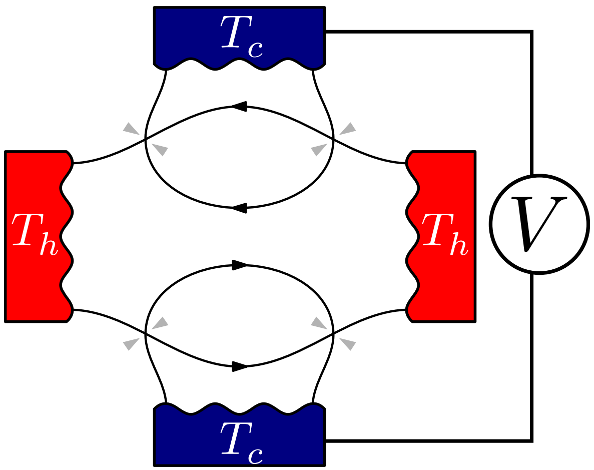
$$I = 23\,\text{pA} \qquad P = 0.5\,\text{fW} \qquad \eta = 77.8\%$$

**Constraints**

- Rotating wave approximation: $E_J \ll \Omega_h, \Omega_c, 2eV$
- Master equation: $\kappa \ll \Omega_c, \Omega_h, 2eV$

# Conclusions & Comparison

**Wave-nature of electrons**

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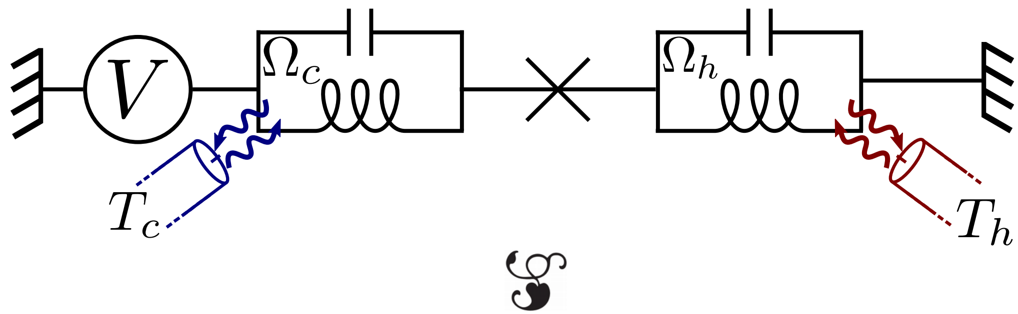
PRB **91**, 195406 (2015)

P. P. Hofer, B. Sothmann

**Particle-nature of Photons**




$$\Delta T \simeq 900\,\mathrm{mK}$$
$$I \simeq 23\,\mathrm{nA}$$
$$P \simeq 0.5\,\mathrm{fW}$$
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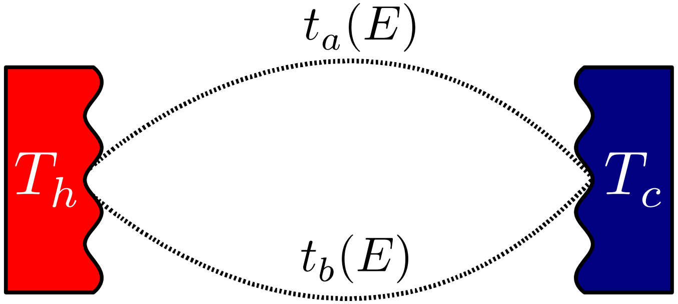
PRB **93**, 041418(R) (2016)

P. P. Hofer, J.-R. Souquet, A. A. Clerk

# Review & Outlook

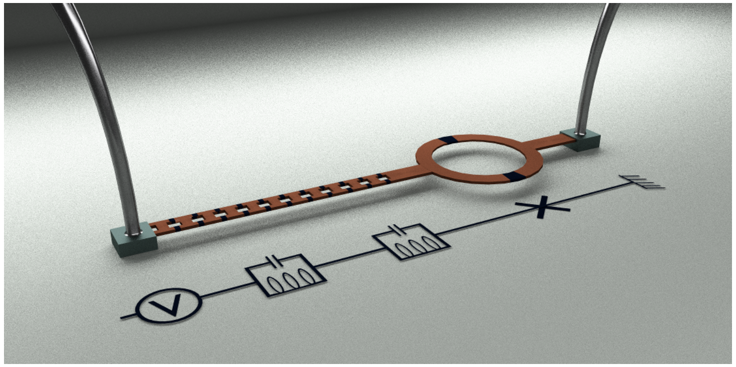



### *Wave-nature of electrons*

- Optimal quantum interference heat engine
  P. Samuelsson, S. Kheradsoud, B. Sothmann, PRL **118**, 256801 (2017)

### *Particle-nature of photons*

- Refrigerator
  P. P. Hofer, M. Perarnau-Llobet, J. B. Brask, R. Silva, M. Huber, N. Brunner, PRB **94**, 235420 (2016)
- Thermometer
  P. P. Hofer, J. B. Brask, M. Perarnau-Llobet, N. Brunner, ArXiv:1703.03719 (accepted in PRL)
- Validity of local Lindblad master equation
  P. P. Hofer, M. Perarnau-Llobet, L. D. M. Miranda, G. Haack, R. Silva, J. B. Brask, N. Brunner, ArXiv:1707.09211



Courtesy of J.-R. Souquet

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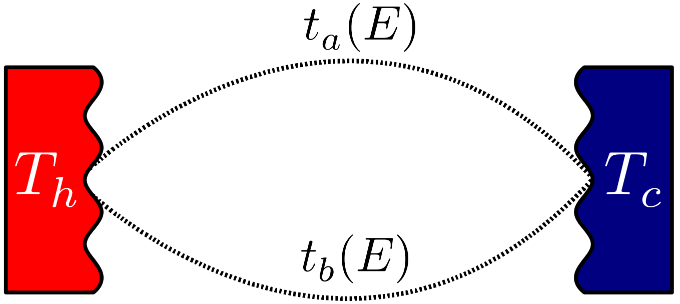


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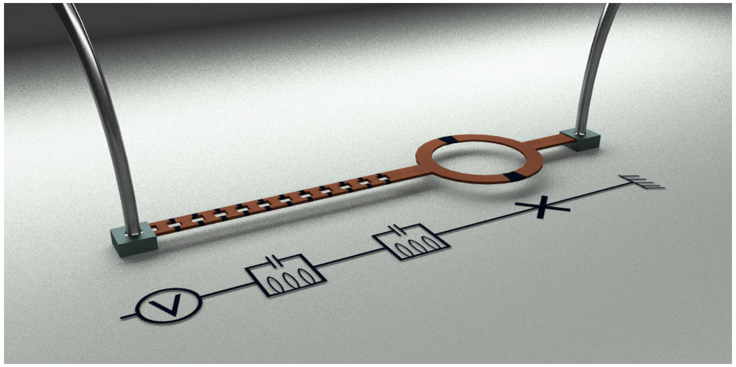
### *Outlook*

- Role of (quantum) fluctuations
- Work extraction from quantum states
  Collaboration with N. Loerch & C. Bruder, University of Basel
- Limitations of classical models



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