Heavy Ion and cosmic ray generators

Klaus Werner

Contents

I Introduction

Before 2010:

Proton-proton scattering:
elementary, understood in terms of pQCD

Heavy ion collisions:
Collective effects, formation of a (flowing) quark-gluon-plasma, macroscopic description

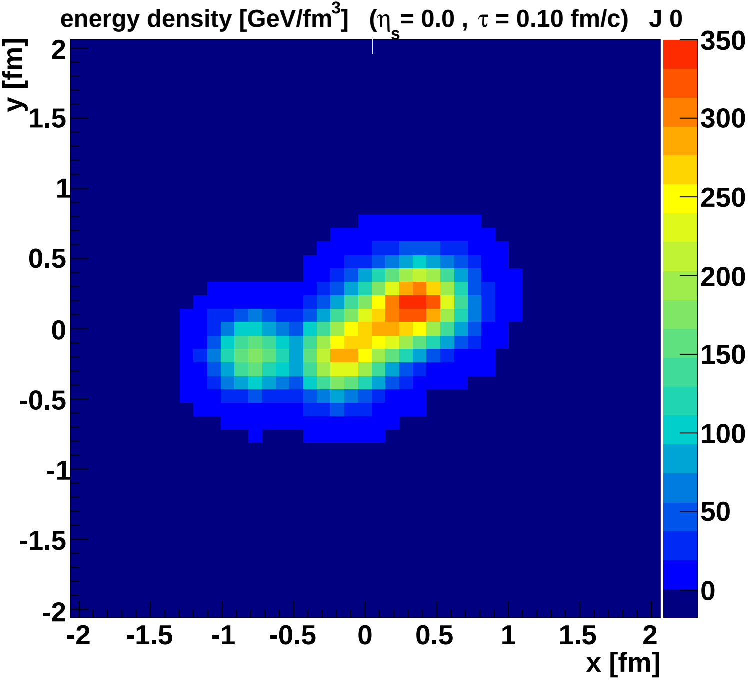
Since 2010: **Incredibly interesting and unexpected pp and pPb results at the LHC (collective effect also in pp?)**

Collective effects means

- **Primary interactions at t = 0**
- **Secondary interactions**
  formation of "matter" which expands collectively, like a fluid

In the following:
An example of a EPOS simulation of expanding matter in pp scattering

pp @ 7TeV EPOS 3.119



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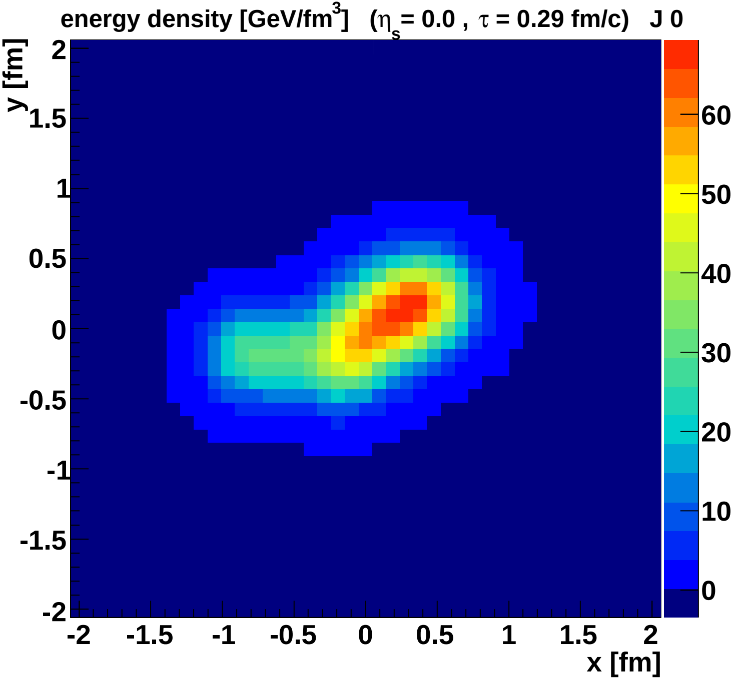

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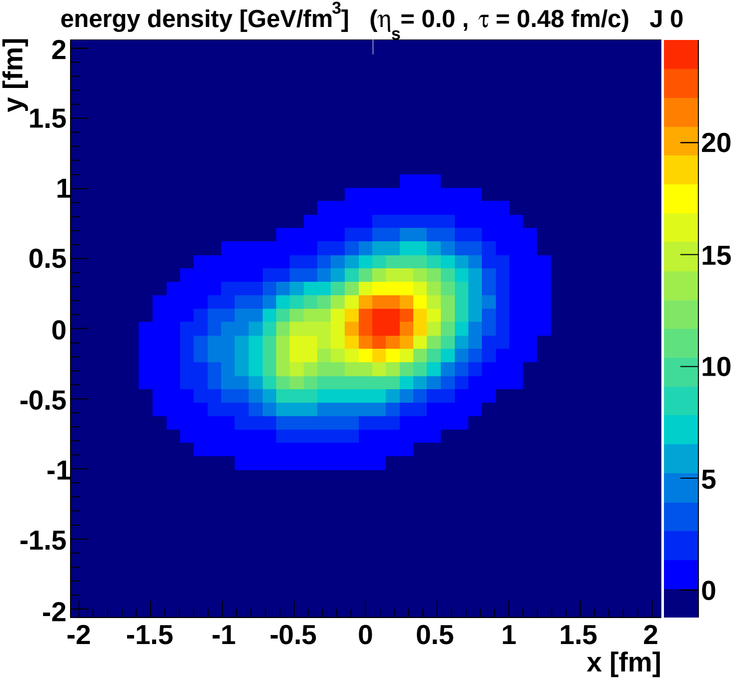

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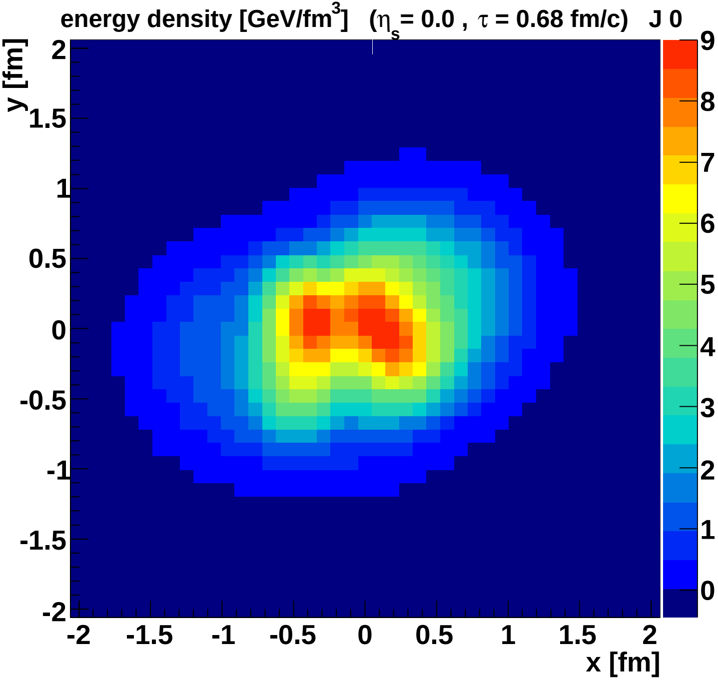

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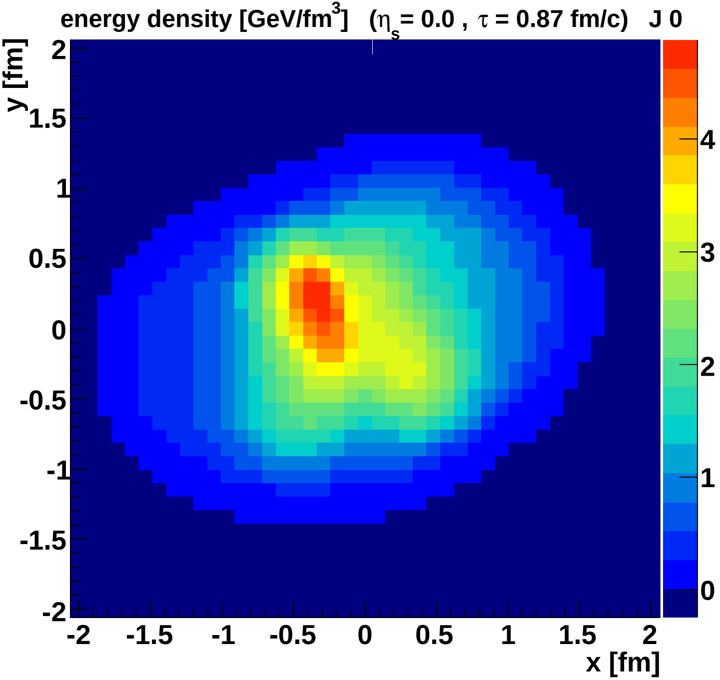

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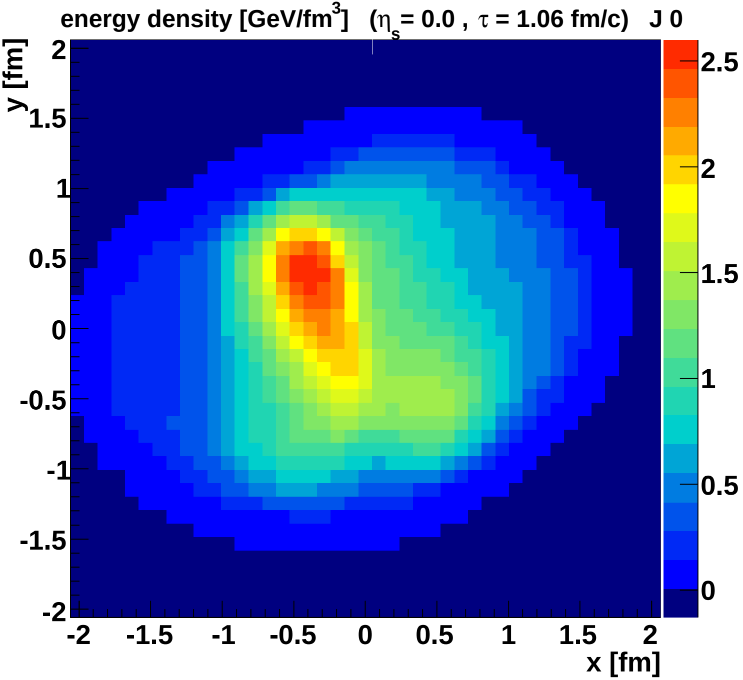

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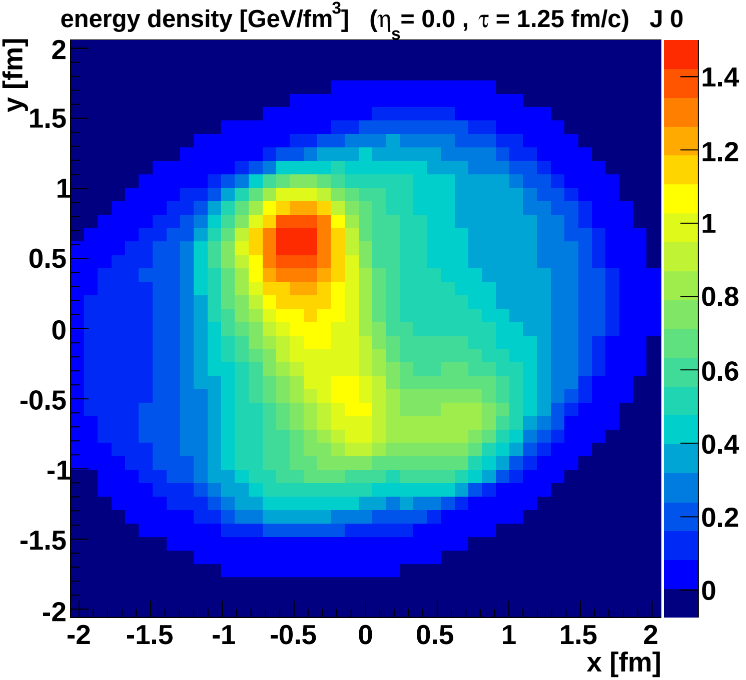

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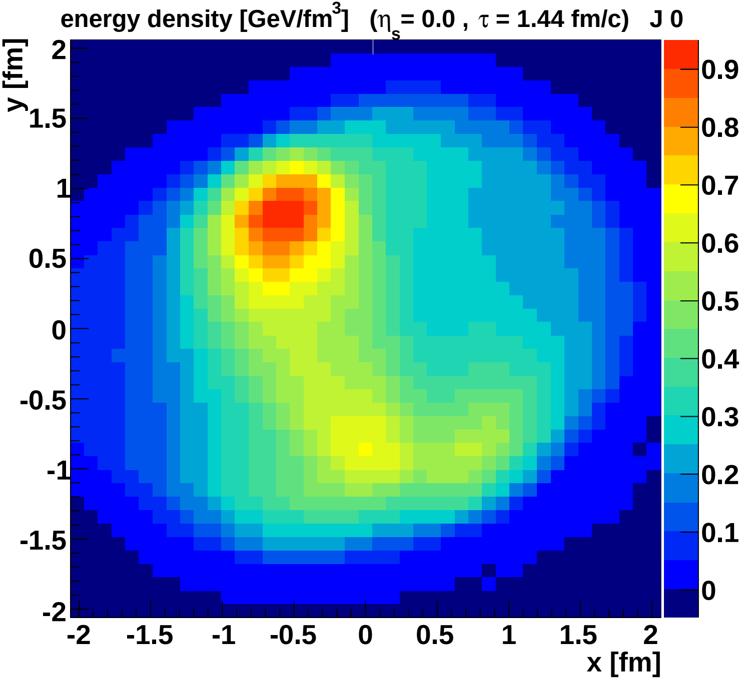

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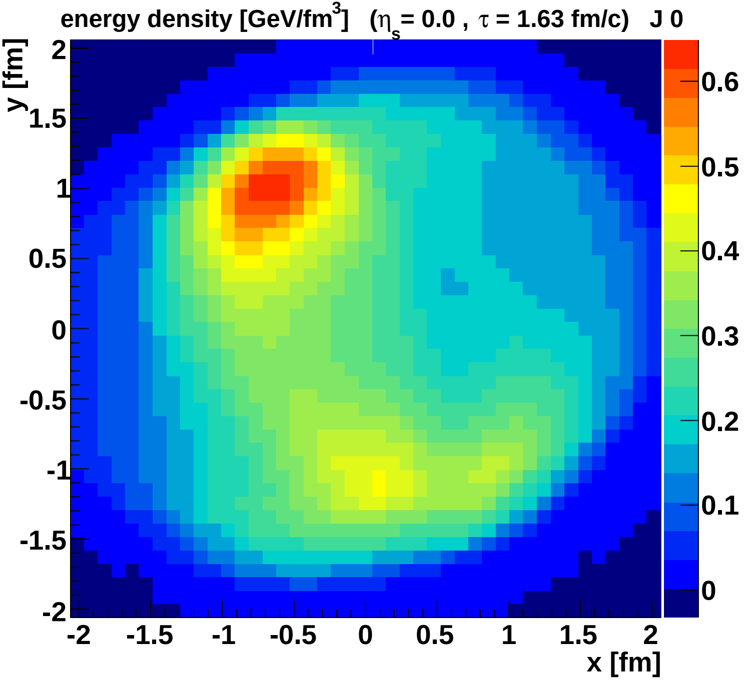

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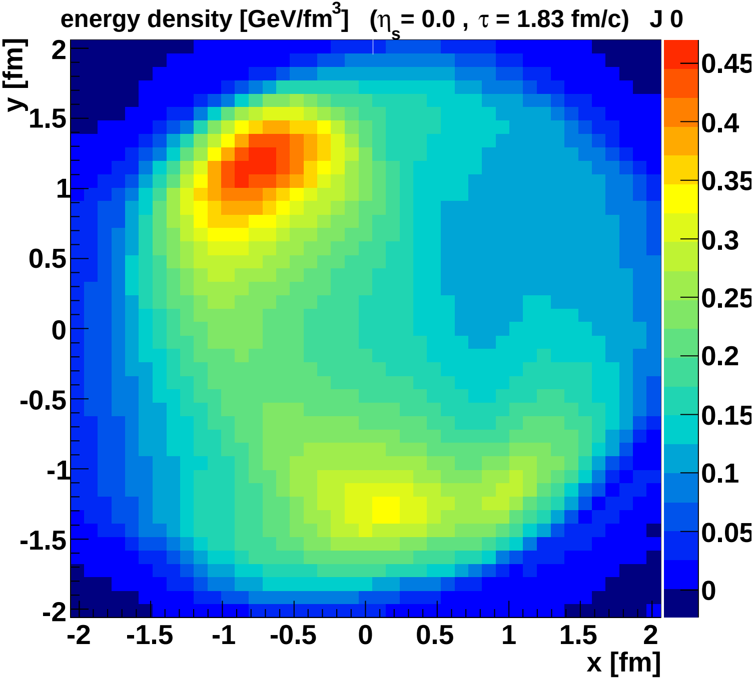

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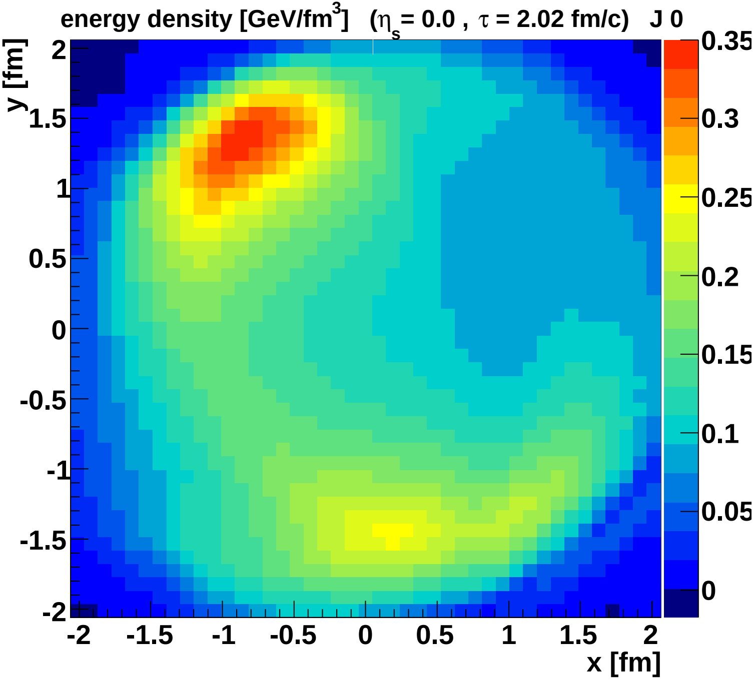

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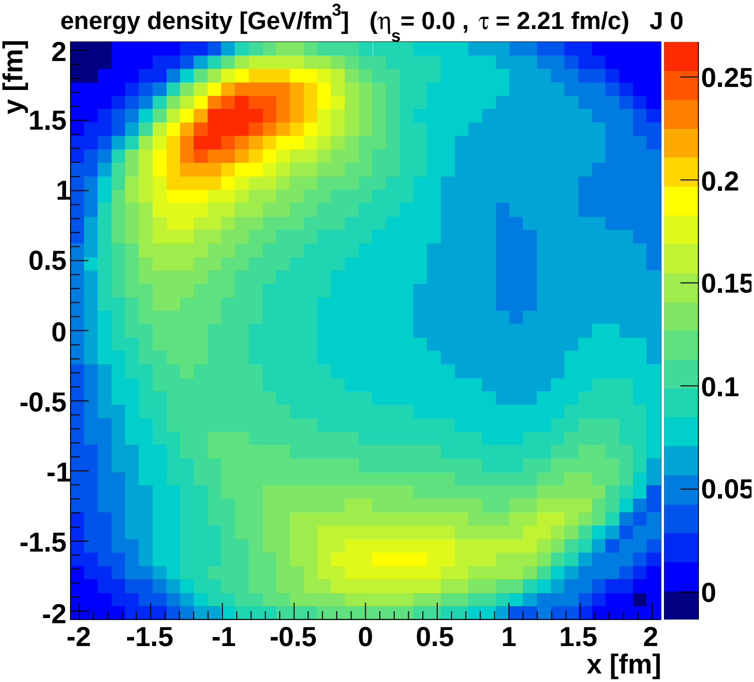

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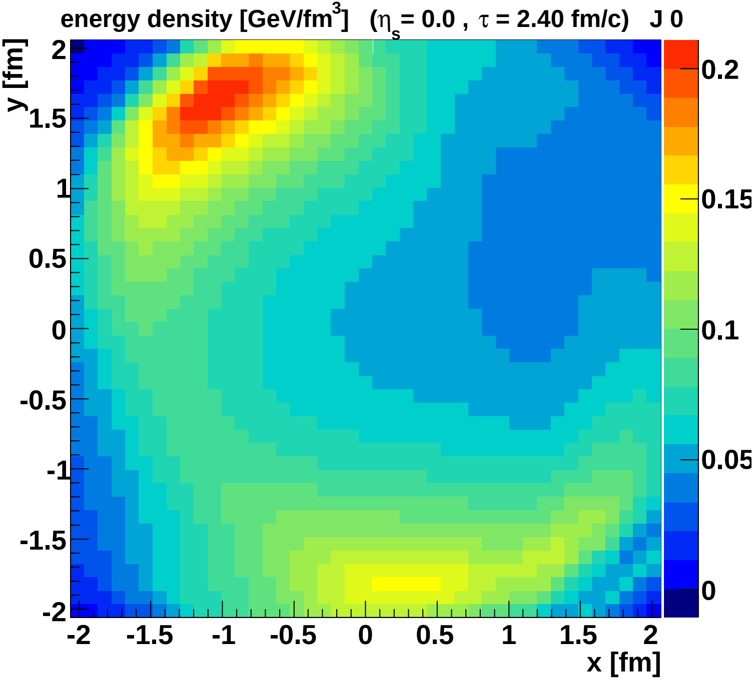

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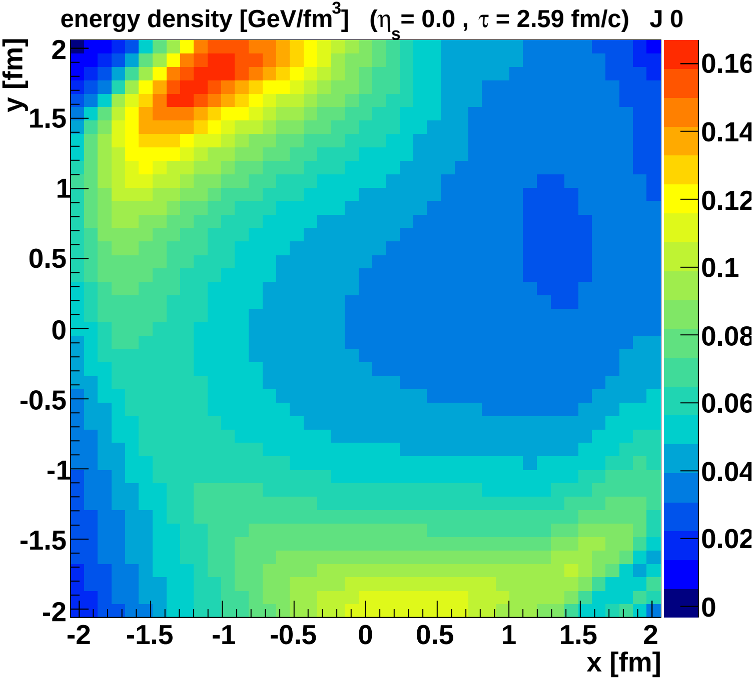
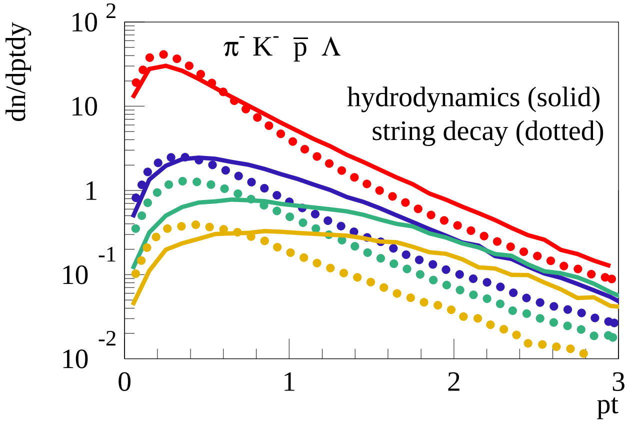


Radial flow visible in particle distributions

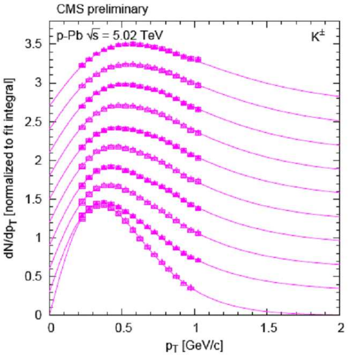
Particle spectra affected by radial flow

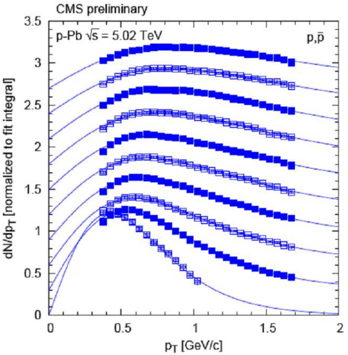


=> mass ordering of $\langle p_t \rangle$, lambda/K increase

pPb at 5TeV

CMS, arXiv:1307.3442



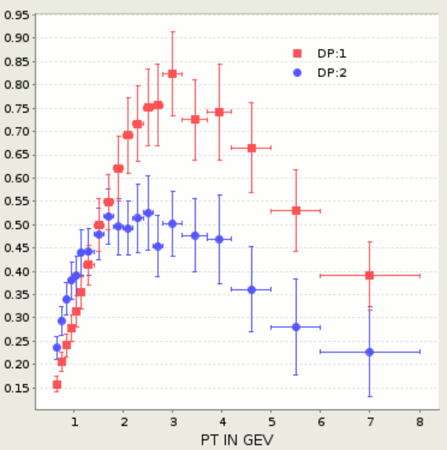


Strong variation of shape with multiplicity

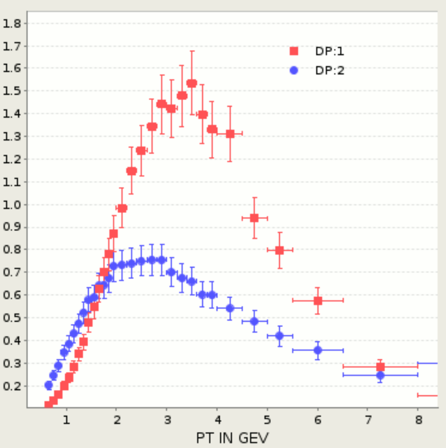
for kaon and even more for proton pt spectra

(flow like)

Λ/K_s versus pT (high compared to low multiplicity) in pPb (left) similar to PbPb (right)



ALICE (**2013**) arXiv:1307.6796



ALICE (**2013**) arXiv:1307.5530
Phys. Rev. Lett. 111, 222301 (2013)

In AA: partially due to flow

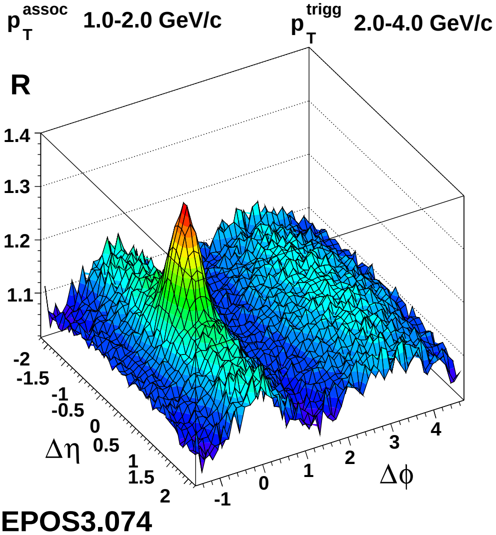
Ridges & flow harmonics

Ridges appear in

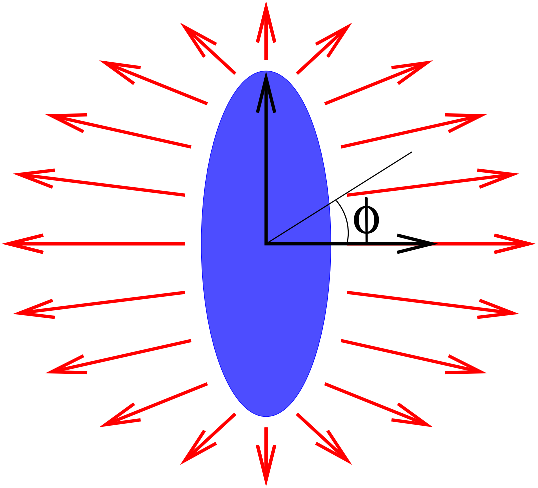
$$R = \frac{1}{N_{\mathrm{trigg}}} \frac{dn}{d\Delta\phi\Delta\eta}$$

due to initial azimuthal anisotropies

(longitudinally invariant)



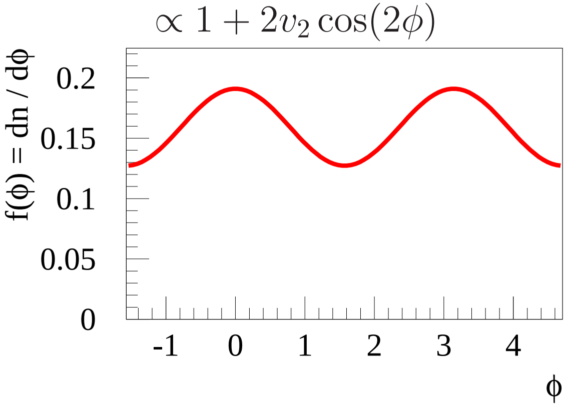
Initial "elliptical" matter distribution:



Preferred expansion along $\phi = 0$ and $\phi = \pi$

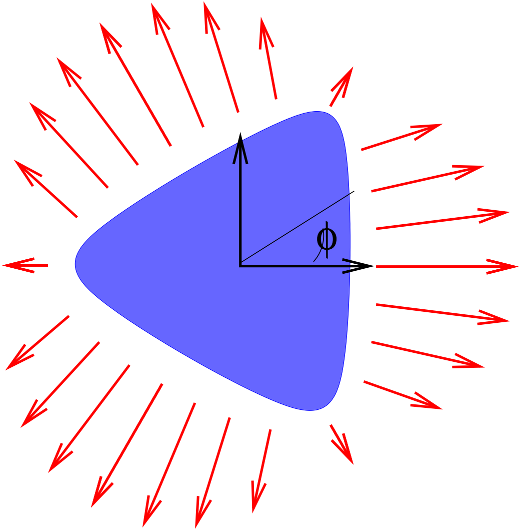
η_s-invariance
same form at any η_s
$\eta_s = \frac{1}{2} \ln \frac{t+z}{t-z}$

Particle distribution: Preferred directions $\phi = 0$ and $\phi = \pi$



Dihadrons:
preferred $\Delta\phi = 0$ and $\Delta\phi = \pi$ (even for big $\Delta\eta$)

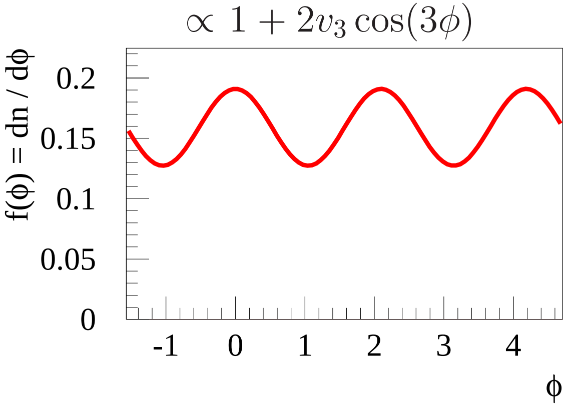
Initial "triangular" matter distribution:



Preferred expansion along $\phi = 0$, $\phi = \frac{2}{3}\pi$, and $\phi = \frac{4}{3}\pi$

η_s-invariance

Particle distribution: Preferred directions $\phi = 0$, $\phi = \frac{2}{3}\pi$, and $\phi = \frac{4}{3}\pi$



Dihadrons:
preferred $\Delta\phi = 0$, and $\Delta\phi = \frac{2}{3}\pi$, and $\Delta\phi = \frac{4}{3}\pi$
(even for large $\Delta\eta$)

In general, superposition of several eccentricities ε_n,

$$\varepsilon_n e^{in\psi_n^{PP}} = -\frac{\int dxdy\, r^2 e^{in\phi} e(x,y)}{\int dxdy\, r^2 e(x,y)}$$

Particle distribution characterized by harmonic flow coefficients

$$v_n e^{in\psi_n^{EP}} = \int d\phi\, e^{in\phi} f(\phi)$$

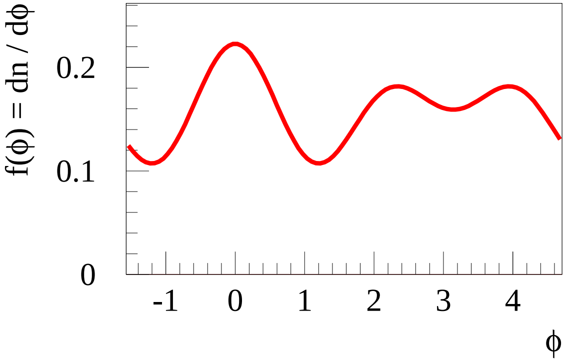
At $\phi = 0$:
The **ridge**

(extended in η)

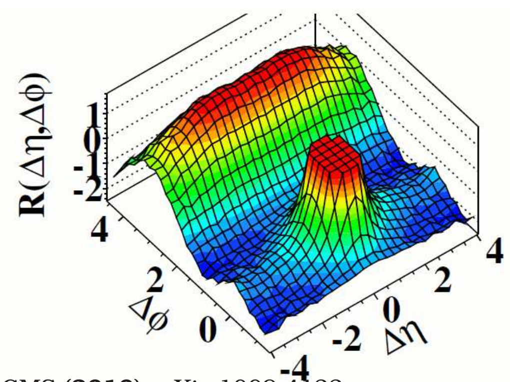
Awayside peak may originate from jets, not the ridge (for large $\Delta\eta$)

Here, v_2 and v_3 non-zero

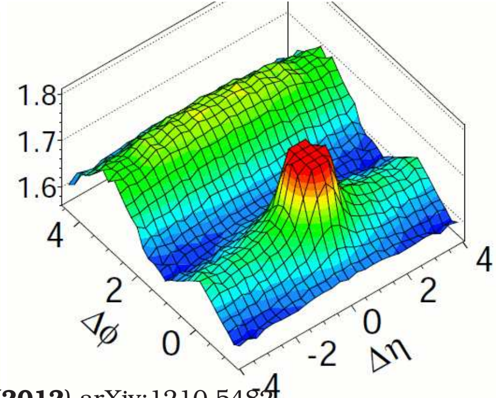
$$\propto 1 + 2v_2 \cos(2\phi) + 2v_3 \cos(3\phi)$$



CMS: Ridges (in dihadron correlation functions) also seen in pp (left) and pPb (right)



CMS (**2010**) arXiv:1009.4122
JHEP 1009:091,2010



CMS (**2012**) arXiv:1210.5482
Phys. Lett. B 718 (2013) 795

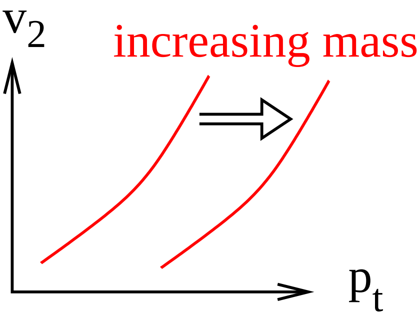
Looks like flow !

Flow harmonics, identified particles

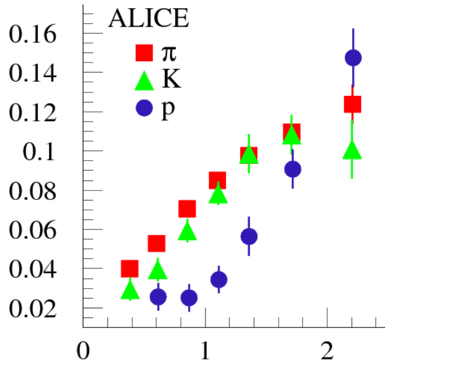
Flow shifts particles to higher p_t

Effect increases wth mass

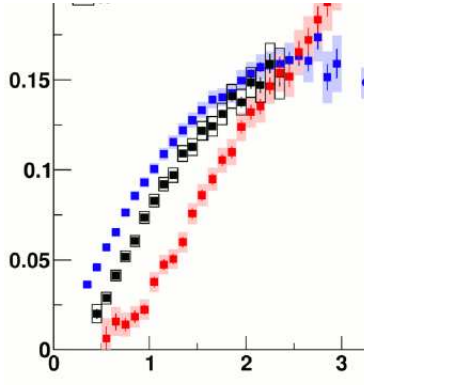
Also true for v_2 vs p_t



ALICE: v2 versus pT: mass splitting (π, K, p) in pPb (left) similar to PbPb (right)



ALICE (**201**3) arXiv:1307.3237



ALICE (**201**2) F. Noferini QM2012

Typical flow result!

So : “Flow-like phenomena” are also seen in pp and pA, therefore:

Heavy ion approach

= primary scattering
+ subsequent fluid evolution

becomes interesting for pp and pA

II Theoretical concepts

concerning primary interactions

providing initial conditions
for secondary interactions

Poles and branch cuts

Even functions $f(x)$ of a **real variable** x
may need to be **continued into the complex plane**,
to understand their properties.

Example
$$f(x) = \sum_{n=0}^{\infty} a_n x^n = \sum_{n=0}^{\infty} \left(\frac{x}{2i}\right)^n .$$

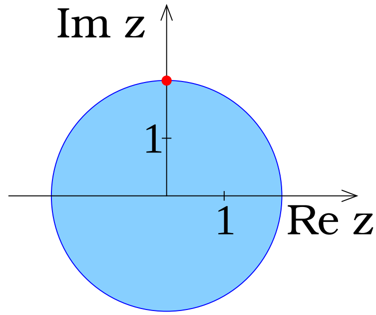
The radius of convergence is

$$\rho = \lim_{n\to\infty} |a_n|^{-1/n} = 2$$

Which is obvious, since f considered as function of a complex variable z, writes

$$f(z) = \frac{1}{1 - z/(2i)}$$

having a **pole** at $z = 2i$,



Blue area: convergence

whereas $f(x)$ has no singularity (for $x \in \mathbb{R}$)

Branch cuts

An example: The logarithm.

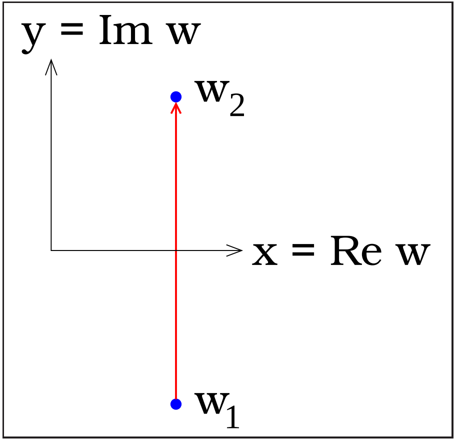
The exponential function defines a mapping M

$$M: \quad \begin{array}{l} \mathbb{C} \to \mathbb{C} \\ w \to z \quad = \exp(w) \end{array}$$

which is well defined in the whole complex plane.

Consider $w = x + iy$, with x fixed and y going from $-\pi$ to π.

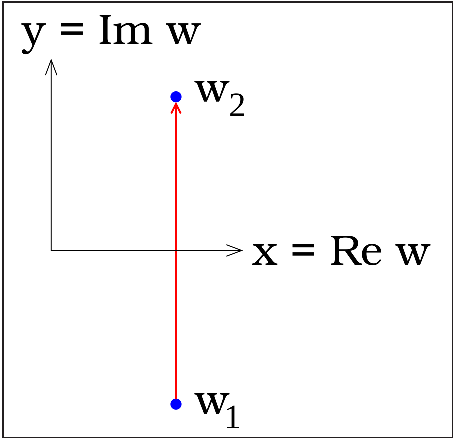
(Trajectory γ going from $w_1 = x - i\pi$ to $w_2 = x + i\pi$)

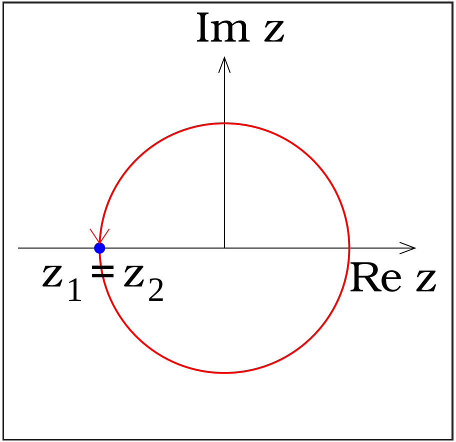


The mapped trajectory $\gamma' = M(\gamma)$ is given as

$$z = \exp(w) = \exp(x)\exp(iy)$$

=> A circle with start and end point $z_1 = z_2 = -e^x$



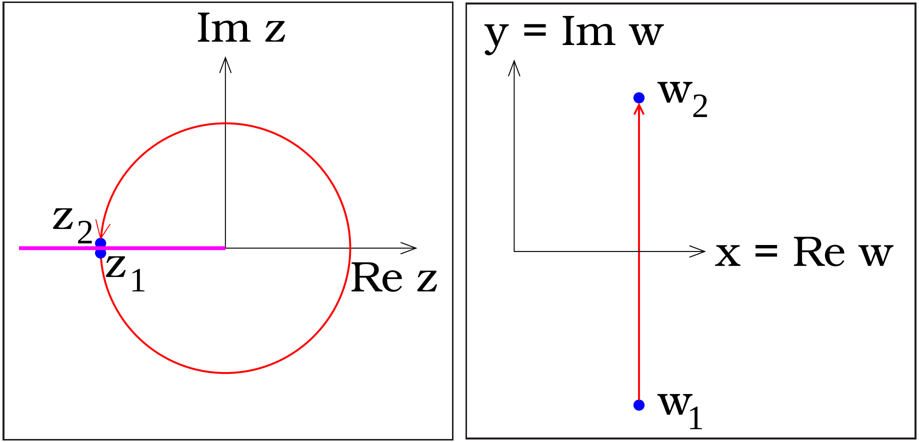


Doing the inverse mapping

$$M^{-1}: \; z \to w = \log(z),$$

we get for $z_1 = z_2$ two different values w_1 and w_2 !!

One has to define log in $\mathbb{C} - \mathbb{R}_{\leq \mathbf{0}}$ (branch).
The negative real axis is called branch cut.



The discontinuity at $z = -e^x$:

$$\log(z + i\epsilon) - \log(z - i\epsilon) = 2\pi i$$

Cut diagrams

The scattering operator $\hat{S}$ is defined via

$$|\psi(t=+\infty)\rangle = \hat{S}\,|\psi(t=-\infty)\rangle$$

Unitarity relation $\hat{S}^\dagger \hat{S} = 1$ gives

$$\begin{aligned} 1 &= \langle i|\,\hat{S}^\dagger \hat{S}\,|i\rangle \\ &= \sum_f \langle i|\,\hat{S}^\dagger\,|f\rangle\,\langle f|\,\hat{S}\,|i\rangle \\ &= \sum_f \langle f|\,\hat{S}\,|i\rangle^*\,\langle f|\,\hat{S}\,|i\rangle \end{aligned}$$

Expressed in terms of the S-matrix:

$$1 = \sum_f S^*_{fi} S_{fi}$$

Using

$$S_{fi} = \delta_{fi} + i(2\pi)^4 \delta(p_f - p_i) T_{fi}$$

dividing by $i(2\pi)^4\delta(0)$:

$$\begin{aligned} \frac{1}{i}\left(T_{ii} - T^*_{ii}\right) &= \sum_f (2\pi)^4 \delta(p_f - p_i)\, |T_{fi}|^2 \\ &= 2w\,\sigma_{\mathrm{tot}} \\ &= 2s\sigma_{\mathrm{tot}} \end{aligned}$$

The l.h.s. :

$$\frac{1}{i}\left(T_{ii} - T_{ii}^{*}\right) = 2\mathrm{Im}T$$

So we get the optical theorem

$$2\mathrm{Im}T_{ii} \;=\; \sum_{f}(2\pi)^{4}\delta(p_{f} - p_{i})\left|T_{fi}\right|^{2} = 2s\,\sigma_{\mathrm{tot}}$$

Assume:

- T_{ii} is Lorentz invariant $\to$ use s, t
- $T_{ii}(s,t)$ is an analytic function of s, with s considered as a complex variable
  (Hermitean analyticity)
- $T_{ii}(s,t)$ is real on some part of the real axis

Using the Schwarz reflection principle, $T_{ii}(s,t)$ first defined for $\mathrm{Im}s \geq 0$ can be continued in a unique fashion via $T_{ii}(s^*,t) = T_{ii}(s,t)^*$.

So:

$$\frac{1}{i}\left(T_{ii}(s,t) - T_{ii}(s,t)^*\right) = \frac{1}{i}\left(T_{ii}(s,t) - T_{ii}(s^*,t)\right)$$

Def:

$$\operatorname{disc} T = T_{ii}(s+i\epsilon,t) - T_{ii}(s-i\epsilon,t)$$

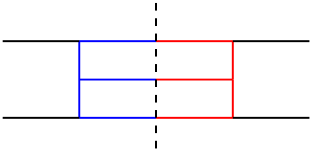
We have finally

$$\frac{1}{\mathrm{i}}\mathrm{disc}\,T \;=\; (2\pi)^4\delta(p_f - p_i)\sum_f |T_{fi}|^2 = 2s\,\sigma_{\mathrm{tot}}$$

Interpretation: $\frac{1}{\mathrm{i}}\mathrm{disc}\,T$ can be seen as a so-called “cut diagram”, with modified Feynman rules, the “intermediate particles” are on mass shell.

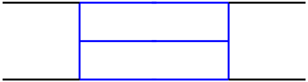
Modified Feynman rules :

- □ Draw a dashed line from top to bottom

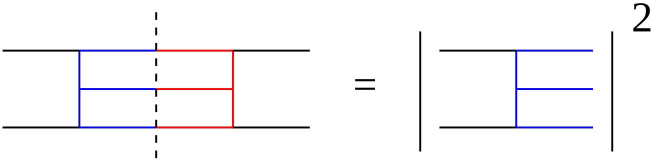


- □ Use "normal" Feynman rules to the left
- □ Use the complex conjugate expressions to the right
- □ For lines crossing the cut: Replace propagators by mass shell conditions $2\pi\theta(p^0)\delta(p^2 - m^2)$

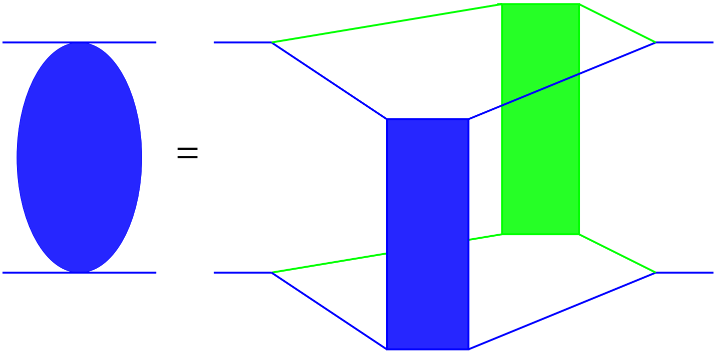
Cutting a diagram representing **elastic** scattering



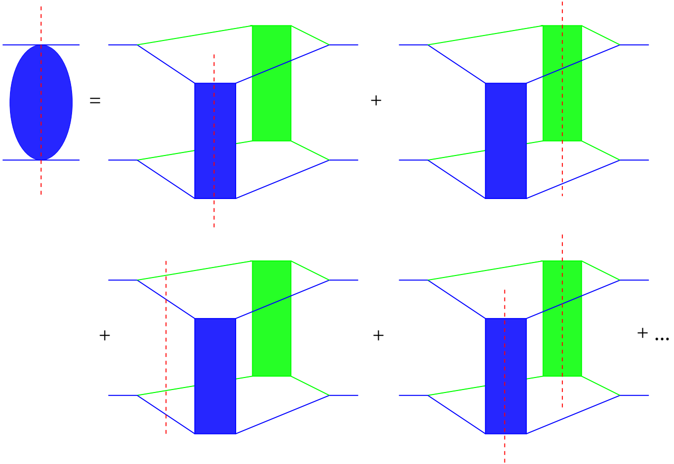
corresponds to **inelastic** scattering



Cutting diagrams is useful in case of substructures:



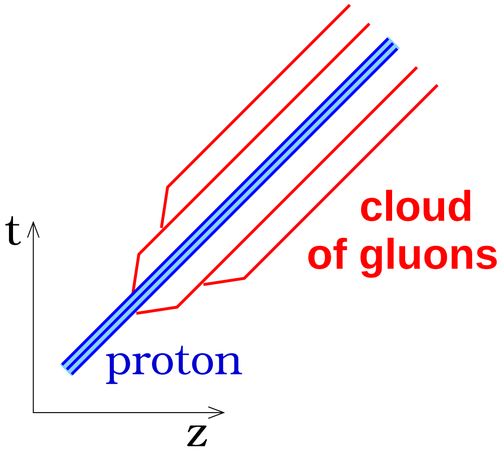
Precisely the multiple scattering structure in EPOS



Cut diagram
= sum of products of cut/uncut subdiagrams
=> Gribov-Regge approach of multiple scattering

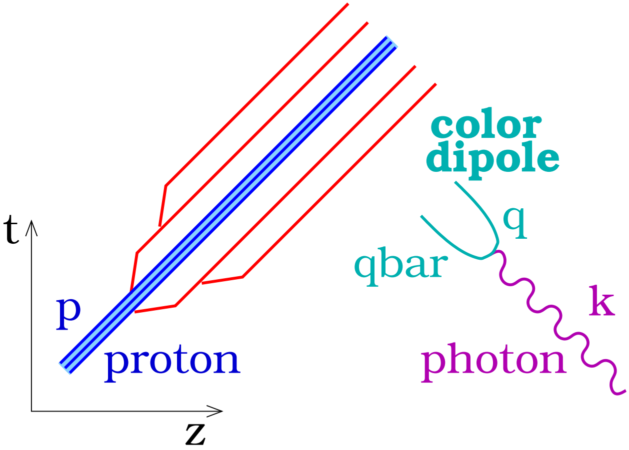
Parton evolution

A fast moving proton



emits successively partons (mainly gluons), quasi-real (large gamma factors)

... which can be probed by a virtual photon (emitted from an electron)



photon splits into q-qbar

→ Color dipole

p and k are proton and photon momentum

What precisely the photon “sees” depends on two kinematic variables,

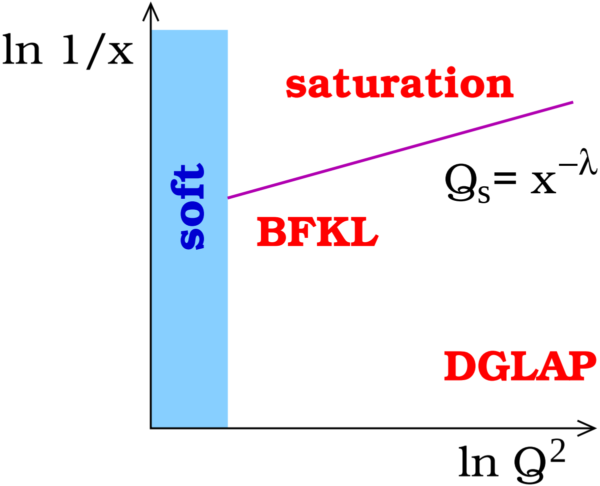
the **virtuality**

$$Q^2 = -k^2$$

and the **Bjorken variable**

$$x = \frac{Q^2}{2pk}$$

which probes partons with momentum fraction x.
It determines also the **approximation scheme** to compute the parton cloud.



DGLAP: summing to all orders of $\alpha_s \ln Q^2$

BFKL: summing to all orders of $\alpha_s \ln \frac{1}{x}$

Linear equations

BFKL (Balitsky, Fadin, Kuraev, and Lipatov):

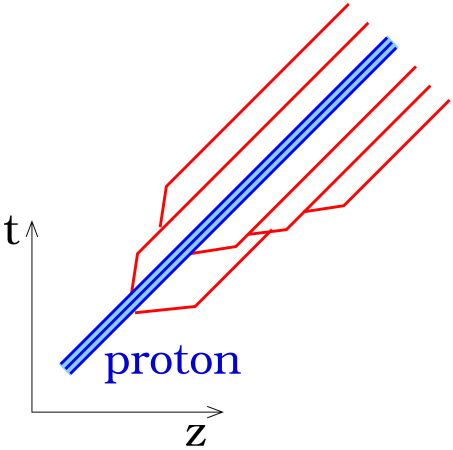
$$\frac{\partial \varphi(x,\boldsymbol{q})}{\partial \ln \frac{1}{x}} \; \frac{\alpha_s N_c}{\pi^2} \int d^2k \, K(\boldsymbol{q},\boldsymbol{k}) \varphi(x,\boldsymbol{k})$$

$$\text{with } xg(x,Q^2) = \int_0^{Q^2} \frac{d^2k}{k^2} \varphi(x,\boldsymbol{k}),$$

DGLAP (Dokshitzer, Gribov, Lipatov, Altarelli and Parisi):

$$\frac{\partial g(x,Q^2)}{\partial \ln q^2} = \int_x^1 \frac{dz}{z} \, \frac{\alpha_s}{2\pi} P(z) g(\frac{x}{z}, Q^2)$$

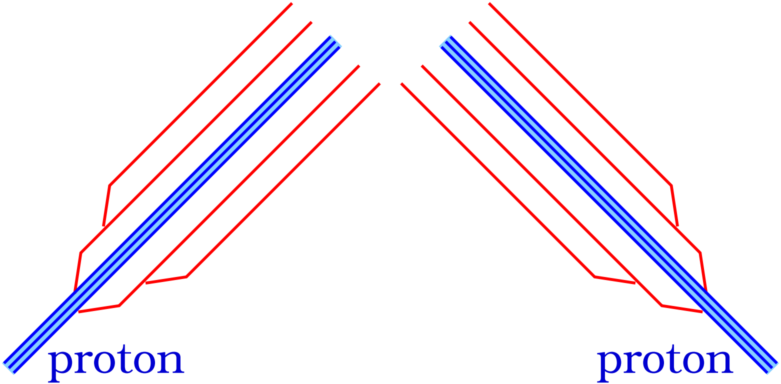
Very large $\ln 1/x$: Saturation domain



Non-linear effects

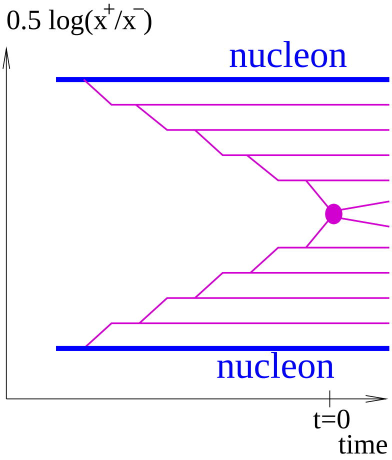
Gluon from one cascade is absorbed by another one

pp scattering (linear domain)



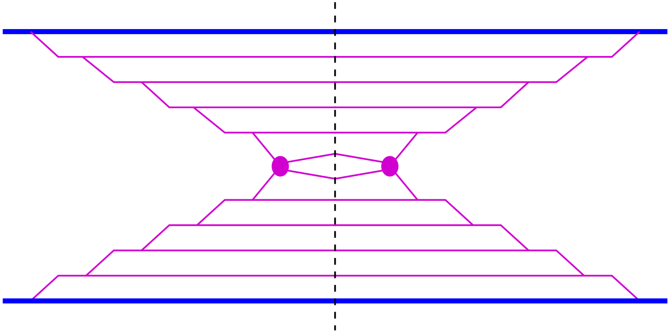
Same evolution as in proton-photon (causality)

Different way of plotting the same reaction



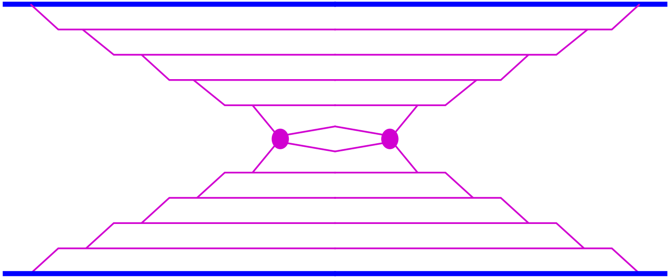
inelastic scattering diagram

Corresponding cut diagram



referred to as "cut parton ladder"
= amplitude squared of the inelastic diagram

Corresponding elastic diagram



referred to as "(uncut) parton ladder"

Soft domain

Very small $\ln Q^2$: No perturbative treatment!

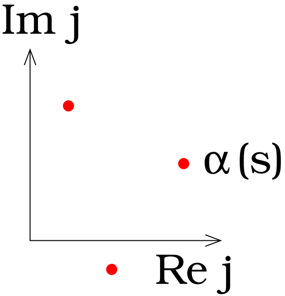
But one may use again the hypothesis of **Lorentz invariance** and **analyticity** of the T-matrix. One starts with a partial wave expansion of the T-matrix (Watson-Sommerfeld transform) :

$$T(t,s) = \sum_{j=0}^{\infty} (2j+1)\mathcal{T}(j,s)P_j(z)$$

with $t \propto z - 1$, $z = \cos\vartheta$, P_j: Legendre polynomials.

With $\alpha(s)$ being the right-most pole of $\mathcal{T}(j,s)$ one gets for $t \to \infty$:

$$T(t,s) \propto t^{\alpha(s)}$$



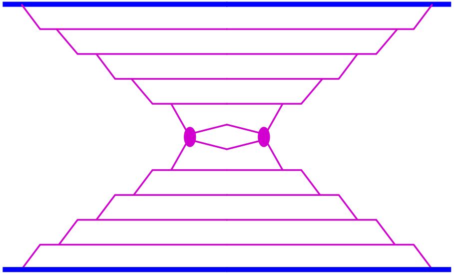
and assuming crossing symmetry one gets the famous asymptotic result

$$T(s,t) \propto s^{\alpha(t)}$$

with the "Regge pole"

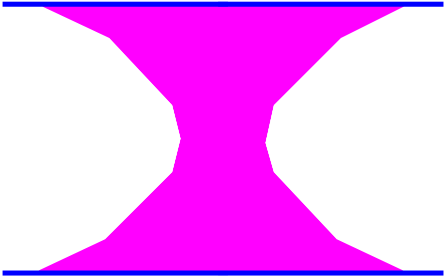
$$\alpha(t) = \alpha(0) + \alpha' t$$

Perturbative:
Parton ladder



T-matrix computed
(DGLAP)

Soft:
Soft Pomeron



gluons fields

T-matrix parametrized

Formulas (see Phys.Rept. 350 (2001) 93-289):

$$T_{\mathrm{soft}}(\hat{s},t) \;=\; 8\pi s_0\, i\, \gamma^2_{\mathrm{Pom-parton}} \left(\frac{\hat{s}}{s_0}\right)^{\alpha_{\mathrm{soft}}(0)}$$

$$\times \exp(\lambda_{\mathrm{soft}}\, t)\,,$$

with

$$\lambda_{\mathrm{soft}} = 2R^2_{\mathrm{Pom-parton}} + \alpha'_{\mathrm{soft}} \ln \frac{\hat{s}}{s_0}.$$

Cut soft Pomeron (Schwarz reflection principle):

$$\frac{1}{i}\mathrm{disc}\,T_{\mathrm{soft}}(\hat{s},t)$$

$$=\frac{1}{i}\left[T_{\mathrm{soft}}(\hat{s}+i0,t)-T_{\mathrm{soft}}(\hat{s}-i0,t)\right]$$

$$=2\mathrm{Im}\,T_{\mathrm{soft}}(\hat{s},t)$$

Interaction cross section,

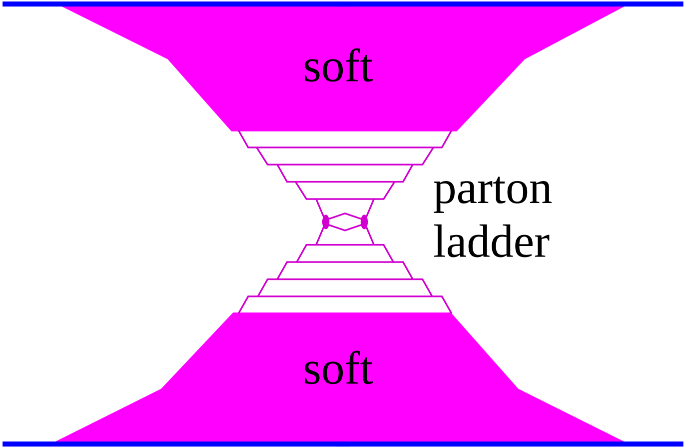
$$\sigma_{\mathrm{soft}}(\hat{s}) = \frac{1}{2\hat{s}} 2\mathrm{Im}\, T_{\mathrm{soft}}(\hat{s}, 0)\,,$$

$$= 8\pi\gamma_{\mathrm{part}}^{2} \left(\frac{\hat{s}}{s_0}\right)^{\alpha_{\mathrm{soft}}(0)-1},$$

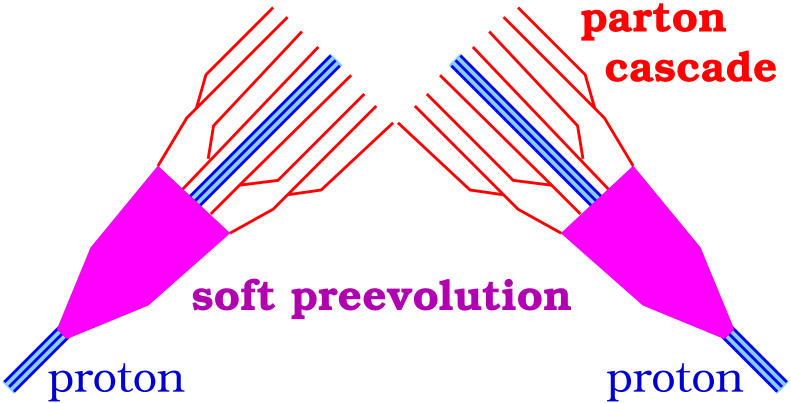
using the optical theorem (with $t = 0$),

which grows faster than data

Semihard Pomeron



Space-time picture of semihard Pomeron



Hard cross section and amplitude (see Phys.Rept. 350 (2001) 93-289) :

$$\sigma^{jk}_{\mathrm{hard}}(\hat{s},Q_0^2) = \frac{1}{2\hat{s}} 2\mathrm{Im}\, T^{jk}_{\mathrm{hard}}(\hat{s},t=0)$$
$$= K \sum_{ml} \int dx_B^+ dx_B^- dp_\perp^2 \frac{d\sigma^{ml}_{\mathrm{Born}}}{dp_\perp^2}(x_B^+ x_B^- \hat{s}, p_\perp^2)$$
$$\times E^{jm}_{\mathrm{QCD}}(x_B^+, Q_0^2, M_F^2)\, E^{kl}_{\mathrm{QCD}}(x_B^-, Q_0^2, M_F^2) \theta\big(M_F^2 - Q_0^2\big)\,,$$

One knows (Lipativ, 86): amplitude is imaginary, and nearly independent on t => (with $R^2_{\mathrm{hard}} \simeq 0$) :

$$T^{jk}_{\mathrm{hard}}(\hat{s},t) = i\hat{s}\, \sigma^{jk}_{\mathrm{hard}}(\hat{s},Q_0^2)\ \exp\left(R^2_{\mathrm{hard}}\, t\right)$$

Semihard amplitude :

$$iT_{\mathrm{semihard}}(\hat{s},t)=\sum_{jk}\int_0^1\frac{dz^+}{z^+}\frac{dz^-}{z^-}$$

$$\times\mathrm{Im}\,T^j_{\mathrm{soft}}\Big(\frac{s_0}{z^+},t\Big)\;\mathrm{Im}\,T^k_{\mathrm{soft}}\Big(\frac{s_0}{z^-},t\Big)\;iT^{jk}_{\mathrm{hard}}(z^+z^-\hat{s},t)$$

(valid for $s\to\infty$ and small parton virtualities except for the ones in the ladder)

Cross sections

(a) Exclusive : a + b $\rightarrow$ c + d

(b) Total : a+b $\rightarrow$ X (sum of (a))

(c) Inclusive : a + b $\rightarrow$ c + X (weighted sum of (a))

There are simple formulas for inclusive cross sections (AGK cancellations), but one needs to go beyond when studying high multiplicity pp.

Consider multiple scattering amplitude

$$iT = \prod iT_{\mathrm{P}}$$

cross section: sum over all cuts.

For each cut Pom:

$$\frac{1}{i}\mathrm{disc}T_{\mathrm{P}} = 2\mathrm{Im}T_{\mathrm{P}} \equiv G$$

For each uncut one:

$$iT_{\mathrm{P}} + \{iT_{\mathrm{P}}\}^{*} = i\,(i\,\mathrm{Im}T_{\mathrm{P}}) + \{i\,(i\,\mathrm{Im}T_{\mathrm{P}})\}^{*} = -2\mathrm{Im}T_{\mathrm{P}} \equiv -G$$

Inclusive cross section: weighted sum over all cuts: The multiplicity for k cut Pomerons is kN, if N is the multiplicity per cut Pomeron.

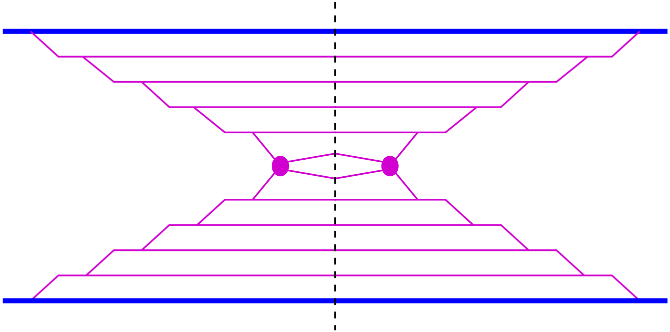
Contribution to the inclusive cross section for n Pomerons:

$$\sigma_{\mathrm{incl}}^{(n)} \propto \sum_{k=0}^{n} kN\, G^{k}\, (-G)^{n-k} \left(\begin{array}{c} n \\ k \end{array}\right) = 0 \text{ for } n > 1$$

Only n=1 contributes (single Pomeron) !!

AGK cancellations for n>1

simple diagram even in case of multiple scattering



corresponds to factorization:

$$\sigma_{\mathrm{incl}} = F \otimes \sigma_{\mathrm{elem}} \otimes F$$

Kind of obvious that **factorization** should hold for inclusive cross sections, so

$$\boldsymbol{\sigma}_{\mathrm{incl}} = \boldsymbol{F} \otimes \boldsymbol{\sigma}_{\mathrm{elem}} \otimes \boldsymbol{F}$$

may be used as starting point, with F taken from DIS (photon-proton).

III Model overview

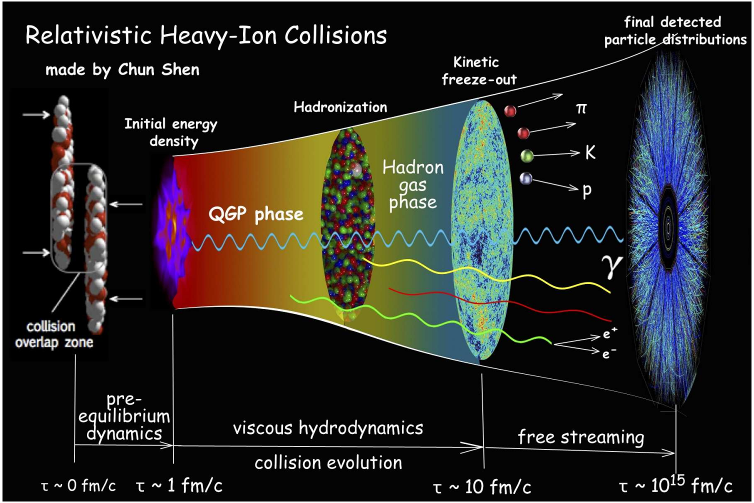
with contributions from T. Pierog, S. Ostapchenko, C. Bierlich, F. Riehn, P. Tribedy, A. Fedynitch

Models for min bias and high multiplicity pp

model	Gribov Regge	Dipole	Facto risation	used for CR	authors
QGSJETII	X			X	Ostapchenko
EPOSLHC	X			X	Pierog, Werner
EPOS3	X				Werner, Pierog
DIPSY		X			Lönnblad, Bierlich
IP-Glasma		X			Tribedy, Schenke
SIBYLL			X	X	Engel, Riehn
DPMJETIII			X	X	Engel, Fedynitch
PYTHIA			X		Sjostrand, Skands
HERWIG			X		Marchesini, Webber

Models for high multipl pp, pA, AA

including collective effects



ohttp://u.osu.edu/vishnu/2014/08/06/sketch-of-relativistic-heavy-ion-collisions/

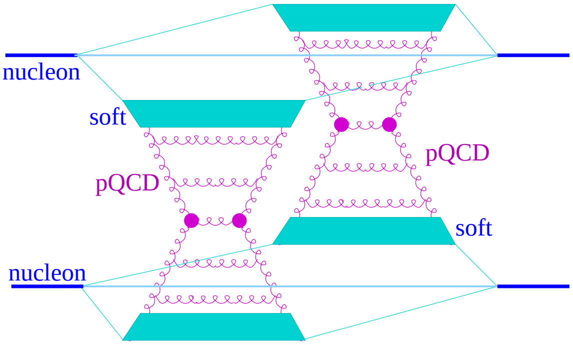
model	primary scatterings	secondary interactions	
EPOS	Gribov Regge	viscous hydrodynamical expansion of QGP	hadronic cascade
IP-Glasma	Dipole model	“	
Supersonic	Wounded nucleon model	“	“
AMPT	Minijets from Pythia	partonic cascade	“

Cascade means:

Successive scatterings $a + b \to c + d$ according to known cross sections

Gribov-Regge multiple scattering approach

EPOS, QGSJETII



S-Matrix based on Pomerons

Pomerons : Parton ladders (initial and final state radiation, DGLAP) + soft

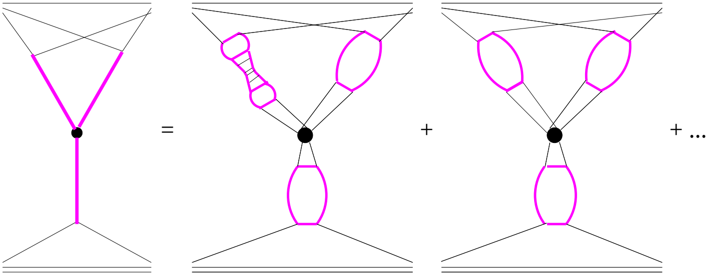
Cutting rules to get inelastic cross sections.

Same principle for pp, pA, AA

more details later

Nonlinear effects in QGSJET

Pomeron-Pomeron coupling



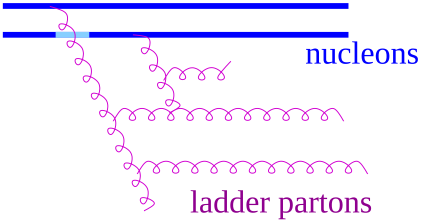
- ☐ Summing of **all orders**
- ☐ No energy conservation
- ☐ (in EPOS full energy conservation, but effective treatment of nonlinear effects)

Nonlinear effects in EPOS

Nonlinear effects (gluon fusion) taken care of via a saturation scale Q_s

Saturation scale depends on Pomeron energy ($\sqrt{x^+x^-s}$) and the environment

Selfconsistent procedure within multiple scattering framework (more later)



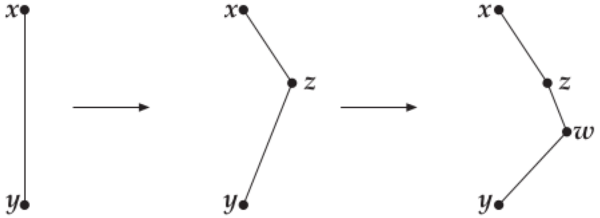
Dipole approach

Initial state radiation in DIPSY (from Christian Bierlich)

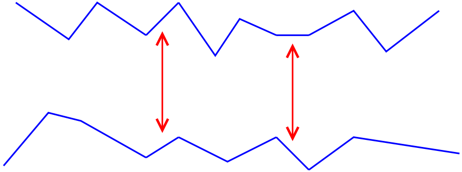
Initial nucleon: Three dipoles

LL BFKL in b-space + corrections: A dipole $(\vec{x}, \vec{y})$ can emit a gluon at position $\vec{z}$ with probability (P) per unit rapidity (Y)

$$\frac{dP}{dY} = \frac{\bar{\alpha}}{2\pi} d^2\vec{z} \frac{(\vec{x}-\vec{y})^2}{(\vec{x}-\vec{z})^2(\vec{z}-\vec{y})^2}$$



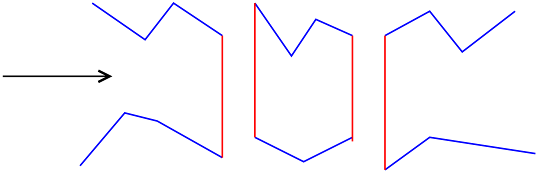
Multiple scattering



Multiple color exchange
between dipoles i and j
with probabilities

$$\frac{\alpha_s^2}{4}\left[\log\left(\frac{(\vec{x}_i-\vec{y}_j)^2(\vec{y}_i-\vec{x}_j)^2}{(\vec{x}_i-\vec{x}_j)^2(\vec{y}_i-\vec{y}_j)^2}\right)\right]^2$$

-> kinky strings



- ☐ **Two “leading“ strings**
- ☐ **Additional strings from loops**
- ☐ **No Remnants**

Many strings:
Lund strings may overlap

=> color ropes
(Larger eff. string tension)

Initial state in IP-Glasma (from Prithwish Tribedy)

IP-Sat dipole model ($r_\perp$ =dipole size):

$$\frac{d\sigma}{d^2b} = 2\left[1 - \exp\left(-F(r_\perp, x, b)\right]\right., \; F \propto r_\perp^2 \alpha_s(\mu^2) xg(x, \mu^2) T(b)$$

$T(b)$: Gaussian profile, $\mu^2 = 4/r_\perp^2 + \mu_0^2$, xg : DGLAP evolution

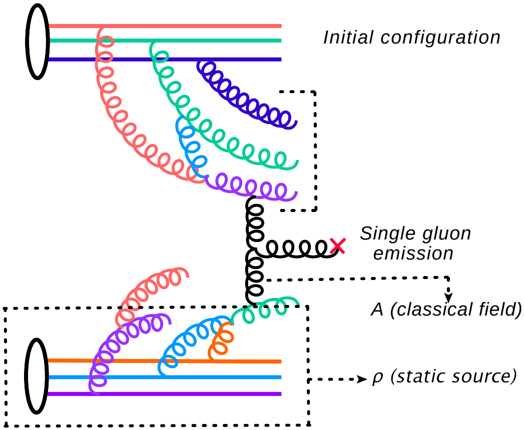
Saturation scale Q_s defined via

$$F\left(r_\perp, x = \frac{2}{Q_s^2}, b\right) = \frac{1}{2}$$

IP-Glasma: Color charge squared for projectile A and target B :

$g^2\mu_A^2 = \sum_{nucleons} g^2\mu_i^2$, with $g^2\mu_i^2 \propto Q_s^2$ with Q_s^2 from IP-Sat model.

Multiple Scattering



Color charge density $\rho_{A/B}$
generated from Gaussian distribution with variance $g^2\mu_A^2$ (contains DGLAP, saturation)

Current
$J^\nu = \delta^{\nu\pm}\rho_{A/B}(x^\mp, x_\perp)$

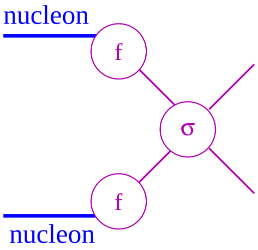
Field from $[D_\mu, F_{\mu\nu}] = J_\nu$
Numerical (lattice) solution, fields can be expressed in terms of initial ones:
$A^i = A^i_A + A^i_B,\ A^\eta = \frac{ig}{2}[A^i_A, A^i_B]$

Multiple scattering:
Nonlinearity in terms of A:
Infinite number of $g + g \to g$ processes

Fields→Gluons→Pythia strings

Models based on factorization

$$\sigma_{\mathrm{jet}} = \int dx_1 dx_2 \int dp_t^2 \sum f_i(x_1, p_t^2)\, f_j(x_2, p_t^2)\, \frac{d\sigma_{ij}}{dp_t^2}(\hat{s}, \hat{t}) \quad (A)$$



PYTHIA
HERWIG
SIBYLL
DPMJETIII

First step: σ_{jet} according to (A)

Second step: Multiple scattering scheme via eikonal formula

$$prob(n) = \frac{\left[\sigma_{\mathrm{jet}}(s)\, T(s,b)\right]^n}{n!} \exp\left(-\sigma_{\mathrm{jet}}(s)\, T(s,b)\right)$$

Multiple scattering in SIBYLL

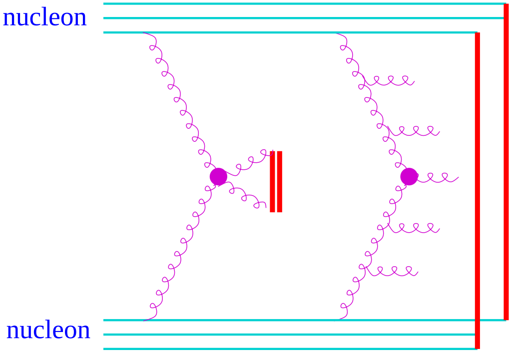
From F. Riehn

Multiple scattering via eikonal model with soft and hard component

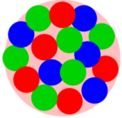
☐ No Remnants

☐ Main scattering
=> qq-q strings

☐ Further scatterings
=> strings between gluon pairs

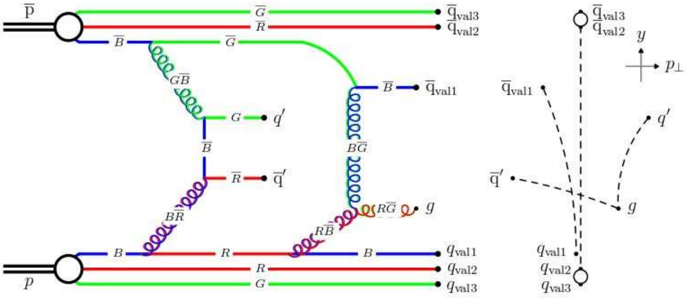


Saturation scale from



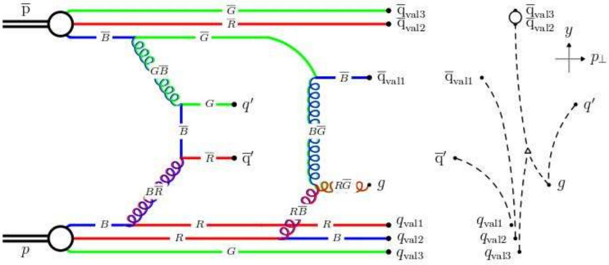
$$\frac{\alpha_s N_c}{Q^2} \times \frac{1}{N_c^2-1}\frac{xG}{\pi R^2} = 1$$

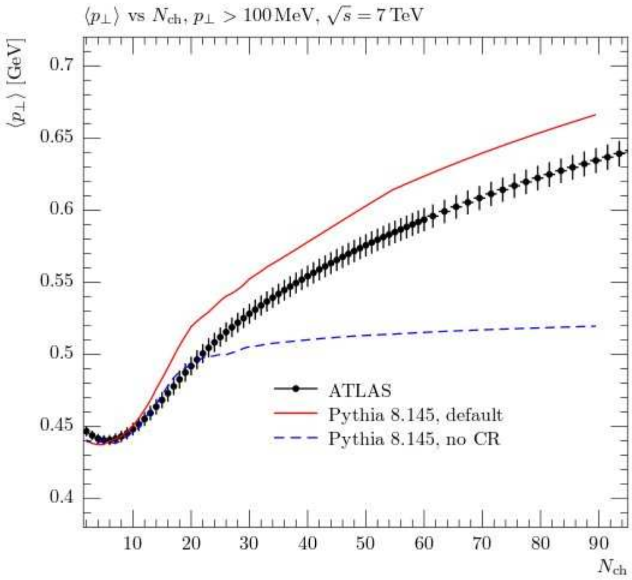
Multiple scattering in Pythia



arXiv:1101.2599

Color reconnections





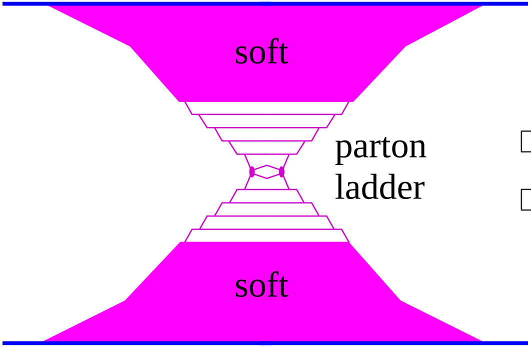
$\langle p_\perp \rangle$ vs N_{ch}, $p_\perp > 100\,\mathrm{MeV}$, $\sqrt{s} = 7\,\mathrm{TeV}$
$\langle p_\perp \rangle$ [GeV]
0.7
0.65
0.6
0.55
0.5
0.45
0.4
10
20
30
40
50
60
70
80
90
N_{ch}
ATLAS
Pythia 8.145, default
Pythia 8.145, no CR

IV Multiple scattering in EPOS

in collaboration with T. Pierog, S. Ostapchenko,
B. Guiot, G. Sophys, , M. Stefaniak

Parton based Gribov-Regge theory. By H.J. Drescher, M. Hladik, S. Ostapchenko, T. Pierog, K. Werner. hep-ph/0007198. Published in Phys.Rept. 350 (2001) 93-289.

Single scattering
(single Pomeron)



- **Parton emission starts long before the actual interaction** (partons are very long-lived due to a large γ).
- **Soft pre-evolution**
- Subsequent parton emissions towards smaller x-values and larger virtualities (from both sides).
- The final partons from either nucleon interact ("hard" collision).

Multiple scattering

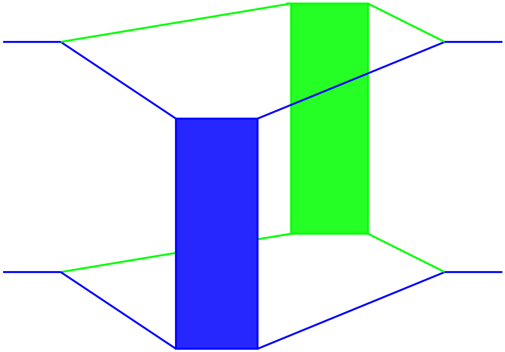
Be T the elastic (pp,pA,AA) scattering T-matrix =>

$$2s\,\sigma_{\mathrm{tot}} = \frac{1}{\mathrm{i}}\mathrm{disc}\,T$$

Basic assumption : Multiple "Pomerons"

$$iT = \sum_k \frac{1}{k!}\left\{iT_{\mathrm{Pom}} \times \ldots \times iT_{\mathrm{Pom}}\right\}$$

Example: 2 “Pomerons”



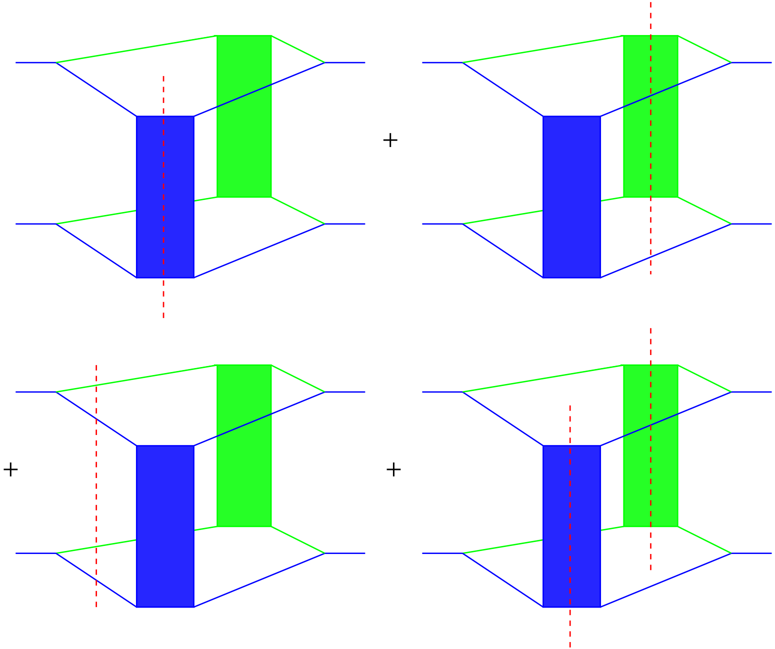
Evaluate

$$\frac{1}{\mathrm{i}}\mathrm{disc}\,\{iT_{\mathrm{Pom}}\times \ldots \times iT_{\mathrm{Pom}}\}$$

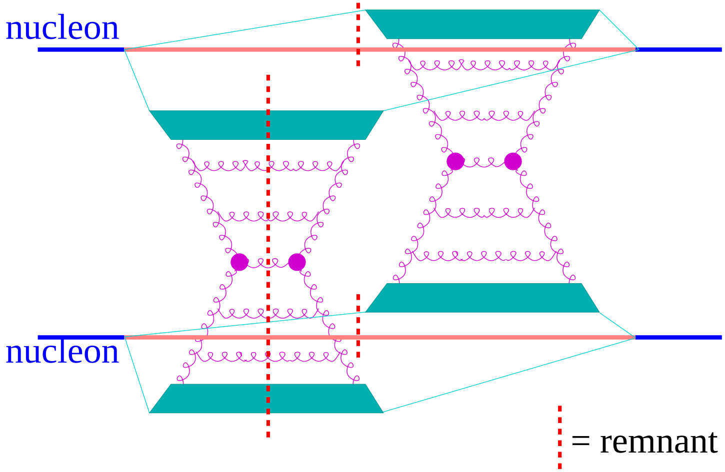
using "cutting rules" :

A "cut" multi-Pomeron diagram amounts to the sum of all possible cuts

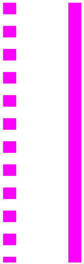
Example of two Pomerons



Using "Pomeron = parton ladder + soft", we have (first diagram)



Using a simplified notation
for “cut” and “uncut” Pomeron



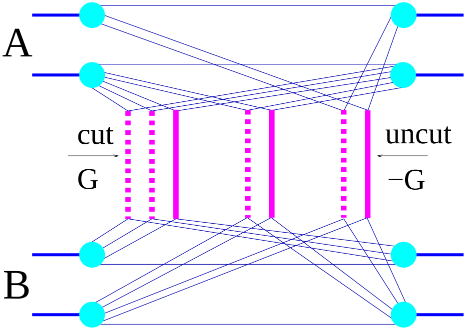
one gets ...

Complete result

(Drescher, Hladik, Ostapchenko, Pierog, and Werner, Phys. Rept. 350, 2001)

For pp, pA, AA:

$$\sigma^{\text{tot}} = \sum_{\text{cut P}} \int \sum_{\text{uncut P}} \int$$



$d\sigma_{\text{exclusive}}$

Dotted lines : Cut Pomerons (parton ladders)

$$\sigma^{\mathrm{tot}} = \int d^2b \int \prod_{i=1}^{A} d^2b_i^A\, dz_i^A\, \rho_A(\sqrt{(b_i^A)^2 + (z_i^A)^2})$$

$$\prod_{j=1}^{B} d^2b_j^B\, dz_j^B\, \rho_B(\sqrt{(b_j^B)^2 + (z_j^B)^2})$$

$$\sum_{m_1 l_1} \ldots \sum_{m_{AB} l_{AB}} (1 - \delta_{0\Sigma m_k}) \int \prod_{k=1}^{AB} \left(\prod_{\mu=1}^{m_k} dx_{k,\mu}^{+} dx_{k,\mu}^{-} \prod_{\lambda=1}^{l_k} d\tilde{x}_{k,\lambda}^{+} d\tilde{x}_{k,\lambda}^{-} \right) \Bigg\{$$

$$\prod_{k=1}^{AB} \left(\frac{1}{m_k!} \frac{1}{l_k!} \prod_{\mu=1}^{m_k} G(x_{k,\mu}^{+}, x_{k,\mu}^{-}, s, |\vec{b} + \vec{b}_{\pi(k)}^{A} - \vec{b}_{\tau(k)}^{B}|) \right.$$

$$\left. \prod_{\lambda=1}^{l_k} -G(\tilde{x}_{k,\lambda}^{+}, \tilde{x}_{k,\lambda}^{-}, s, |\vec{b} + \vec{b}_{\pi(k)}^{A} - \vec{b}_{\tau(k)}^{B}|) \right)$$

$$\prod_{i=1}^{A} \left(1 - \sum_{\pi(k)=i} x_{k,\mu,}^{+} - \sum_{\pi(k)=i} \tilde{x}_{k,\lambda}^{+} \right)^{\alpha} \prod_{j=1}^{B} \left(1 - \sum_{\tau(k)=j} x_{k,\mu}^{-} - \sum_{\tau(k)=j} \tilde{x}_{k,\lambda}^{-} \right)^{\alpha} \Bigg\}$$

- **Complicated with energy sharing included**

  (10,000,000-dimensional intergrals)

- **but doable**
  - **Parameterizations for** $G(x^+, x^-, s, b)$
  - **Analytical integrations**
  - **Employing Markov chain techniques**

Step 1:

□ We compute **partial cross sections** σ_K for particular configurations K via analytical integration

□ K is a multi-dimensional variable
for example for double scattering in pp with two Pomerons involved: $K = \left\{x_1^+, x_1^-, \vec{p}_{t1}, x_2^+, x_2^-, \vec{p}_{t2}\right\}$

□ Configurations K in AA scattering may be quite complex

Step 2:

The partial cross sections σ_K can be

- □ interpreted as **probability distributions**,
- □ enabling us to use Monte Carlo techniques to **generate configurations** K.

Since we are dealing with multidimensional probability distributions, we have to employ very sophisticated

Markov chain techniques

to generate configurations according to Ω.

Configurations via Markov chains
(the heart of EPOS, see Phys. Rept. 350, 2001)

Consider a sequence of multidimensional random numbers

$$x_1\,,\ x_2\,,\ x_3, \ldots$$

with f_t being the law for x_t.

A homogeneous Markov chain is defined as

$$f_t(x) = \sum_{x'} f_{t-1}(x')p(x' \to x).$$

with $p(x' \to x)$ being the transition probability (or matrix). Normalization : $\sum_x p(x' \to x) = 1$.

Let f be the law for x_t. The law for x_{t+1} is

$$\sum_a f(a)\,p(a \to b)\,.$$

One defines an operator T (comme Translation)

$$Tf(b) = \sum_a f(a)\,p(a \to b)\,.$$

So Tf is the law for x_{t+1} when f is the law for x_t.

A law is called stationary if $Tf = f$.

> Theorem: If a stationary law $Tf = f$ exists, then $T^k f_1$ converges towards f (which is unique) for any f_1.

So to generate (multidimensional) random numbers according to some (given) law f,

- □ one constructs a T such that $Tf = f$
- □ and then iterates $T^k f_1$

One needs, for a given law f,
to **find a transition matrix p such that $Tf = f$**

Sufficient condition (detailed balance):

$$f(a)\,p(a \to b) = f(b)\,p(b \to a)\,,$$

Proof :

$$\begin{aligned} Tf(b) &= \sum_a f(a)\,p(a \to b) \\ &= \sum_a f(b)\,p(b \to a) \\ &= f(b) \sum_a p(b \to a) \\ &= f(b)\,. \end{aligned}$$

Metropolis alorithm

Definitions:

$$\begin{aligned} p_{ab} &= p(a \to b)\,, \\ f_a &= f(a)\,. \end{aligned}$$

Take

$$p_{ab} = w_{ab}\, u_{ab}\,. \qquad (a \neq b)\,.$$

with

w_{ab} : proposal matrix ($\sum_b w_{ab} = 1$)

u_{ab} : acceptance matrix ($u_{ab} \leq 1$)

This is NOT the simple acceptance-rejection method!!

Detailed balance:

$$f_a\, p_{ab} = f_b\, p_{ba}$$

amounts to

$$f_a\, w_{ab}\, u_{ab} = f_b\, w_{ba}\, u_{ba}\ ,$$

or

$$\frac{u_{ab}}{u_{ba}} = \frac{f_b}{f_a}\,\frac{w_{ba}}{w_{ab}}\ .$$

$$\frac{u_{ab}}{u_{ba}} = \frac{f_b}{f_a}\,\frac{w_{ba}}{w_{ab}}\,.$$

is solved by

$$u_{ab} = F\left(\frac{f_b}{f_a}\,\frac{w_{ba}}{w_{ab}}\right),$$

with a function F with

$$\frac{F(z)}{F(\frac{1}{z})} = z\,.$$

$$\mathrm{Proof:\ With}\ z \equiv \frac{f_b}{f_a}\,\frac{w_{ba}}{w_{ab}}\ \mathrm{one\,finds\ :}\ \frac{u_{ab}}{u_{ba}} = \frac{F(z)}{F(\frac{1}{z})} = z = \frac{f_b}{f_a}\,\frac{w_{ba}}{w_{ab}}.$$

The F according to Metropolis is

$$F(z) = \min(z, 1)\,.$$

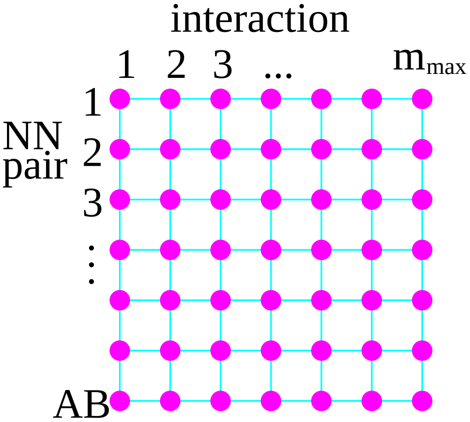
One finds indeed

$$\frac{F(z)}{F(\frac{1}{z})} = \frac{\min(z,1)}{\min(\frac{1}{z},1)} = \left\{ \begin{array}{lll} z/1 & \text{pour} & z \leq 1 \\ 1/\frac{1}{z} & \text{pour} & z > 1 \end{array} \right\} = z\,.$$

So one proposes for each iteration a new configuation b according to some w_{ab}, and accepts it with probability

$$u_{ab} = \min\left(\frac{f_b}{f_a}\frac{w_{ba}}{w_{ab}}, 1\right).$$

Configuration lattice, define w_{ab} such that b changes w.r.t. a only on one lattice site (like Ising model Metropolis)



Long iterations, but allows to generate very complex configurations according to very complex laws.

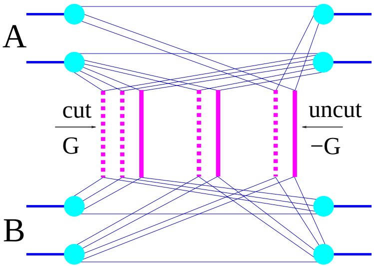
Particle production

Generating “configurations” is only half the story:

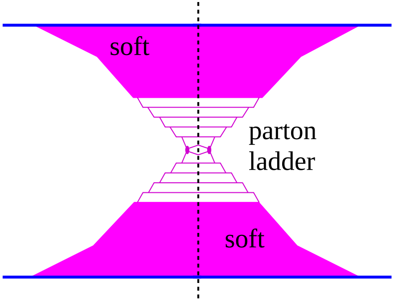
How do we obtain the corresponding partons which “make” the ladder, and finally the hadrons?

(for a given ladder, given momenta and flavors at the endpoints)

For particle production, only the cut Pomerons plays a role



the uncut ones have been summed over



Reminder: in order to compute the contribution of a cut Pomeron to a partial cross section, we sum over emitted partons, integrate over all momenta.

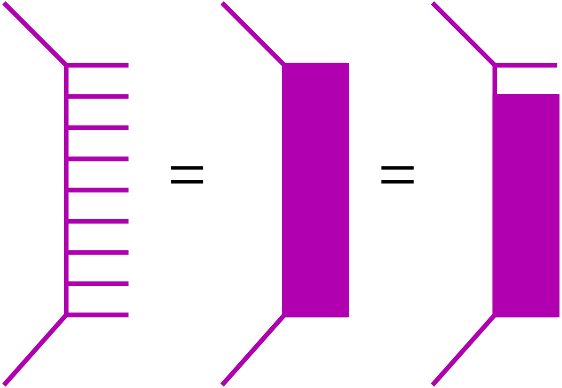
Consistency requires to use these same formulas to obtain probability distributions for the parton emissions (what we do).

Realization: big tables with pre-calculated cross sections, to be used via interpolation to generate partons according to iterative equations.

$$\begin{aligned}
\sigma^{ij}_{\mathrm{hard}}(\hat{s},Q_1^2,Q_2^2) &= \sum_k \int \frac{dQ^2}{Q^2} \int d\xi\, \Delta^i(Q_1^2,Q^2)\, \frac{\alpha_s}{2\pi}\, P_i^k(\xi)\, \sigma^{kj}_{\mathrm{hard}}(\xi\hat{s},Q^2,Q_2^2) \\
&+ \sigma^{ji}_{\mathrm{ord}}(\hat{s},Q_2^2,Q_1^2)
\end{aligned}$$

$$\begin{aligned}
\sigma^{ij}_{\mathrm{ord}}(\hat{s},Q_1^2,Q_2^2) &= \sum_k \int \frac{dQ^2}{Q^2} \int d\xi\, \Delta^i(Q_1^2,Q^2)\, \frac{\alpha_s}{2\pi}\, P_i^k(\xi)\, \sigma^{kj}_{\mathrm{ord}}(\xi\hat{s},Q^2,Q_2^2) \\
&+ \sigma^{ij}_{\mathrm{Born}}(\hat{s},Q_1^2,Q_2^2)
\end{aligned}$$

$$\begin{aligned}
\sigma^{ij}_{\mathrm{Born}}(\hat{s},Q_1^2,Q_2^2) &= K \int dp_\perp^2\, \frac{d\sigma^{ij}_{\mathrm{Born}}}{dp_\perp^2}(\hat{s},p_\perp^2) \\
&\times\ \Delta^i(Q_1^2,M_\mathrm{F}^2)\, \Delta^j(Q_2^2,M_\mathrm{F}^2)\, \Theta\big(M_\mathrm{F}^2 - \max[Q_1^2,Q_2^2]\big)
\end{aligned}$$



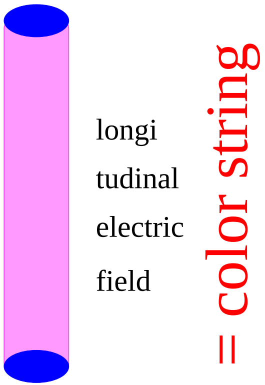
Probability of single emission:

$$prob(\xi, Q^2) = \frac{dQ^2}{Q^2}\,\Delta^i(Q_1^2, Q^2)\,\frac{\alpha_s}{2\pi}\,P_i^k(\xi)\,\sigma_{\mathrm{hard}}^{kj}(\xi\hat{s}, Q^2, Q_2^2)$$

From partons to strings:

For $t > 0$, a (cut) Pomeron represents actually a (mainly) **longitudinal color field**,

where the ladder rungs (gluons) represent small transverse momentum components[1].



[1] Lund model idea, first e+e-, then generalized to pp, see also CGC

Realization:

One-dimensional character of the fields

=> classical string theory
(which does not use much more than some general symmetries)

- ☐ **Mapping: parton ladders -> kinky strings**
  (parton momentum = kink)

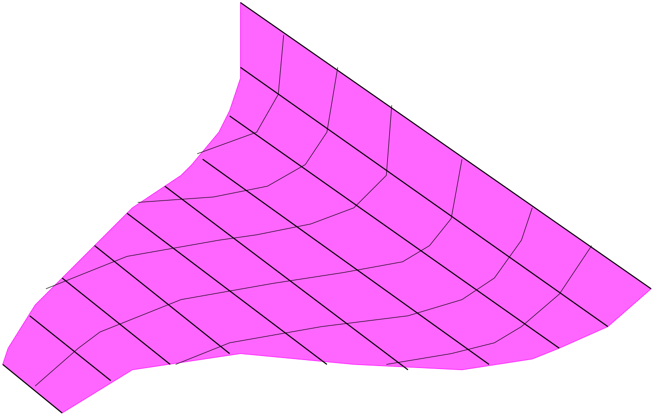
- ☐ **Classical string evolution + decay via area law**

String:

two-dimensional surface

$$x(\sigma,\tau)$$

in Minkowski space



Break probability :

$$dP = p_B \, dA,$$

In detail: The string surface is given as

$$x^\mu(\sigma,\tau) = x_0 + \frac{1}{2}\int_{\sigma-\tau}^{\sigma+\tau} g^\mu(\xi)d\xi,$$

so it is completely given in terms of some function $g^\mu(\xi)$ with

$$g^\mu(\sigma) = \dot{x}^\mu(\sigma,\tau=0).$$

We consider only strings with a piecewise constant initial velocity g, which are called kinky strings.

- □ **This string is characterized by a sequence of σ intervals $[\sigma_k, \sigma_{k+1}]$, and the corresponding constant values (say v_k) of g in these intervals.**

Such an interval with the corresponding constant value of g is referred to as “kink”.

A parton ladder represents a **sequence of partons** of the type $q - g ... - g - \bar{q}$, with soft "end partons" q and $\bar{q}$, and hard inner gluons g.

The mapping "partons →string" is done such that we **identify a parton sequence with a kinky string**

by requiring "parton = kink",

$$\text{with} \quad \boldsymbol{\sigma_{k+1} - \sigma_k} = \textbf{energy of parton } \boldsymbol{k}$$

$$\textbf{and } \boldsymbol{v_k} = \textbf{momentum of parton } \boldsymbol{k} \; / \; \boldsymbol{E_k}.$$

What is really done (PR 232, pp 87-299, 1993, PR 350, pp 93-289, 2001):

A string represents a two-dimensional surface in Minkowski space

$$x = x(\sigma, \tau),$$

with σ being a space-like and τ a time-like parameter.

In order to obtain the equations of motion, we need a Lagrangian. It is obtained by demanding the invariance of the action with respect to gauge transformations. This way one finds the Lagrangian of Nambu-Goto:

$$L = -\kappa\sqrt{(x'\dot{x})^2 - x'^2\dot{x}^2},$$

with “dot” and “prime” referring to the partial derivatives with respect to σ and τ, and with κ being the string tension.

With this Lagrangian we get the Euler-Lagrange equations of motion:

$$\frac{\partial}{\partial\tau}\frac{\partial L}{\partial\dot{x}_\mu}+\frac{\partial}{\partial\sigma}\frac{\partial L}{\partial x'_\mu}=0.$$

We use the gauge fixing

$$x'^2+\dot{x}^2=0 \text{ and } x'\dot{x}=0,$$

which provides a very simple equation of motion, namely a wave equation,

$$\frac{\partial^2 x_\mu}{\partial\tau^2}-\frac{\partial^2 x_\mu}{\partial\sigma^2}=0,$$

with the boundary conditions:

$$\partial x_\mu/\partial\sigma=0,\ \sigma=0,\pi.$$

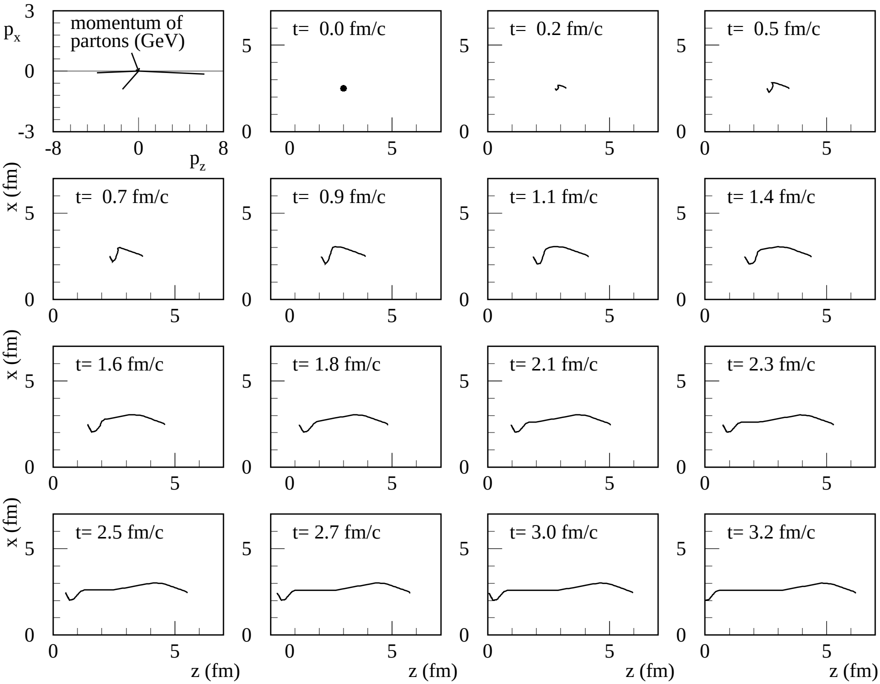
The solution of the equation of motion (with initial extension zero) is

$$x^{\mu}(\sigma,\tau) = x_0 + \frac{1}{2}\left(\int_{\sigma-\tau}^{\sigma+\tau} g^{\mu}(\xi)d\xi\right),$$

where g is the initial velocity, $g(\sigma) = \dot{x}(\sigma,\tau)_{\tau=0}$.

Strings are classified according to the function g. Strings with piecewise constant g are called kinky strings, each segment being called kink, finally identified with perturbative partons.

In the following figure, we show the evolution of a string generated in electron-positron annihilation (4 internal kinks).



momentum of partons (GeV)
p_x
p_z
3
0
-3
-8
0
8
t= 0.0 fm/c
t= 0.2 fm/c
t= 0.5 fm/c
t= 0.7 fm/c
t= 0.9 fm/c
t= 1.1 fm/c
t= 1.4 fm/c
t= 1.6 fm/c
t= 1.8 fm/c
t= 2.1 fm/c
t= 2.3 fm/c
t= 2.5 fm/c
t= 2.7 fm/c
t= 3.0 fm/c
t= 3.2 fm/c
x (fm)
z (fm)
0
5

Hadron production

is finally realized via string breaking, such that string fragments are identified with hadrons.

Hypothesis: the string breaks within an infinitesimal area dA on its surface with a probability which is proportional to this area,

$$\boldsymbol{dP = p_B\, dA},$$

where p_B is the fundamental parameter of the procedure. [1]

[1] Elegant realization, making use of the dynamics of strings with piecewise constant initial conditions.

A string break is realized via **quark-antiquark** or **diquark-antidiquark** pair production with probability

$$p_{i(j)} = \frac{1}{Z} \exp\left(-\pi \frac{M_{i(j)}^2}{\kappa}\right)$$

with

$$M_{ij} = M_i + M_j + c_i c_j M_0$$

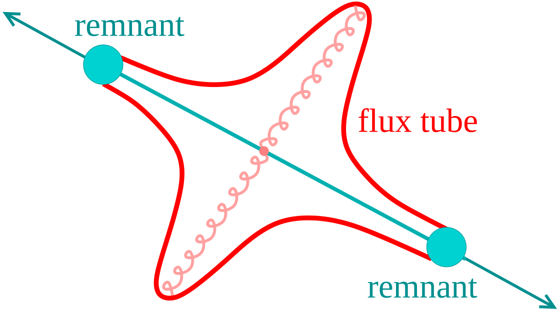
Transverse momenta $\vec{p}_t$ and $-\vec{p}_t$ are generated at each breaking, according to

$$f\left(k\right) \propto e^{-|\vec{p}_t|/2\bar{p}_t}\,, \tag{1}$$

with a parameter $\bar{p}_t$.

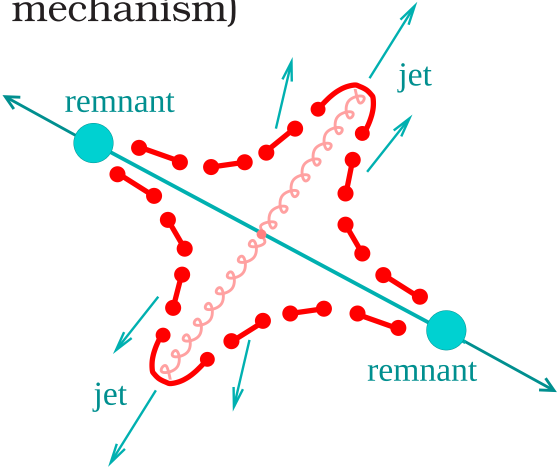
Jets:

Parton ladder = color flux tubes = **kinky strings**



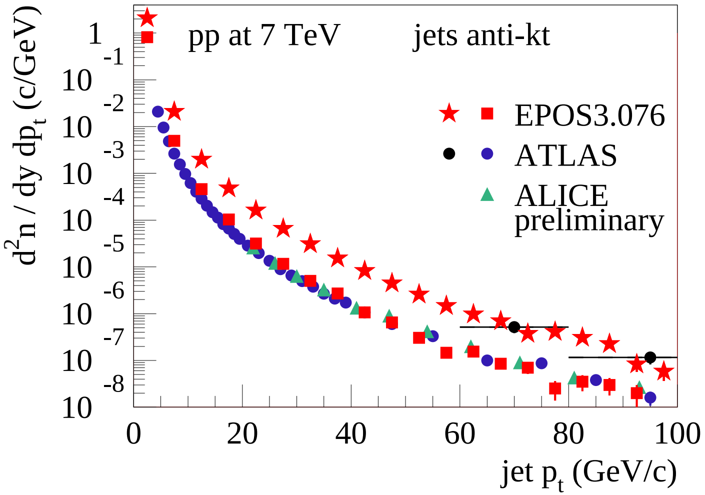
(here no IS radiation, only hard process producing two gluons)

which expand and break
via the production of quark-antiquark pairs
(Schwinger mechanism)

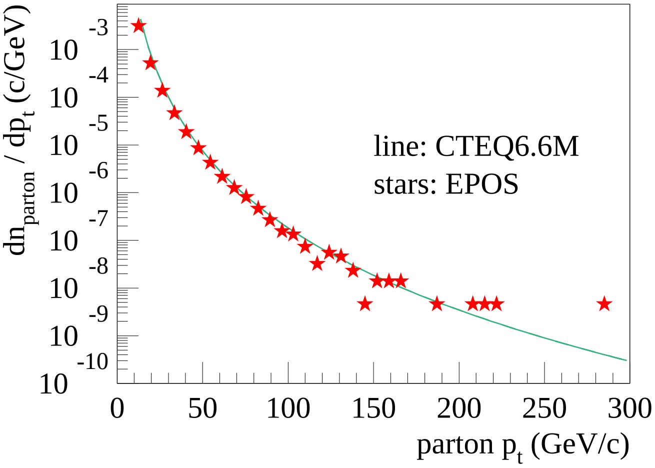


String segment = hadron. Close to “kink”: jets

Check: jet production in pp at 7 TeV



Comparison with parton model calculation using CTEQ PDFs for pp at 7 TeV



V Collectivity in EPOS

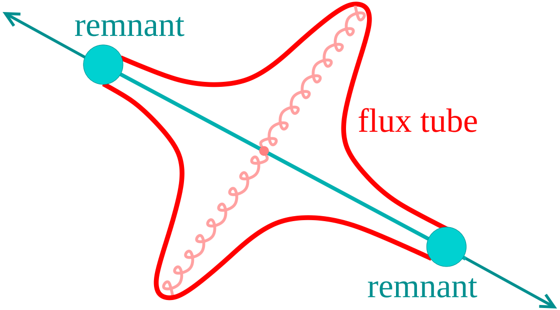
Heavy ion collisions
or high energy & high multiplicity pp events:

- the usual procedure has to be modified, since the density of strings will be so high that they cannot possibly decay independently

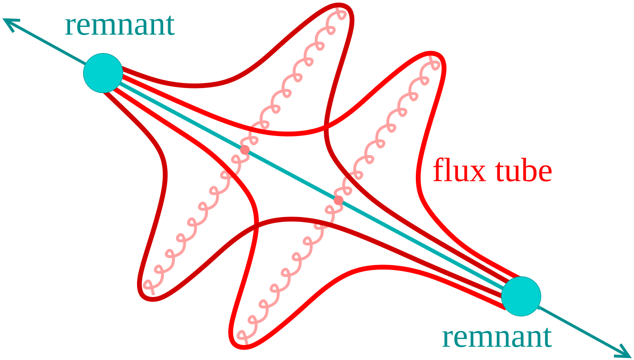
Some string pieces will constitute bulk matter, others show up as jets

These are the same strings (all originating from hard processes at LHC) which constitute BOTH jets and bulk !!

again: single scattering => 2 color flux tubes



... two scatterings => 4 color flux tubes



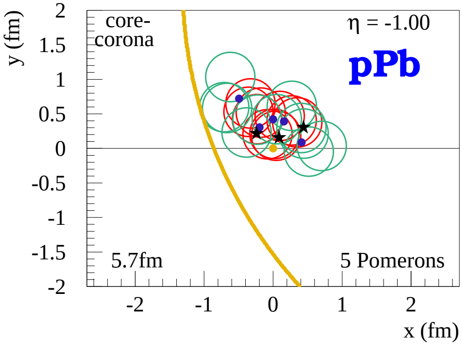
... many scatterings (AA) => many color flux tubes

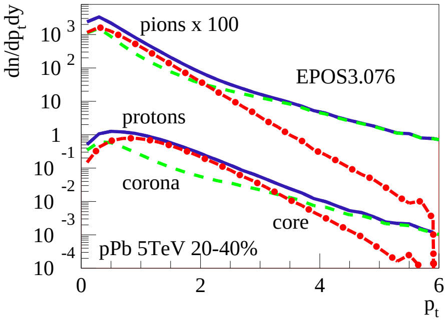
=> matter + escaping pieces (jets)

Core-corona procedure (for pp, pA, AA):

Pomeron => parton ladder => flux tube (kinky string)

String segments with high pt escape => **corona**,
the others form the **core** = initial condition for hydro
depending on the local string density



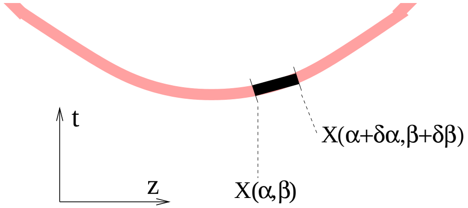


Core:

(we use α and β rather than σ and τ)

We split each string into a sequence of string segments, corresponding to widths $\delta\alpha$ and $\delta\beta$ in the string parameter space

Picture is schematic: the string extends well into the transverse dimension, correctly taken into account in the calculations



Energy momentum tensor and the flavor flow vector at some position x at initial proper time $\tau = \tau_0$:

$$T^{\mu\nu}(x) = \sum_i \frac{\delta p_i^\mu \delta p_i^\nu}{\delta p_i^0} g(x - x_i),$$

$$N_q^\mu(x) = \sum_i \frac{\delta p_i^\mu}{\delta p_i^0} q_i \, g(x - x_i),$$

$q \in u, d, s$: net flavor content of the string segments

$\delta p = \left\{ \frac{\partial X(\alpha,\beta)}{\partial \beta} \delta\alpha + \frac{\partial X(\alpha,\beta)}{\partial \alpha} \delta\beta \right\}$: four-momenta of the segments.

g: Gaussian smoothing kernel with a transverse width $\sigma_\perp$

The Lorentz transformation into the comoving frame provides the energy density ε and the flow velocity components v^i.

The evolution of the system for $\tau \geq \tau_0$ treated **macroscopicly**, solving the equations of **relativistic hydrodynamics**:

Three equations concerning conserved currents:

$$\boldsymbol{\partial_\nu N_q^\nu = 0}$$

with

$$N_q^\nu = n_q \, u^\nu$$

and n_q ($q = u, d, s$) representing (net) quark densities, u^ν is the velocity four vector.

Four equations concerning energy-momentum conservation:

$$\partial_\nu T^{\mu\nu} = 0.$$

The energy-momentum tensor $T^{\mu\nu}$ is

- □ the flux of the μth component of the momentum vector
- □ across a surface with constant ν coordinate (using four-vectors)

T^{00}: **Energy density** $dE/dx^1dx^2dx^3$ (x^0 const)

T^{01}: **Energy flux** $dE/dx^0dx^2dx^3$ (x^1 const)

T^{i0}: **Momentum density**

T^{ij}: **Momentum flux**

The equation

$$\partial_\nu T^{\mu\nu} = 0$$

is very general, no need for thermal equilibrium, no need for particles.

The energy-momentum tensor is

the **conserved Noether current**

associated with **space-time translations**.

□ We have $4+n_f$ equations, so we should express T in terms of 4 quantities (unknowns)

□ and/or find additional equations

□ which means additional assumptions

First approach: **Ideal Fluid**

In the local rest frame of a fluid cell:

- □ $T^{00} = \varepsilon$ (energy density in LRF)
- □ $T^{0i} = 0$ (no energy flow)
- □ $T^{0i} = 0$ (no momenum in LRF)
- □ $T^{ij} = \delta_{ij} p$ (p = isotropic pressure)

In arbitrary frame:

$$T^{\mu\nu} = (\varepsilon + p)u^{\mu}u^{\nu} - pg^{\mu\nu}$$

+ Equation of state $p = p(\varepsilon)$ of QGP from lQCD

=> 4 equations for 4 unknowns (ε, velocity)

Other way of writing T:

$$\boxed{T^{\mu\nu} = \varepsilon u^{\mu} u^{\nu} - p\Delta^{\mu\nu}}$$

with Δ being the projector $\perp$ to u ($\Delta^{\mu\nu} u_{\nu} = 0$):

$$\Delta^{\mu\nu} = g^{\mu\nu} - u^{\mu} u^{\nu}$$

Including viscous effects, following Landau:

Navier Stokes equations (with shear and bulk viscosity η, ζ):

$$T^{\mu\nu} = \varepsilon u^{\mu} u^{\nu} - (p + \Pi)\, \Delta^{\mu\nu} + \pi^{\mu\nu}$$

$$\pi^{\mu\nu} = \pi^{\mu\nu}_{NS} = 2\eta\, \nabla^{\langle\mu} u^{\nu\rangle},$$

$$\Pi = \Pi_{NS} = -\zeta\, \nabla_{\alpha} u^{\alpha}$$

$$A_{\langle\mu} B_{\nu\rangle} = \tfrac{1}{2}\left(\Delta^{\alpha}_{\mu}\Delta^{\beta}_{\nu} + \Delta^{\alpha}_{\nu}\Delta^{\beta}_{\mu} - \tfrac{2}{3}\Delta^{\alpha\beta}\Delta_{\mu\nu}\right) A_{\alpha} B_{\beta},\ \nabla^{\mu} = \Delta^{\mu\nu}\partial_{\nu}$$

$\pi^{\mu\nu}$, Π shear stress tensor, bulk pressure

NS does not work:

□ instabilities due to acausal behavior

Solution (Israel-Steward):

$$T^{\mu\nu} = \varepsilon u^{\mu} u^{\nu} - (p + \Pi)\,\Delta^{\mu\nu} + \pi^{\mu\nu}$$

$$\pi^{\mu\nu} = \pi^{\mu\nu}_{\mathbf{NS}} + \tau_{\pi}\left(-D\pi^{\mu\nu} + I^{\mu\nu}_{\pi}\right),$$

$$\Pi = \Pi_{\mathbf{NS}} + \tau_{\Pi}\left(-D\Pi + I_{\Pi}\right)$$

$$\textbf{with } D = u^{\mu}\partial_{\mu}$$

Different choices for the I. Implemented in EPOS3 by Y. Karpenko:
$I^{\mu\nu}_{\pi} = -\frac{4}{3}\pi^{\mu\nu}\partial_{\gamma}u^{\gamma} - [u^{\nu}\pi^{\mu\beta} + u^{\mu}\pi^{\nu\beta}]u^{\lambda}\partial_{\lambda}u_{\beta}, \quad I_{\Pi} = -\frac{4}{3}\Pi\partial_{\gamma}u^{\gamma}$

EPOS implementation (Yuri Karpenko)

Milne coordinates:

$$\begin{aligned}\eta &= \frac{1}{2}\ln\frac{t+z}{t-z}\\ \tau &= \sqrt{t^2-z^2}\end{aligned}$$

Metric tensor:

$$g^{\mu\nu} = \mathrm{diag}(1,-1,-1,-1/\tau^2).$$

Nonzero Christoffel symbols:

$$\Gamma^{\eta}_{\tau\eta} = \Gamma^{\eta}_{\eta\tau} = 1/\tau, \quad \Gamma^{\tau}_{\eta\eta} = \tau.$$

The hydrodynamic equations (using covariant drivatives):

$$\partial_{;\nu}T^{\mu\nu} = \partial_{\nu}T^{\mu\nu} + \Gamma^{\mu}_{\nu\lambda}T^{\nu\lambda} + \Gamma^{\nu}_{\nu\lambda}T^{\mu\lambda} = 0$$

Freeze out

happens at a hypersurface defined by $T = T_H$ (for given T_H).

Hyper-surface: $x^\mu = x^\mu(\tau, \varphi, \eta)$:

$$x^0 = \tau \cosh \eta, \; x^1 = r \cos \varphi, \; x^2 = r \sin \varphi, \; x^3 = \tau \sinh \eta,$$

with $r = r(\tau, \varphi, \eta)$.

The hypersurface element is

$$d\Sigma_\mu = \varepsilon_{\mu\nu\kappa\lambda} \frac{\partial x^\nu}{\partial \tau} \frac{\partial x^\kappa}{\partial \varphi} \frac{\partial x^\lambda}{\partial \eta} d\tau d\varphi d\eta,$$

(with $\varepsilon^{0123} = 1$)

Computing the derivatives, one gets:

$$\begin{aligned}
d\Sigma_0 &= \left\{ -r\frac{\partial r}{\partial \tau}\tau \cosh\eta + r\frac{\partial r}{\partial \eta}\sinh\eta \right\} d\tau d\varphi d\eta, \\
d\Sigma_1 &= \left\{ \qquad \frac{\partial r}{\partial \varphi}\tau \sin\varphi + r\,\tau\cos\varphi \right\} d\tau d\varphi d\eta, \\
d\Sigma_2 &= \left\{ \qquad -\frac{\partial r}{\partial \varphi}\tau \cos\varphi + r\,\tau\sin\varphi \right\} d\tau d\varphi d\eta, \\
d\Sigma_3 &= \left\{ \quad r\frac{\partial r}{\partial \tau}\tau \sinh\eta - r\frac{\partial r}{\partial \eta}\cosh\eta \right\} d\tau d\varphi d\eta.
\end{aligned}$$

Cooper-Frye hadronization amounts to calculating

$$E\frac{dn}{d^3p} = \int d\Sigma_\mu p^\mu f(up),$$

with u being the flow four-velocity in the global frame, related to Milne fram via

$$\begin{aligned} u^0 &= \tilde{u}^{\,0}\cosh\eta + \tilde{u}^{\,3}\sinh\eta\,, \\ u^1 &= \tilde{u}^{\,1}\,, \\ u^2 &= \tilde{u}^{\,2}\,, \\ u^3 &= \tilde{u}^{\,0}\sinh\eta + \tilde{u}^{\,3}\cosh\eta\,. \end{aligned}$$

Similarly p expressed in terms of $\tilde{p}$ in the Milne frame.

f is the Bose-Einstein or Fermi-Dirac distribution.

Hadronic afterburner: UrQMD

After "hadronization" hadrons follow straight and may still interact via

$$h_1 + h_2 \to \sum_j h'_j$$

We use "UrQMD".

M. Bleicher et al., J. Phys. G25 (1999) 1859;

H. Petersen, J. Steinheimer, G. Burau, M. Bleicher and H. Stocker, Phys. Rev. C78 (2008) 044901

VI Flow in small systems

=> comparing models

with / without collectivity built in

pPb results (more results: arXiv:1312.1233)

We will compare EPOS3 with data

and also with

EPOS LHC

LHC tune of EPOS1.99, :
same GR, but uses **parameterized flow**
T. Pierog et al, arXiv:1306.5413

AMPT

Parton + hadron cascade **-> some collectivity**
Z.-W. Lin, C. M. Ko, B.-A. Li, B. Zhang and S. Pal, Phys. Rev. C 72, 064901 (2005).

QGSJET

GR approach, **no flow**
S. Ostapchenko, Phys. Rev. D74 (2006) 014026

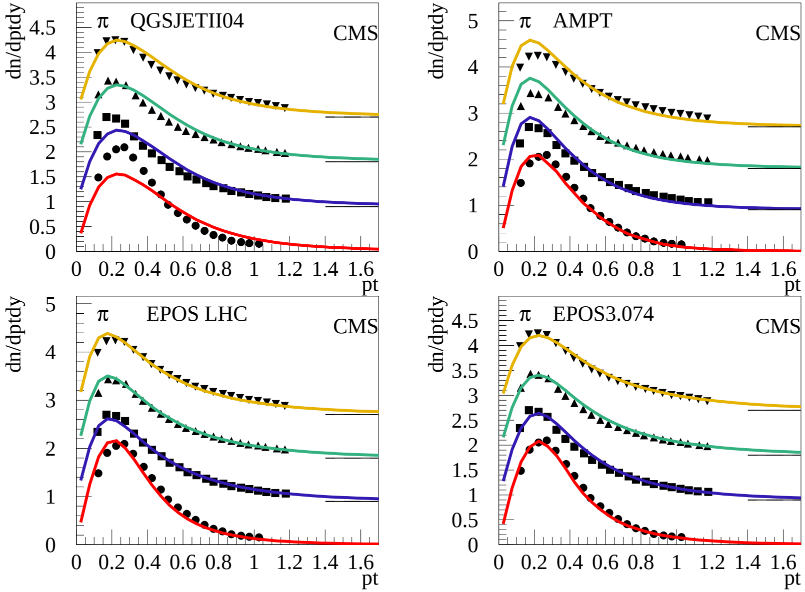
CMS: Multiplicity dependence
of pion, kaon, proton pt spectra

CMS, arXiv:1307.3442

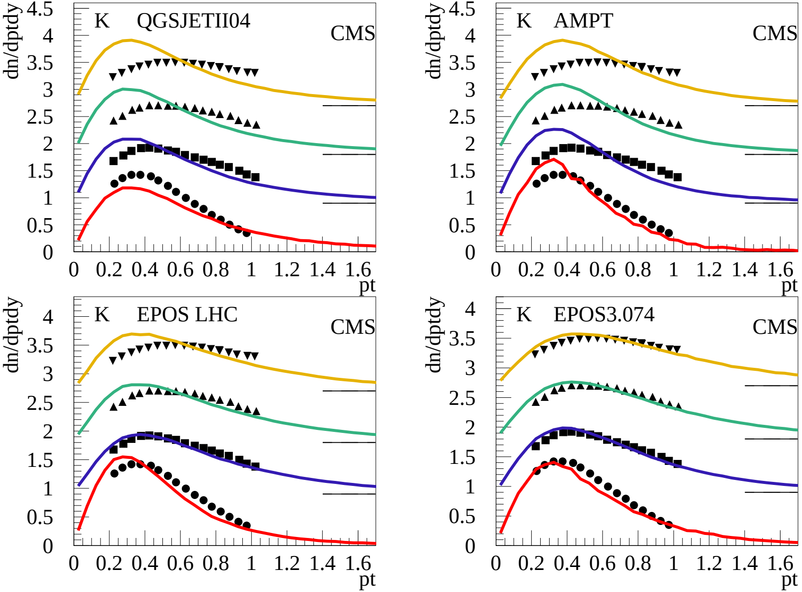
We plot 4 centrality classes:

$\langle N_{\mathbf{trk}}^{\mathbf{offline}} \rangle$ = 8, 84, 160, 235 (in $|\eta| < 2.4$)

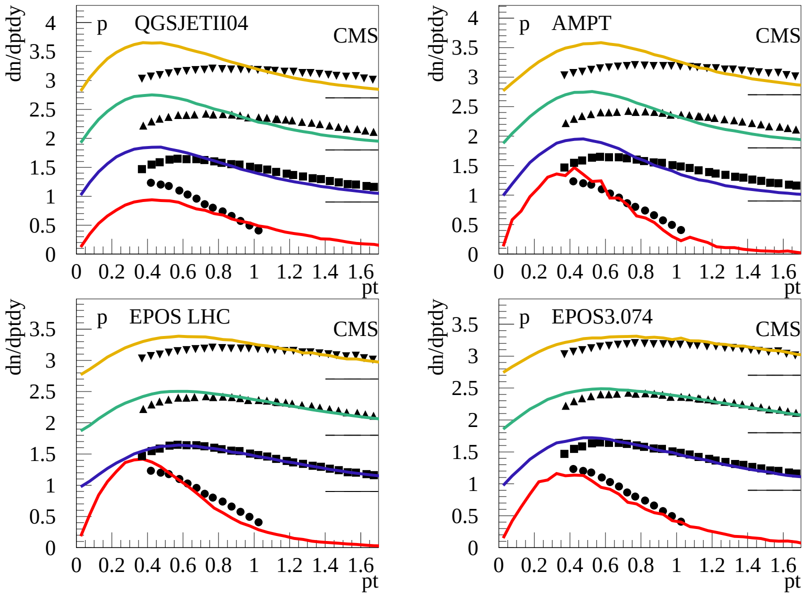
Multiplicity = centrality measure



Little change with multiplicity for pions



Kaon spectra change with multiplicity

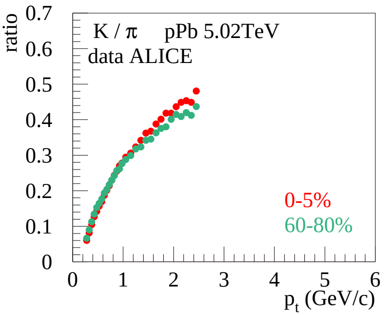


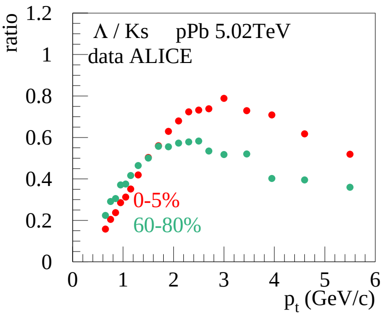
Strong variation of proton spectra => flow helps

ALICE: compare pt spectra for identified particles in different multiplicity classes: 0-5%,...,60-80%

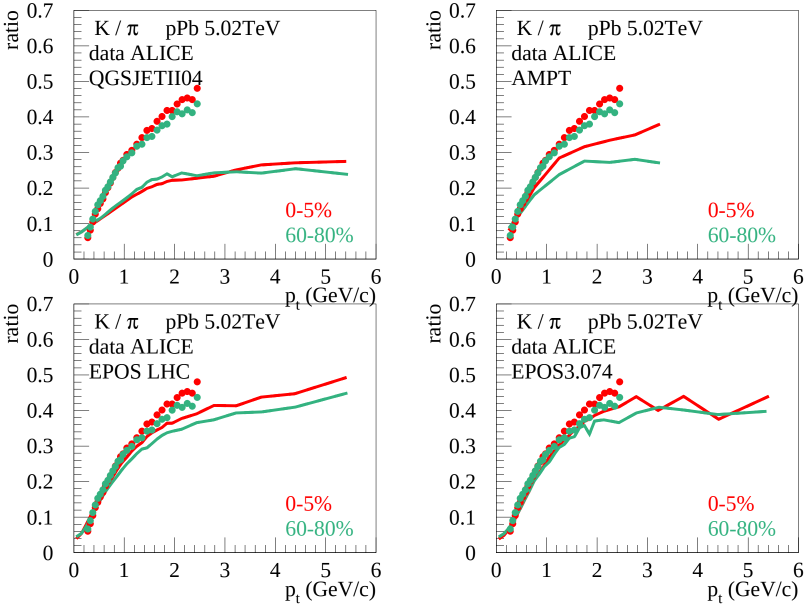
(in $2.8 < \eta_{\text{lab}} < 5.1$) From R. Preghenella, ALICE, talk Trento workshop 2013

Useful : ratios (K/pi, p/pi...)

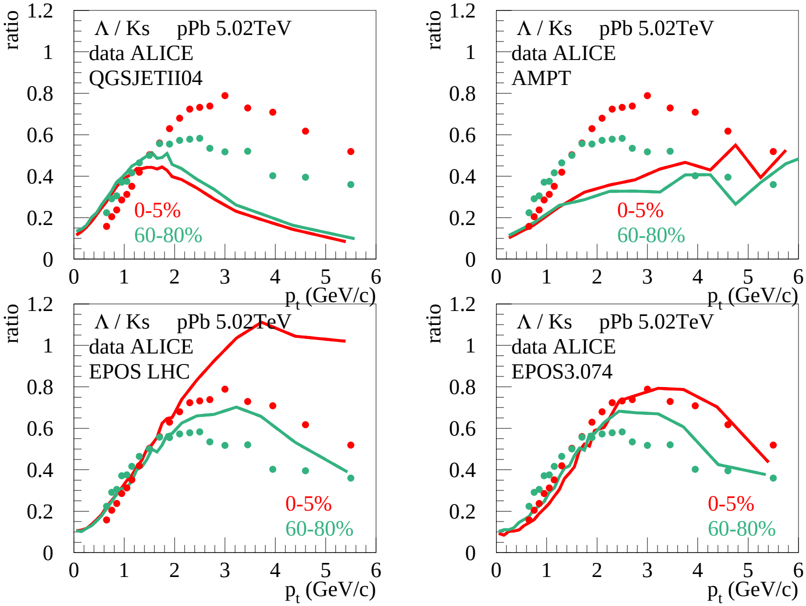




Significant variation of lambda/K – like in PbPb



No multiplicity dependence (not trivial to get the peripheral right)



Significant multiplicity dependence. Flow helps

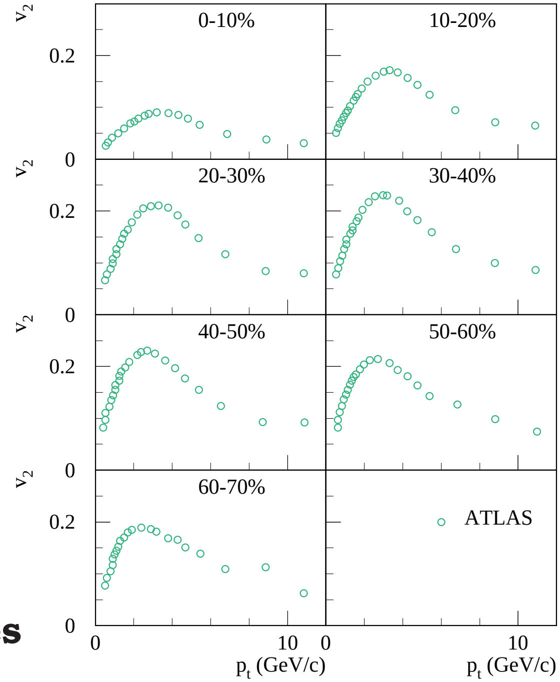
v_2 in PbPb

from central to peripheral

Changes smoothly towards peripheral

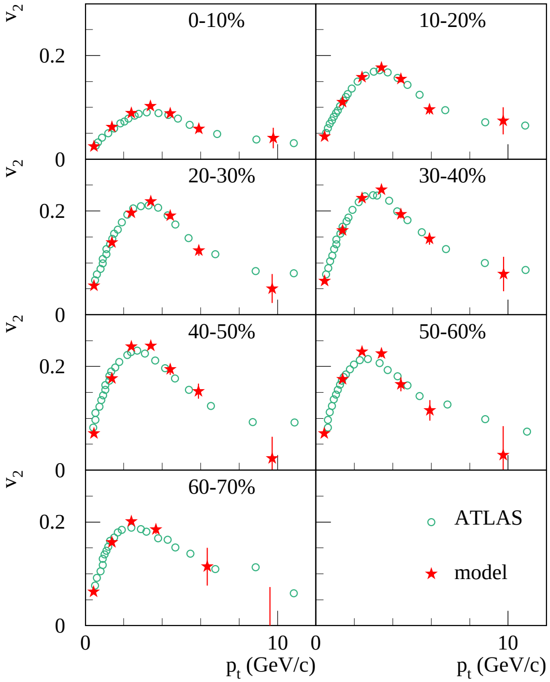
=>

physics changes smoothly



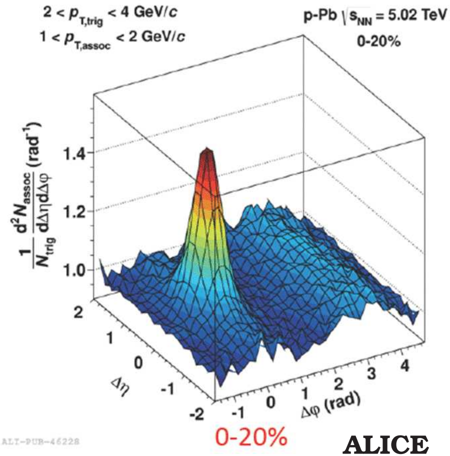
with EPOS simulations

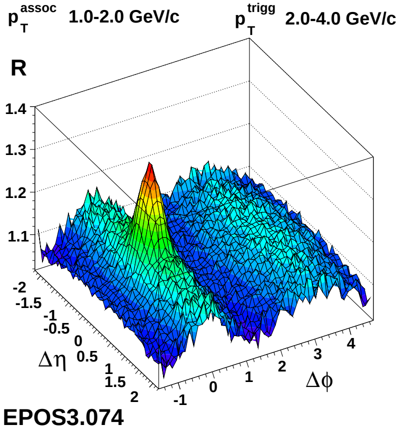
Flow is needed even for peripheral collisions!



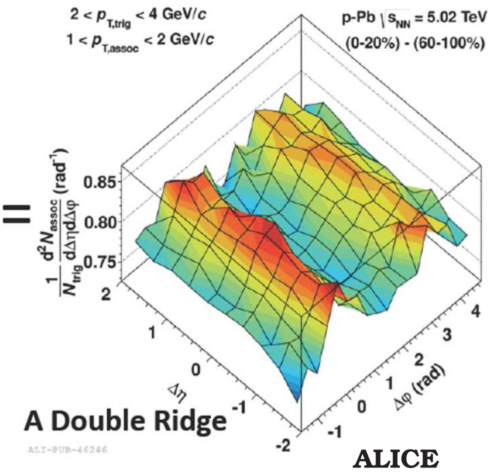
“Ridges” in pA

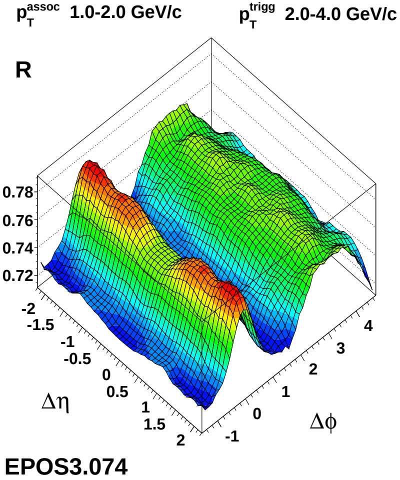
ALICE, arXiv:1212.2001, arXiv:1307.3237



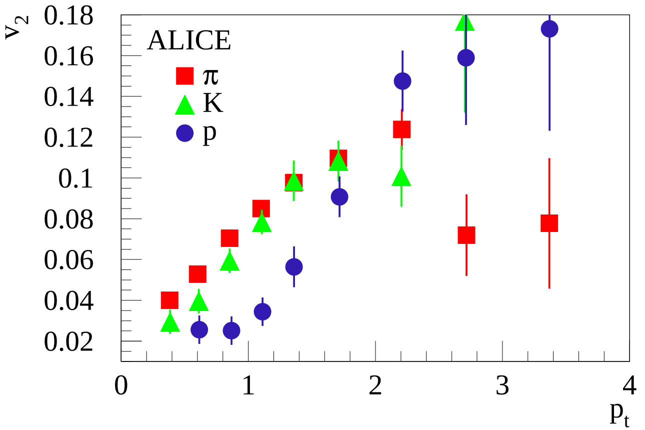


Central - peripheral (to get rid of jets)





Identified particle v2

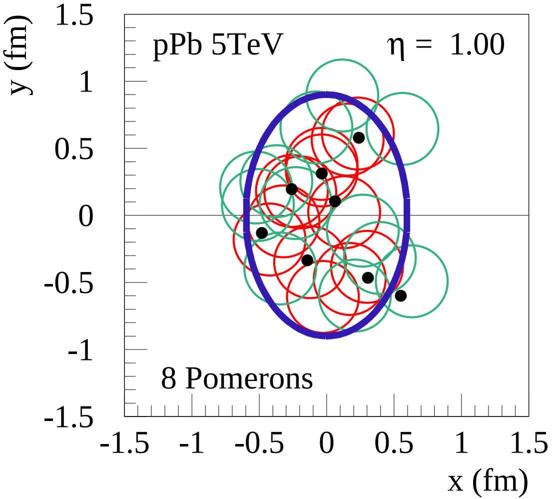


mass splitting, as in PbPb !!!

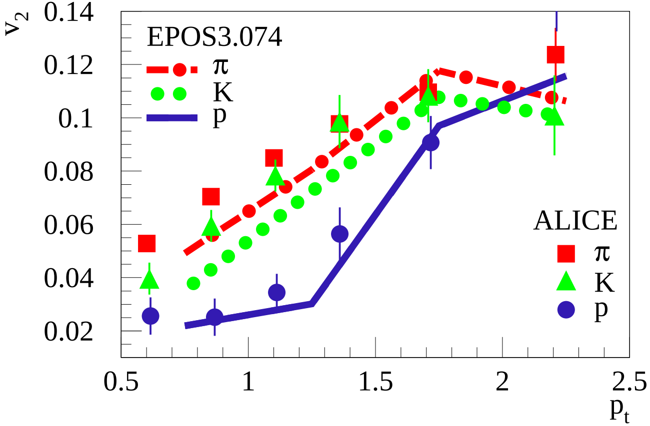
pPb in EPOS3:

Pomerons (number and positions) characterize geometry (P. number $\propto$ multiplicity)

random
azimuthal
asymmetry
=>
asymmetric flow
seen at higher pt
for heavier ptls



v2 for π, K, p clearly differ



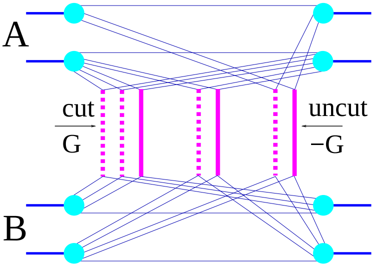
mass splitting, due to flow

VII Recent developments

(Saturation, strangeness and charm enhancement with multiplicity)

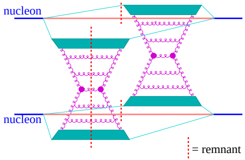
Reminder :

$$\sigma^{\text{tot}} = \sum_{\text{cut P}} \int \sum_{\text{uncut P}} \int$$



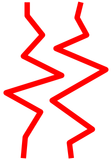
$d\sigma_{\text{exclusive}}$

=



parton ladders

=> kinky strings

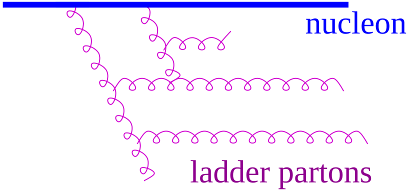


Non-linear effects

Computing the expressions G for single Pomerons:
A cutoff Q_0 is needed (for the DGLAP integrals).

Taking Q_0 constant leads to a power law increase of cross sections vs energy (=> wrong)

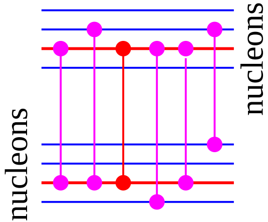
because non-linear effects like gluon fusion are not taken into account



Solution: Instead of a constant Q_0, use a dynamical saturation scale for each Pomeron:

$$Q_s = Q_s(N_{\mathbb{P}}, s_{\mathbb{P}})$$

with



$N_{\mathbb{P}}$ **= number of Pomerons connected to a given Pomeron (whose probability distribution depends on Q_s)**

$s_{\mathbb{P}}$ **= energy of considered Pomeron**

We get $Q_s(N_{\mathbb{P}}, s_{\mathbb{P}})$ from fitting

- □ the energy dependence of elementary quantities (σ_{tot}, σ_{el}, σ_{SD}, $dn^{\mathrm{ch}}/d\eta(0)$) for pp
- □ the multiplicity dependence of dn^{π}/dp_t at large p_t for pp at 7 TeV

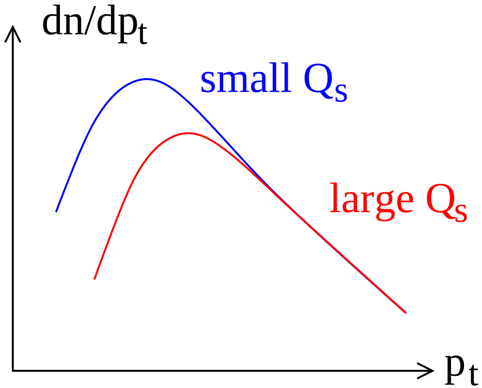
We find

$$\boldsymbol{Q_s \propto \sqrt{N_{\mathbb{P}}} \times (s_{\mathbb{P}})^{0.30}}$$

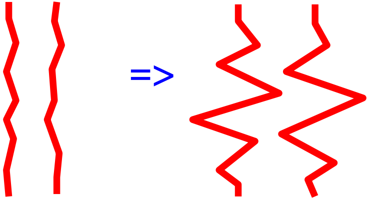
CGC for AA:

$$Q_s \propto N_{\mathrm{part}} \times (1/x)^{0.30}$$

Parton distributions



Increasing multiplicity
=> increasing $N_{\mathbf{Pom}}$
=> Increasing Q_s
=> harder Pomerons
=> harder strings



=> more high pt particles

=> Strong increase of $\langle p_t \rangle$ with multiplicity

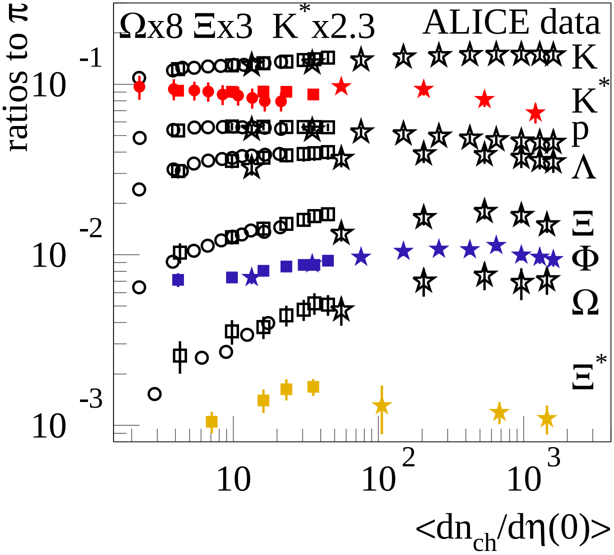
These saturation effects concern the corona !

What about multiplicity dependence of core-corona separation ?

- **First check particle ratios**
  (core-corona)
- **Then mean pt vs multiplicity**
  (core-corona+saturation)

We compare simulations to ALICE data

Particle ratios to pions vs $\left\langle \frac{dn_{ch}}{d\eta}(0) \right\rangle$



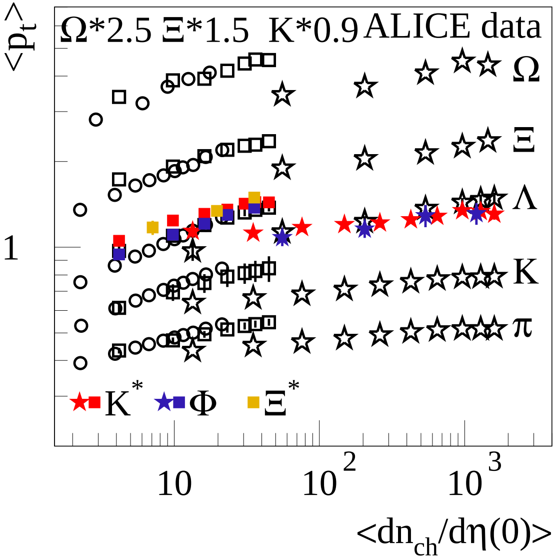
circles = pp (7TeV)

squares = pPb (5TeV)

stars = PbPb (2.76TeV)

Refs: next slide

Mean p_t vs $\left\langle \frac{dn_{ch}}{d\eta}(0) \right\rangle$



circles = pp (7TeV)

squares = pPb (5TeV)

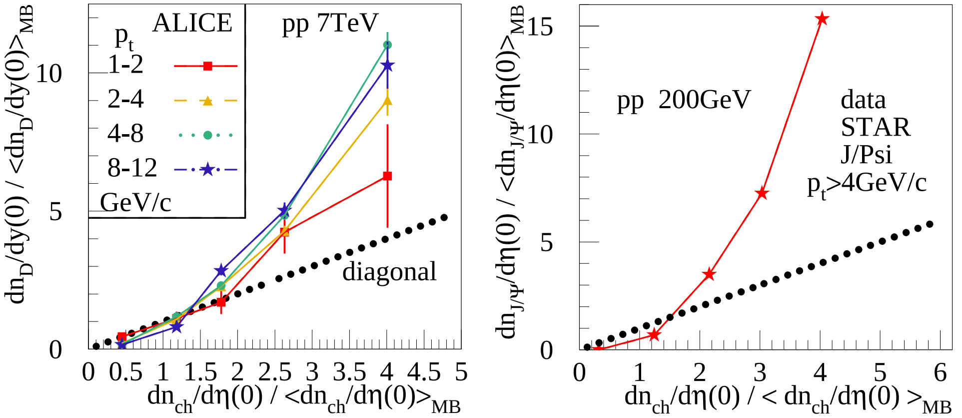
stars = PbPb (2.76TeV)

Data partly collected by A. G. Knospe

Refs:

<dNch/deta> in Pb+Pb: Phys. Rev. Lett. 106 032301 (2011)
pi+-, K+-, and (anti)protons in Pb+Pb: Phys. Rev. C 88 044910 (2013)
Lambda in Pb+Pb: Phys. Rev. Lett. 111 222301 (2013)
Xi- and Omega in p+Pb: Phys. Lett. B 758 389-401 (2016)
pi+-, K+-, (anti)protons, and Lambda in p+Pb: Phys. Lett. B 728 25-38 (2014)
<dNch/deta> in p+Pb: Eur. Phys. J. C 76 245 (2016)
Xi- and Omega in p+Pb: Phys. Lett. B 758 389-401 (2016)
<dNch/deta> in p+p 7 TeV: Eur. Phys. J. C 68 345-354 (2010)
pi+-, K+-, and (anti)protons in p+p 7 TeV: Eur. Phys. J. C 75 226 (2015)
Xi- and Omega in p+p 7 TeV: Phys. Lett. B 712 309 (2012)
and data points from Rafael Derradi de Souza, SQM2016

D or J/Ψ multiplicity vs $\frac{dn_{ch}}{d\eta}(0)$ in pp



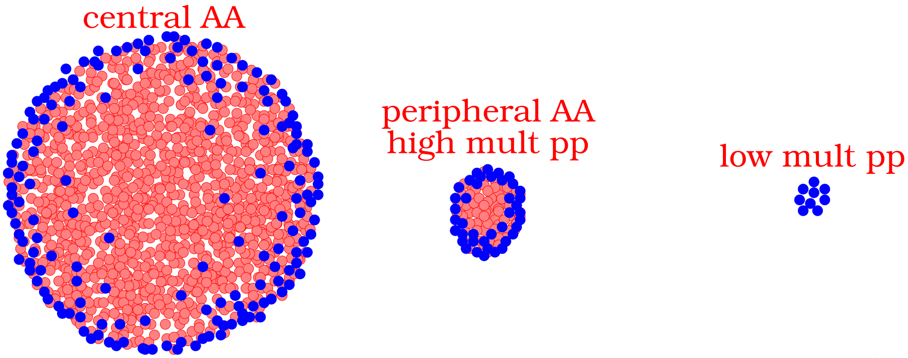
ALICE JHEP 09 (2015) 148,
arXiv:1505.00664v1

STAR, shown at MPI2016

strongly nonlinear increase

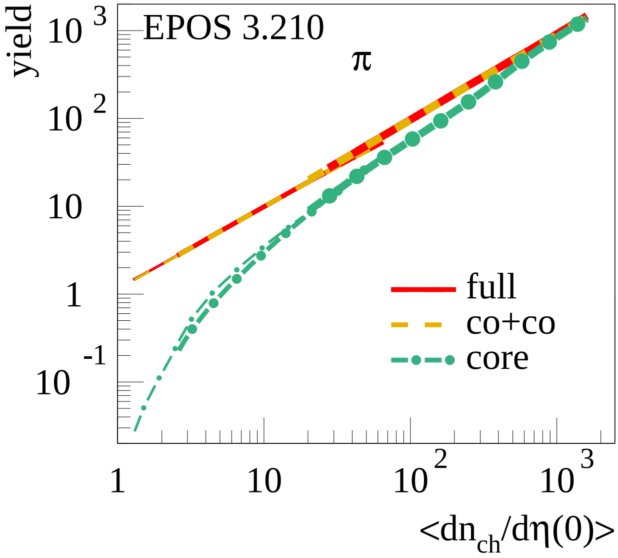
Core-corona picture in EPOS

Gribov-Regge approach => (Many) kinky strings
=> core/corona separation (based on string segments)



core => hydro => statistical decay ($\mu = 0$)
corona => string decay

Pion yields: core / corona contribution



thick lines

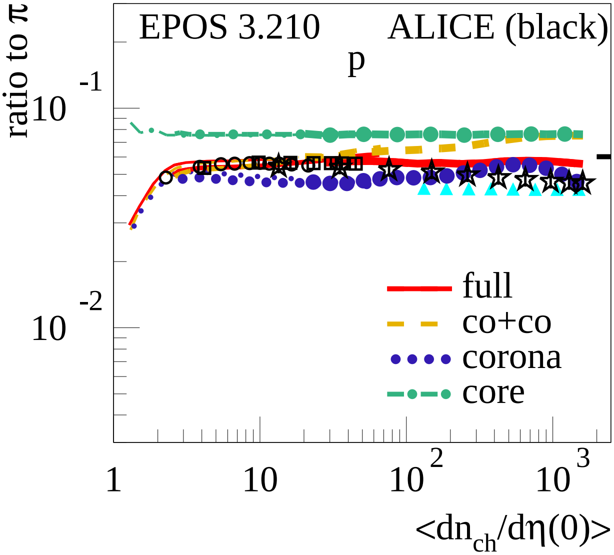
= pp (7TeV)

thin lines
= pPb (5TeV)

hc = hadronic cascade

(UrQMD)

Proton to pion ratio

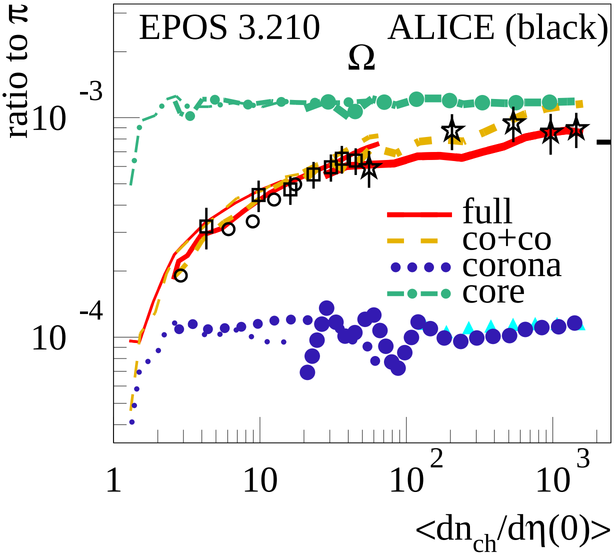


core hadronization:
$T = 164\,\mathrm{MeV},\ \mu_B = 0$

statistical model fit
(horizontal black line)
A. Andronic et al.,
arXiv:1611.01347
$T = 156.5\,\mathrm{MeV}, \mu_B = 0.7\,\mathrm{MeV}$

thick lines = pp (7TeV)
thin lines = pPb (5TeV)
circles = pp (7TeV)
squares = pPb (5TeV)
stars = PbPb (2.76TeV)

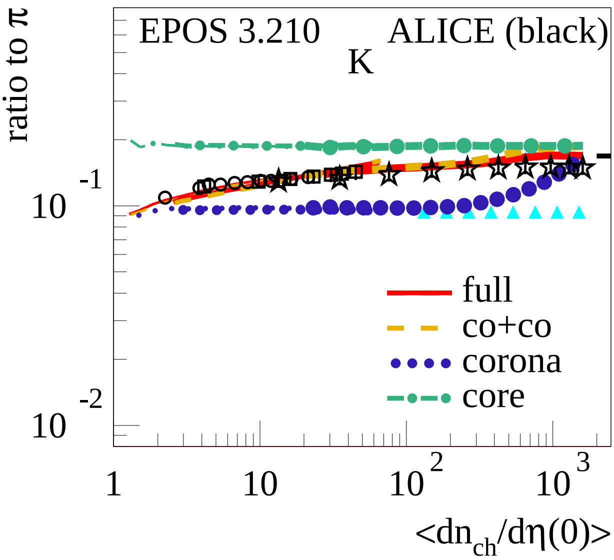
Omega to pion ratio



thick lines = pp (7TeV)
thin lines = pPb (5TeV)

circles = pp (7TeV)
squares = pPb (5TeV)
stars = PbPb (2.76TeV)

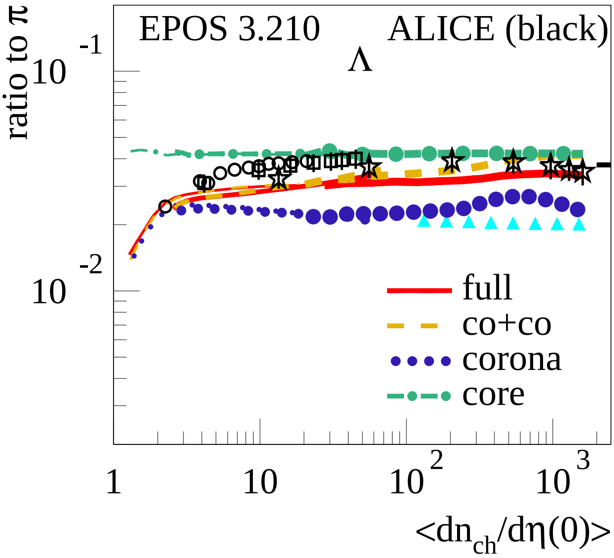
Kaon to pion ratio



thick lines = pp (7TeV)
thin lines = pPb (5TeV)

circles = pp (7TeV)
squares = pPb (5TeV)
stars = PbPb (2.76TeV)

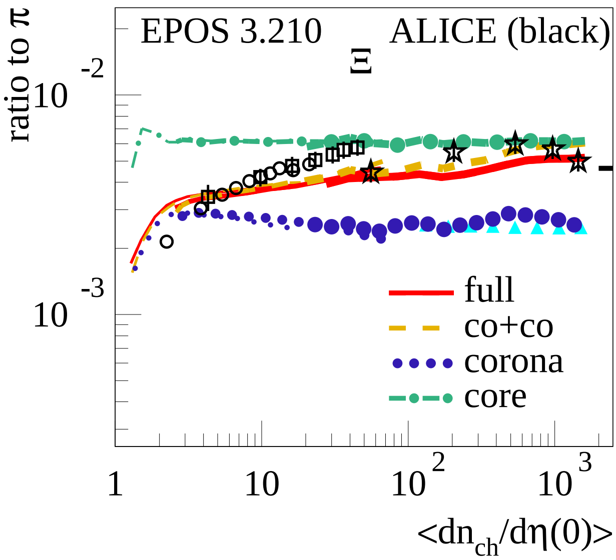
Lambda to pion ratio



thick lines = pp (7TeV)
thin lines = pPb (5TeV)

circles = pp (7TeV)
squares = pPb (5TeV)
stars = PbPb (2.76TeV)

Xi to pion ratio



thick lines = pp (7TeV)
thin lines = pPb (5TeV)

circles = pp (7TeV)
squares = pPb (5TeV)
stars = PbPb (2.76TeV)

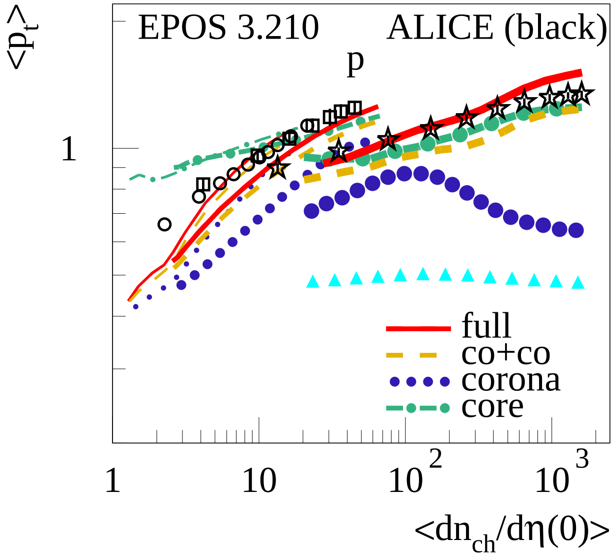
Ratios h/π for $h = p, K, \Lambda, \Xi, \Omega$ vs $\left\langle \frac{dn}{d\eta}(0) \right\rangle$:

Core and corona contributions separately roughly constant

Difference (core - corona) increasing for $p \rightarrow K \rightarrow \Lambda \rightarrow \Xi \rightarrow \Omega$

=> inceasing slope
(not enough for Ξ, Ω)

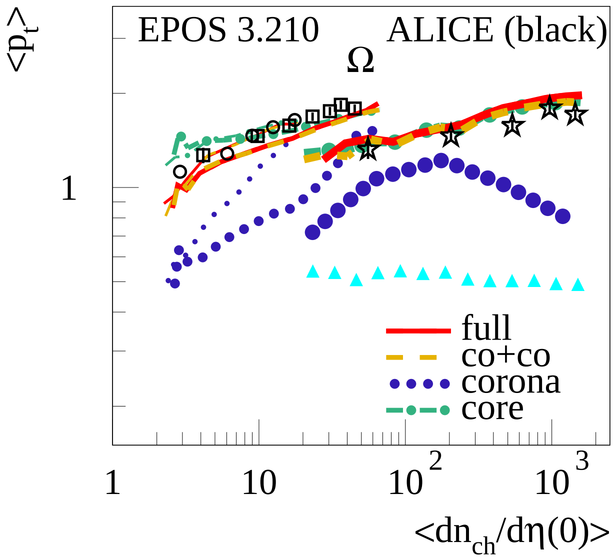
Average p_t of protons



thin lines = pp (7TeV)
intermediate lines = pPb (5TeV)
thick lines = PbPb (2.76TeVVV)

circles = pp (7TeV)
squares = pPb (5TeV)
stars = PbPb (2.76TeV)

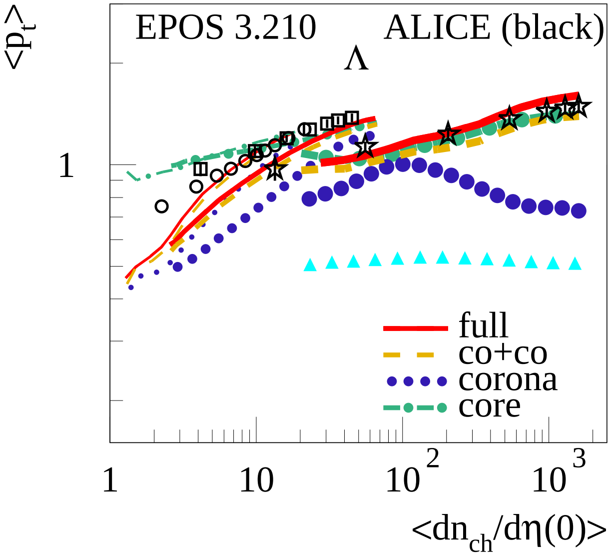
Average p_t of Omegas



thin lines = pp (7TeV)
intermediate lines = pPb (5TeV)
thick lines = PbPb (2.76TeVVV)

circles = pp (7TeV)
squares = pPb (5TeV)
stars = PbPb (2.76TeV)

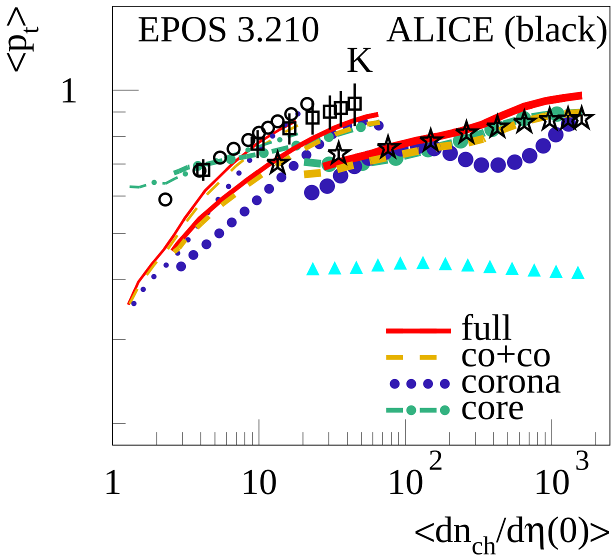
Average p_t of lambdas



thin lines = pp (7TeV)
intermediate lines = pPb (5TeV)
thick lines = PbPb (2.76TeVVV)

circles = pp (7TeV)
squares = pPb (5TeV)
stars = PbPb (2.76TeV)

Average p_t of kaons



thin lines = pp (7TeV)
intermediate lines = pPb (5TeV)
thick lines = PbPb (2.76TeVVV)

circles = pp (7TeV)
squares = pPb (5TeV)
stars = PbPb (2.76TeV)

Average p_t of $K, p, \Lambda, \Xi, \Omega$ vs $\left\langle \frac{dn}{d\eta}(0) \right\rangle$:

Moderate increase of core contribution
(same for pp and pPb, similar to PbPb)

Strong increase of corona contribution
(stronger for pp than for pPb, much stronger than for PbPb)

Slope(pp) > slope(pPb) >> slope(PbPb)

K, π **: pp-pPb splitting**

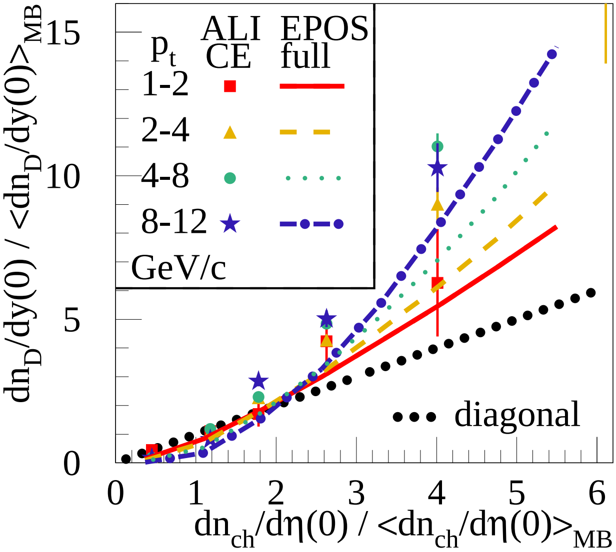
The multiplicity dependence of the corona contribution is crucial

Very closely related to this discussion:

The multiplicity dependence of charm production (D, J/Ψ,...)

The "ultimate tool" to test multiple scattering (and the implementation of Q_S)

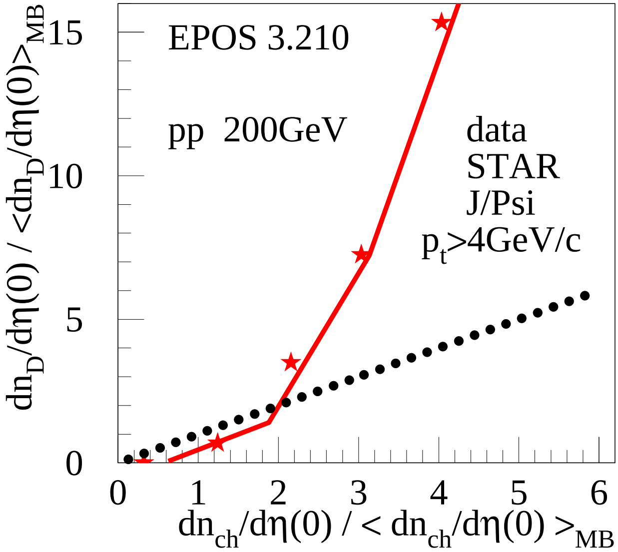
EPOS 3 compared to ALICE data



hadronic cascade on/off has no effect

hydro on/off has small effect

EPOS 3 compared to RHIC data

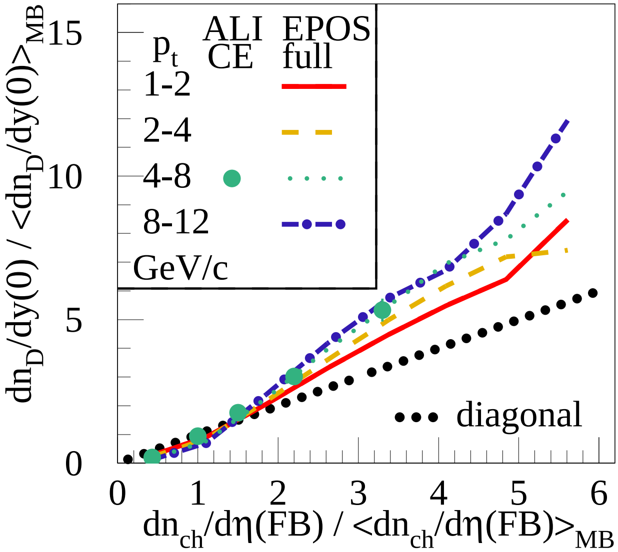


Calculations:
D mesons

Data: J/Ψ

Increase stronger than at LHC

Multiplicity at FB rapidity (LHC)



FB = forward/backward rapidity range:

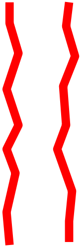
$$2.8 < \eta < 5.1$$
and
$$-3.7 < \eta < -1.7$$

Smaller increase

Low multiplicity (LM)

Small N_{Pom}

few soft $\mathbb{P}$'s



$\mathbb{P}$ = Pomeron

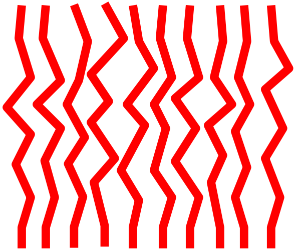
"Hardness" increases with N_{Pom}

(larger Q_s)

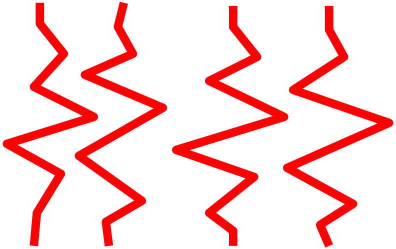
High multiplicity (HM)

many hard $\mathbb{P}$'s on avg

(A) more $\mathbb{P}$'s, but less hard



(B) fewer $\mathbb{P}$'s, but harder



LM $\rightarrow$ HM:

Pomerons get harder (larger Q_s)

$\rightarrow$ favors high pt or large masse production

in particular due to case B (fewer IP's, but harder) for highest pt bins !

Bigger effect at RHIC due to much narrower N_{Pom} distribution (harder IP's are needed)

Smaller effect for $\frac{dn}{d\eta}(FB)$ as multipl. variable

(case B is replaced by case C: fewer IP's, but more covering the FB rapidity range)