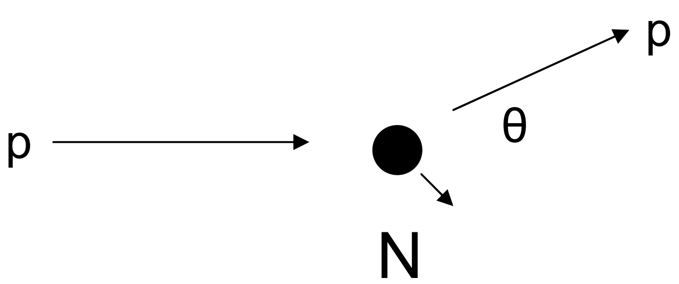
Crystal-Particle Interactions WG - 3 June 2009

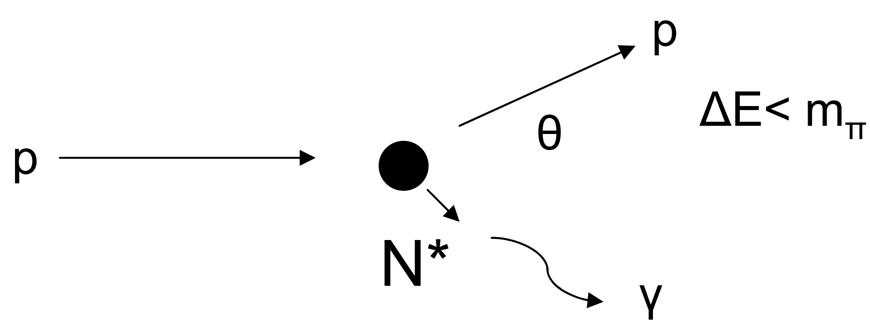
R. Noble, J. Spencer, A. Seryi, G. Stupakov, A. Taratin, G. Smirnov, M. Silari, UNM collaborator Jim Ellison, IHEP collaborator Igor Yazynin (others are welcome to join us!)

Noble and Spencer, First preliminary results

Revise and improve fast formulas for scattering angle-energy loss that Igor uses in his CRYST-AP code and provide these to Said Hasan for CRYM crystal emulation program. We are not doing full Monte-Carlo simulations, so we take some liberties to simplify calculations.

Start with Nuclear Elastic and Quasi-elastic p + N.





But elastic scattering actually has two parts: Coulomb (EM) and Nuclear interaction

(When $\Delta E > m_\pi$, this becomes Diffractive Nuclear/Nucleon Excitation→ not in this presentation)

Elastic low-t scattering: p p → p p

$$d\sigma_{el}/dt = \pi |f_N + f_C|^2$$

Invariant momentum transfer squared = $|t| \approx p^2 \theta^2$

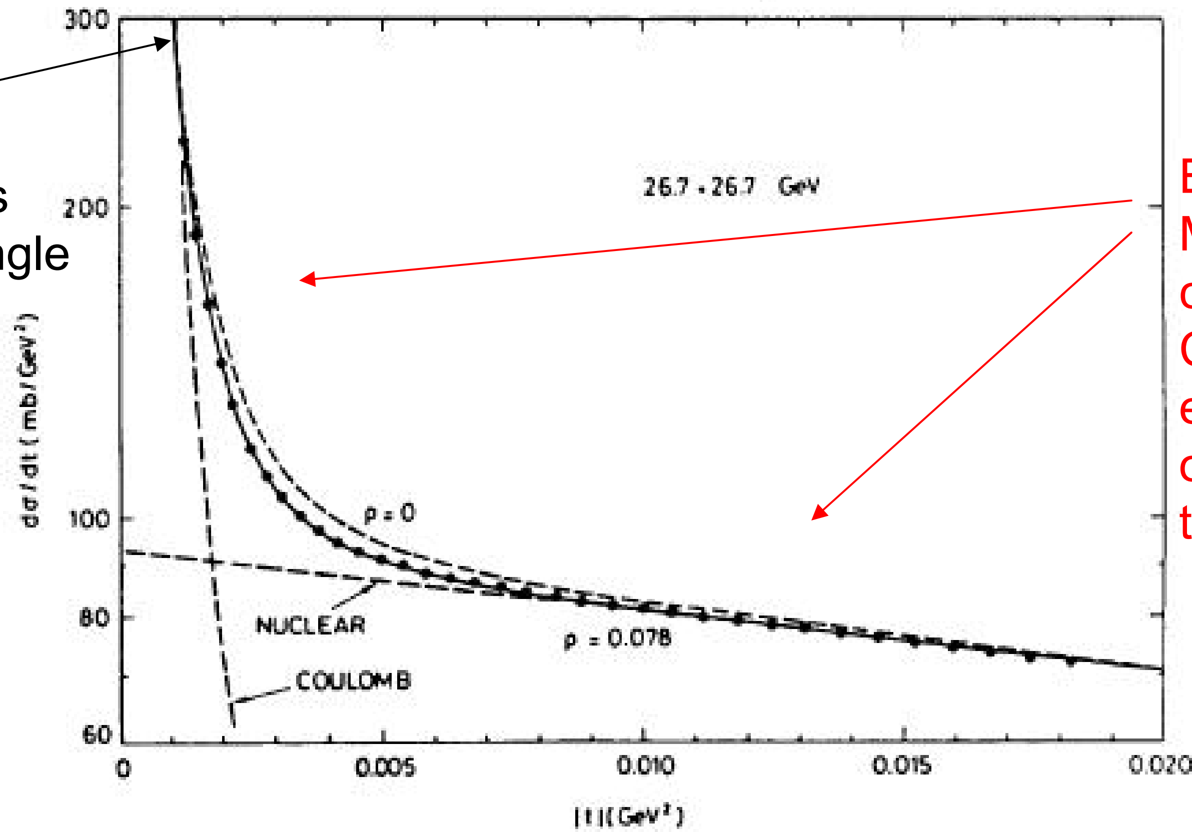
Note: $d\sigma/dt \approx (\pi/p^2)\ d\sigma/d\Omega$ if no φ dep.

where the nuclear and Coulomb scattering amplitudes have the forms

$$f_N = \frac{\sigma_T}{4\pi}(\rho + i)\, e^{b/2\, t} \sim \exp(-\text{const}\ p^2 \theta^2)\ ,\ \rho \text{ small}$$

$$f_C = -\frac{2\alpha}{t} G_n(t)\, G_p(t)\, e^{i\alpha\phi}\ . \sim 1/p^2 \theta^2$$

$1/p^4 \theta^4$ mostly contributes to small angle MCS



Except for detailed Monte Carlo sims, we can separate Coulomb and Nuclear elastic contributions over essentially entire t - range

Fig. 9. Small t pp elastic differential cross section at $\sqrt{s} = 53.4$ GeV [10]. The full line is a fit to the data using formula (22). The Coulomb and nuclear contributions are also shown separately.

Differential Cross Section

Rutherford Scattering:
pN → pN
(mostly small angle MCS, $\sim Z^2$)

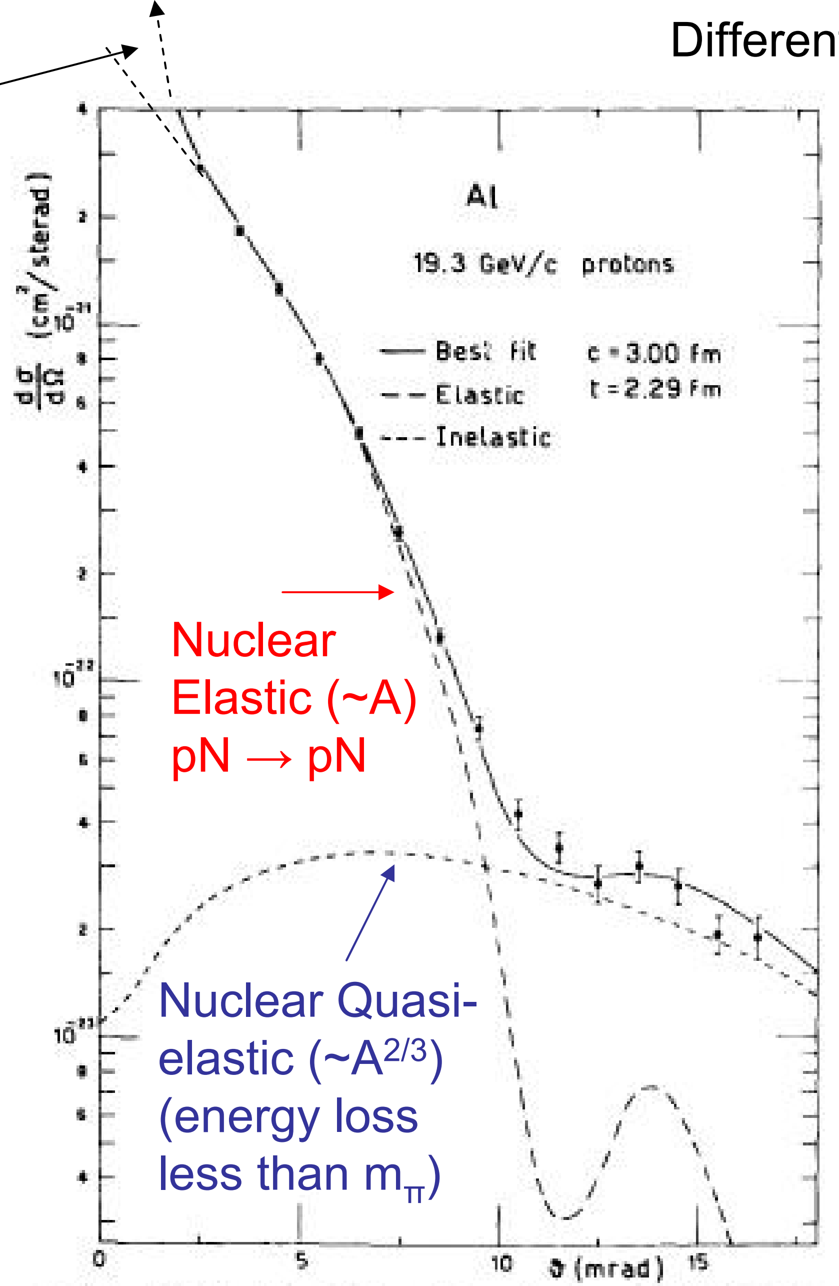


Fig. 5. The experimental data of ref. [4] on the scattering of 19.3 GeV/c protons by Al are shown together with the result of the best fit. Elastic and inelastic contributions are shown separately.

Aluminum and Silicon are similar

Al: A=27, Z=13
Si: A=28, Z=14

A. Van Ginneken fits of Schiz et al data for dσ /dt ≈ (π/ p^2) dσ/dΩ at 175 GeV/c for different nuclei.

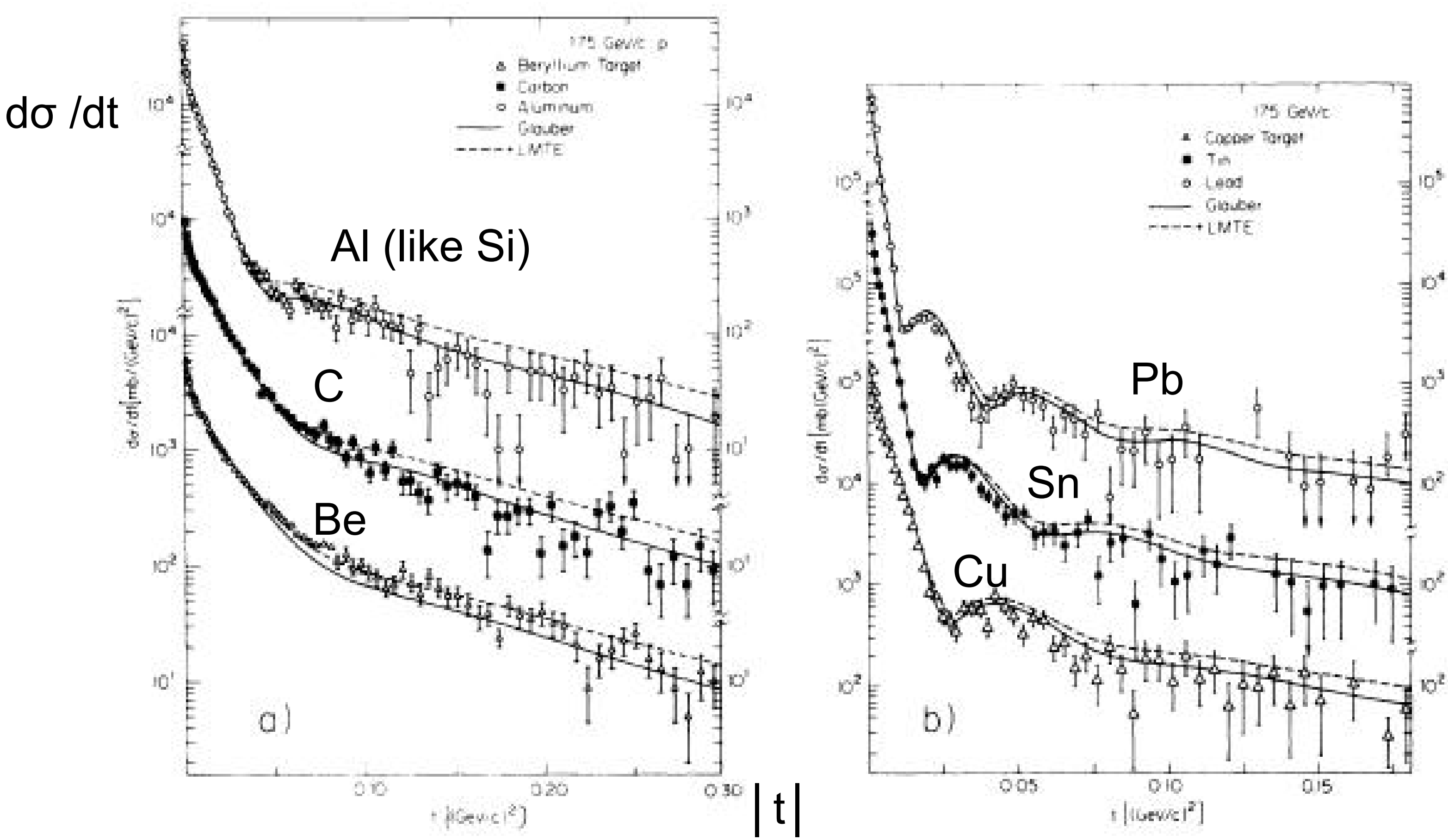


FIG. 3. Fits of Glauber theory to *dσ /dt* data of Ref. 10 for elastic and quasielastic scattering (solid lines) of 175-GeV/*c* protons on various nuclear targets. Dashed lines include diffractive low-mass target excitation.

We can use data like this to calculate <θ^2> for different processes.

For example, using Al which is like Si :

For each sub-process which we may treat independently, the contribution to mean square angle deviation is:

$$\langle\theta^2\rangle = (1/\sigma) \int \theta^2 \, d\sigma/d\Omega \, d\Omega$$

Nuclear elastic w/o Rutherford peak:
$\langle\theta^2\rangle^{1/2} \approx 85$ mrad / p(GeV/c)

Nuclear quasi-elastic
$\langle\theta^2\rangle^{1/2} \approx 298$ mrad / p(GeV/c)

(to get projected rms angle on x or y plane, divide by $\sqrt{2}$)

Since cross section is divided out, these are approx. same for nearby nuclei and for proton momenta over which σ changes little.

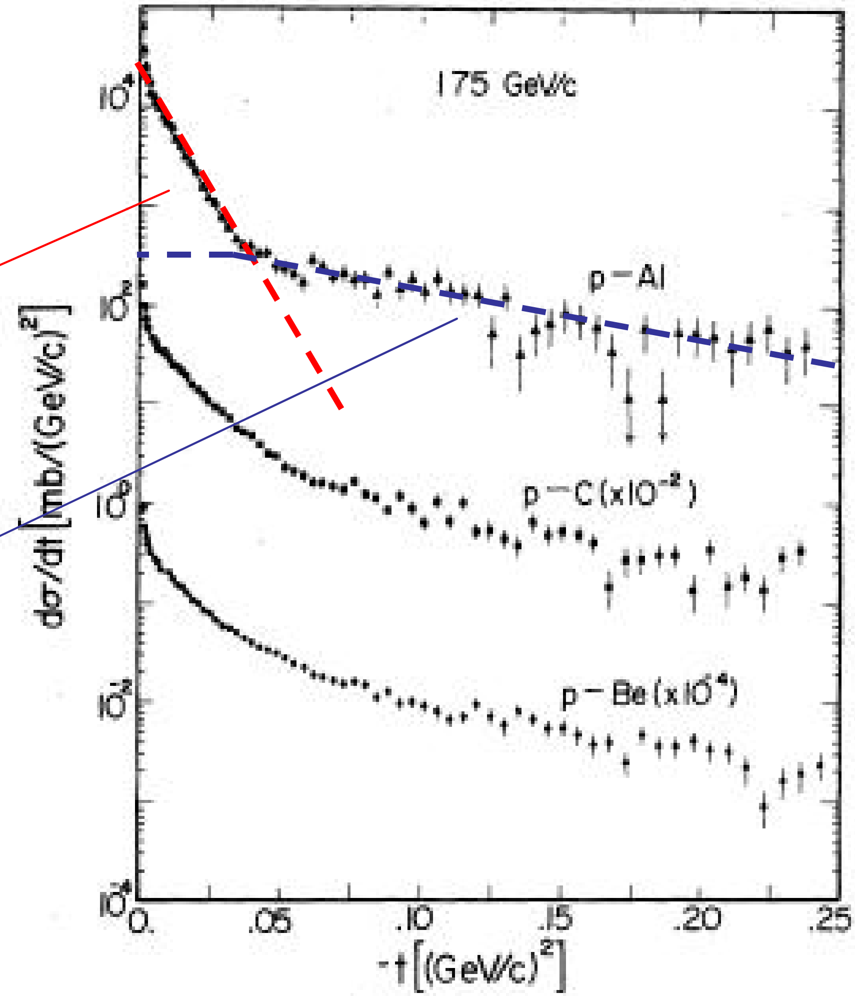


FIG. 3. $d\sigma/dt$ for elastic scattering at incident beam momentum of 175 GeV/c for the following: p-Be, p-C, p-Al; solid lines present results of a fit of the data to Eq. (4) (see text and footnote 18).

Schiz et al, PRD21 (1980)

Characteristic length (mean free path) for any process:

$1/\lambda = \sigma n = \sigma(cm^2)\,\rho(grams/cc)\,N_A / A(grams)$ = 1 / interaction length in cm

1. Nuclear elastic (w/o Rutherford peak) for Al at 175 GeV/c: $\sigma_{nuc\text{-}el} \approx 143$ mb

   $\lambda_{nuc\text{-}el}$ (Al) = 118 cm (for Si with 2.33 g/cc, $\lambda_{nuc\text{-}el}$ = 134 cm)

(1 mb $= 10^{-27}$ cm^2)

2. Nuclear Quasi-elastic for Al: $\sigma_{nuc\text{-}quasi} \approx 29.3$ mb

   $\lambda_{nuc\text{-}quasi}$ (Al) = 589 cm (for Si with 2.33 g/cc, $\lambda_{nuc\text{-}quasi}$ = 670 cm)

Nuclear Elastic Angle simulation:
IF (RNDM< DZ / 134 cm),
THEN $\Delta\theta_x$, $\Delta\theta_y$ = 85 mrad / p(GeV/c) /√2 * RANDOM GAUSSIAN * RANDOM (+ -)

Nuclear Quasi-Elastic Angle simulation:
IF (RNDM< DZ / 670 cm),
THEN $\Delta\theta x$, $\Delta\theta y$ = 298 mrad / p(GeV/c) /√2 * RANDOM GAUSSIAN * RANDOM (+ -)