

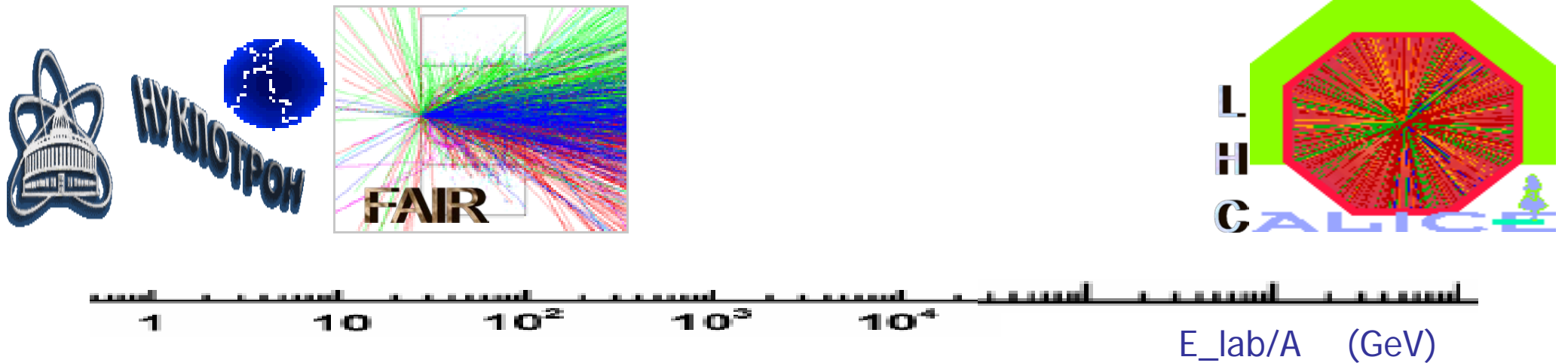
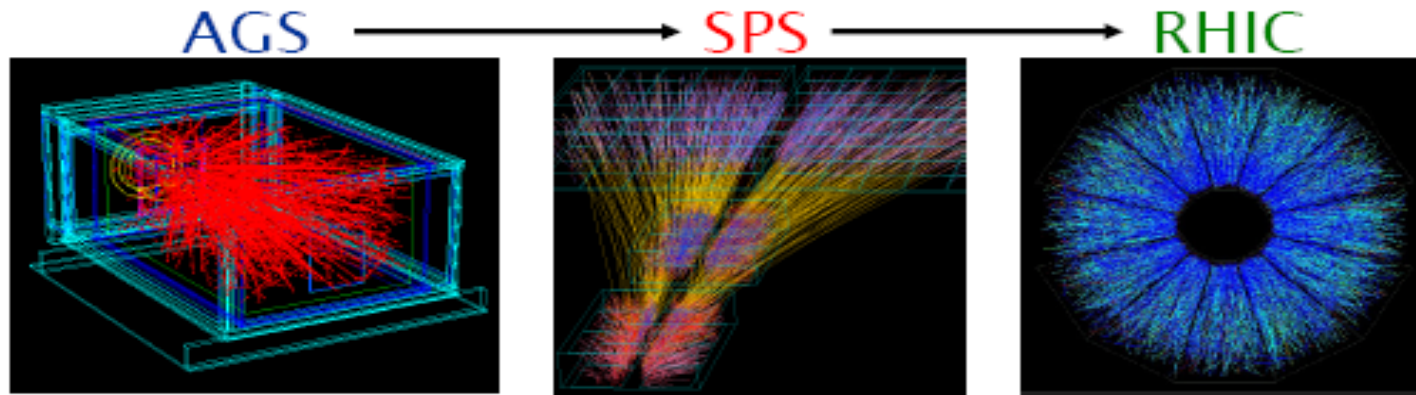


Space-time picture of ultrarelativistic nuclear collisions

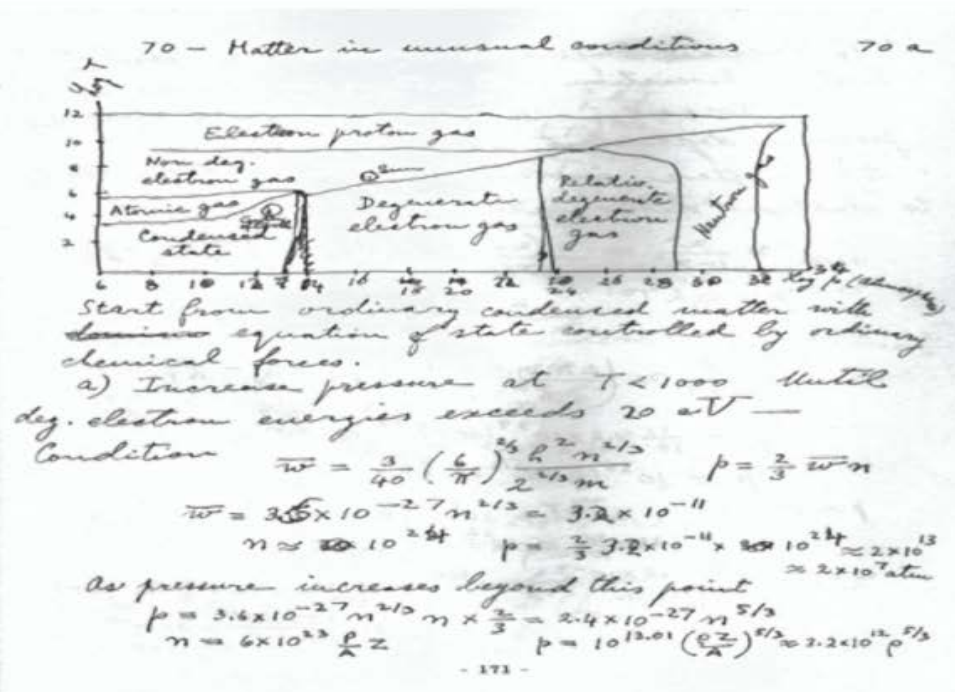
Yuri Sinyukov

Part 2

Heavy Ion Experiments

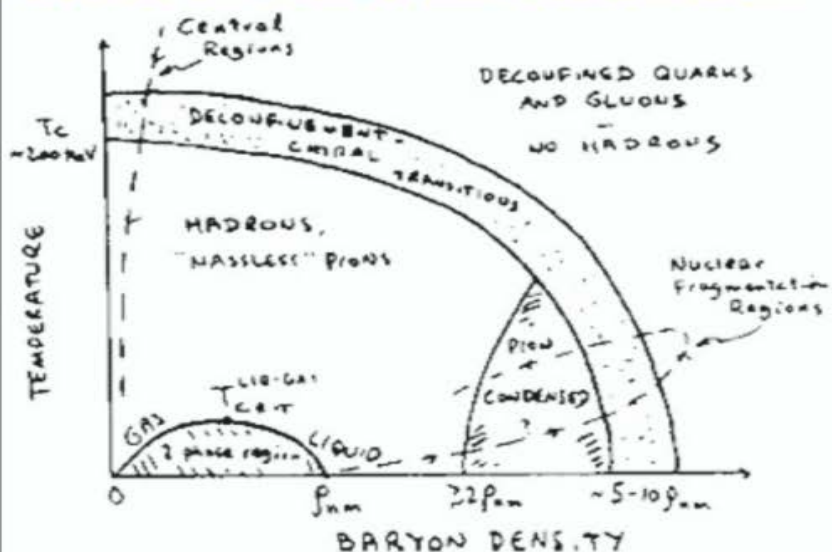


The thermodynamic arias occupied by different forms of the matter

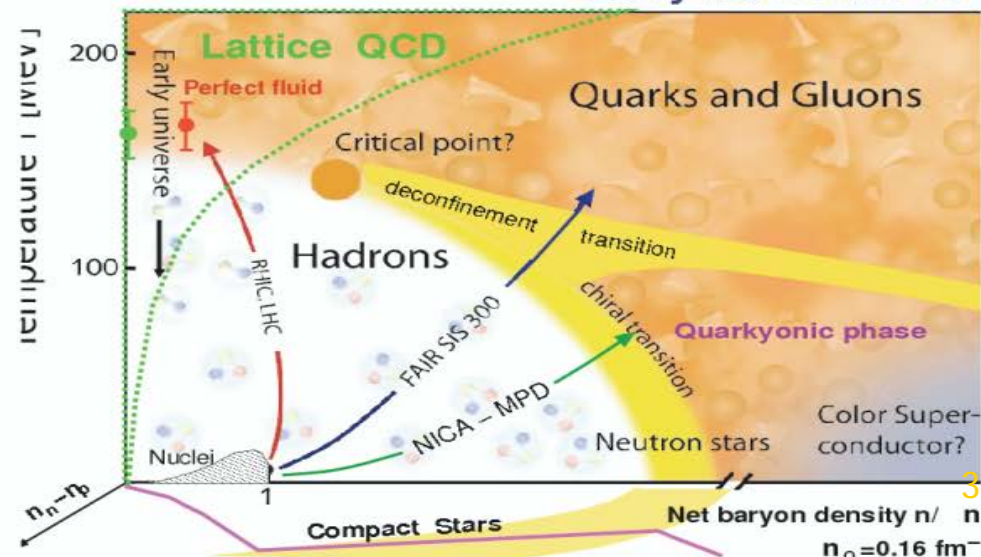


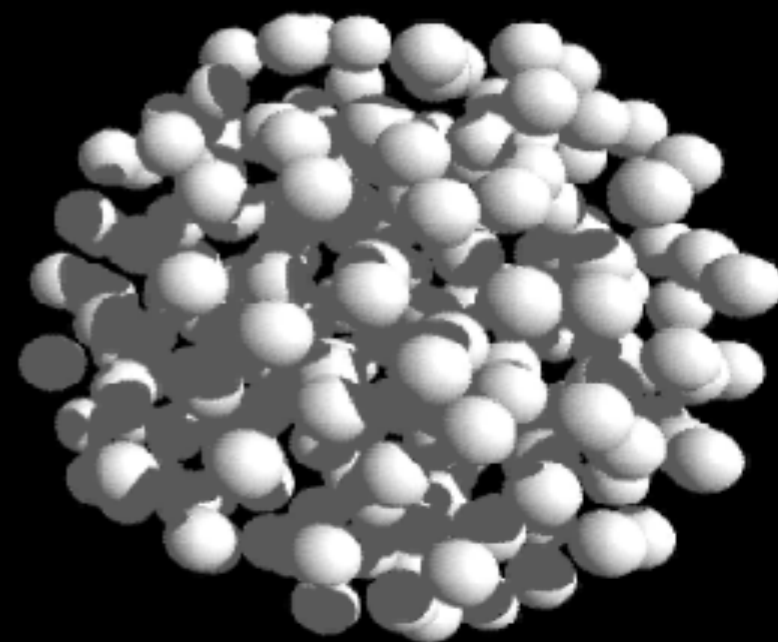
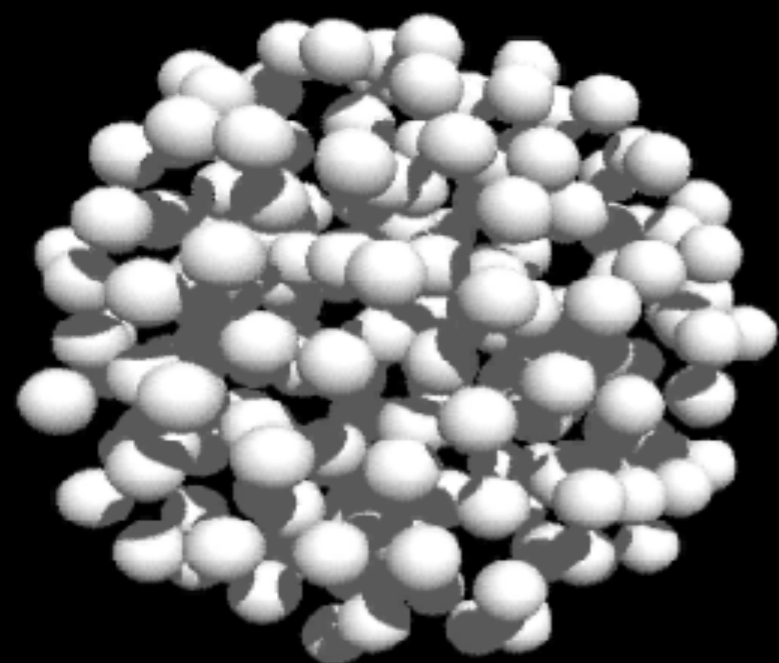
1953 Enrico Fermi

1983 US long range plan, Gordon Baym



Larry McLerran '09



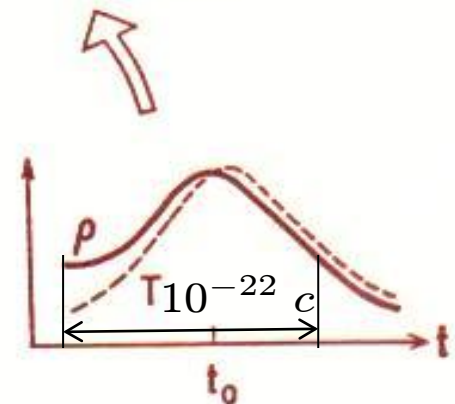
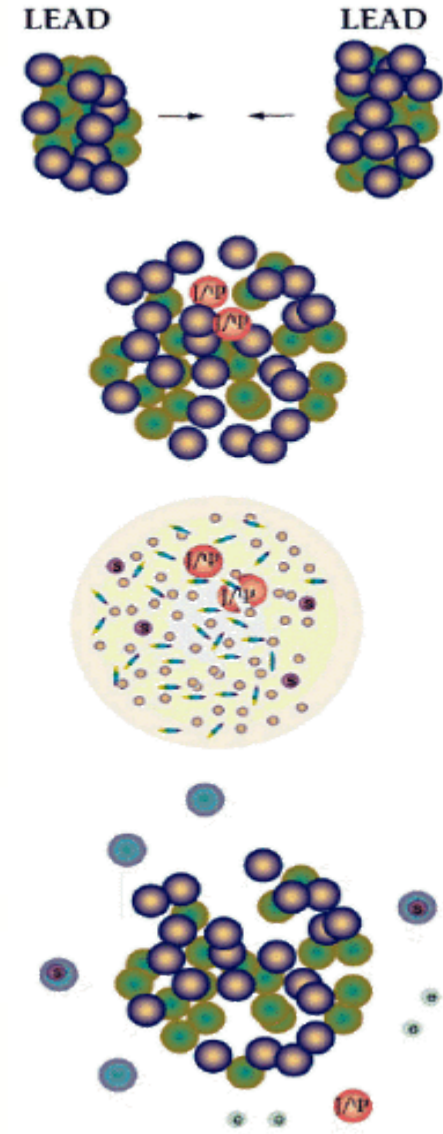
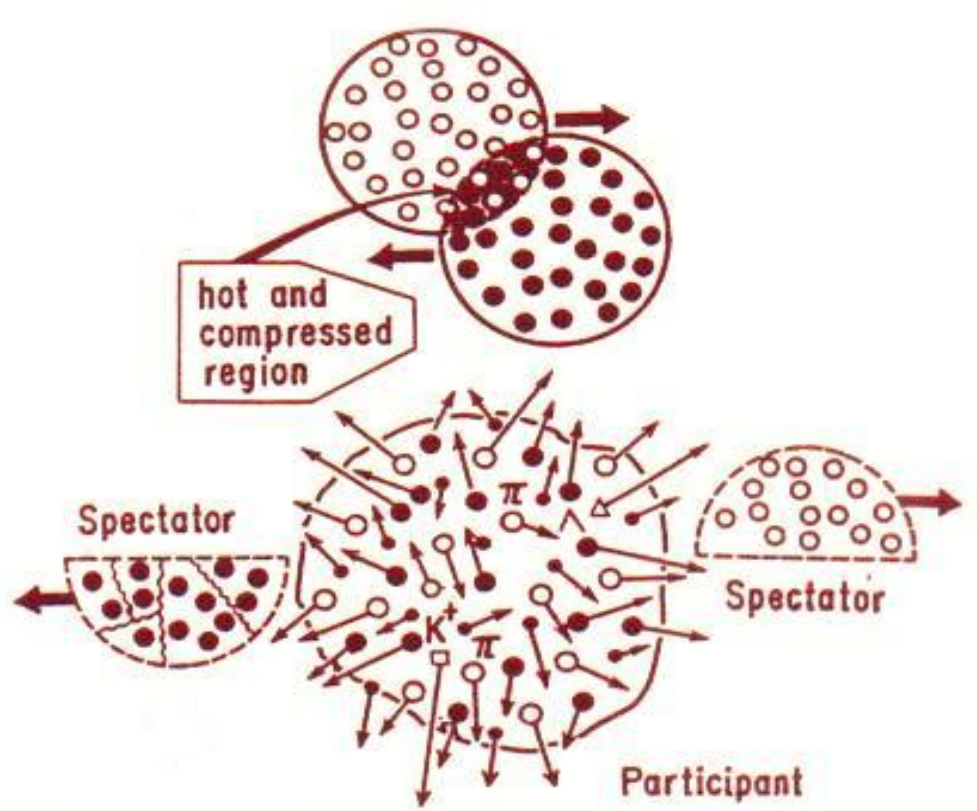


The cartoon of relativistic heavy ion collisions

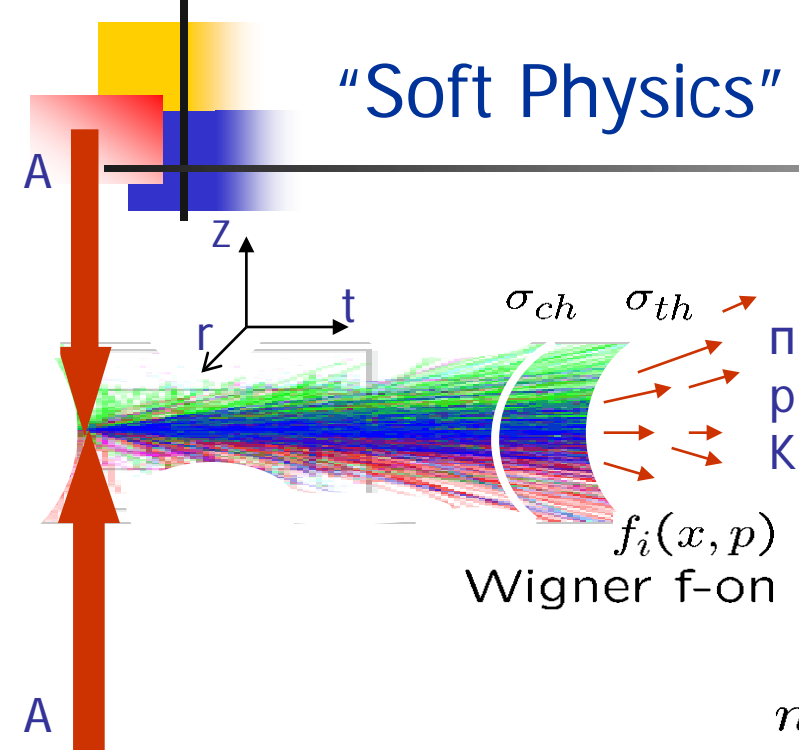
At the start =====>

At the end =====>

Temporal dependence
of the pressure and
temperature =====>



"Soft Physics" measurements



$T_{f.o.}$
Landau, 1953
 $\sigma_{f.o.}$

$$N_i = \int \frac{d^3 p}{p^0} d\sigma_{\mu} p^{\mu} f_i(x, p)$$

$$n_i(p) \equiv p^0 \frac{d^3 N_i}{d^3 p} = \int d\sigma_{\mu} p^{\mu} f_i(x, p) \sim e^{-\sqrt{m_i^2 + p_T^2} / T_{eff,i}}$$

$$n_i(p_1, p_2) \equiv p_1^0 p_2^0 \frac{d^6 N_i}{d^3 p_1 d^3 p_2} = C(p, q) n(p_1) n(p_2)$$

$$p = (p_1 + p_2) / 2$$

$$q = p_1 - p_2$$

$\left\{ \frac{N_i}{N_j} \right\} \rightarrow T_{ch}$ and μ_{ch} soon after hadronization (chemical f.o.)

$$\frac{d^3 N}{dp, dy d\varphi} = \frac{d^2 N}{dp, dy} \frac{1}{2\pi} (1 + 2v_1 \cos(\varphi) + 2v_2 \cos(2\varphi) + \dots)$$

Directed flow

Elliptic flow

$$1 + \lambda \exp(-R_L(p)^2 q_L^2 - R_S(p)^2 q_S^2 - R_O(p) q_O^2)$$

Radial flow

$$\rightarrow T_{eff,i} \approx T_{f.o.} + m_i \frac{\langle v_T^2 \rangle}{2}$$

Inverse of spectra slope

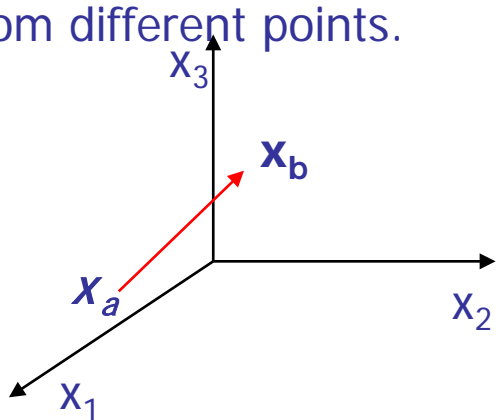
QS Correlation function

Space-time structure of the matter evolution, e.g.,

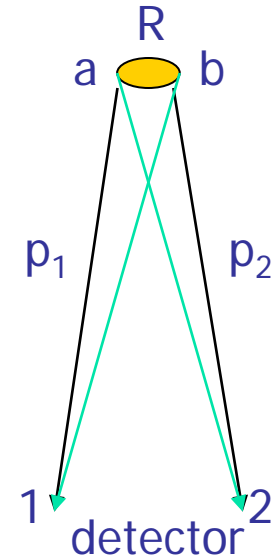
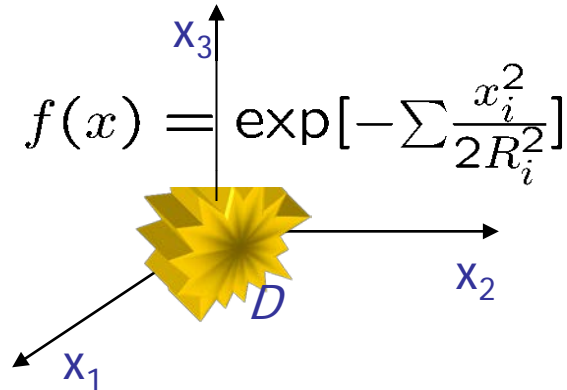
$$\tau \approx R_L \sqrt{\frac{m_T}{T_{f.o.}}}$$

Interferometry microscope (Kopylov / Podgoretcky - 1971)

The idea of the correlation femtoscopy is based on an impossibility to distinguish between registered identical particles emitted from different points.



$t=0$



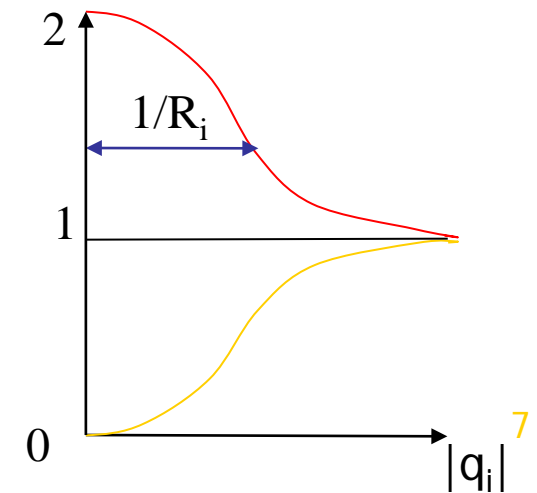
Momentum representation

$$\Psi_{x_a, x_b}(p_1, p_2) = \frac{1}{\sqrt{2}} [e^{-ip_1 \cdot x_a} e^{-ip_2 \cdot x_b} \pm e^{-ip_2 \cdot x_a} e^{-ip_1 \cdot x_b}]$$

Probabilities:

$$W_{x_a, x_b}(p_1, p_2) = |\Psi_{x_a, x_b}(p_1, p_2)|^2 = 1 \pm \cos [(\underbrace{p_1 - p_2}_{\mathbf{q}}) \cdot (x_a - x_b)]$$

$$W_D(p_1, p_2) = \int d^3x_a d^3x_b f(x_a) f(x_b) W_{x_a, x_b}(p_1, p_2) = 1 \pm \left| \int d^3x f(x) e^{i\mathbf{q} \cdot \mathbf{x}} \right|^2 = 1 \pm \exp \left[-\sum q_i^2 R_i^2 \right]$$



THE DEVELOPMENT OF THE FEMTOSCOPY

Even ultra small systems can have an internal structure.

Then the distribution function $f(x,p)$ and emission function of such an objects are inhomogeneous and, typically, correlations between the momentum p of emitted particle and its position x appear.

In this case and in general the interferometry microscope measure the homogeneity lengths in the systems [Yu. Sinyukov , 1986, 1993-1995].

$$\left| \frac{f(p, x_0 + \bar{\lambda}) - f(p, x_0)}{f(p, x_0)} \right| = \frac{1}{2} \quad \text{at} \quad \left. \frac{\partial f(p, x)}{\partial x_i} \right|_{x_0(p)} = 0 \quad \longrightarrow \quad \lambda_i^2 = \frac{f(x_0, p)}{|f''_{x_i}(x_0, p)|}$$

In hydrodynamic situation the distribution functions on x and p are not factorized. They are entangled, correlated,

$$f(x, p) = f(p) \exp\left(-\sum \frac{x_i^2}{2R_i^2}\right) \longrightarrow \frac{1}{(2\pi)^3} \left[\exp\left(\frac{p^\mu u_\mu(x)}{T(x)} - \frac{\mu(x)}{T(x)}\right) - 1 \right]^{-1}$$

Correlations functions are defined through thermal Wick`s theorem

$$p^0 \frac{dN}{dp} = \langle a_p^+ a_p \rangle, \quad p_1^0 p_2^0 \frac{dN}{dp_1 dp_2} = \langle a_{p_1}^+ a_{p_2}^+ a_{p_1} a_{p_2} \rangle$$

$$C(p_1, p_2) = 1 + \frac{\langle a_{p_1}^+ a_{p_2} \rangle \langle a_{p_2}^+ a_{p_1} \rangle}{\langle a_{p_1}^+ a_{p_1} \rangle \langle a_{p_2}^+ a_{p_2} \rangle} \quad \langle a_{p_1}^+ a_{p_2} \rangle = \int_{\sigma_{out}} d\sigma_\mu(x) p^\mu \exp(iqx) f_{esc}(x, p)$$

THE DEVELOPMENT OF THE FEMTOSCOPY

To provide calculations analytically one should use the saddle point method and Boltzmann approximation to Bose-Einstein distribution function. Then the single particle spectra are proportional to homogeneity volume:

$$p^0 \frac{d^3 N}{d^3 p} \propto \prod_i \lambda_i(p)$$

and just these homogeneity lengths forms exponent in correlation function

$$C = 1 + \exp \left[-\sum q_i^2 R_i(p)^2 \right]$$

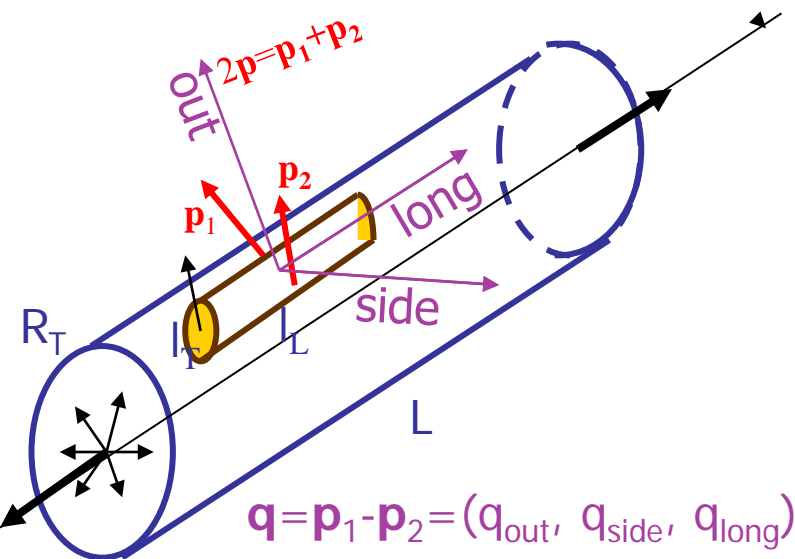
Interferometry radii:

$$R_L(p_T) \approx \lambda_L = \tau \sqrt{\frac{T_{f.o.}}{m_T}} / \cosh(y), \quad m_T = \sqrt{m^2 + p_T^2}$$

$$R_S \approx \lambda_T = R_T / \sqrt{1 + I m_T / T_{f.o.}}, \quad I \propto \langle v_T^2 \rangle$$

$$R_O^2 \approx \lambda_T^2 + v^2 \langle \Delta t^2 \rangle_p - 2v \langle \Delta x_o \Delta t \rangle_p, \quad v = \frac{p_{out}}{p_0}$$

$$C(p, q) = \frac{d^6 N / d^3 p_1 d^3 p_2}{d^3 N / d^3 p_1 d^3 N / d^3 p_2} \approx 1 + e^{R_L^2(p) q_L^2 + R_S^2(p) q_S^2 + R_O^2(p) q_O^2}$$



QGP $\rightarrow R_{out}/R_{side} \gg 1$ Exp : $R_{out}/R_{side} \approx 1$ RHIC HBT PUZZLE

Correlation femtoscopy of nucleus-nucleus collisions

- The femtoscopy analysis is used by Collaborations at SPS, RHIC and LHC. They provide the measurements of the space-time scales in the expanding matter with accuracy 10^{-15} m and 10^{-23} c.

Some basic points

«Sinyukov-Makhlin formula”
for an estimate of the
duration of “Little Bang”

$$\tau \approx R_L \sqrt{\frac{m_T}{T_{f.o.}}}$$

$$R_i^2(k_T) = \tau^2 \lambda^2 \left(1 + \frac{3}{2} \lambda^2 \right)$$

2015

$$\lambda^2 = \frac{T}{m_T} (1 - \bar{v}_T^2)^{1/2}$$

Femto “homogeneity lengths”
general interpretation of the
femtoscopy measurements as
“homogeneity lengths”

$$\lambda_i^2 = \frac{f(x_0, p)}{|f''_{x_i}(x_0, p)|}$$

$$\bar{v}_T^2$$

“Bowler–Sinyukov
treatment”

extracts the femtoscopy
correlations from effects of
the Coulomb interactions and long-
lived resonances

$$C(\mathbf{q}) = [(1-\lambda) + \lambda K(q_{inv})(1+G(\mathbf{q}))]$$

The evidences of space-time evolution of the thermal matter in A+A collisions:

Rough estimate of the fireball lifetime for Au+Au $\sqrt{s} = 200$ GeV:

$$R_L(p_T, y = 0) \approx \tau \sqrt{\frac{T_{f.o.}}{m_T}} \quad (m_T \gg T_{f.o.}) \quad \longrightarrow \quad \tau > 10 \text{ fm}/c$$

for $m_T = \sqrt{m^2 + p_T^2} = 1.75 \text{ GeV}$

In p+p all femto-scales are of order 1 fm ! \longrightarrow

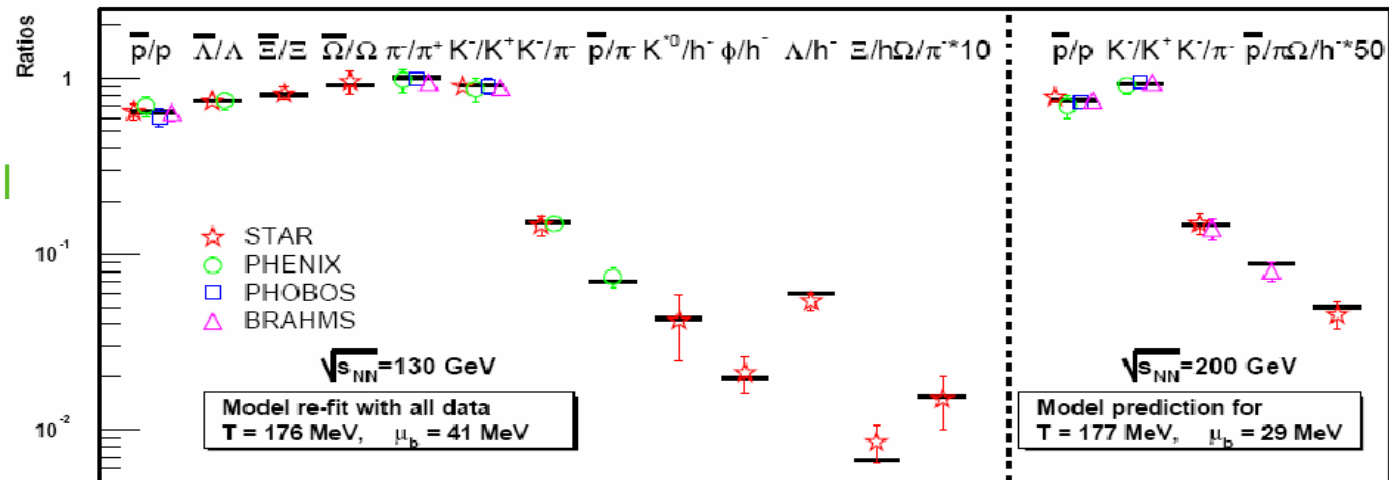
A+A is not some kind of superposition of the individual collisions of nucleons of nuclei

The phenomenon of space-time evolution of the strongly interacting matter in A+A collisions

What is the nature of this matter at the early collision stage?

Whether does the matter becomes thermal?

Particle number ratios are well reproduced in thermal gas model with 2 parameters: T , μ_B for collision energies from AGS to RHIC:
thermal+chemical equilibrium



Collective expansion of the fireball.

- Observation of the longitudinal expansion:

$$R_L(p_T, y) \approx \tau \sqrt{\frac{T_{f.o.}}{m_T}} / \cosh(y)$$

It was conformed by NA35/NA49 Collaborations (CERN), 1995 !

- Observation of transverse (radial) collective flows:

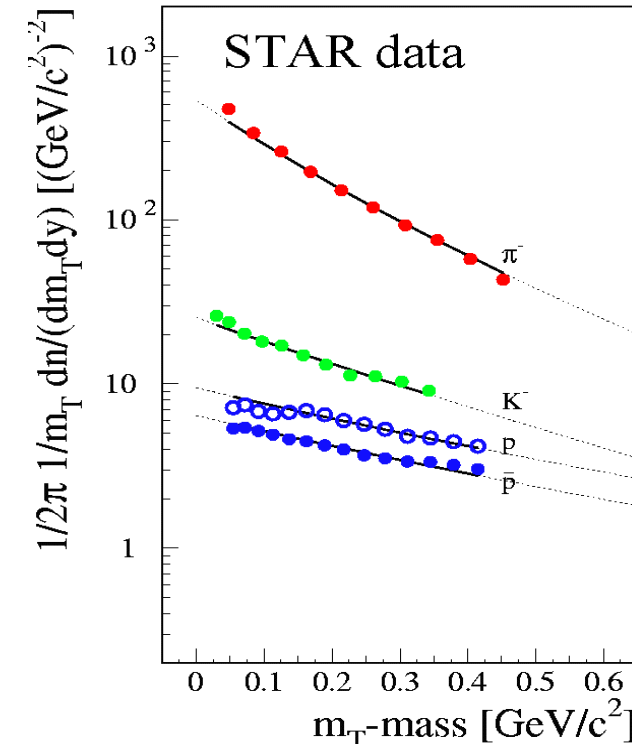
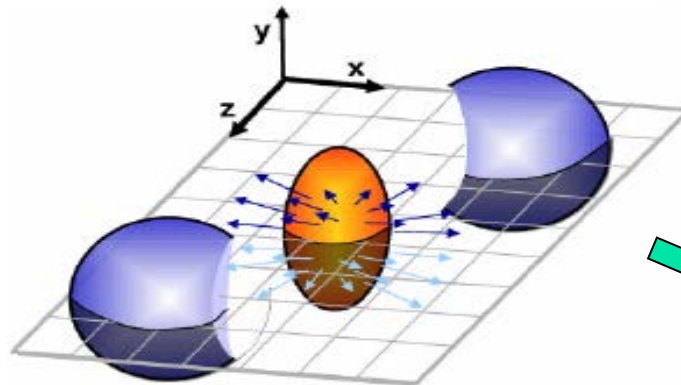
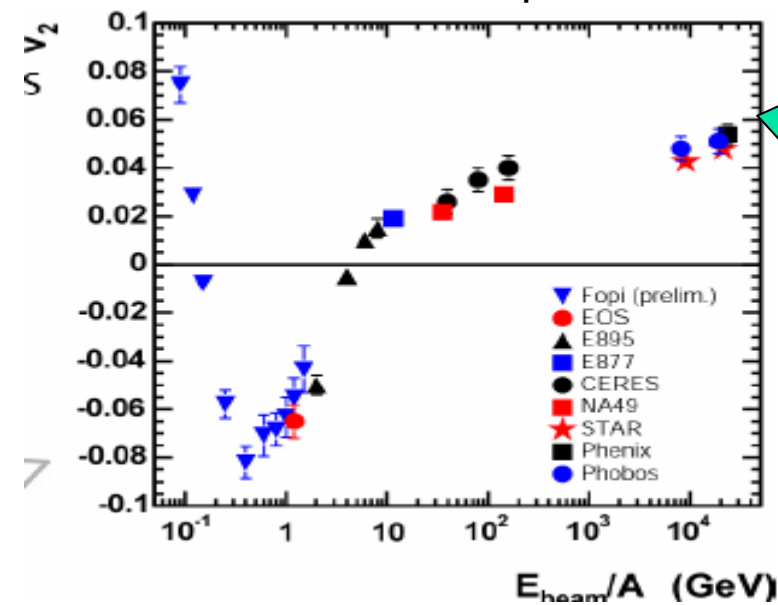
Effective temperature for different particle species

(inverse of the slope) $p_T < m$:

radial collective flow

$$T_{eff} = T_{f.o.} + \frac{m \langle v^2 \rangle}{2}$$

- Observation of elliptic flows:



HYDRODYNAMICS !

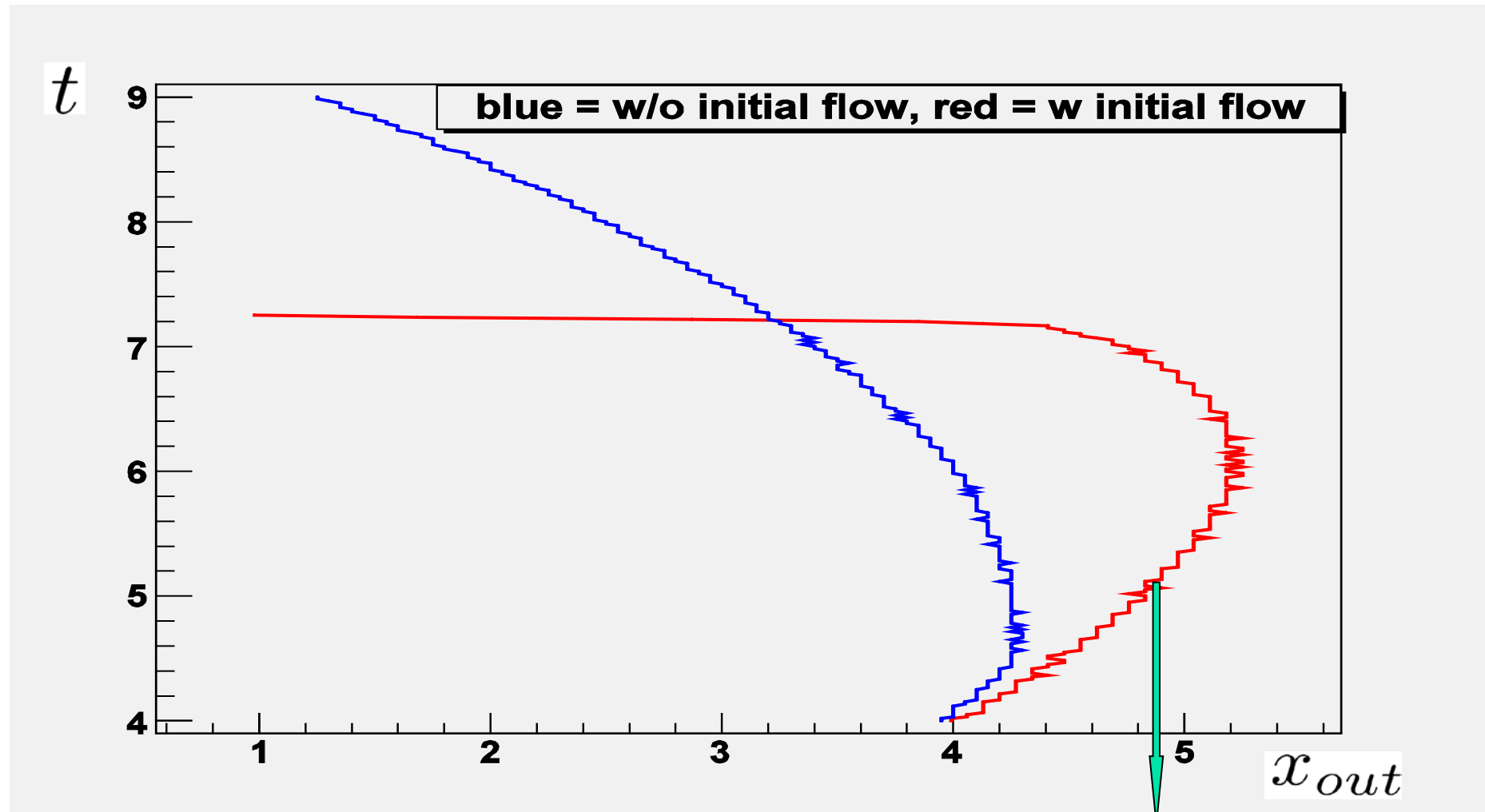
Empirical observations and theoretical problems

- The creation of the superdense matter with $\epsilon > 100 \epsilon_{nucl}$ is observed at RHIC and LHC
- The thermalization of such a matter is seen in the particle yields, spectra and correlations.
- Hydrodynamics describe well the soft physics (bulk matter observables).
- The letter means an existence of a new form of thermal matter at the temperatures $T=155-350$ MeV : asymptotically free QGP \longrightarrow strongly coupled sQGP, or quark-gluon fluid.

THEORETICAL PROBLEM

- An satisfying description of elliptic flows at RHIC requires the earlier thermalization, $\tau_{th} \simeq 0.4 \text{ fm}/c$ and almost perfect fluid . At the same time the most optimistic estimates give thermalization time 1-1.5 fm/c.
- QGP and experimental ratio $R_{out}/R_{side} \approx 1 \longrightarrow$ **HBT Puzzle**

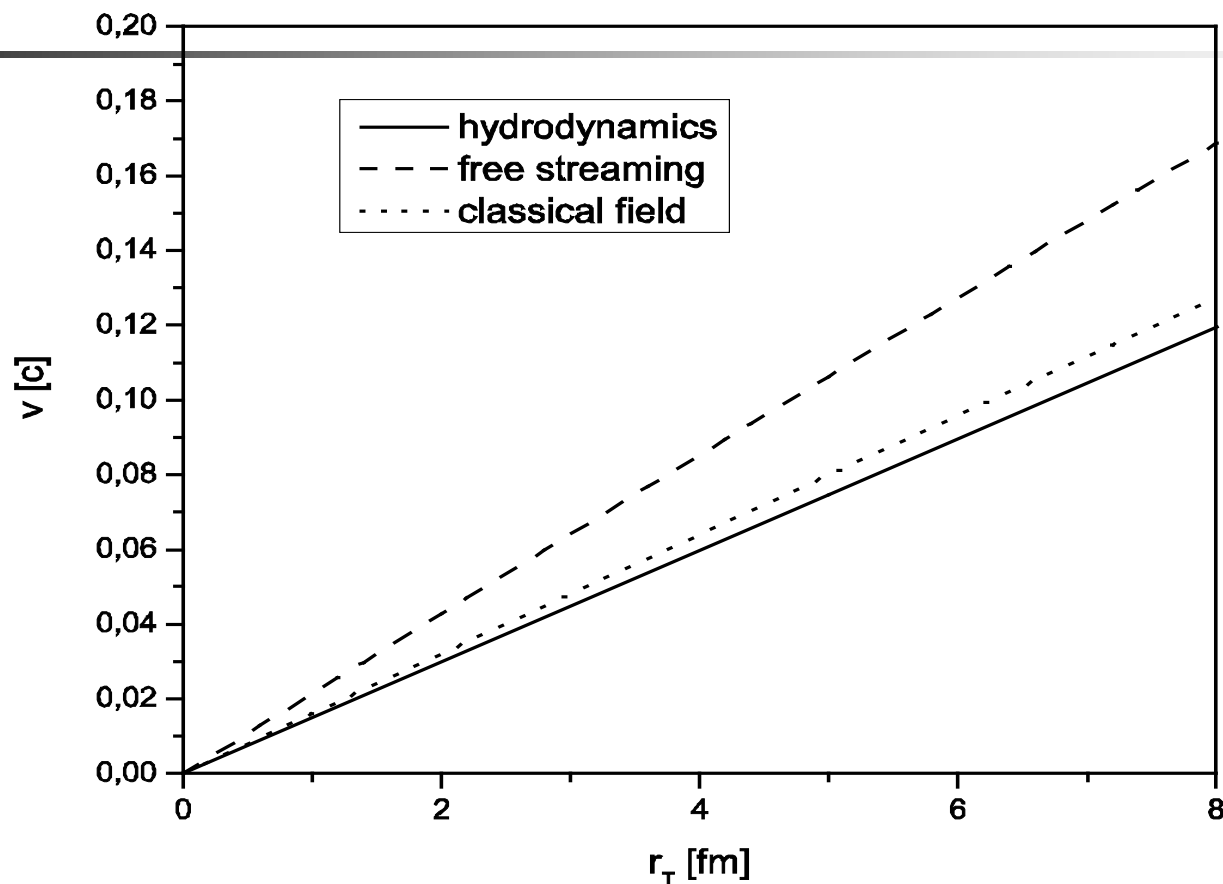
Initial flows and Ro/Rs ratio ($t_0=1-2$ fm/c)



$$R_{out}^2 \approx R_{side}^2 + v^2 \langle \Delta t^2 \rangle_p - 2v \langle \Delta x_{out} \Delta t \rangle_p, v = \frac{p_T}{p_0}$$

Pre-thermal transverse flow

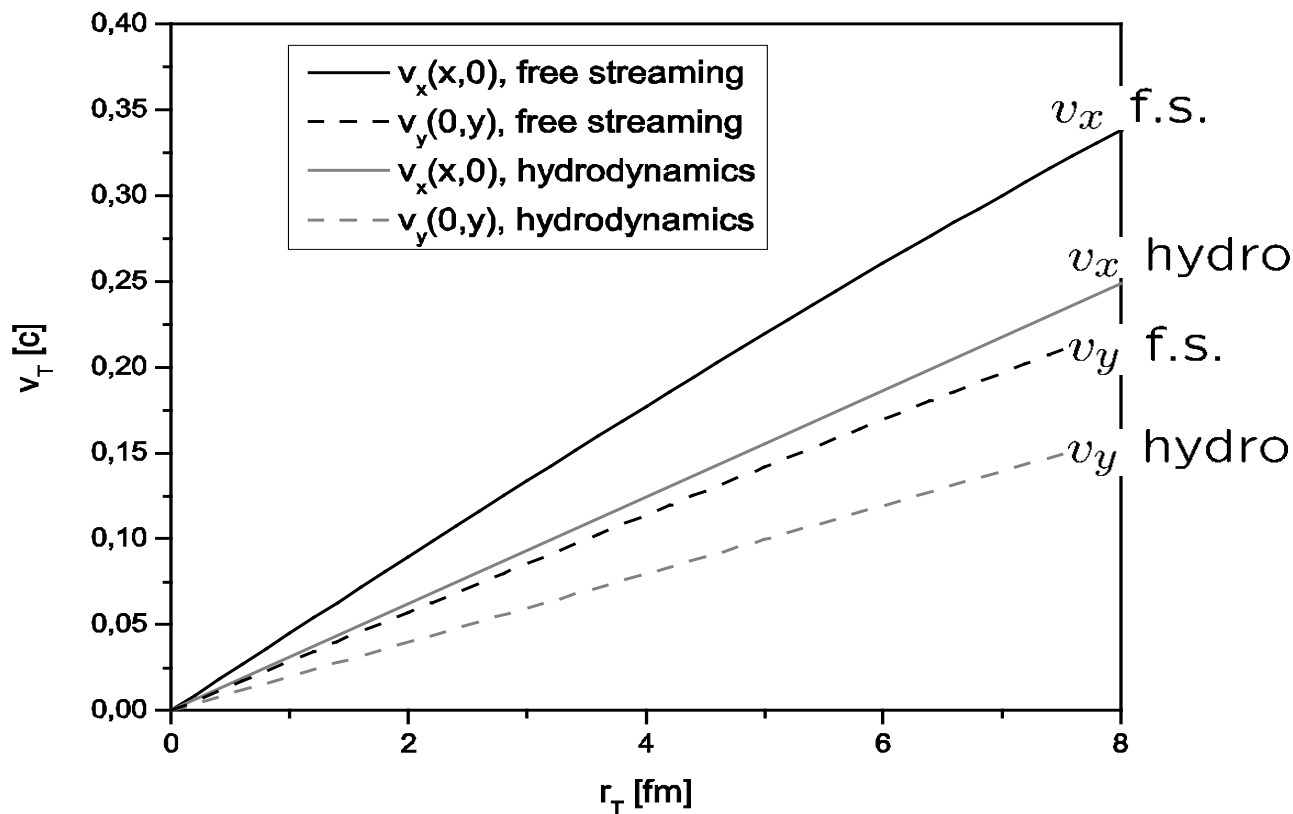
Collective velocities developed between $\tau_0=0.3$ and $\tau=1.0$ fm/c



Central collisions

Collective velocity developed at pre-thermal stage from proper time $\tau_0 = 0.3$ fm/c by supposed thermalization time $\tau_{th} = 1$ fm/c for scenarios of partonic free streaming and free expansion of classical field. The results are compared with the hydrodynamic evolution of perfect fluid with hard equation of state $p = 1/3$ epsilon started at τ_0 . Impact parameter $b=0$.

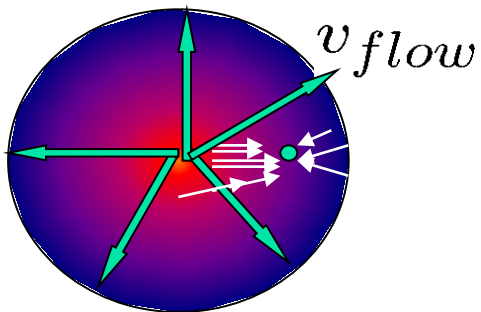
Collective velocities and their anisotropy developed between $\tau_0=0.3$ and $\tau = 1.0$ fm/c



Non-central collisions
b=6.3 fm

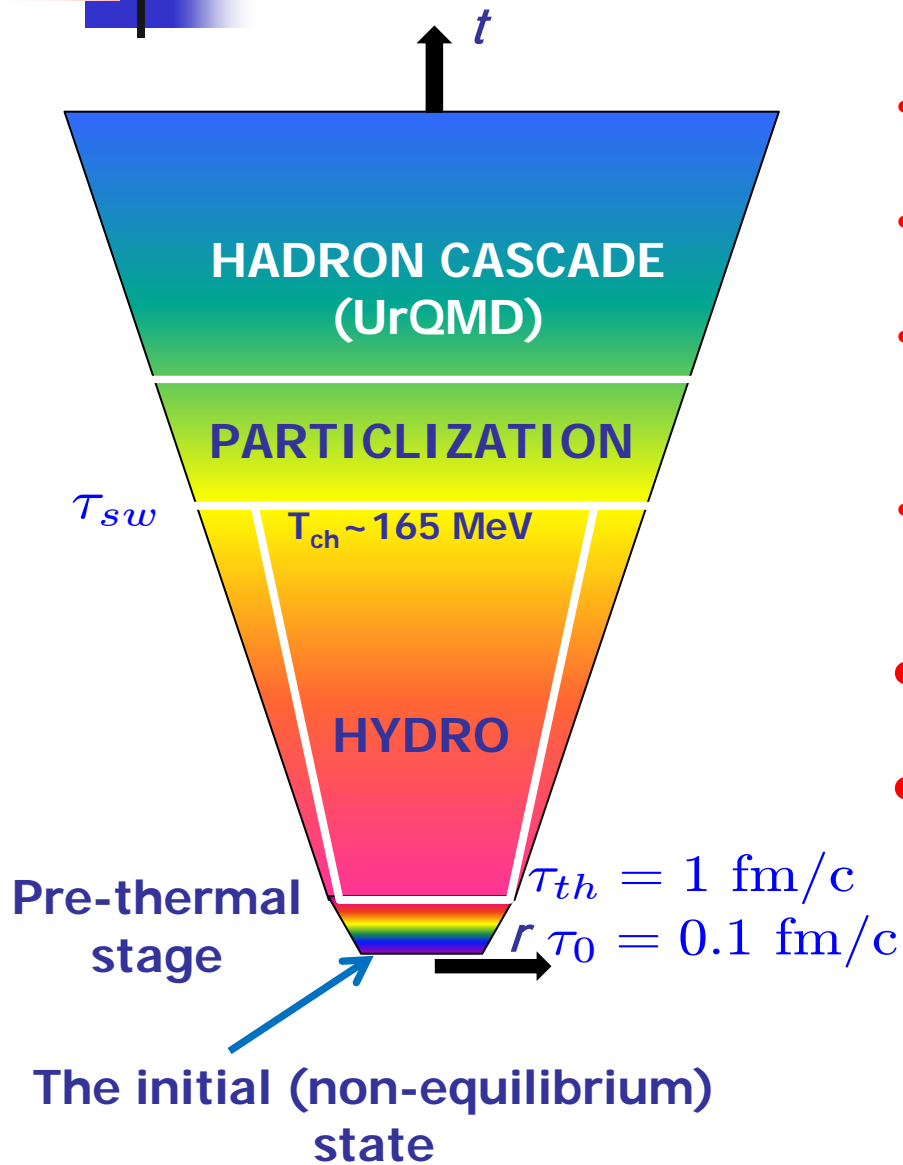
Collective velocity developed at pre-thermal stage from proper time $\tau_0=0.3$ fm/c by supposed thermalization time $\tau_i = 1$ fm/c for scenarios of partonic free streaming. The results are compared with the hydrodynamic evolution of perfect fluid with hard equation of state $p = 1/3 \epsilon$ started at τ_0 . Impact parameter $b=6.3$ fm.

- The initial transverse flow in thermal matter as well as its anisotropy are developed at pre-thermal - either partonic, string or classical field (glasma) - stage with even more efficiency than in the case of very early perfect hydrodynamics.
- Such radial and elliptic flows develop no matter whether a pressure already established. The general reason for them is an essential finiteness of the system in transverse direction.
- The anisotropy of the flows transforms into asymmetry of the transverse momentum spectra only of (partial) thermalization happens.
- So, the results, first published in 2006, show that whereas the assumption of (partial) thermalization in relativistic A + A collisions is really crucial to explain soft physics observables, the hypotheses of early thermalization at times less than 1 fm/c is not necessary.



$$v_{flow,i} \sim r_{it} / \lambda_{homog,i}^2$$

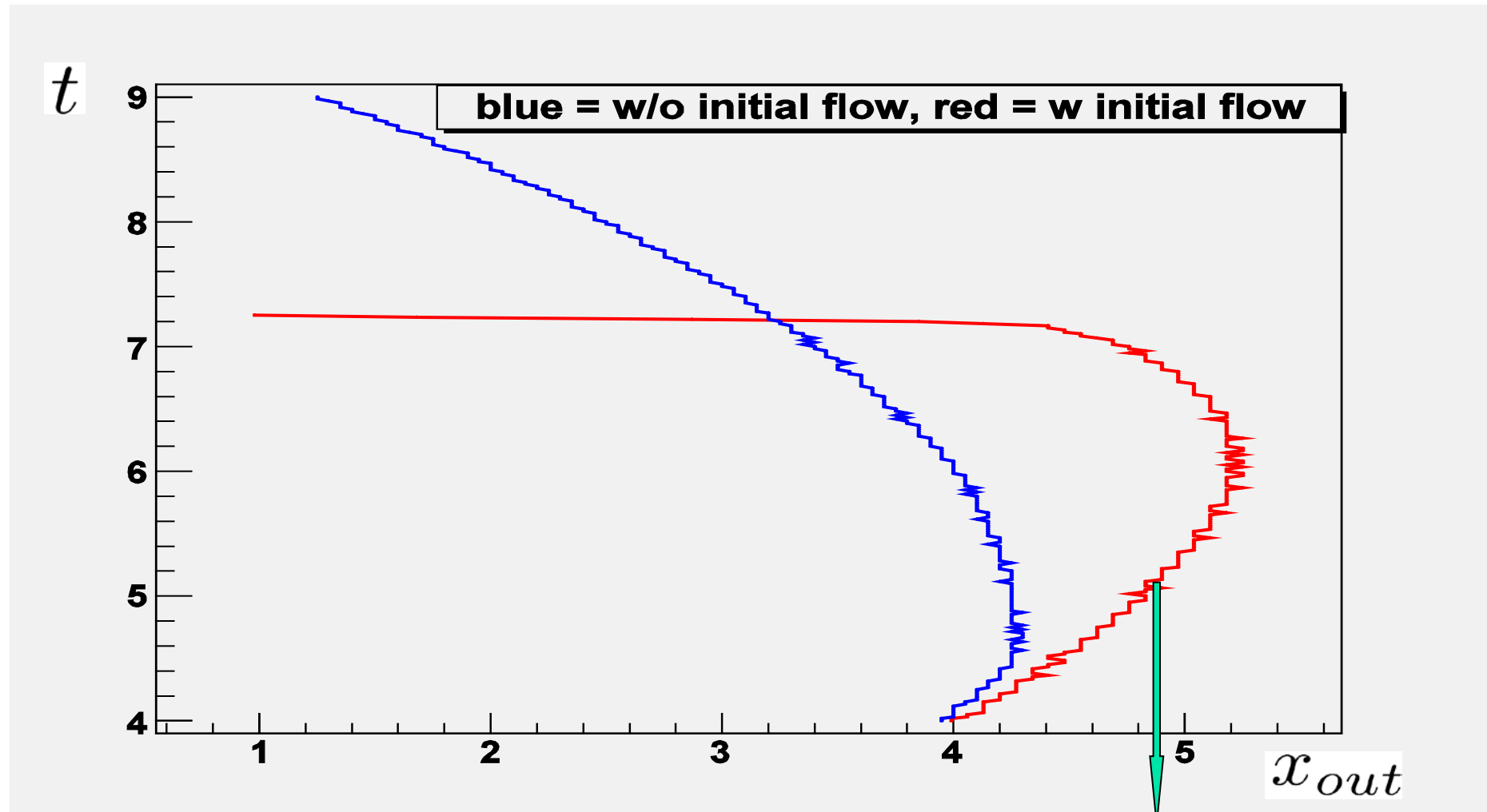
Integrated HydroKinetic model: HKM → iHKM



Complete algorithm incorporates the stages:

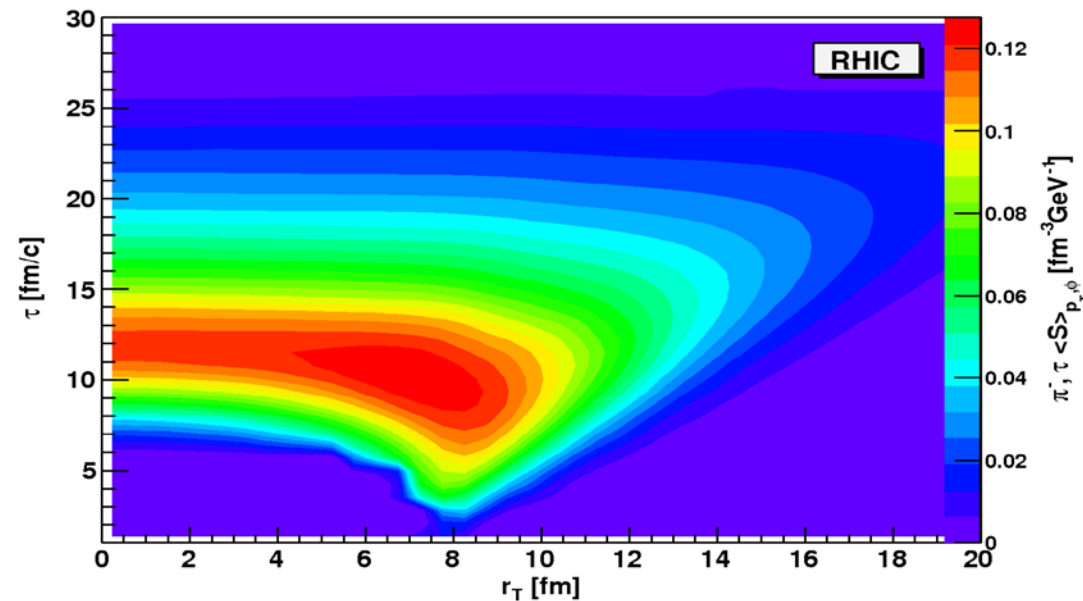
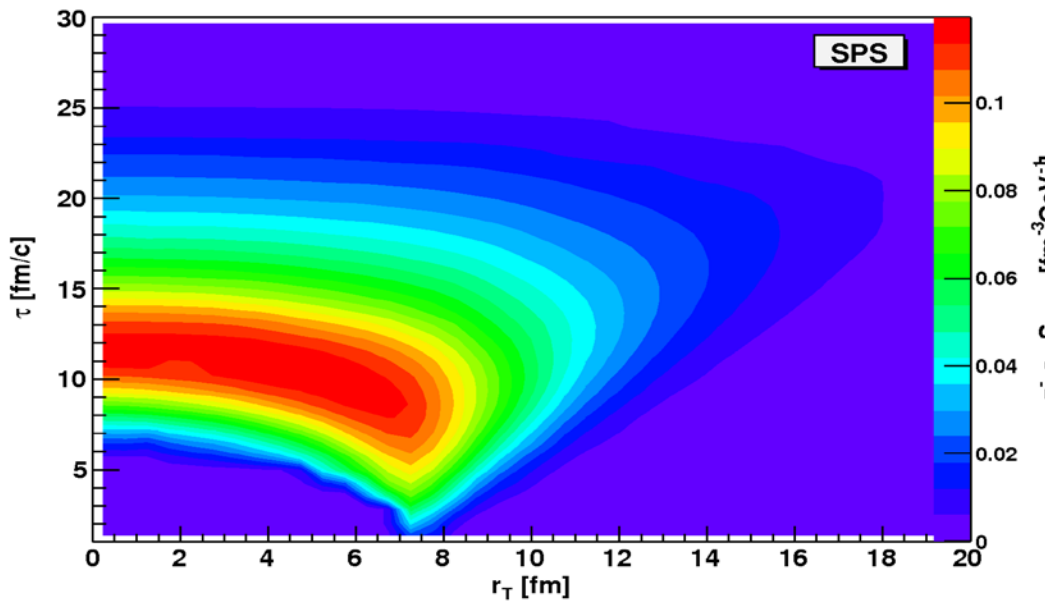
- generation of the initial states;
 - thermalization of initially non-thermal matter;
 - viscous chemically equilibrated hydrodynamic expansion;
 - sudden (with option: continuous) particlization of expanding medium;
 - a switch to UrQMD cascade with near equilibrium hadron gas as input;
 - simulation of observables.
- Yu.S., Akkelin, Hama: PRL 89 (2002) 052301;
 ... + Karpenko: PRC 78 (2008) 034906;
 Karpenko, Yu.S. : PRC 81 (2010) 054903;
 ... PLB 688 (2010) 50;
 Akkelin, Yu.S. : PRC 81 (2010) 064901;
 Karpenko, Yu.S., Werner: PRC 87 (2013) 024914;
 Naboka, Akkelin, Karpenko, Yu.S. : PRC **91** (2015) 014906;
 Naboka, Karpenko, Yu.S. Phys. Rev. C **93** (2016) 024902.

Initial flows and Ro/Rs ratio ($t_0=1-2$ fm/c)

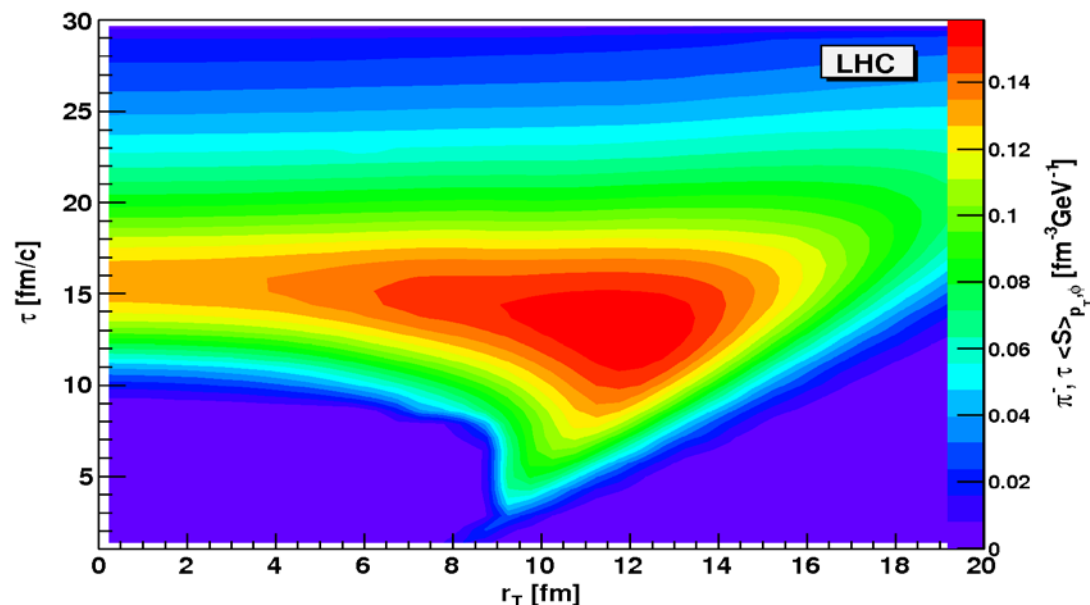


$$R_{out}^2 \approx R_{side}^2 + v^2 \langle \Delta t^2 \rangle_p - 2v \langle \Delta x_{out} \Delta t \rangle_p, v = \frac{p_T}{p_0}$$

Emission functions for top SPS, RHIC and LHC energies



$$R_{out}^2 \approx R_{side}^2 + v^2 \langle \Delta t^2 \rangle_p - 2v \langle \Delta x_{out} \Delta t \rangle_p, v = \frac{p_T}{p^0}$$



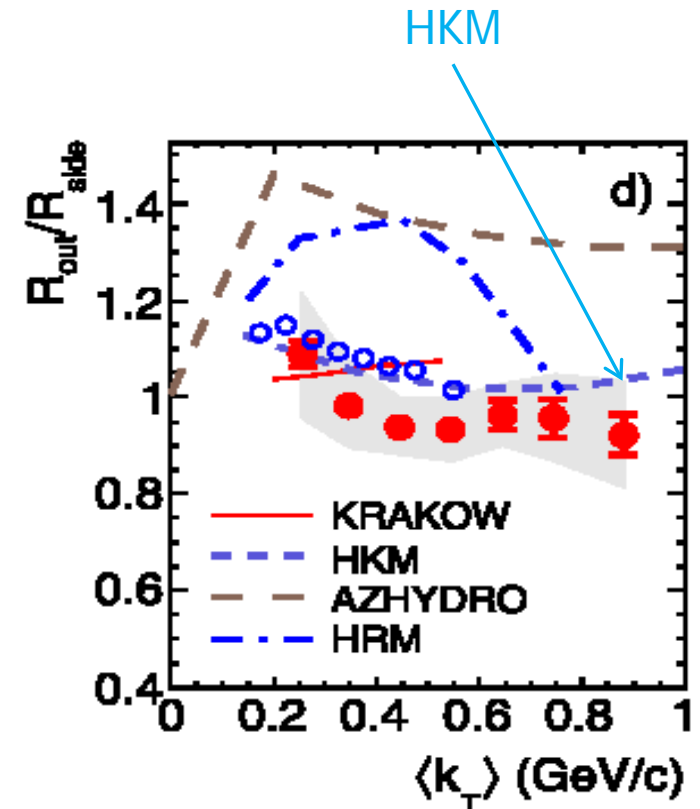
HKM prediction: solution of the HBT Puzzle

Two-pion Bose–Einstein correlations in central Pb–Pb collisions
at $\sqrt{s_{NN}} = 2.76$ TeV[☆] ALICE Collaboration Physics Letters B 696 (2011) 328.



Quotations:

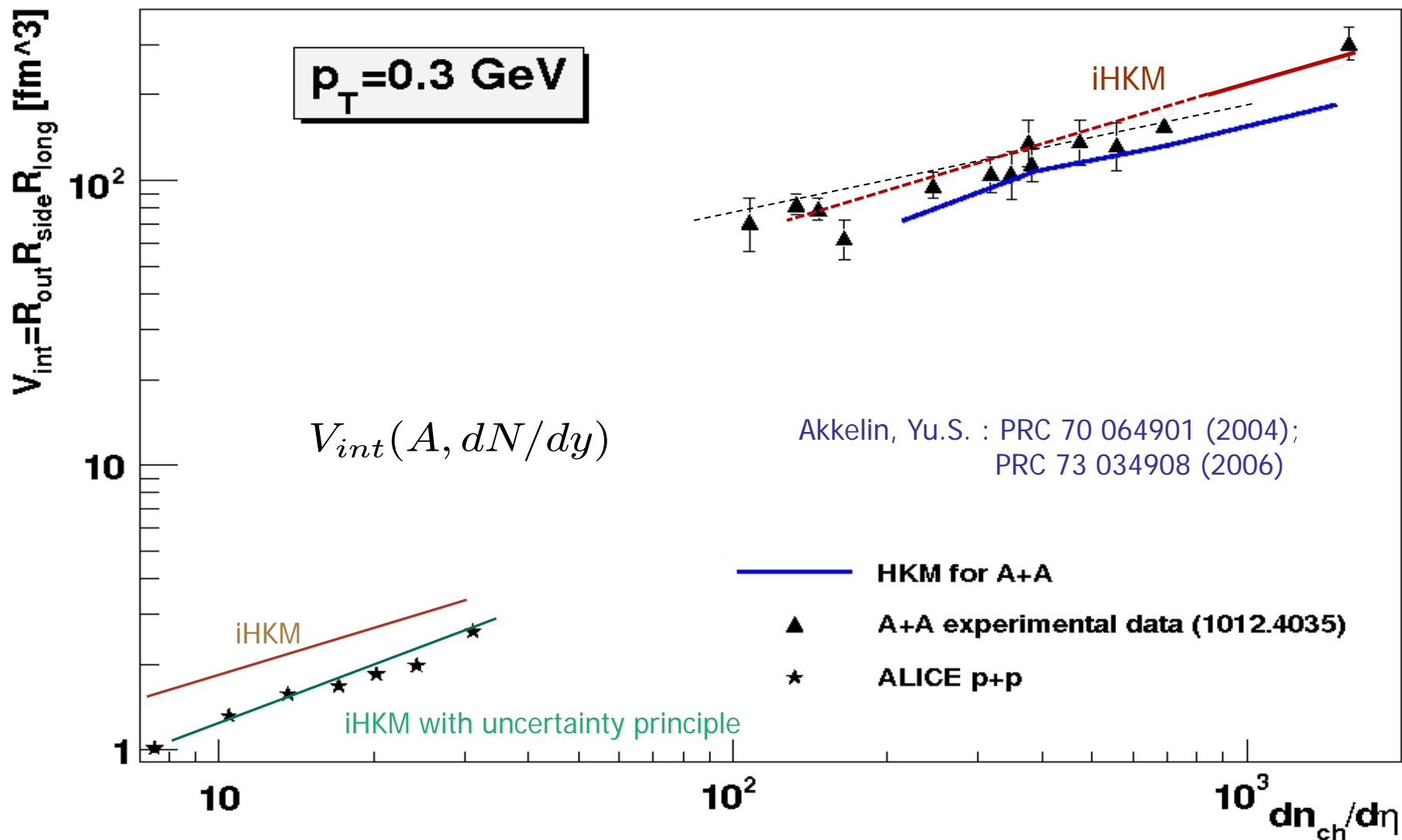
Available model predictions are compared to the experimental data in Figs. 2-d and 3. Calculations from three models incorporating a hydrodynamic approach, AZHYDRO [45], KRAKOW [46,47], and HKM [48,49], and from the hadronic-kinematics-based model HRM [50,51] are shown. An in-depth discussion is beyond the scope of this Letter but we notice that, while the increase of the radii between RHIC and the LHC is roughly reproduced by all four calculations, only two of them (KRAKOW and HKM) are able to describe the experimental R_{out}/R_{side} ratio.



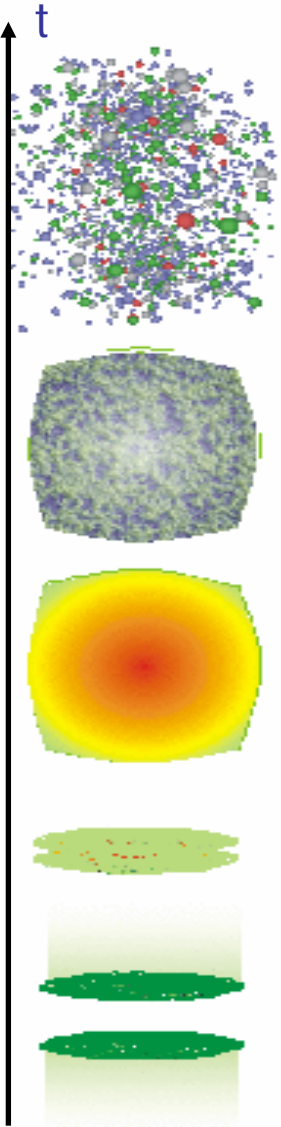
[48] I.A. Karpenko, Y.M. Sinyukov, Phys. Lett. B 688 (2010) 50.

[49] N. Armesto, et al. (Eds.), J. Phys. G 35 (2008) 054001.

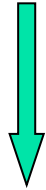
Role of a dissipative stage in formation of large Vint at LHC



Expecting Stages of Evolution in Ultrarelativistic A+A collisions



Relatively small space-time scales,
 $R_{\text{out}} / R_{\text{side}} \sim 1$ (HBT puzzle)

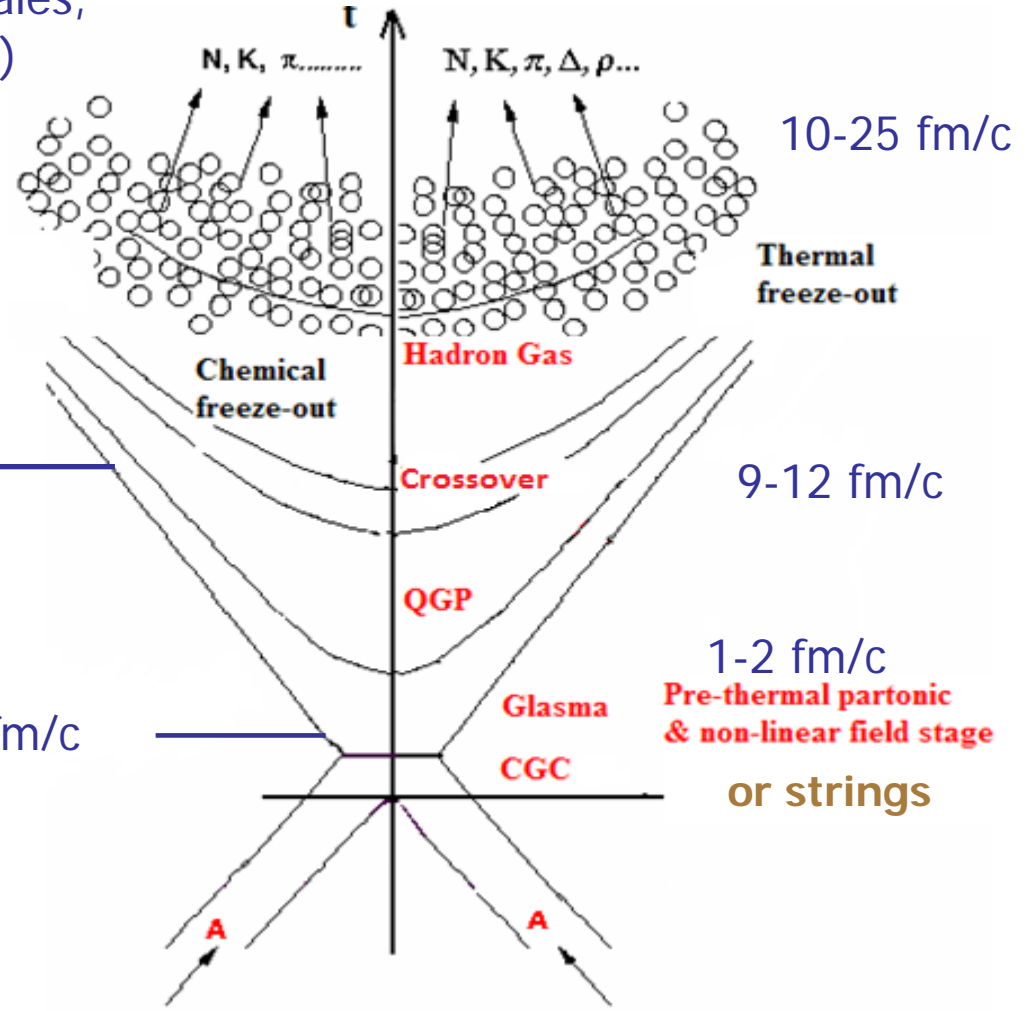


Early thermal freeze-out:
 $T_{\text{th}} \approx T_{\text{ch}} \approx 150 \text{ MeV}$

Elliptic flows



Early thermalization at $\approx 0.5 \text{ fm/c}$



Continuous freeze-out vs sudden freeze-out

- Thermal models of particle production vs dynamic/evolutionary approaches

Kinetic/thermal freeze-out

Sudden freeze-out

Cooper-Frye prescription

$$p^0 \frac{d^3 N_i}{d^3 p} = \int_{\sigma_{th}} d\sigma_\mu p^\mu f_i(x, p)$$

The σ_{th} is typically isotherm.

Continuous freeze-out

$$p^0 \frac{d^3 N_i}{d^3 p} = \int d^4 x S_i(x, p) \approx \int_{\sigma(p)} d\sigma_\mu p^\mu f_i(x, p)$$

The $\sigma(p)$ is piece of hypersurface where the particles with momentum near p has a maximal emission rate.

Yu.S. Phys. Rev. C78,

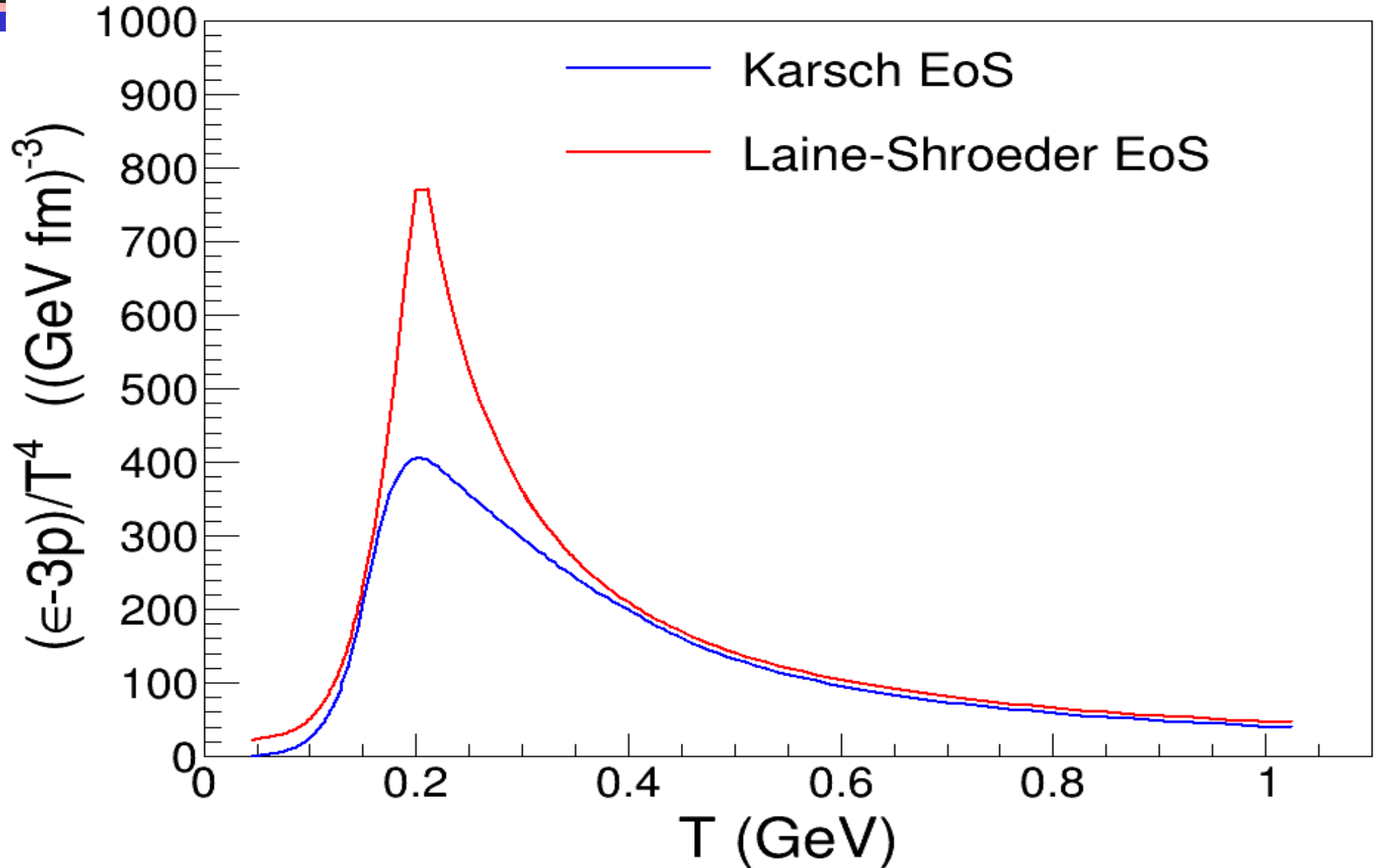
Chemical freeze-out

$$N_i = \int_p \int_{\sigma_{ch}} \frac{d^3 p}{p^0} d\sigma_\mu p^\mu f_i\left(\frac{p^\mu u_\mu(x)}{T_{ch}}, \frac{\mu_{i,ch}}{T_{ch}}\right)$$

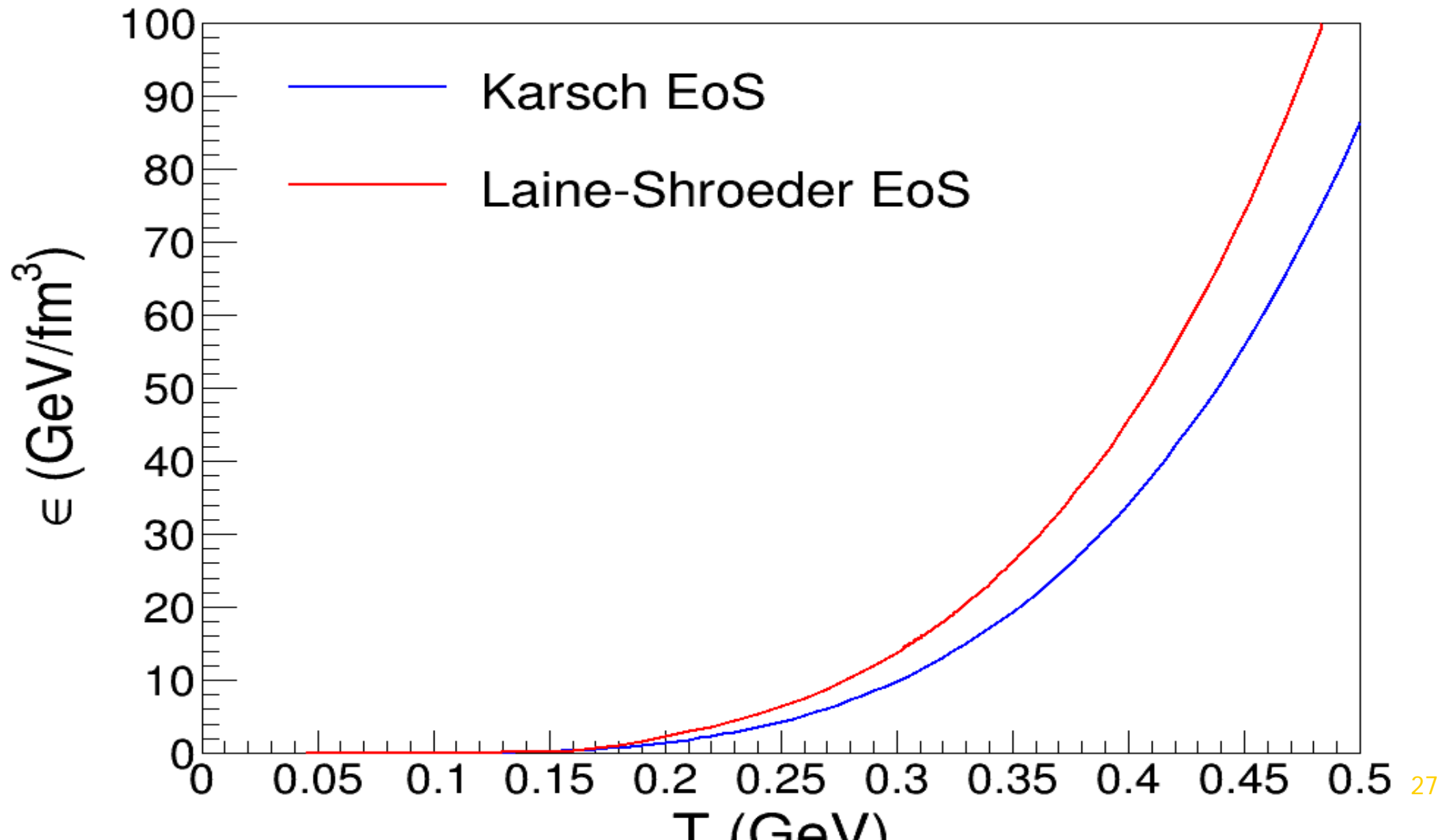
The numbers of quasi-stable particles is defined from N_i with taking into account the resonance decays but **not** inelastic re-scattering.

The T_{ch} is the minimal temperature when the expanding system is still (near) in local thermal and chemical equilibrium. Below the hadronic cascade takes place: $T_{ch} \rightarrow T_{part}$. The inelastic reactions, annihilation processes in hadron-resonance gas change the quasi-particle yields in comparison with sudden chem. freeze-out.

Equation of State - 1



Equation of state -2



Thermal models vs evolutionary approach

Basic matter properties:
thermodynamic **EoS**

Thermal models

Chemical freeze-out at

$$T_{ch} \approx T_h$$

Particle number ratios

$$\left\{ \frac{N_i}{N_j} \right\}$$

L.-S. \longrightarrow Karsch, Fodor (lattice QCD)

Evolutionary models

$$\frac{dN_{charge}}{d\eta}(c)$$

$$\frac{dN_\pi}{p_T dp_T}$$

High dense matter formation time τ_0

Max. energy density $\epsilon(\tau_0) \equiv \epsilon_0$

At the particlization temperature $T_{part} \approx T_h$ hydrodynamic evolution transforms (suddenly or continuously) into interact. hadron gas evolution

EoS:

$$T_h = 165 \text{ MeV} \longrightarrow 156 \text{ MeV}$$

$$\tau_0 = 0.1 \text{ fm/c} \longrightarrow 0.15 \text{ fm/c}$$

$$\epsilon_0 = 679 \text{ GeV/fm}^3 \longrightarrow 495 \text{ GeV/fm}^3$$

iHKM

Kinetic freeze-out

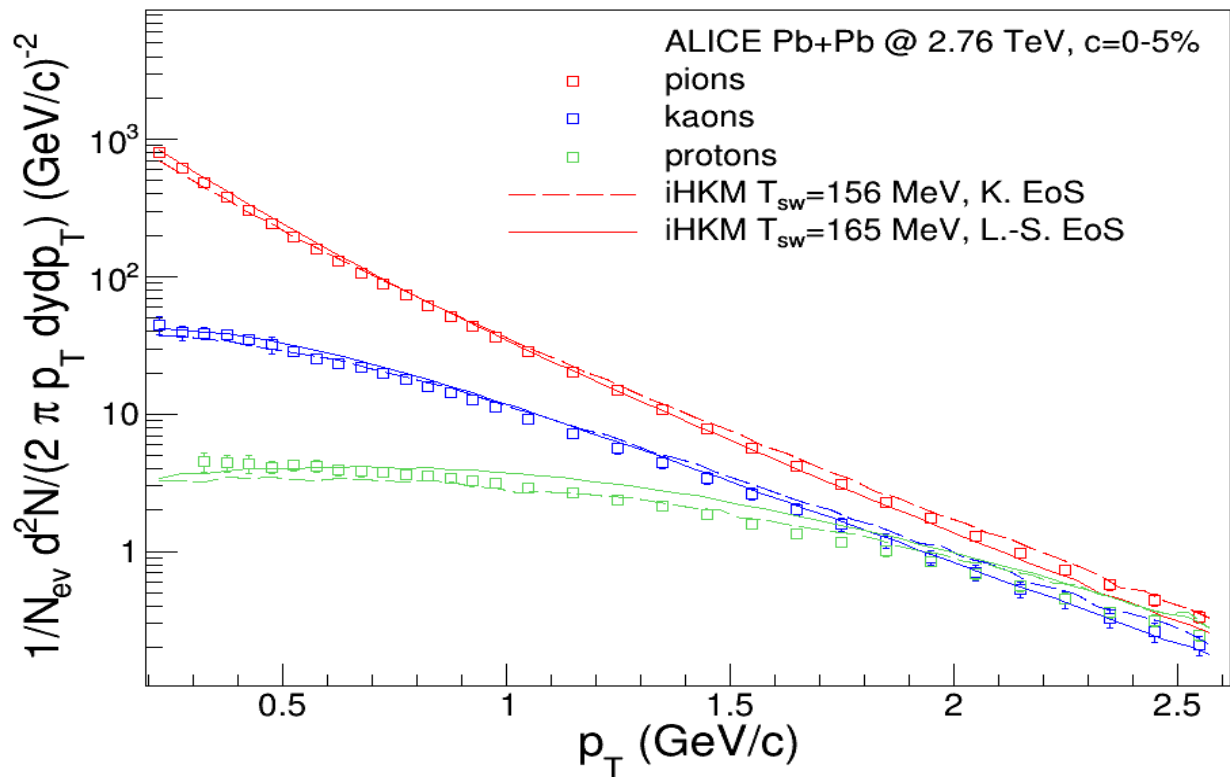
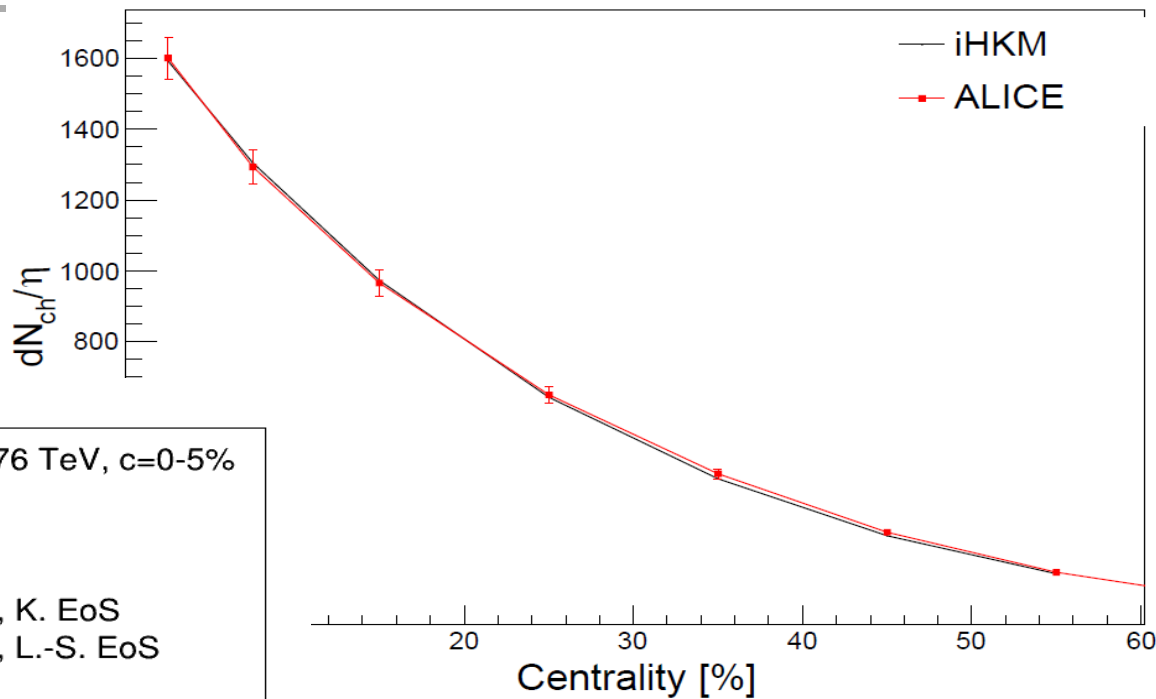
«Blast-wave» parametrization of sharp freeze-out hypersurface and transverse flows on it. Spectra

$$\frac{dN_i}{p_T dp_T} \longrightarrow T_{th}$$

Kinetic freeze-out is continuous, lasts more than 5 fm/c. «Effective temperature» of maximal emission: $T_{th}(p)$.

Multiplicity dependence of all charged particles on centrality and spectra for LHC energy

$\sqrt{s_{NN}} = 2.76 \text{ TeV}$

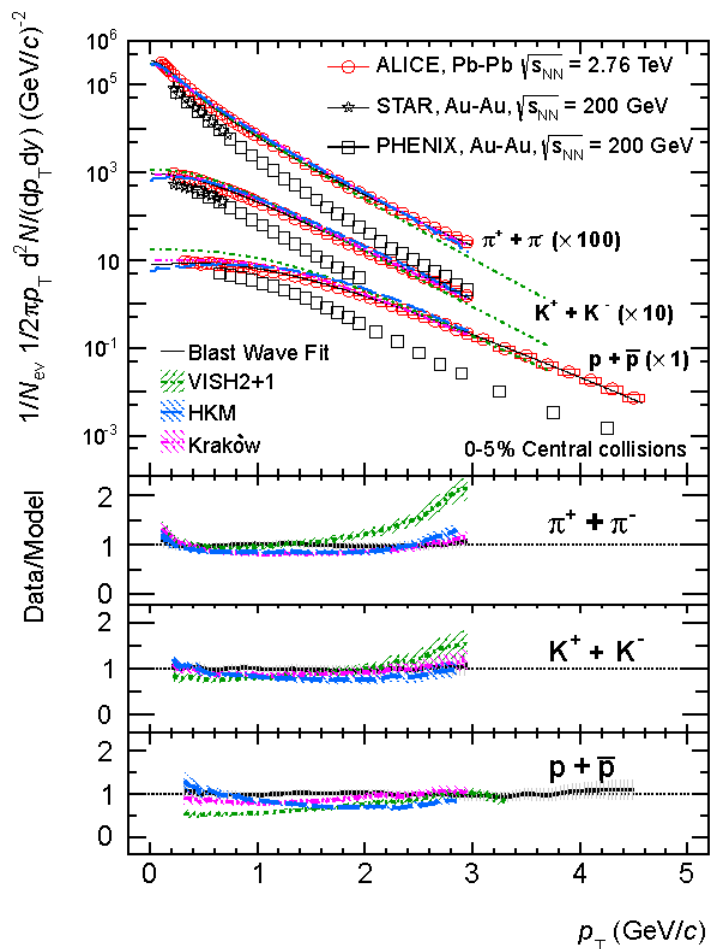


Predictions for particle yield at LHC in central collisions from HKM

Pion, Kaon, and Proton Production in Central Pb–Pb Collisions at

$$\sqrt{s_{NN}} = 2.76 \text{ TeV}$$

The ALICE Collaboration *Phys. Rev. Lett.* **109**, 252301 (2012)



Quotations

This interpretation is supported by the comparison with HKM [39, 40], a similar model in which, after the hydrodynamic phase, particles are injected into a hadronic cascade model (UrQMD [41, 42]), which further transports them until final decoupling. The hadronic phase builds additional radial flow, mostly due to elastic interactions, and affects particle ratios due to inelastic interactions. HKM yields a better description of the data. At the LHC, hadronic final state interactions, and in particular antibaryon-baryon annihilation, may therefore be an important ingredient for the description of particle yields [43, 40], contradicting the scenario of negligible abundance-changing processes in the hadronic phase. The third model shown in Fig. 1 (Kraków [44, 45]) introduces non-equilibrium corrections due to viscosity at the transition from the hydrodynamic description to particles, which change the effective T_{ch} , leading to a good agreement with the data. In the region $p_T \lesssim 3 \text{ GeV}/c$ (Kraków) and $p_T \lesssim 1.5 \text{ GeV}/c$ (HKM) the last two models reproduce the experimental data within $\sim 20\%$, supporting a hydrodynamic interpretation of the transverse momentum spectra at the LHC. These models also describe correctly other features of the space-time evolution of the system, as measured by ALICE with charged pion correlations [46].

[39] Y. Karpenko and Y. Sinyukov, *J.Phys.* **G38**, 124059 (2011), nucl-th/1107.3745.

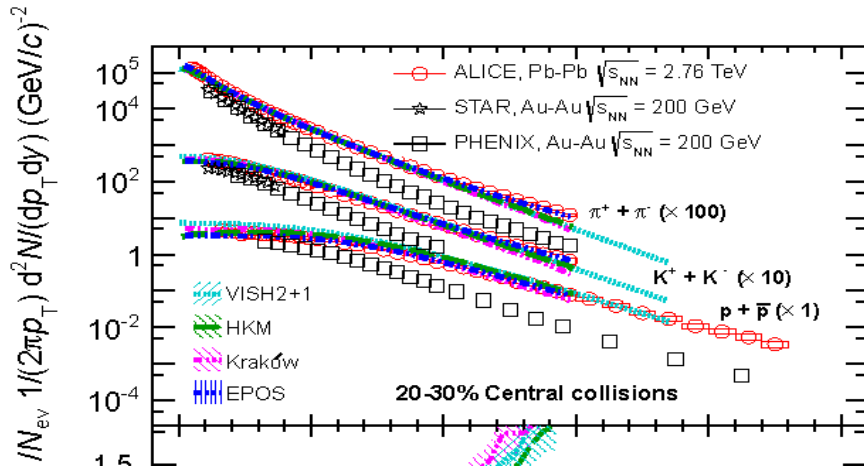
[40] Y. Karpenko, Y. Sinyukov, and K. Werner, (2012), nucl-th/1204.5351.

Predictions for particle spectra at LHC in non-central collisions

Centrality Dependence of π , K, p in Pb–Pb at $\sqrt{s_{NN}} = 2.76$ TeV

ALICE Collaboration

arXiv:1303.0737v1 [hep-ex]



Quotations:

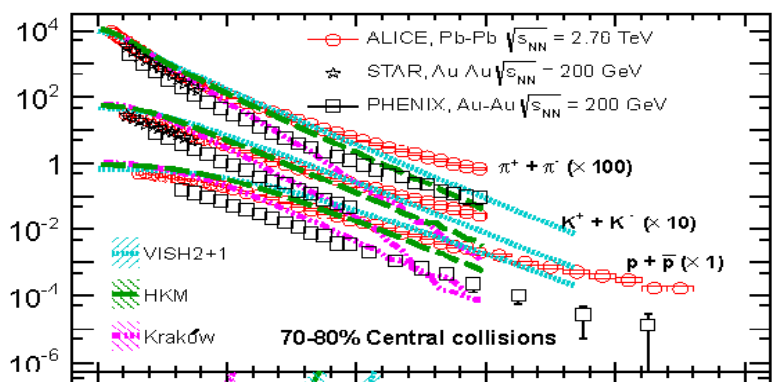
The difference between VISH2+1 and the data are possibly due to the lack of an explicit description of the hadronic phase in the model. This idea is supported by the comparison with HKM [47, 50]. HKM is a model similar to VISH2+1, in which after the hydrodynamic phase particles are injected into a hadronic cascade model (UrQMD), which further transports them until final decoupling. The hadronic phase builds up additional radial flow and affects particle ratios due to the hadronic interactions. As can be seen, this model yields a better description of the data. The protons at low p_T , and hence their total number, are rather well reproduced, even if the slope is significantly smaller than in the data. Antibaryon-baryon annihilation is an important ingredient for the description of particle yields in this model [47, 50].

.....

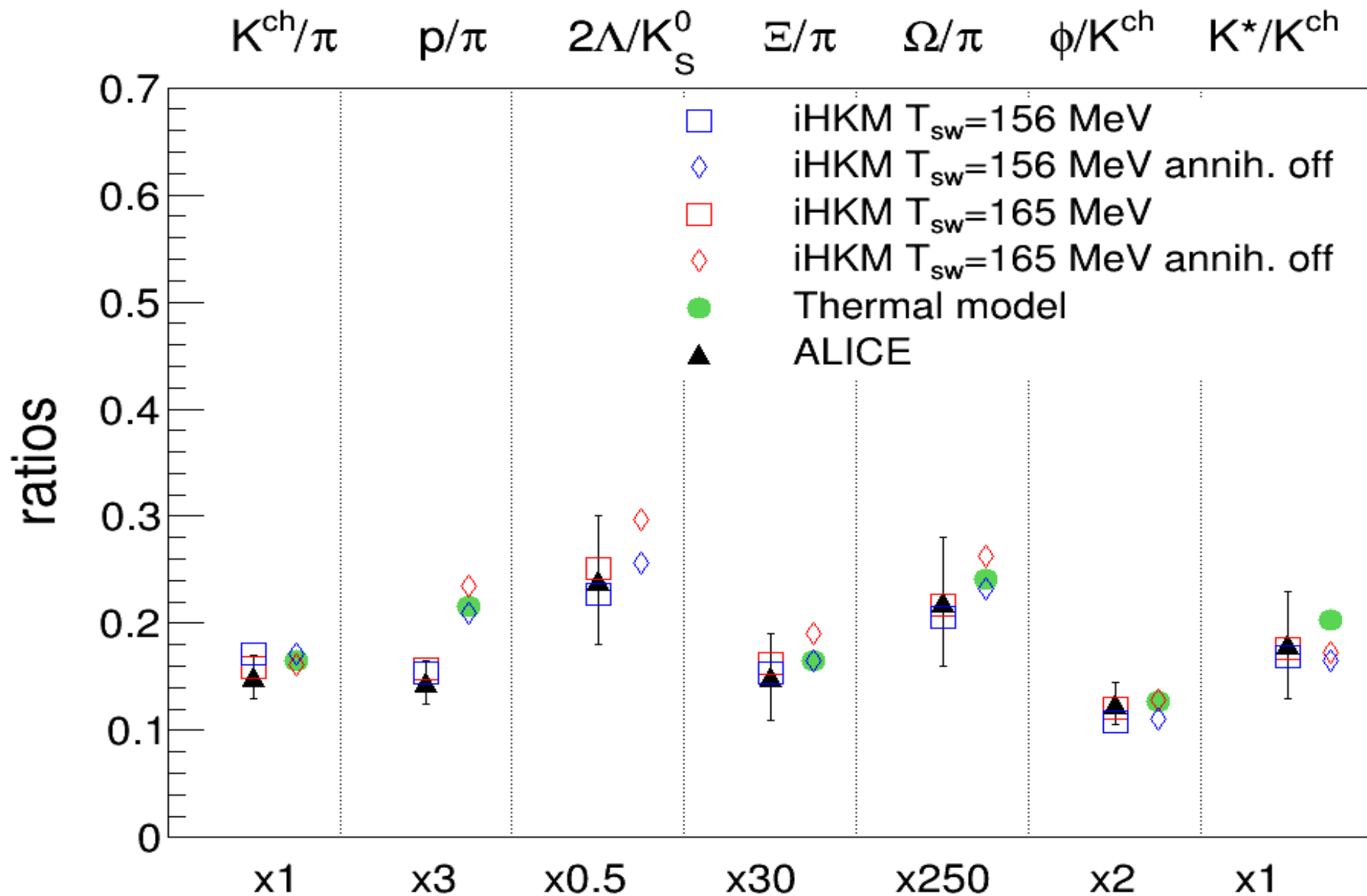
Phys. Rev. C 87, 024914 (2013)

[47] Y. Karpenko, Y. Sinyukov, and K. Werner, (2012), arXiv:1204.5351 [nucl-th]

[50] Y. Karpenko and Y. Sinyukov, J.Phys.G **G38**, 124059 (2011).



Particle number ratios at the LHC



Space-time picture of the emission and K^{*0} probes.

$K^* \rightarrow K\pi$ $K^{*0} \rightarrow K^+\pi^-$ Lifetime 4.2 fm/c

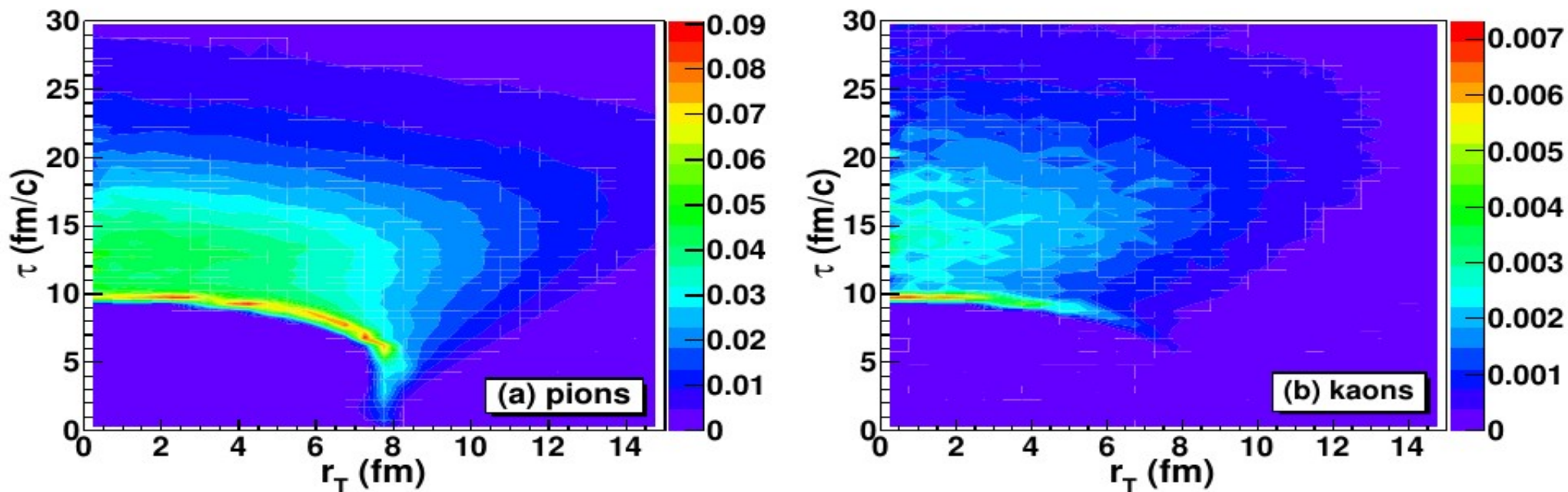
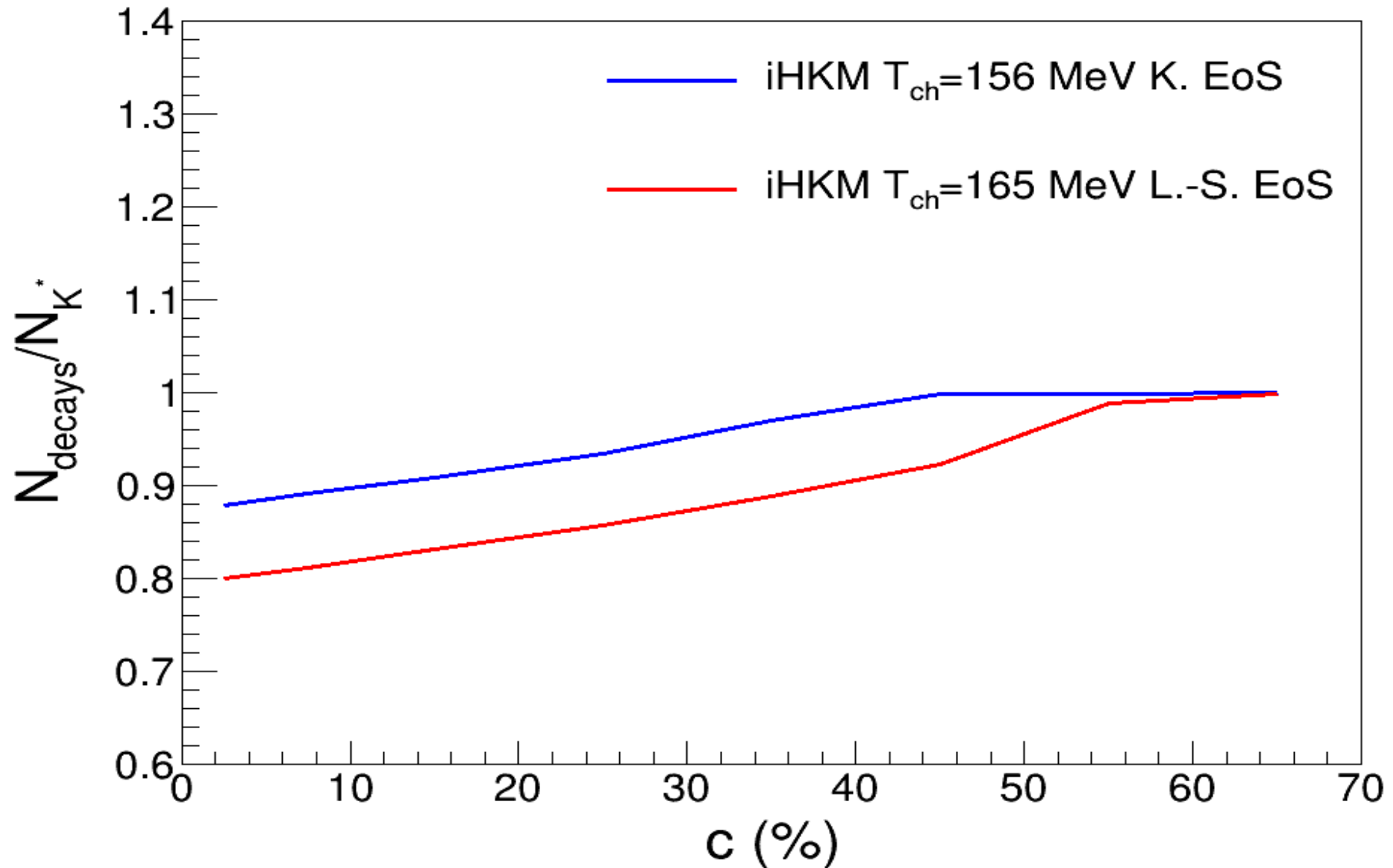
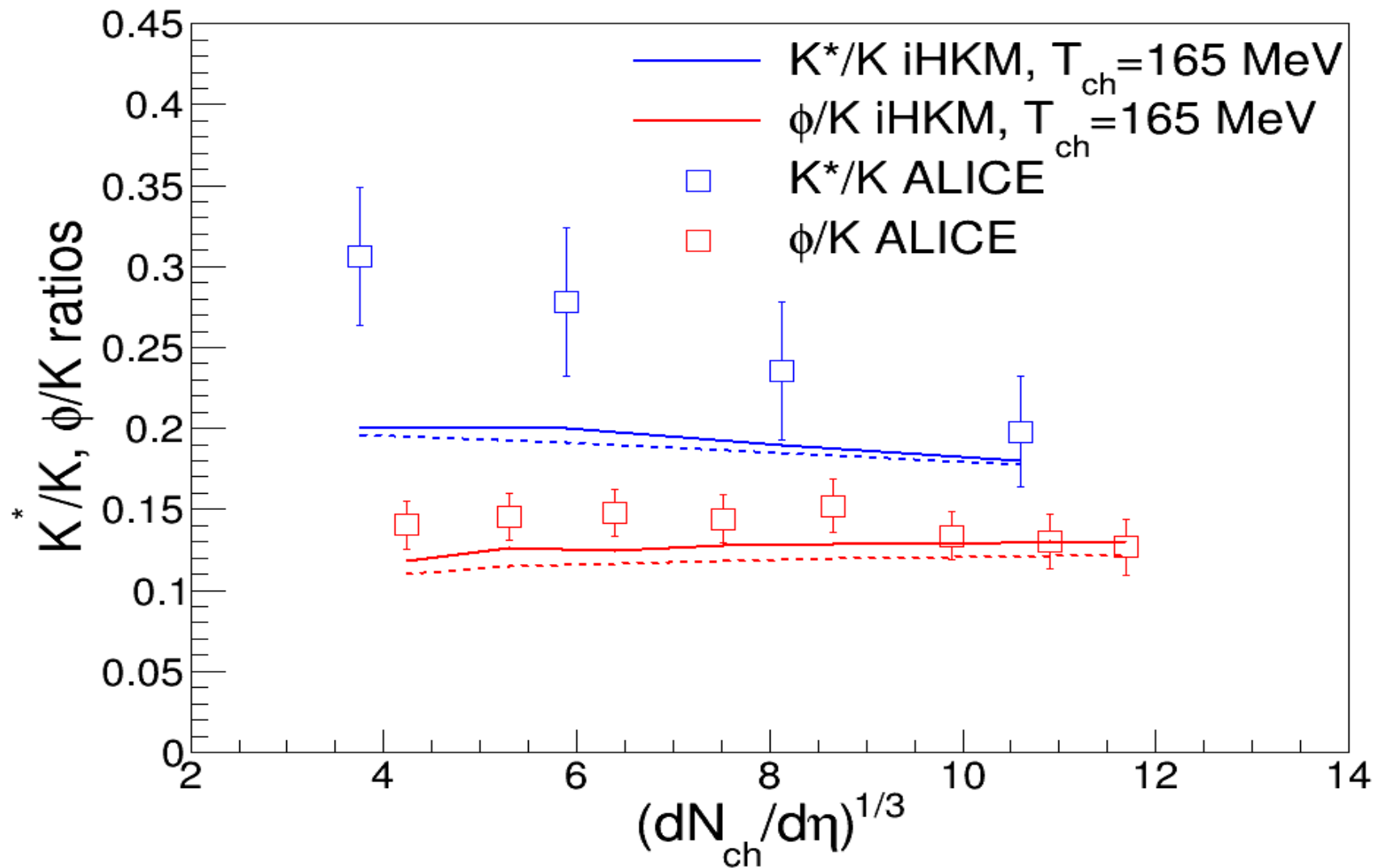


FIG. 4. The momentum angle averaged emission functions per units of space-time and momentum rapidities $g(\tau, r_T, p_T)$ [fm^{-3}] (see body text) for pions (a) and kaons (b) obtained from the HKM simulations of Pb+Pb collisions at the LHC $\sqrt{s_{NN}} = 2.76$ GeV, $0.2 < p_T < 0.3$ GeV/c, $|y| < 0.5$, $e = 0 - 5\%$.

Suppression of K_{S}^{*0} to continuous thermal freeze-out



Ratio



Some conclusions about particle productions

- Nor chemical neither thermal freeze-out cannot be considered as sudden at some corresponding temperatures.
- Particle yield probe ($\frac{dN_i}{d\eta} / \frac{dN_j}{d\eta}$ as well as absolute values $\frac{dN_i}{d\eta}$!) demonstrate that even at the minimal hadronization temperature $T_{ch} = T_h = 156$ MeV, the annihilation and other non-elastic scattering reactions play an important role in formation particle number ratios, especially, such where protons and pions are participating.
- It happens that the results for small and relatively large T_h are quite similar. It seems that inelastic processes (other than the resonance decays), that happen at the matter evolution below T_h , play a role of the compensatory mechanism in formation of $\frac{dN_i}{d\eta} / \frac{dN_j}{d\eta}$. Chemical f.-o. is continuous.
- As for the thermal freeze-out, the $K^{*0}(892)$ probes demonstrate that even 4-5 fm/c (proper time!) after hadronization at least 20% of decay products are still scattered.



Acknowledgement

Щиро дякую усіх присутніх за їх
присутність та увагу !

Thank you





Initial states

The most commonly used models of initial state are:

High Energies

MC-G (Monte Carlo Glauber)
MC-KLN (Monte Carlo Kharzeev-Levin-Nardi)
EPOS (parton-based Gribov-Regge model)
EKRT (perturbative QCD + saturation model)
IP-Glasma (Impact Parameter dependent Glasma)

Low Energies

MC-G (Monte Carlo Glauber) - ?
UrQMD (Ultra-Relativistic Molecular Dynamics) - ?

PROBLEM:

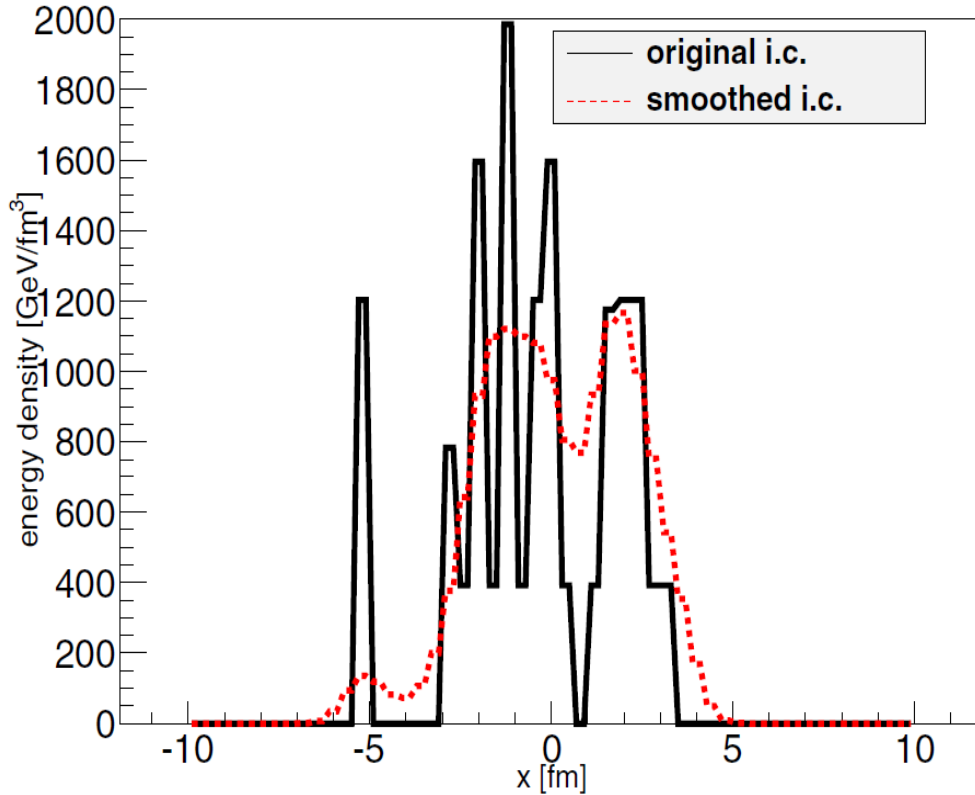
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**the biggest experimental discovery for a few decades is that hydrodynamics is the basis
of the "Standard Model " of high energy A+A collisions**

MC-G Initial State (IS) attributed to

$$\tau_0 = 0.1 \text{ fm}/c$$

GLISSANDO 2



- The initial state (IS) is highly inhomogeneous.
- It is not locally equilibrated.
- The IS most probable is strongly momentum anisotropic (result from CGC)

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MC-G Hybrid for ensemble of ISs :

$$\epsilon(b; \tau_0, \mathbf{r}_T) = \epsilon_0 \frac{(1 - \alpha) N_W(b, \mathbf{r}_T)/2 + \alpha N_{bin}(b, \mathbf{r}_T)}{(1 - \alpha) N_W(b = 0, \mathbf{r}_T = 0)/2 + \alpha N_{bin}(b = 0, \mathbf{r}_T = 0)}$$

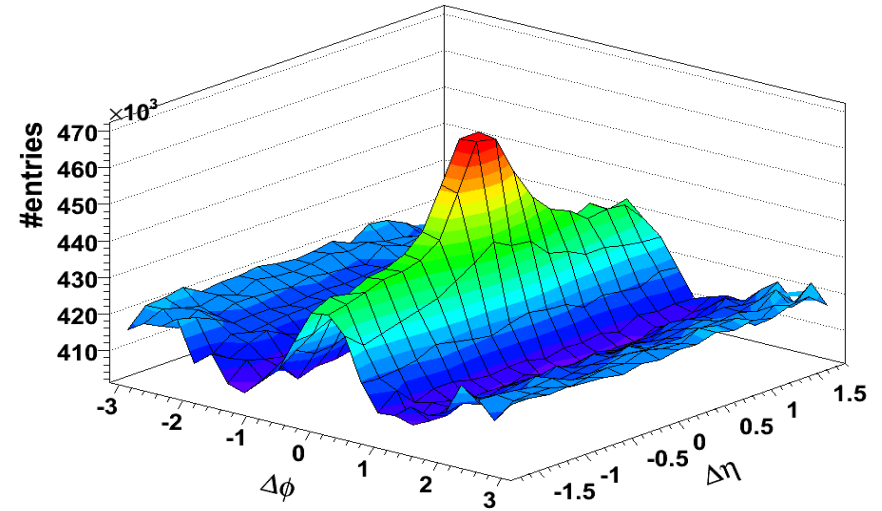
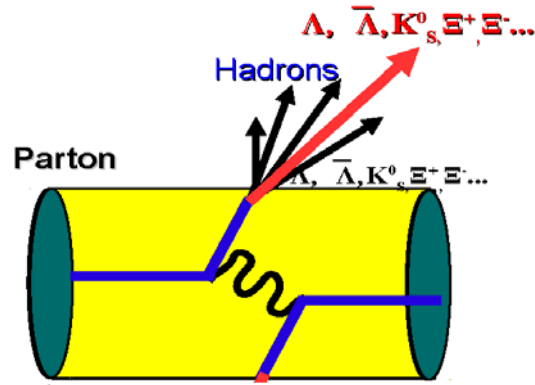
Parameters of IS

$$\Lambda = \lambda_\perp / \lambda_\parallel$$

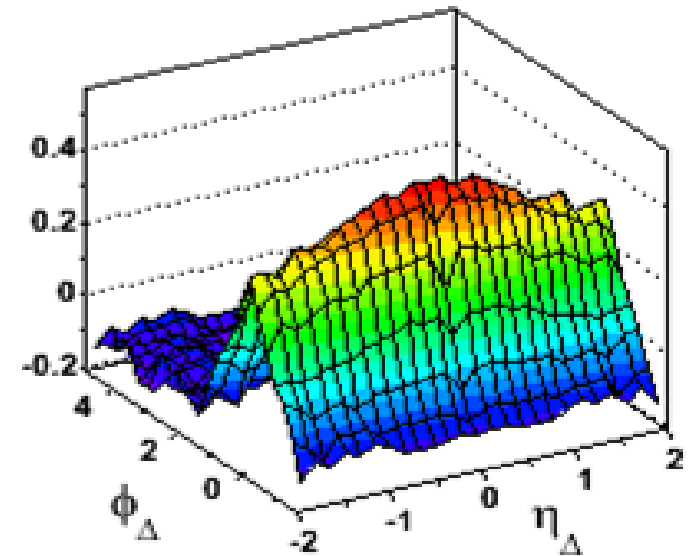
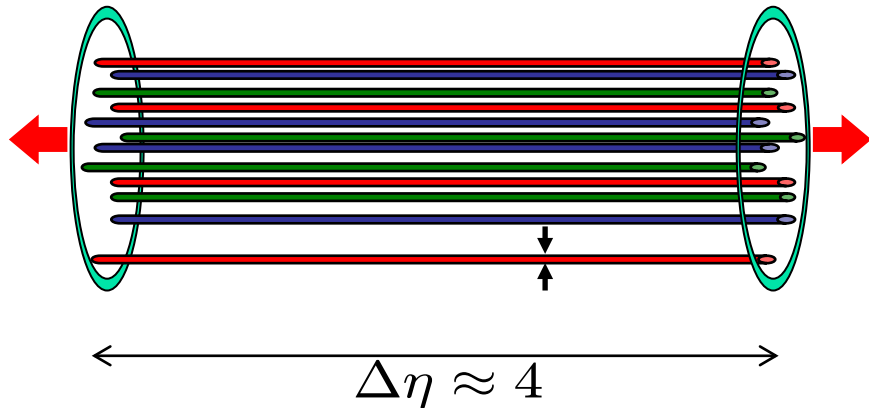
$$\epsilon_0, \alpha = 0.24^{39}$$

Ridges as probes of jet formation and fluctuating initial conditions (IC)

Hard ridge is formed by hard parton processes resulting in jets

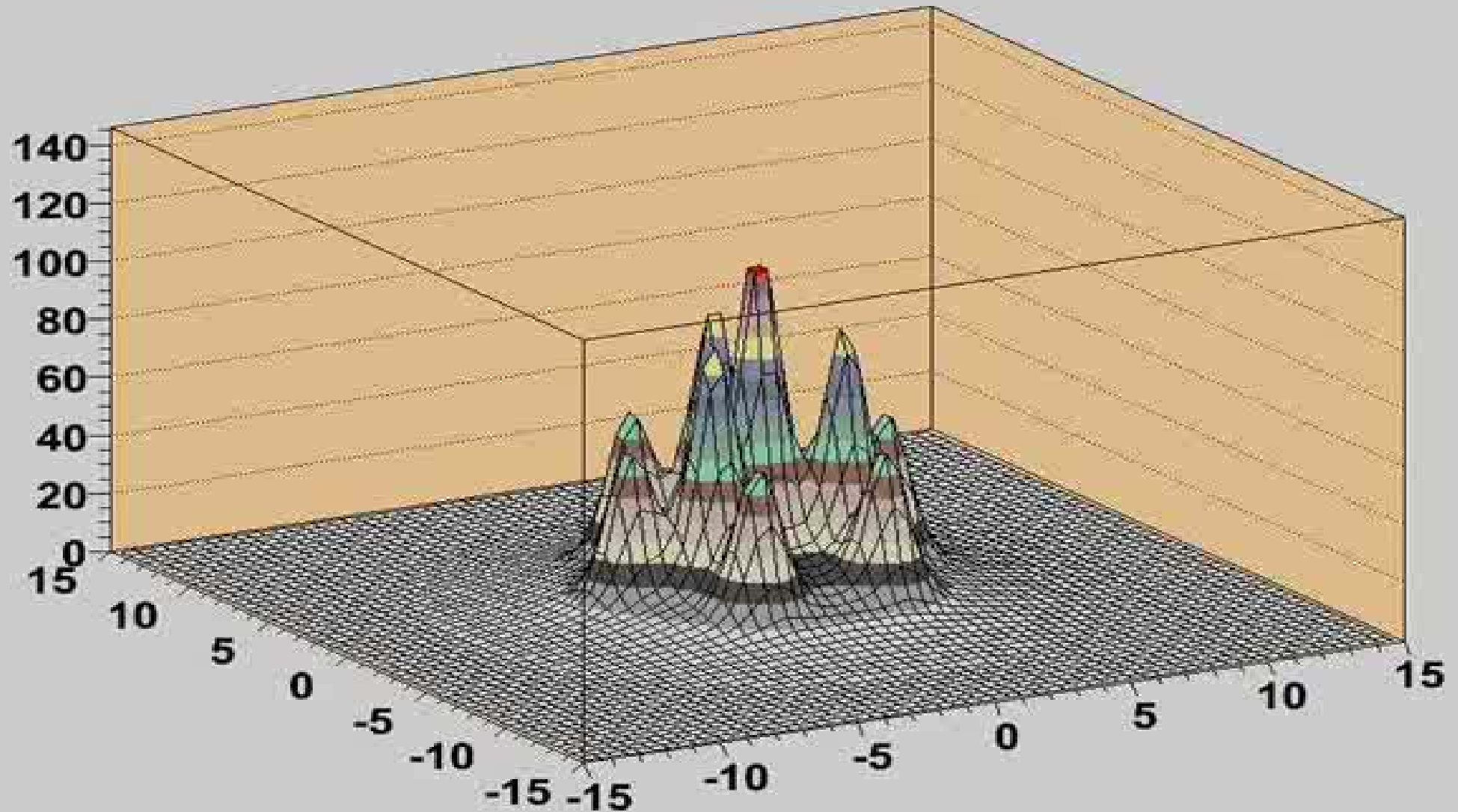


Soft ridges
Causality reason → it is formed by IC



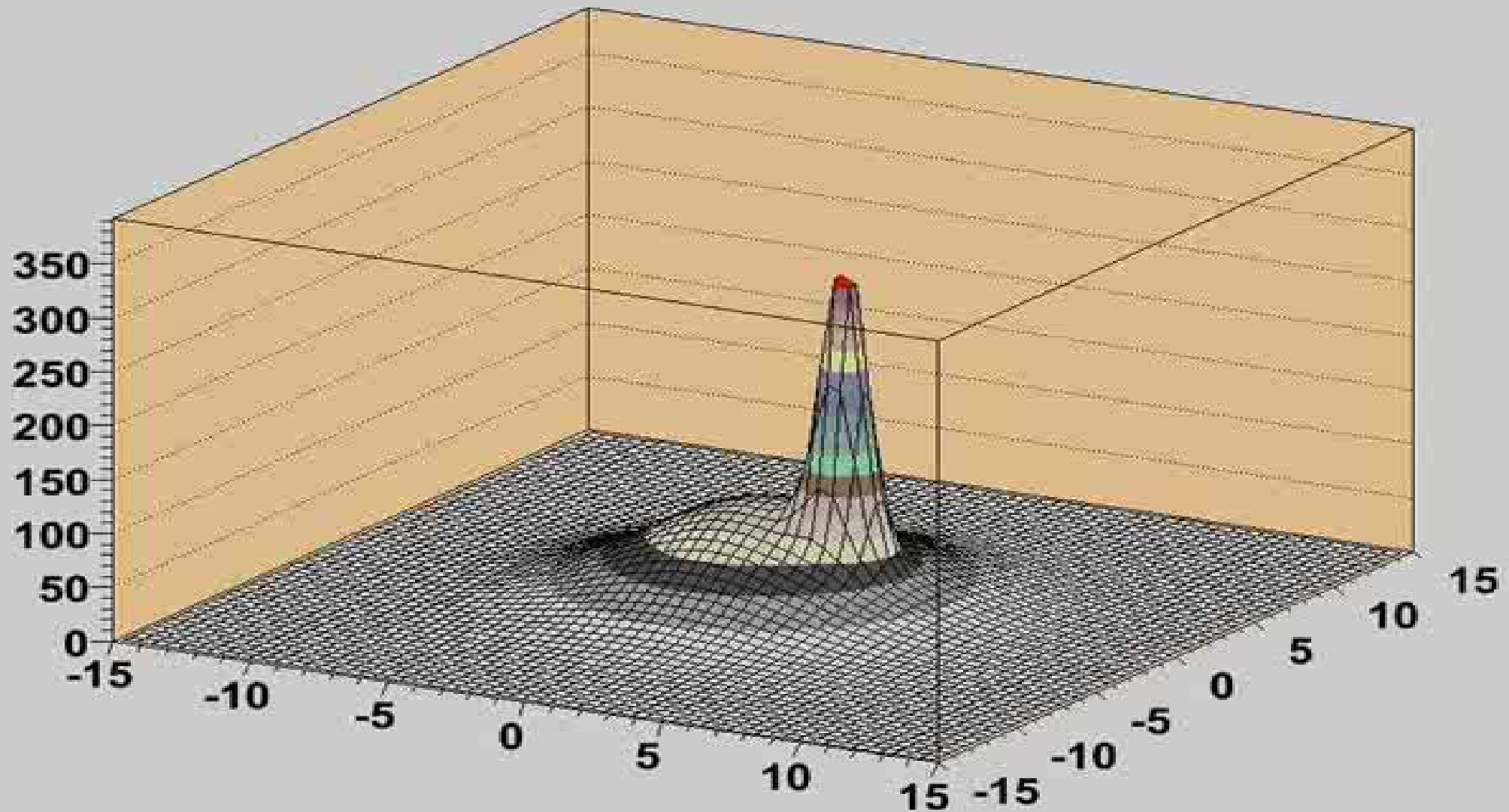
Hydrodynamics with initial tube-like fluctuations
(formation of ridges, initially 10 small tubes)

$e(t)$, $t[0] = 0.200000$ fm/c



Hydrodynamics with initial tube-like fluctuations
(formation of ridges, initially 1 shifted small tube)

$e(t)$, $t[0] = 0.200000$ fm/c

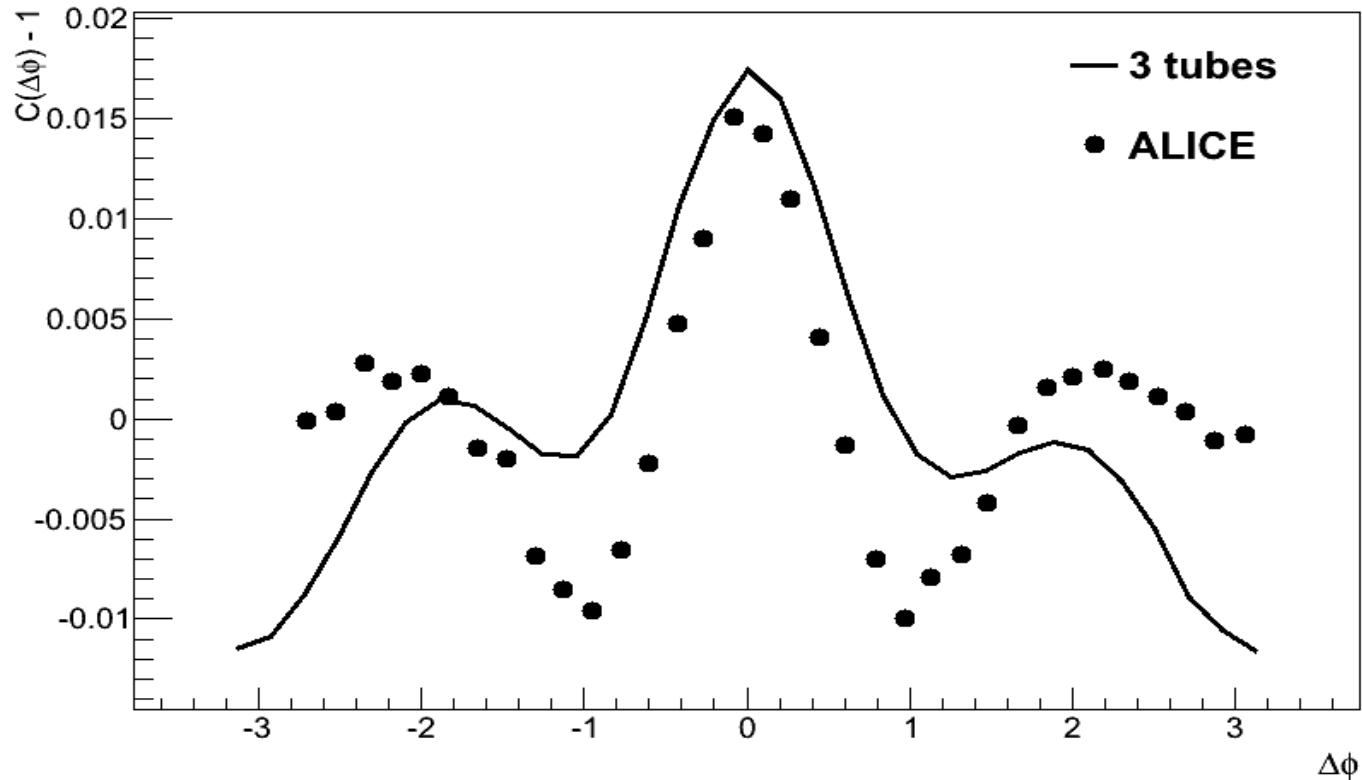


Ridges in HydroKinetic Model (HKM)

Tubular initial conditions and ridge formation

M.S. Borysova[†], O.D. Borysov^b, Yu.O. Karpenko^{†,¶}, V.M. Shapoval[†] and
Yu.M. Sinyukov[†],

“Advances in High Energy Physics”, 2015)





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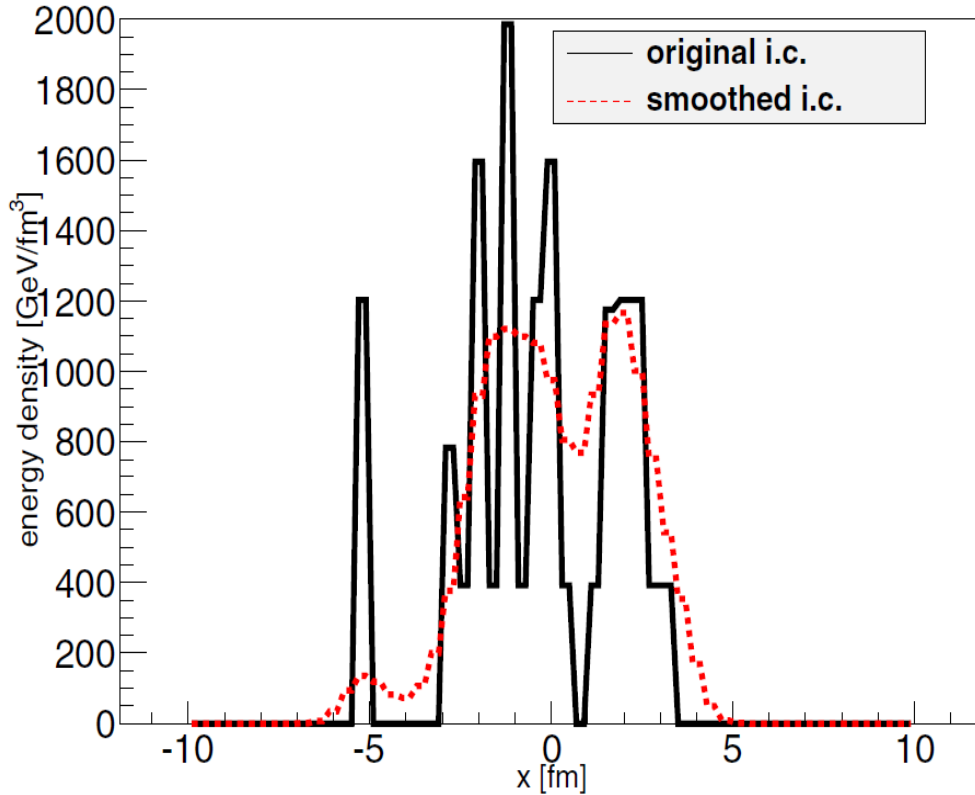
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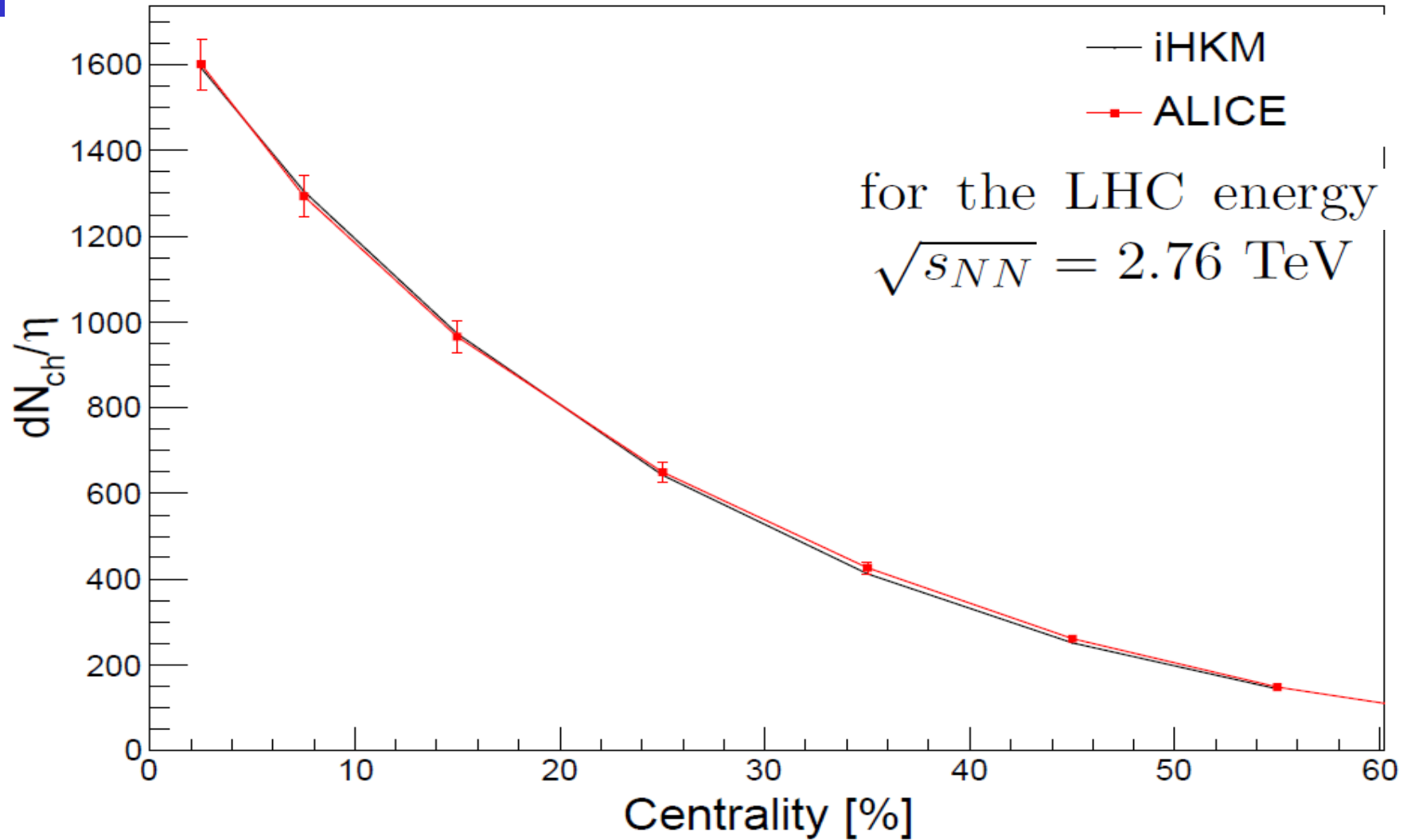
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Parameters of IS

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$$\epsilon_0, \alpha = 0.24^{45}$$

Multiplicity dependence of all charged particles on centrality



parameter values : $\tau_0 = 0.1$ fm/c, $\tau_{rel} = 0.25$ fm/c, $\eta/s = 0.08$, $\Lambda = 100$

Pre-thermal stage (thermalization)

Akkelin, Yu.S. :PRC **81** (2010); Naboka, Akkelin, Karpenko, Yu.S. : PRC **91** (2015).

Non-thermal state $\tau_0 = 0.1 \text{ fm}/c \longrightarrow$ locally near equilibrated state $\tau_{th} = 1 \text{ fm}/c$

Boltzmann equation in
relaxation time approximation
(integral form)

MAIN OBJECT

$$\mathcal{P}_\sigma(x, p) = \exp\left(-\int_t^{t_\sigma} \frac{d\bar{t}}{\tau_{rel}(\bar{x}, p)}\right)$$

$$\bar{x} \equiv (\bar{t}, \mathbf{x}_\sigma + (\mathbf{p}/p_0)(\bar{t} - t_\sigma))$$

MAIN ANSATZ with minimal number of parameters: $\tau_0, \tau_{th}, \tau_{rel}$

$$\mathcal{P}_{\tau_0 \rightarrow \tau}(\tau) = \left(\frac{\tau_{th} - \tau}{t_{th} - \tau_0}\right)^{\frac{\tau_{th} - \tau_0}{\tau_{rel}(\tau_0)}} \longrightarrow T^{\mu\nu}(x) = T_{free}^{\mu\nu}(x)\mathcal{P}(\tau) + T_{hyd}^{\mu\nu}(x)(1 - \mathcal{P}(\tau))$$

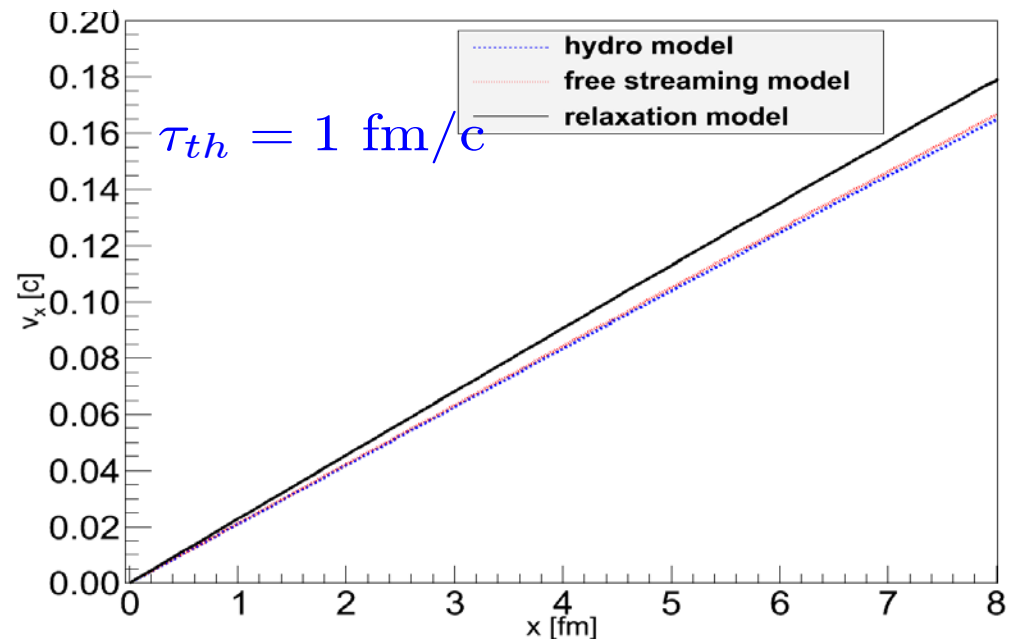
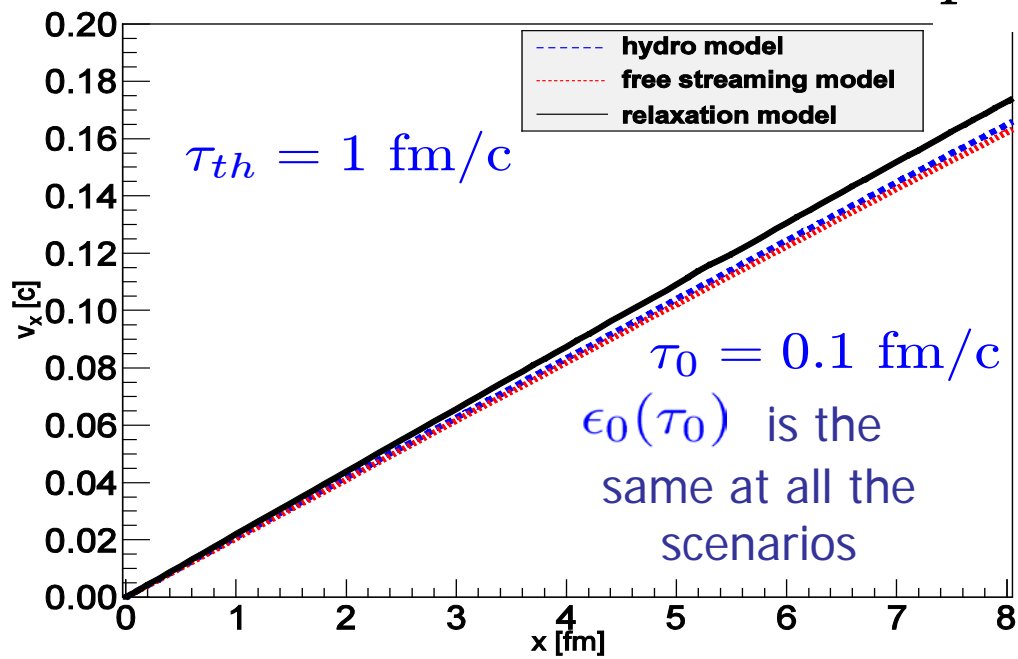
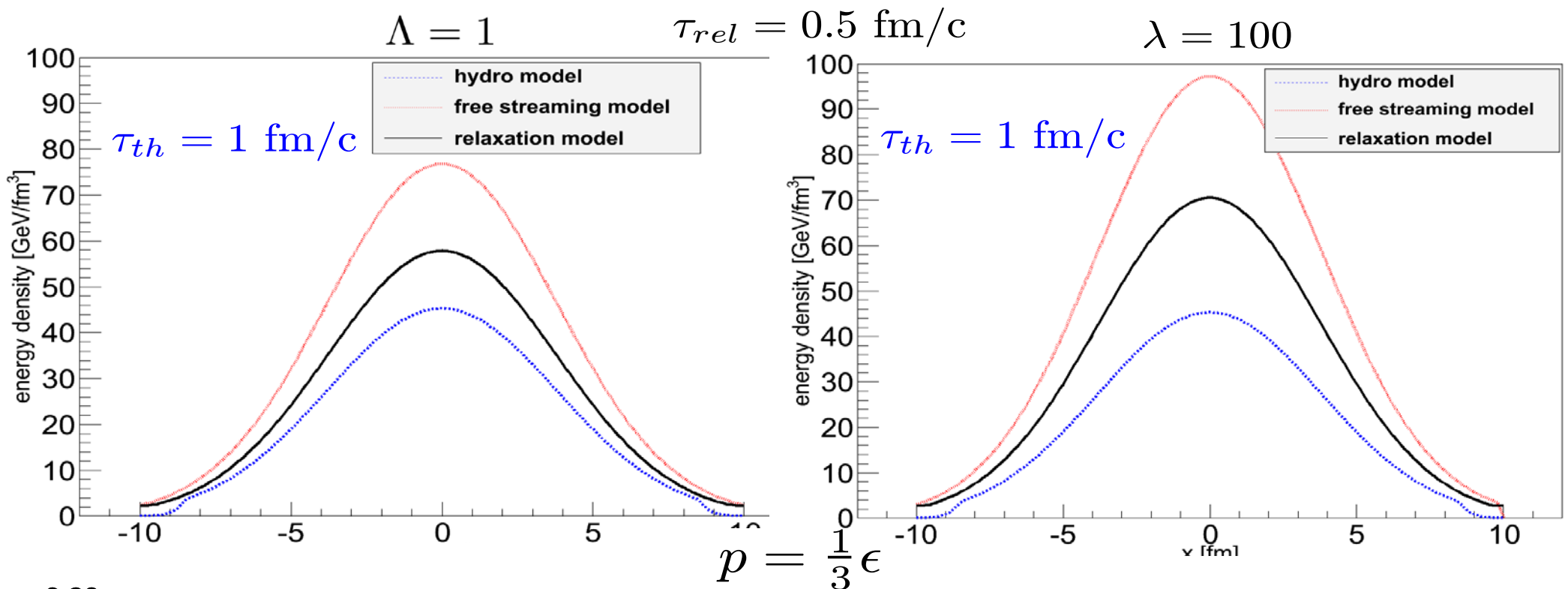
$$\longrightarrow 0 \leq \mathcal{P}(\tau) \leq 1, \mathcal{P}(\tau_0) = 1, \mathcal{P}(\tau_{th}) = 0, \partial_\mu \mathcal{P}(\tau)_{\tau_{th}} = 0$$

MAIN EQUATIONS

$$\partial_{;\mu} \tilde{T}_{hyd}^{\mu\nu}(x) = -T_{free}^{\mu\nu}(x) \partial_{;\mu} \mathcal{P}(\tau)$$

where $\left\{ \begin{array}{l} \tilde{T}_{hyd}^{\mu\nu} = [1 - \mathcal{P}(\tau)] T_{hyd}^{\mu\nu} \\ \tilde{\pi}^{\mu\nu} = \pi^{\mu\nu} (1 - \mathcal{P}) \end{array} \right.$

$$(1 - \mathcal{P}(\tau)) \left\langle u^\gamma \partial_{;\gamma} \frac{\tilde{\pi}^{\mu\nu}}{(1 - \mathcal{P}(\tau))} \right\rangle = -\frac{\tilde{\pi}^{\mu\nu} - (1 - \mathcal{P}(\tau)) \pi_{NS}^{\mu\nu}}{\tau_\pi} - \frac{4}{3} \tilde{\pi}^{\mu\nu} \partial_{;\gamma} u^\gamma$$



The other stages: Hydro evolution, particlization, hadronic cascade

- **Hydro evolution:** $\tau \leq \tau_{th}$ $T^{\mu\nu}(x) = T_{\text{free}}^{\mu\nu}(x)\mathcal{P}(\tau) + T_{\text{hyd}}^{\mu\nu}(x)(1 - \mathcal{P}(\tau)) \xrightarrow{\tau \geq \tau_{th}} T_{\text{hyd}}^{\mu\nu}(x)$
 $= (\epsilon_{\text{hyd}}(x) + p_{\text{hyd}}(x) + \Pi)u_{\text{hyd}}^{\mu}(x)u_{\text{hyd}}^{\nu}(x) - (p_{\text{hyd}}(x) + \Pi)g^{\mu\nu} + \pi^{\mu\nu}.$

IC is the result of pre-thermal evolution reached at τ_{th}

Solving of Israel-Stewart Relativistic Viscous Fluid Dynamics with $\Pi=0$

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at the isotherm hypersurface $T=165$ MeV

energy density $\epsilon = 0.5$ GeV/fm³ for the Laine-Schroeder EoS

Switching hypersurface build with help of Cornelius routine.

For particle distribution the Grad's 14 momentum ansatz is used:

$$\frac{d^3 \Delta N_i}{dp^* d(\cos\theta) d\phi} = \frac{\Delta\sigma_{\mu}^* p^{*\mu}}{p^{*0}} p^{*2} f_{eq}(p^{*0}; T, \mu_i) \left[1 + (1 \mp f_{eq}) \frac{p_{\mu}^* p_{\nu}^* \pi^{*\mu\nu}}{2T^2(\epsilon + p)} \right]$$

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For identical bosons (in smoothness approximation) with only Coulomb FSI

Y. Sinyukov, R. Lednicky, S.V. Akkelin, J. Pluta, B. Erazmus, Phys. Lett. B 432 (1998) 248.

$$C(p, q) = 1 - \lambda + \lambda \left\langle |\psi_{-\mathbf{k}^*}^c(\mathbf{r}^*)|^2 \right\rangle (1 + \langle \cos(qx) \rangle) \rightarrow 1 + \lambda \langle \cos(qx) \rangle$$

$\mathbf{k}^* = \mathbf{q}^*/2$

where $\langle \cos(qx) \rangle = \exp(-q_{out}^2 R_{out}^2 - q_{side}^2 R_{side}^2 - q_{long}^2 R_{long}^2 - \dots c.t.)$ $\mathbf{r}^* = \mathbf{x}_1^* - \mathbf{x}_2^*$

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where $\langle \cos(qx) \rangle = \exp(-q_{out}^2 R_{out}^2 - q_{side}^2 R_{side}^2 - q_{long}^2 R_{long}^2 - \dots c.t.)$ $\mathbf{r}^* = \mathbf{x}_1^* - \mathbf{x}_2^*$

Correlation functions in semi-classical events generators BE correlation:

$$C(\vec{q}) = \frac{\sum_{i \neq j} \delta_{\Delta}(\vec{q} - p_i + p_j) (1 + \cos(p_j - p_i)(x_j - x_i))}{\sum_{i \neq j} \delta_{\Delta}(\vec{q} - p_i + p_j)}$$

where $\delta_{\Delta}(x) = 1$ if $|x| < \Delta p/2$ and 0 otherwise, with Δp being the bin size in histograms.

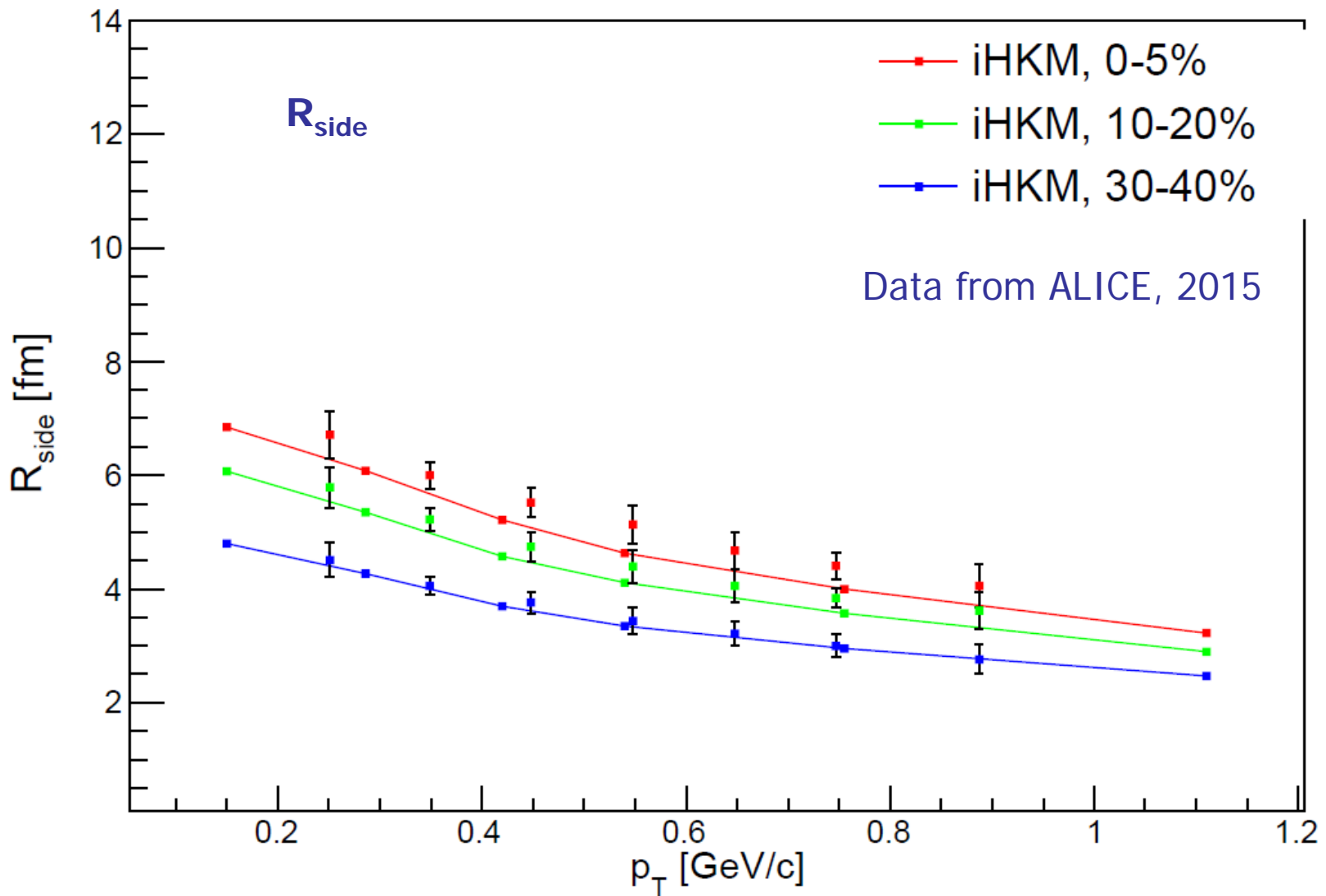


FIG. 9. The R_{side} dependence on transverse momentum for different centralities in the iHKM scenario under the same conditions as in Fig. 1. The experimental data are from [33].

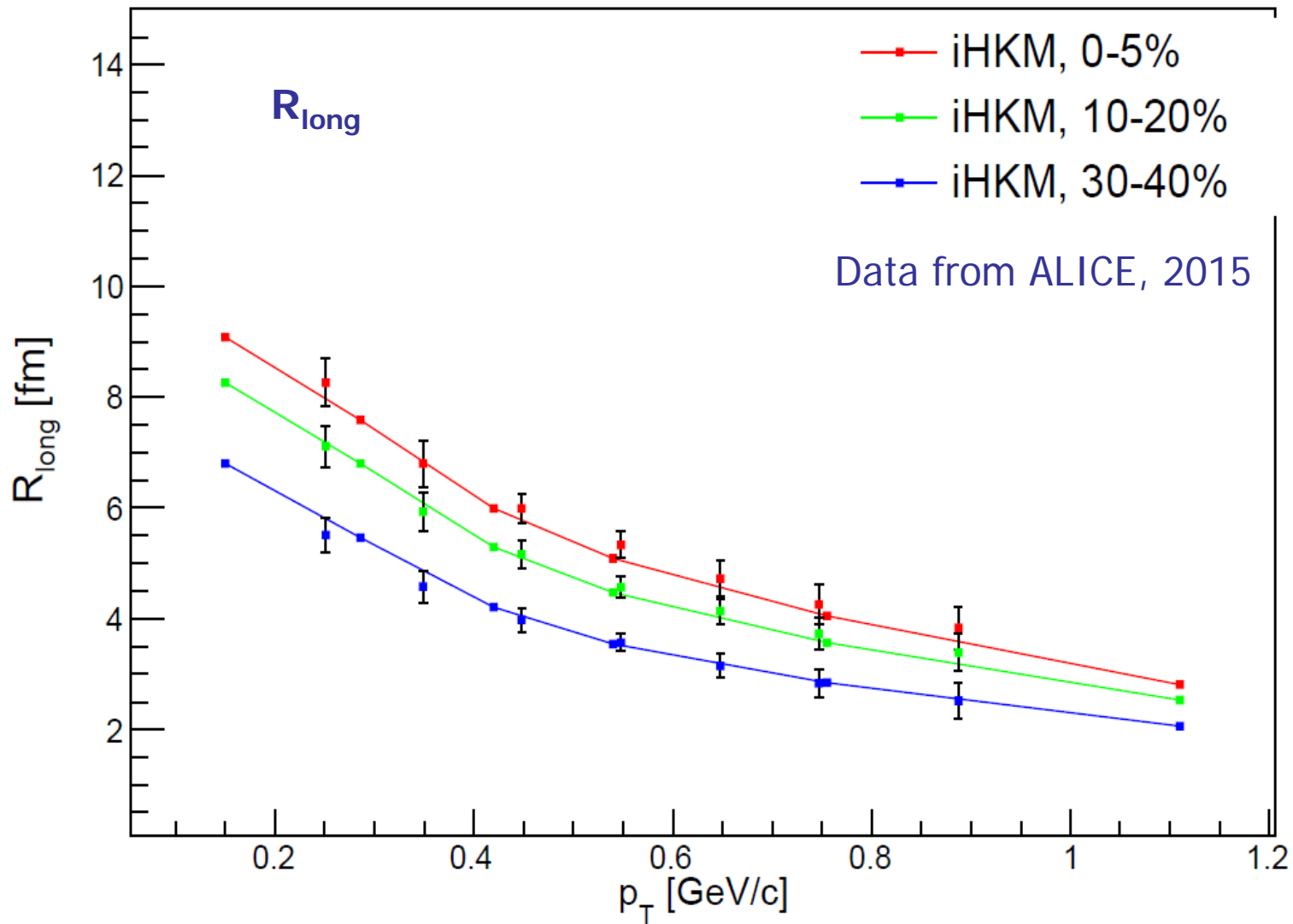


FIG. 10. The R_{long} dependence on transverse momentum for different centralities in the iHKM basic scenario -the same conditions as in Fig. 1. The experimental data are from [33].

STAR Collaboration
arXiv:1302.3168 February 2013

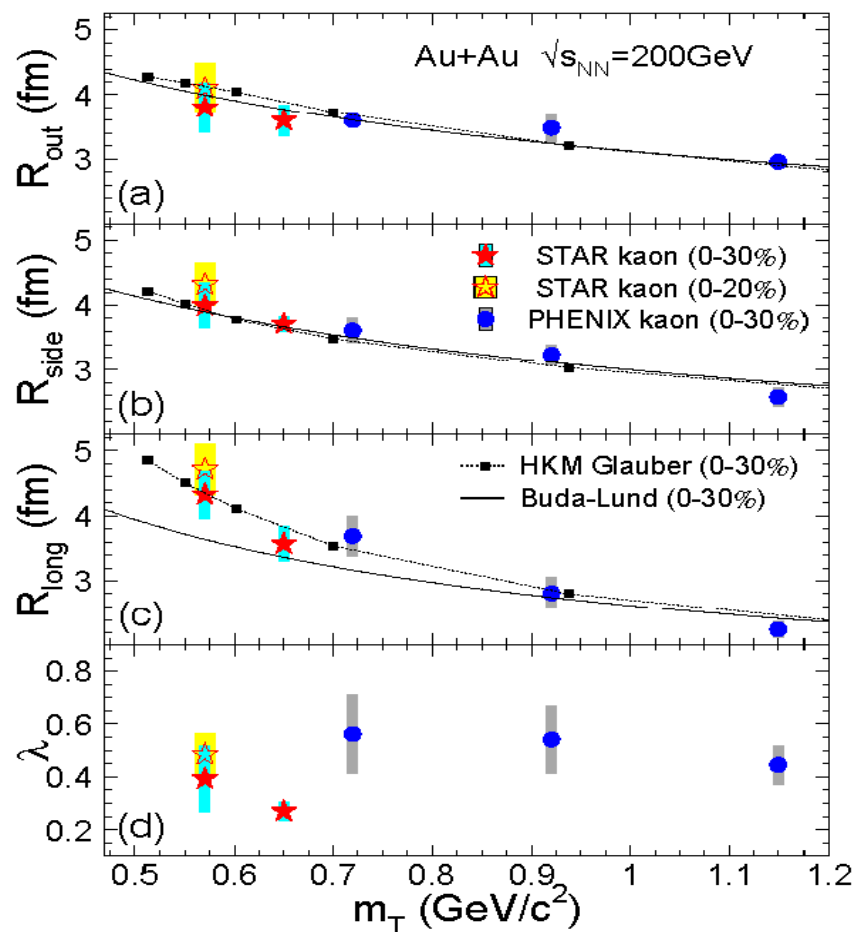


FIG. 4. Transverse mass dependence of Gaussian radii (a) R_{out} , (b) R_{side} and (c) R_{long} for mid-rapidity kaon pairs from the 30% most central Au+Au collisions at $\sqrt{s_{NN}}=200$ GeV. STAR data are shown as solid stars; PHENIX data [10] as solid circles (error bars include both statistical and systematic uncertainties). Hydro-kinetic model [23] with initial Glauber condition and Buda-Lund model [22] calculations are shown by solid squares and solid curves, respectively.

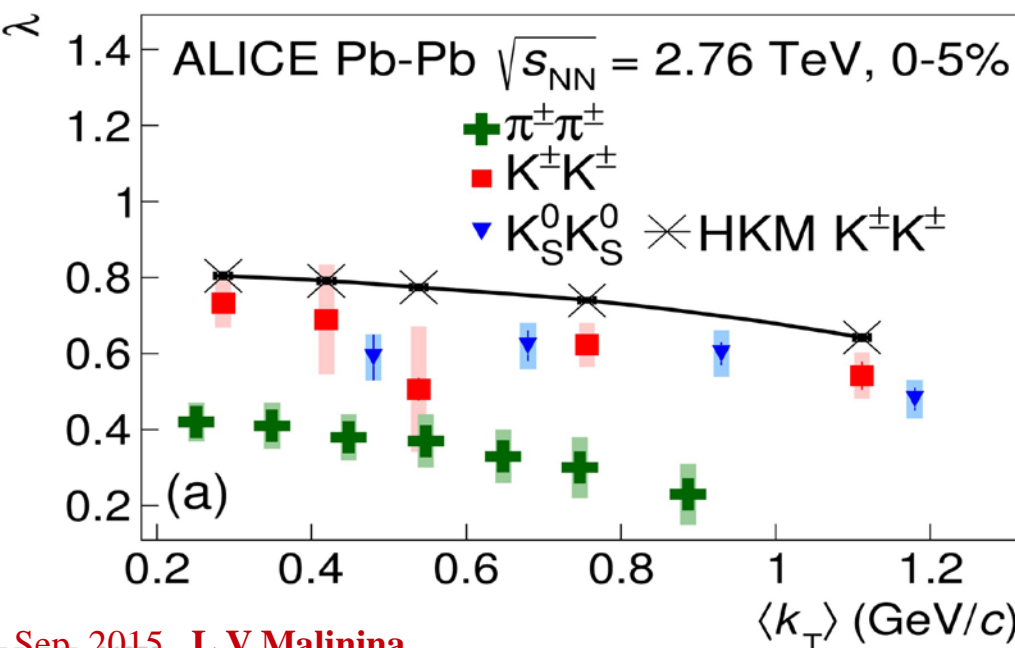
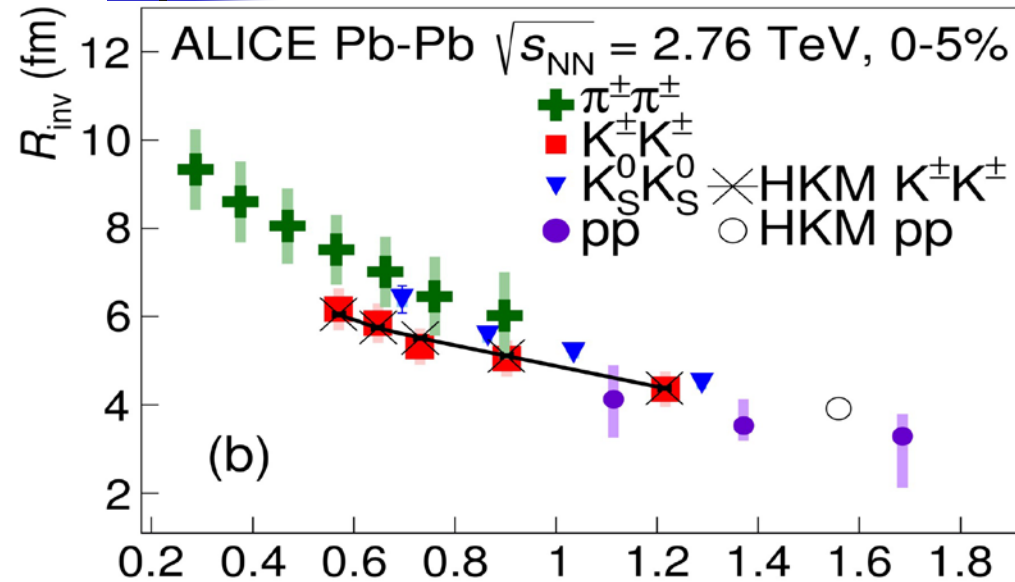
Quotations:

Our measurement at $0.2 \leq k_T \leq 0.36$ GeV/c clearly favours the HKM model as more representative of the expansion dynamics of the fireball.

In the outward and side-ward directions, this decrease is adequately described by m_T -scaling. However, in the longitudinal direction, the scaling is broken. The results are in favor of the hydro-kinetic predictions [23] over pure hydrodynamical model calculations.

[23] I. A. Karpenko and Y. M. Sinyukov, Phys. Rev. C **81** (2010) 054903.

$K^\pm K^\pm$ and $K^0 K^0$ in Pb-Pb: HKM model

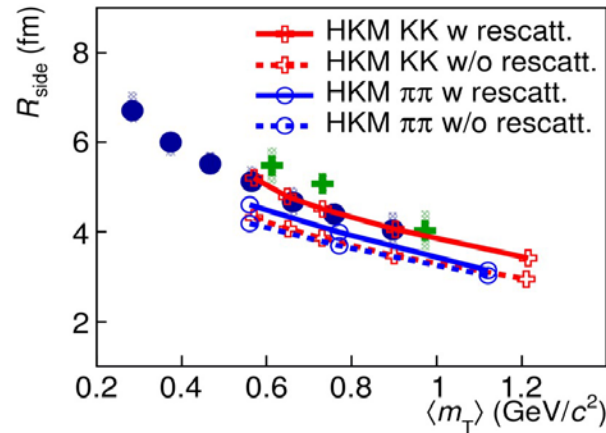
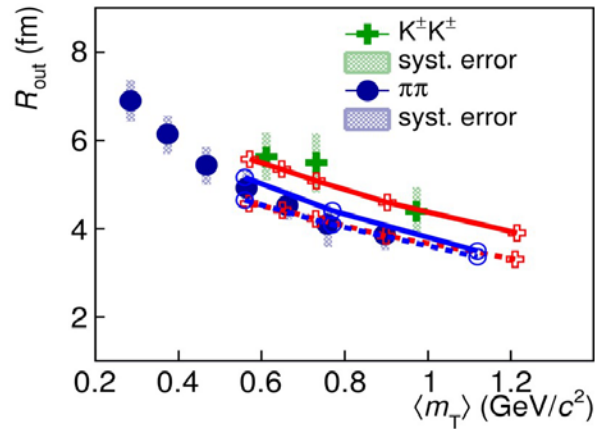


New results from [ArXiv.org:1506.07884](https://arxiv.org/abs/1506.07884)

- R and λ for $\pi^\pm \pi^\pm$, $K^\pm K^\pm$, $K^0 K^0$, pp for 0-5% centrality
- Radii for kaons show good agreement with HKM predictions for $K^\pm K^\pm$ (V. Shapoval, P. Braun-Munzinger, Yu. Sinyukov Nucl.Phys.A929 (2014))

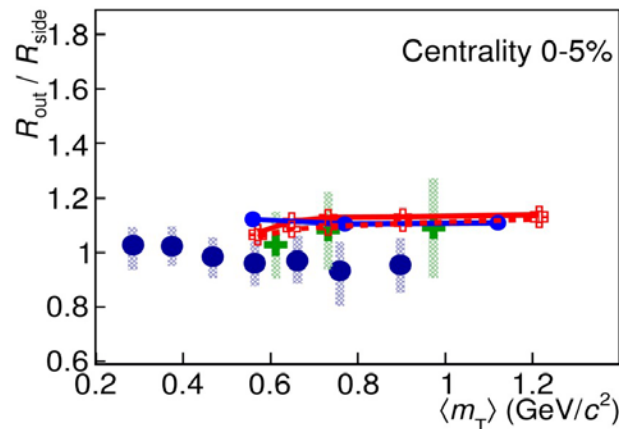
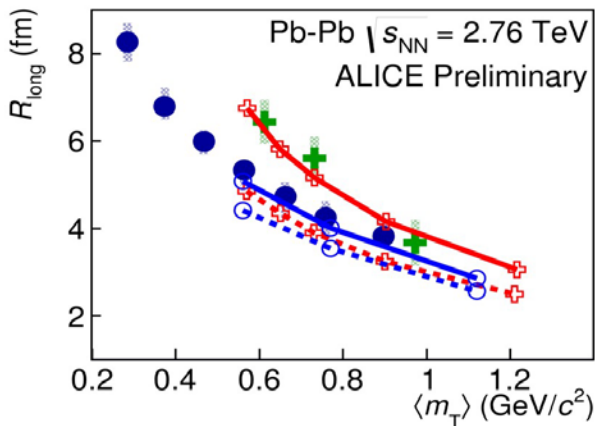
- λ decrease with k_T , both data and HKM
- HKM prediction for λ slightly overpredicts the data
- Λ_π are lower Λ_K due to the stronger influence of resonances

Comparison with HKM for 0-5% centrality



- HKM model with re-scatterings (M. Shapoval, P. Braun-Munzinger, Iu.A. Karpenko, Yu.M. Sinyukov, Nucl.Phys. A 929 (2014) 1.) describes well ALICE π & K data.

● HKM model w/o re-scatterings demonstrates



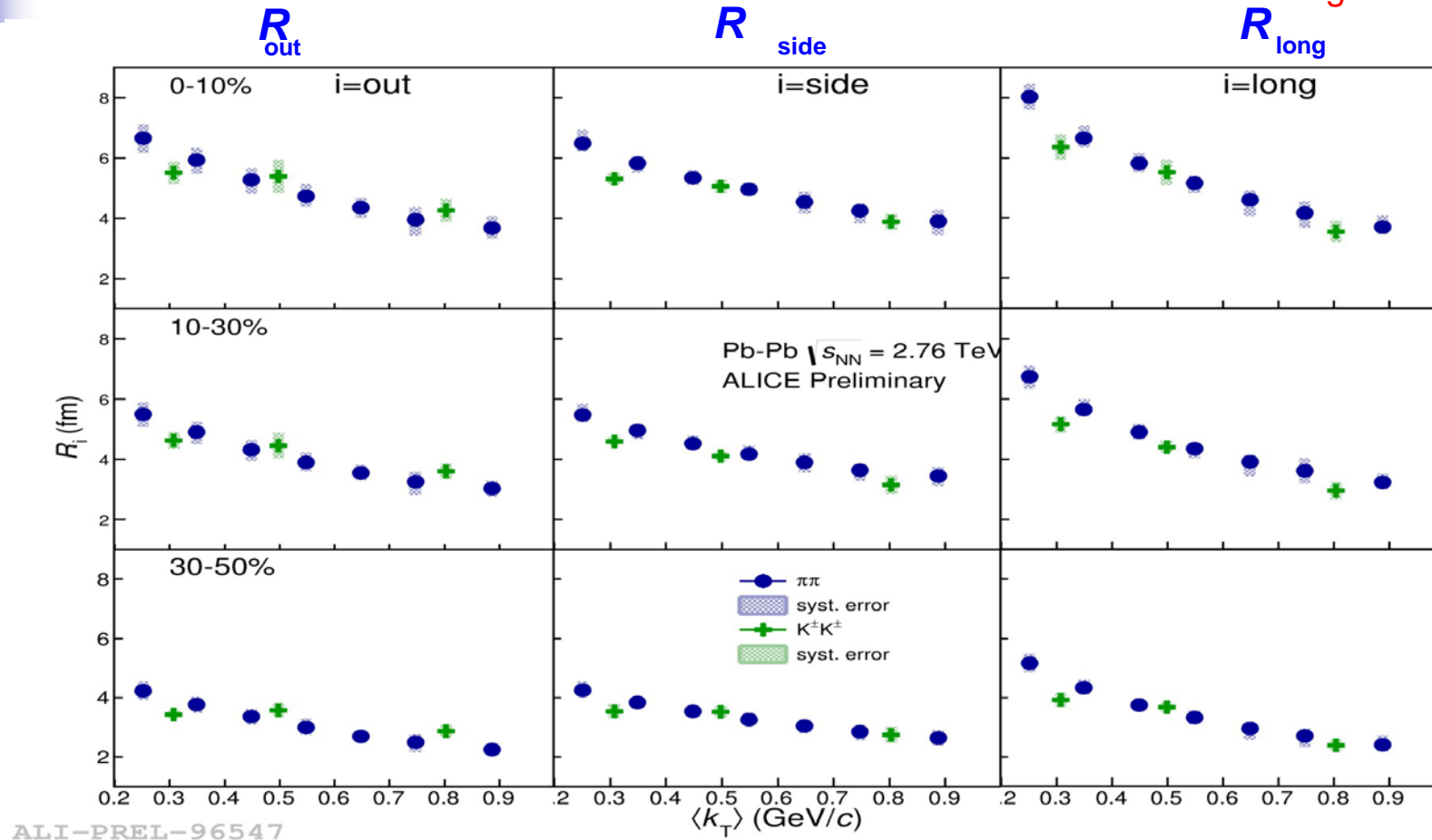
● approximate m_T scaling for π & K, but does not describe ALICE π & K data

- The observed deviation from m_T scaling is explained in (M. Shapoval, P. Braun-Munzinger, Iu.A. Karpenko, Yu.M. Sinyukov, Nucl.Phys. A 929 (2014) 1.) by essential transverse flow & re-scattering phase.

● HKM model slightly underestimates R_{side} and overestimates $R_{\text{out}} / R_{\text{side}}$ ratio for π

3D $K^\pm K^\pm$ & $\pi\pi$ radii versus k_T

Pion results from [ArXiv.org:1507.06842](https://arxiv.org/abs/1507.06842)



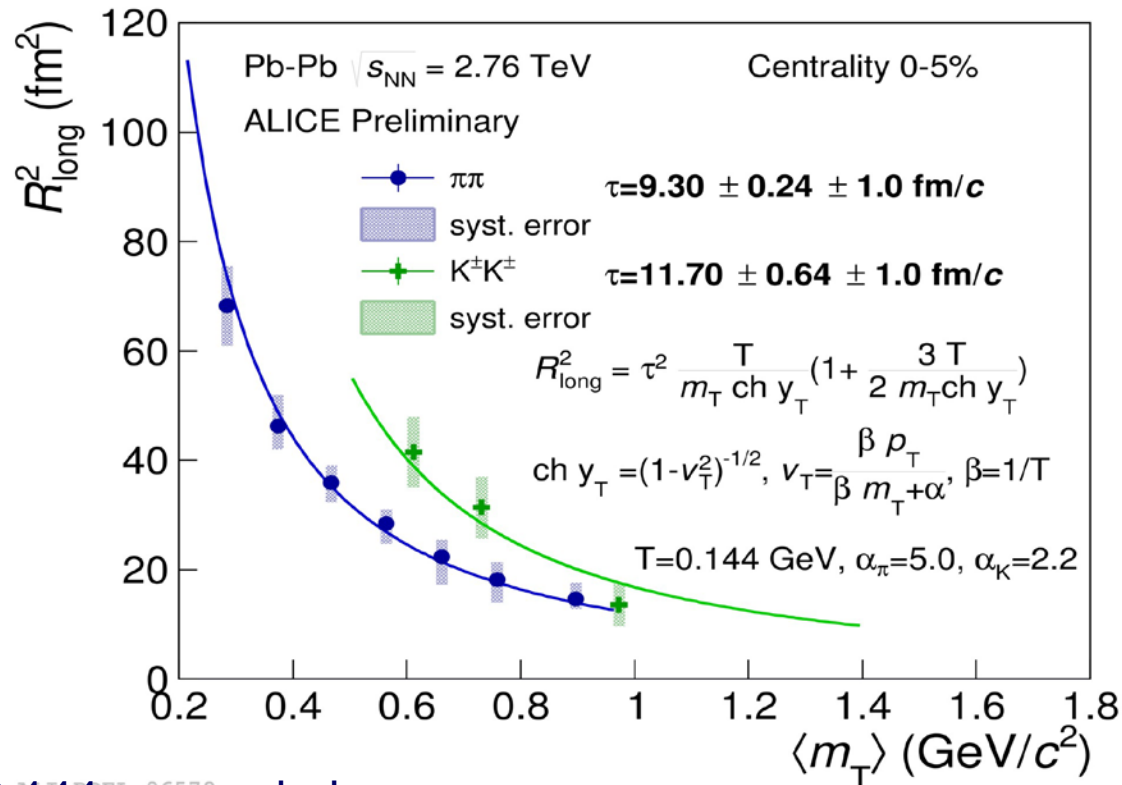
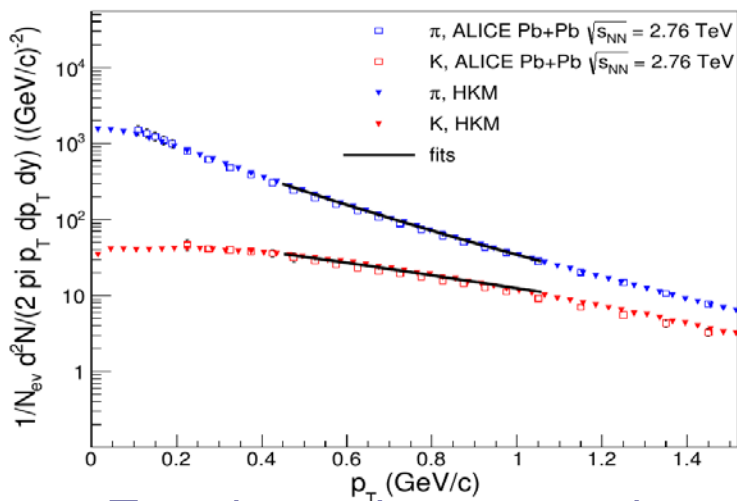
ALI-PREL-96547

- Radii scale better with k_T than with m according with HKM predictions
(V. Shapoval, P. Braun-Munzinger, Iu.A. Karpenko, Yu.M. Sinyukov, Nucl.Phys. A 929 (2014) 1);
- Similar observations were reported by PHENIX at RHIC ([arxiv:1504.05168](https://arxiv.org/abs/1504.05168)).

Extraction of emission time from fit R_{long}

The new formula for extraction of the maximal emission time for the case of strong transverse flow was used (Yu. S., Shapoval, Naboka, Nucl. Phys. A 946 (2016) 227)

The parameters of freeze-out: T and “intensity of transverse flow” α were fixed by fitting π and K spectra (arxiv:1508.01812)



To estimate the systematic errors: $T = 0.144$ was varied

on ± 0.03 GeV & free α_{π} , α_K , were used; systematic errors ~ 1 fm/c

Indication: $\tau_{\pi} < \tau_K$. Possible explanations (arxiv:1508.01812): HKM includes re-scatterings (UrQMD cascade): e.g. $K\pi \rightarrow K^*(892) \rightarrow K\pi$, $KN \rightarrow K^*(892)X$; ($K^*(892)$ lifetime 4-5 fm/c) [$\pi N \rightarrow N^*(\Delta)X$, $N^*(\Delta) \rightarrow \pi X$ (N^* 's (Δ s)- short lifetime)]

Space-time picture of the pion and kaon emission

$$R_l^2(k_T) = \underbrace{\tau^2 \lambda^2 \left(1 + \frac{3}{2} \lambda^2\right)}_{2015} \approx \text{w/o transv. expansion} \approx \underbrace{\tau^2 \frac{T}{m_T}}_{1987} \underbrace{\frac{K_2\left(\frac{m_T}{T}\right)}{K_1\left(\frac{m_T}{T}\right)}}_{1995}$$

where

$$\lambda^2 = \frac{T}{m_T} \left(1 - \frac{\overbrace{k_T^2}^{\bar{v}_T^2}}{(m_T + \alpha T)^2}\right)^{1/2}$$

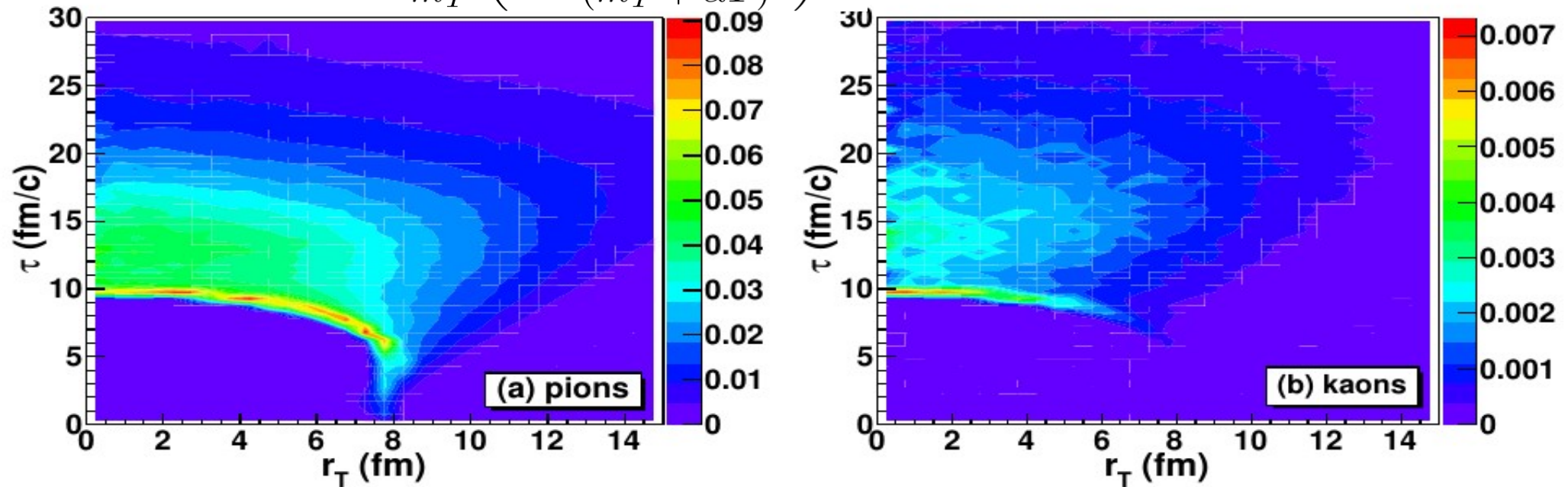


FIG. 4. The momentum angle averaged emission functions per units of space-time and momentum rapidities $g(\tau, r_T, p_T)$ [fm^{-3}] (see body text) for pions (a) and kaons (b) obtained from the HKM simulations of Pb+Pb collisions at the LHC $\sqrt{s_{NN}} = 2.76$ GeV, $0.2 < p_T < 0.3$ GeV/c, $|y| < 0.5$, $e = 0 - 5\%$. From Yu.S., Shapoval, Naboka, Nucl. Phys. A 946 (2016) 247 ([arXiv:1508.01812](https://arxiv.org/abs/1508.01812))

Source function $S(\mathbf{r}^*)$

Integrated in time distribution of the pairs on relative distance between particles in the pairs in the rest frames of the pairs

The correlation function in smoothness approximation

$$C(p, q) = 1 + \frac{\int d^4x_1 d^4x_2 g_1(x_1, p_1) g_2(x_2, p_2) (|\psi(\tilde{q}, r)|^2 - 1)}{\int d^4x_1 g_1(x_1, p_1) \int d^4x_2 g_2(x_2, p_2)}$$

where $\psi(\tilde{q}, r)$ is reduced Bether-Salpeter amplitude, $r = x_1 - x_2$, $R = (x_1 + x_2)/2$

$$q = p_1 - p_2, p = (p_1 + p_2)/2 \quad \tilde{q} = q - p(qp)/p^2$$

The relative distance distribution function

$$\begin{aligned} s(r, p_1, p_2) &= \frac{\int d^4R g_1(R + r/2, p_1) g_2(R - r/2, p_2)}{\int d^4R g_1(R, p_1) \int d^4R g_2(R, p_2)} = \text{Main contribution to } C(p_1, p_2) \text{ is at } \mathbf{v}_1 \approx \mathbf{v}_2 \\ &= \int d^4R \tilde{g}_1\left(R + r/2, \frac{2m_1}{m_1 + m_2} p\right) \tilde{g}_2\left(R - r/2, \frac{2m_2}{m_1 + m_2} p\right) = s(r, p) \end{aligned}$$

$$\underbrace{C(\mathbf{q}^*, \mathbf{p}=\mathbf{0}) - 1}_{R(\mathbf{q})} = \int d^3r^* \int \underbrace{dt^* s(r^*, \mathbf{0})}_{S(\mathbf{r}^*)} (|\psi(\mathbf{r}^*, \mathbf{q}^*)|^2 - 1) = \int d^3r^* \underbrace{S(\mathbf{r}^*)}_{e^{-r^2/4r_0^2}} \underbrace{K(\mathbf{r}^*, \mathbf{q}^*)}_{f_0 d_0 a}$$

m_T scaling of source radii for meson and baryon pairs because in pair rest frame the transverse flow are absent

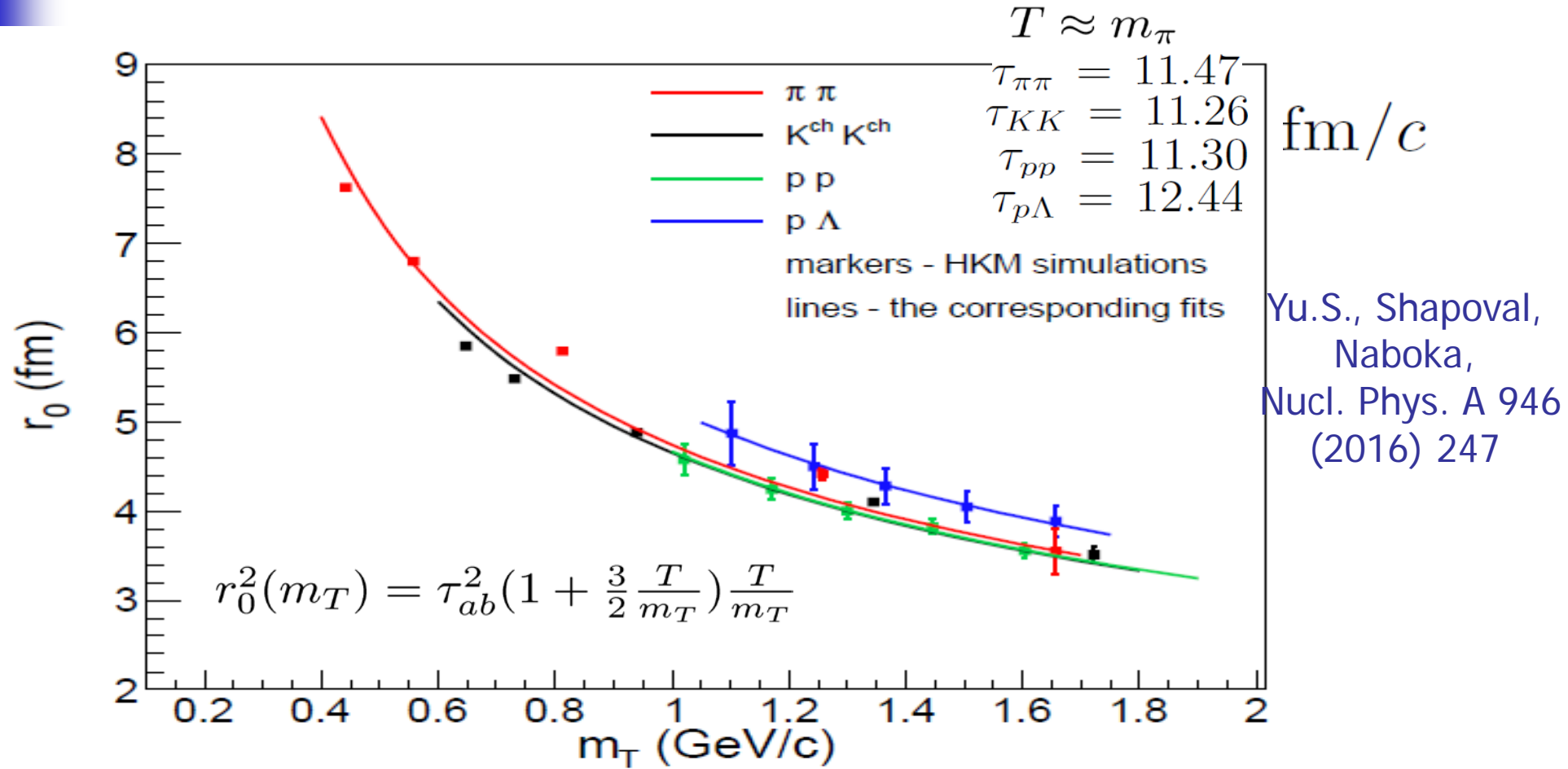
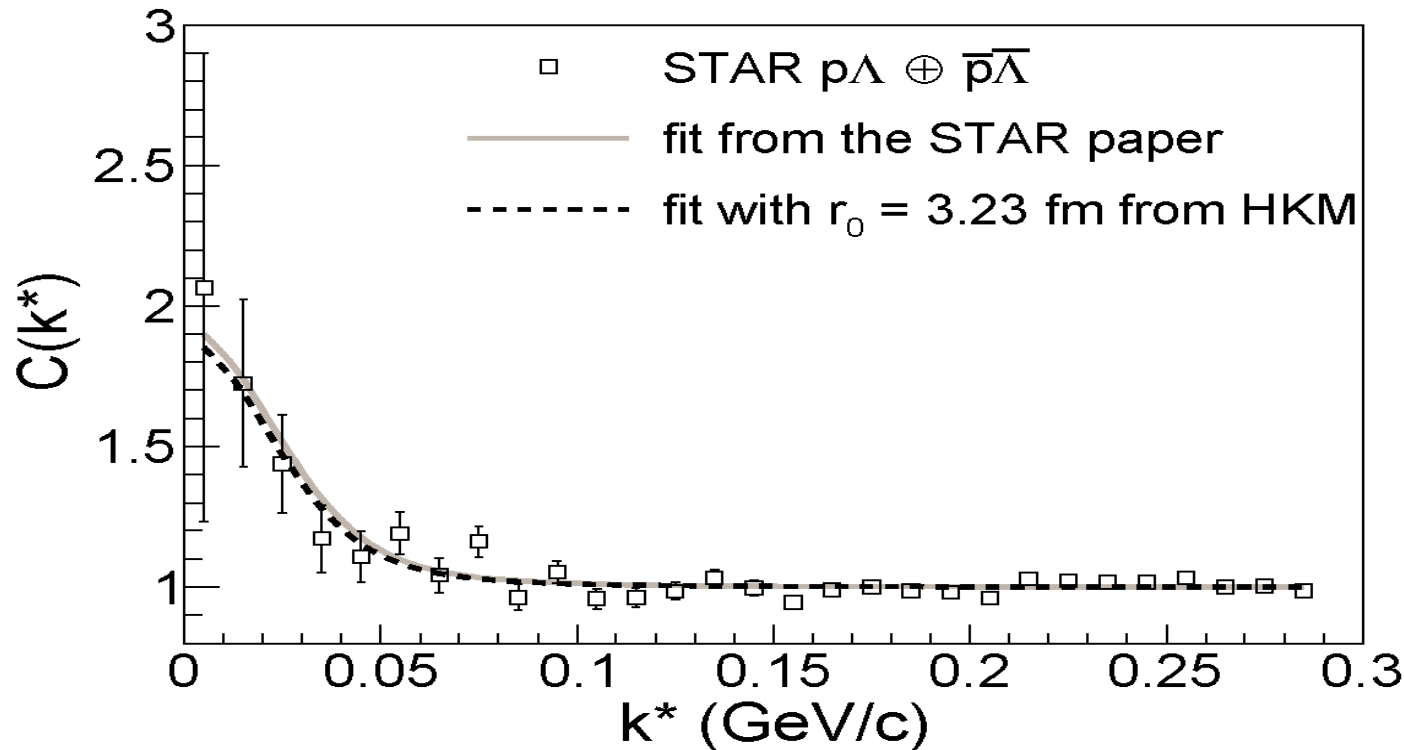


FIG. 7. The m_T dependencies of $\pi\pi$, $K^{ch}K^{ch}$, pp and $p\Lambda$ source radii r_0 extracted from corresponding angle-averaged source functions calculated in HKM for $\sqrt{s_{NN}} = 2.76$ GeV Pb+Pb collisions at the LHC, $c = 0 - 5\%$, $|\eta| < 0.8$. Transverse momentum ranges are $0.14 < p_T < 1.5$ GeV/c for pions and kaons, $0.7 < p_T < 4.0$ GeV/c for protons and $0.7 < p_T < 5.0$ GeV/c for Lambdas.

The $p - \Lambda \oplus \bar{p} - \bar{\Lambda}$ correlation function RHIC

F. Wang, S. Pratt, Phys. Rev. Lett. 83, 3138 (1999)

$$f_0^s = 2.88 \text{ fm}, f_0^t = 1.66 \text{ fm}, d_0^s = 2.92 \text{ fm}, d_0^t = 3.78 \text{ fm}$$



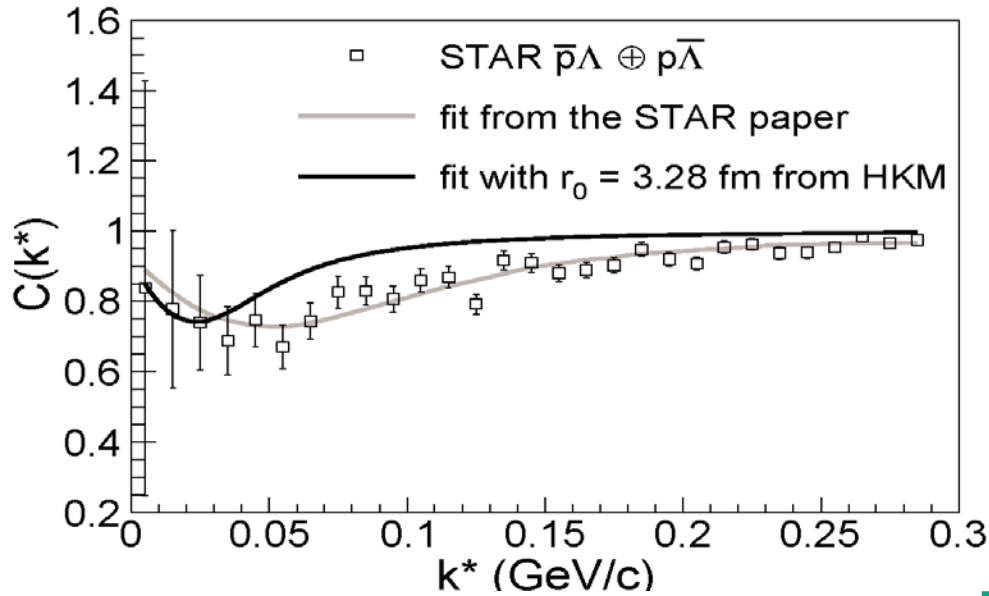
Scattering lengths f_0

"Effective radius" d_0

Shapoval, Erazmus,
Lednický, and Yu.S.
Phys. Rev. C **92**,
034910 (2015)

The $p - \Lambda \oplus \bar{p} - \bar{\Lambda}$ correlation function measured by STAR (open markers), the corresponding fit according to (6) with parameters fixed as in the STAR paper [4] within the Lednický and Lyuboshitz analytical model [1] (light solid line) and our fit within the same model with the source radius r_0 extracted from the HKM calculations (dark dashed line).

The $\bar{p} - \Lambda \oplus p - \bar{\Lambda}$ correlation function RHIC



STAR: $\Re f = -2.03 \pm 0.96_{-0.12}^{+1.37}$ fm,
 $\Im f = 1.01 \pm 0.92_{-1.11}^{+2.43}$ fm,
 $r_0 = 1.50 \pm 0.05_{-0.12}^{+0.10} \pm 0.3$ fm

$$C_{meas}(k^*) = \lambda(k^*)C(k^*) + (1 - \lambda(k^*))$$

Residual correlations



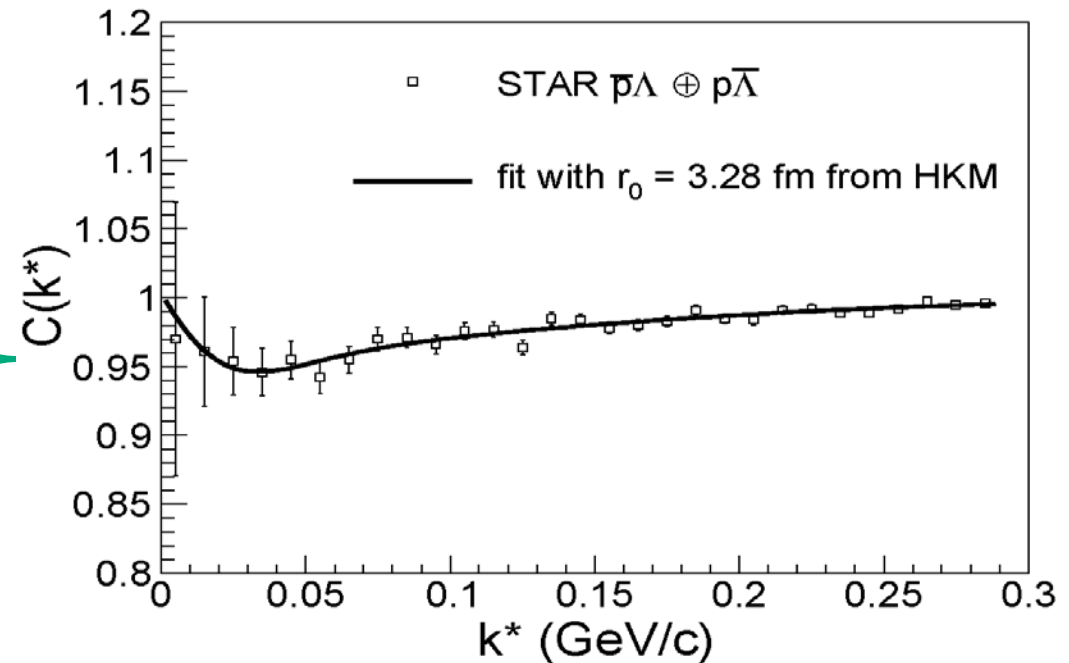
$$C_{uncorr}(k^*) = 1 + \lambda(k^*)(C(k^*) - 1) + \alpha(k^*)(C_{res}(k^*) - 1)$$

$$C_{res}(k^*) = 1 - \tilde{\beta}e^{-4k^{*2}R^2} \underbrace{\tilde{\alpha}(1 - \lambda(k^*))}_{\text{residual correlation}}$$

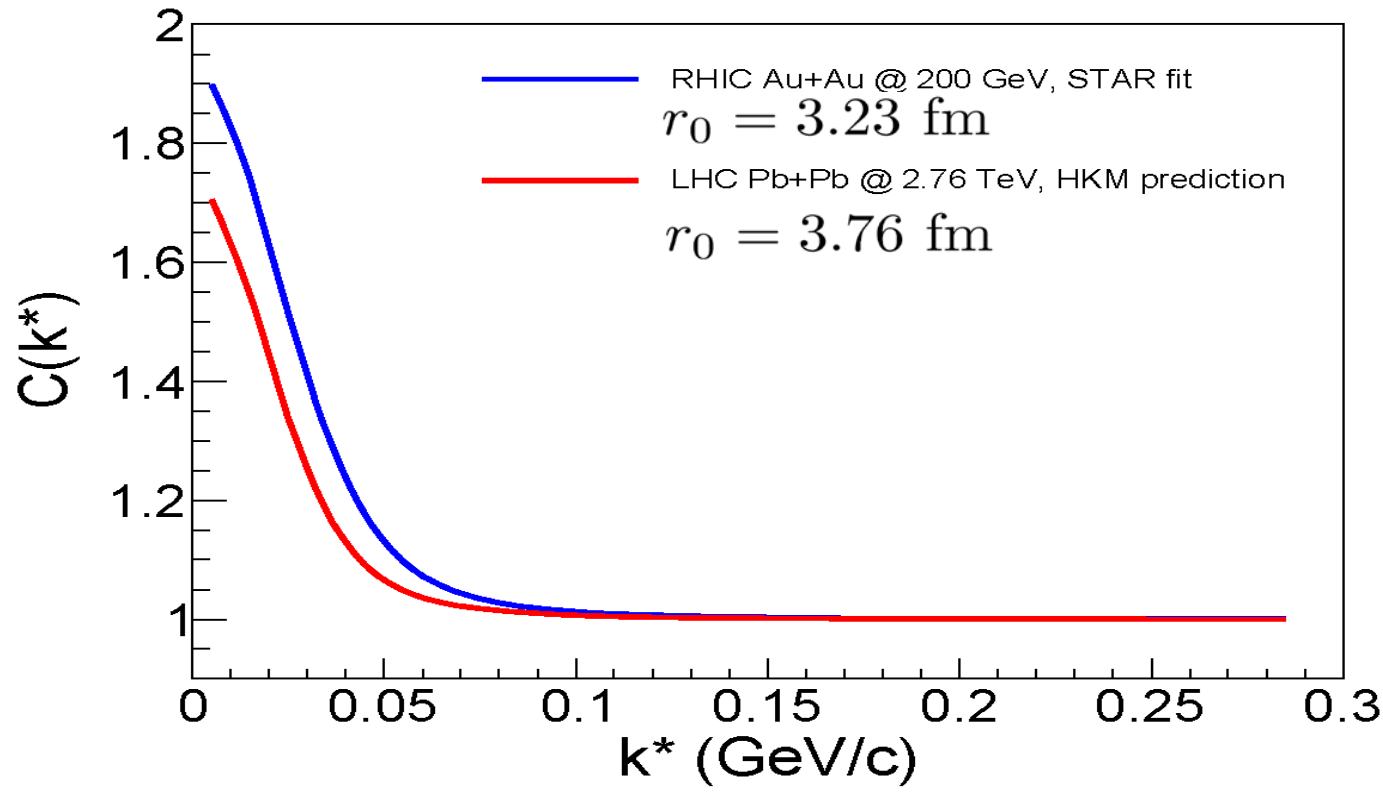
$$\Re f_0 = 0.14 \pm 0.66 \text{ fm,}$$

$$\Im f_0 = 1.53 \pm 1.31 \text{ fm, } \beta = \tilde{\alpha}\tilde{\beta} = 0.034 \pm 0.005$$

$$\text{and } R = 0.48 \pm 0.05 \text{ fm, with } \chi^2/\text{ndf} = 0.87.$$



The $p - \Lambda \oplus \bar{p} - \bar{\Lambda}$ correlation function (prediction for LHC)



Shapoval, Yu. S. , Naboka
 Phys. Rev. C **92**, 044910
 (2015)

Scattering lengths f_0
 "Effective radius" d_0
 are the same as at RHIC

FIG. 2. The HKM prediction for purity corrected $p - \Lambda \oplus \bar{p} - \bar{\Lambda}$ correlation function in the LHC Pb+Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV, $c = 0 - 5\%$, $|\eta| < 0.8$, with $0.7 < p_T < 4$ GeV/c for protons and $0.7 < p_T < 5$ GeV/c for lambdas (red line). The LHC source radius value calculated in HKM is $r_0 = 3.76$ fm. The Lednický-Lyuboshitz fit to the top RHIC energy correlation function, corresponding to the STAR experiment [8], with $r_0 = 3.23$ fm extracted from the HKM source function is presented for comparison (blue line).

The $\bar{p} - \Lambda \oplus p - \bar{\Lambda}$ correlation function (prediction for LHC)

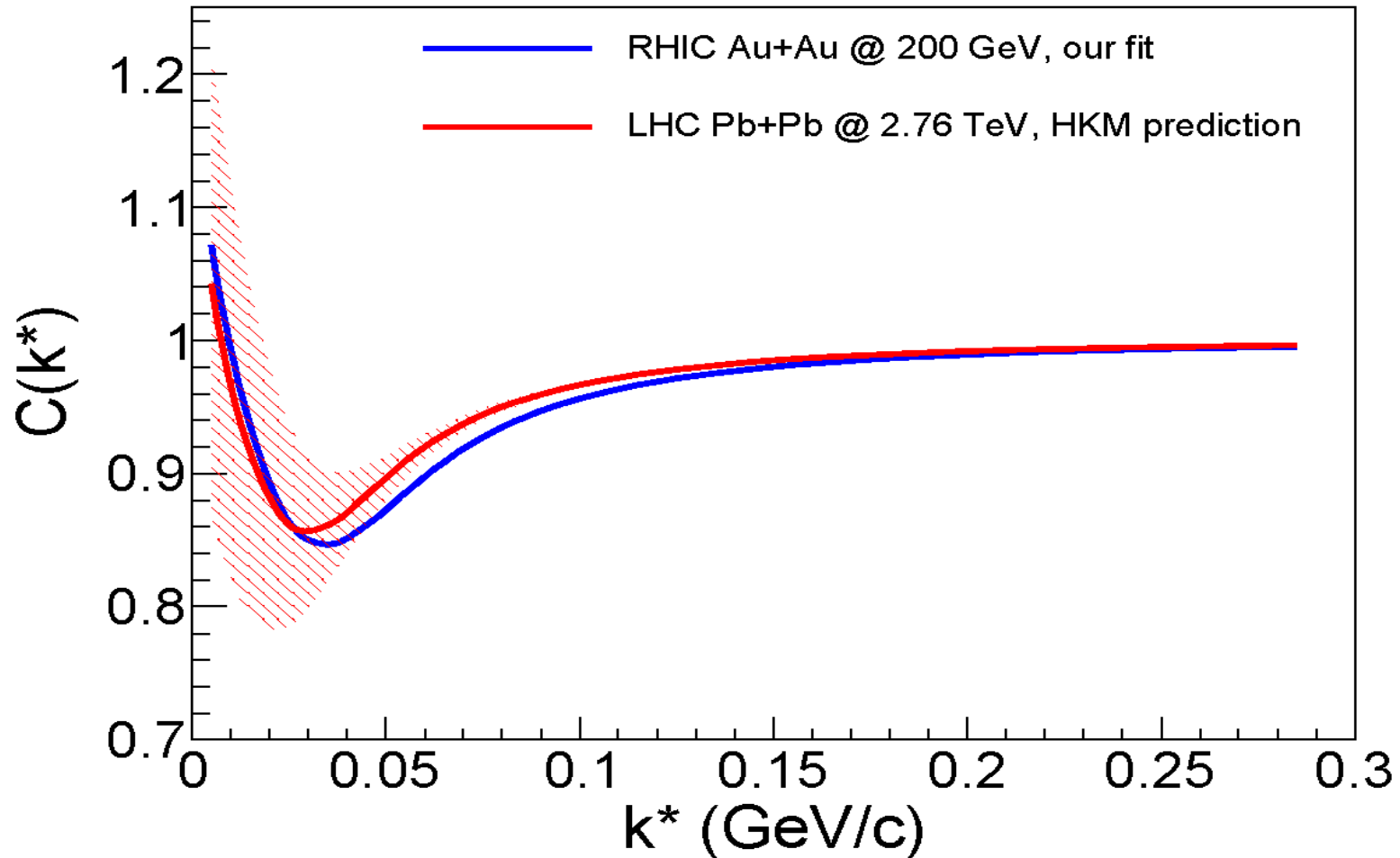
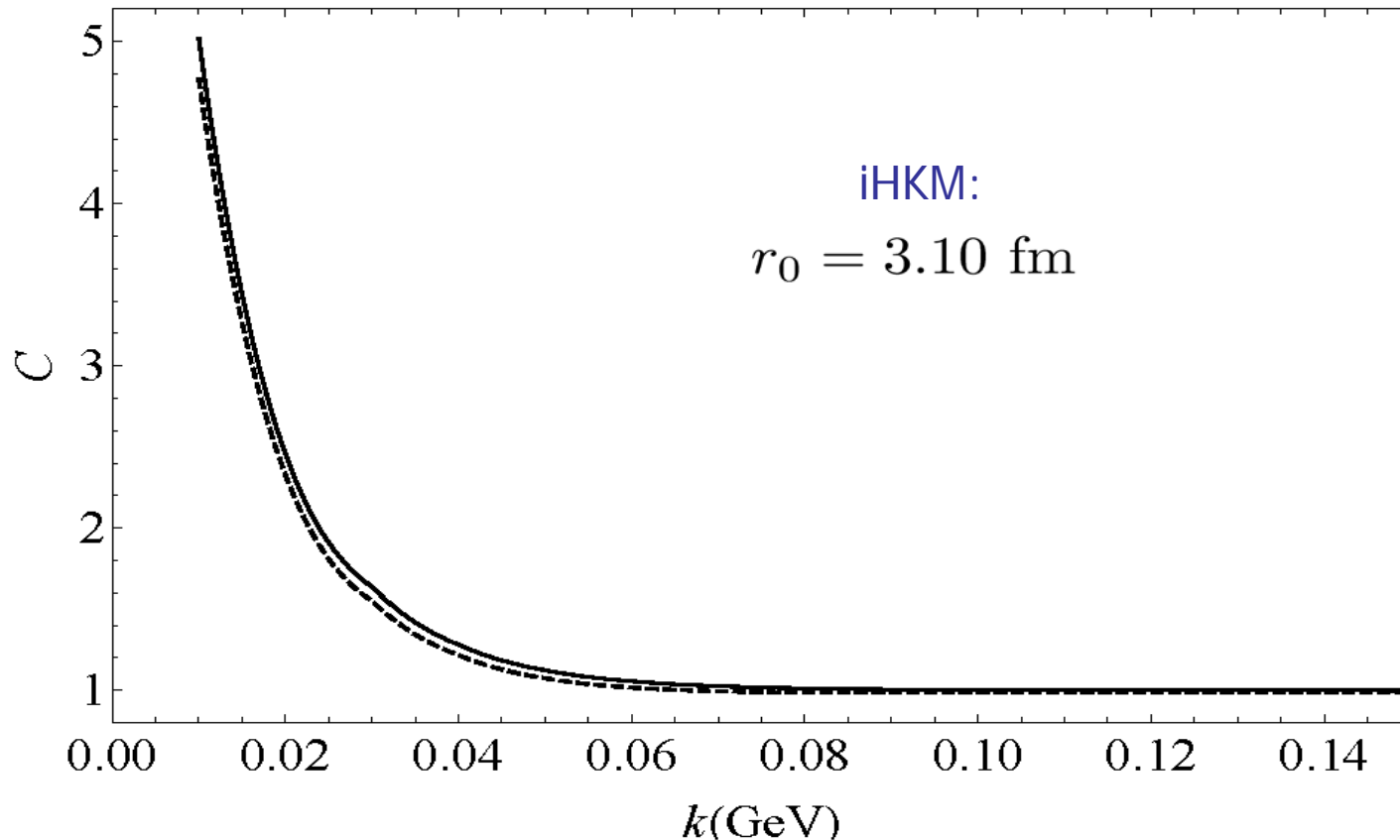


FIG. 5. The same as in previous two figures, but the correlation functions are corrected for purity and residual correlations, i. e. $C(k^*) = 1 + (C_{uncorr}(k^*) - 1)/\lambda(k^*) - \alpha(k^*)(C_{res}(k^*) - 1)/\lambda(k^*)$.

"Pure" correlation function for $p - \Xi^- \oplus \bar{p} - \Xi^+$ (prediction for the LHC)



Scattering lengths f_0

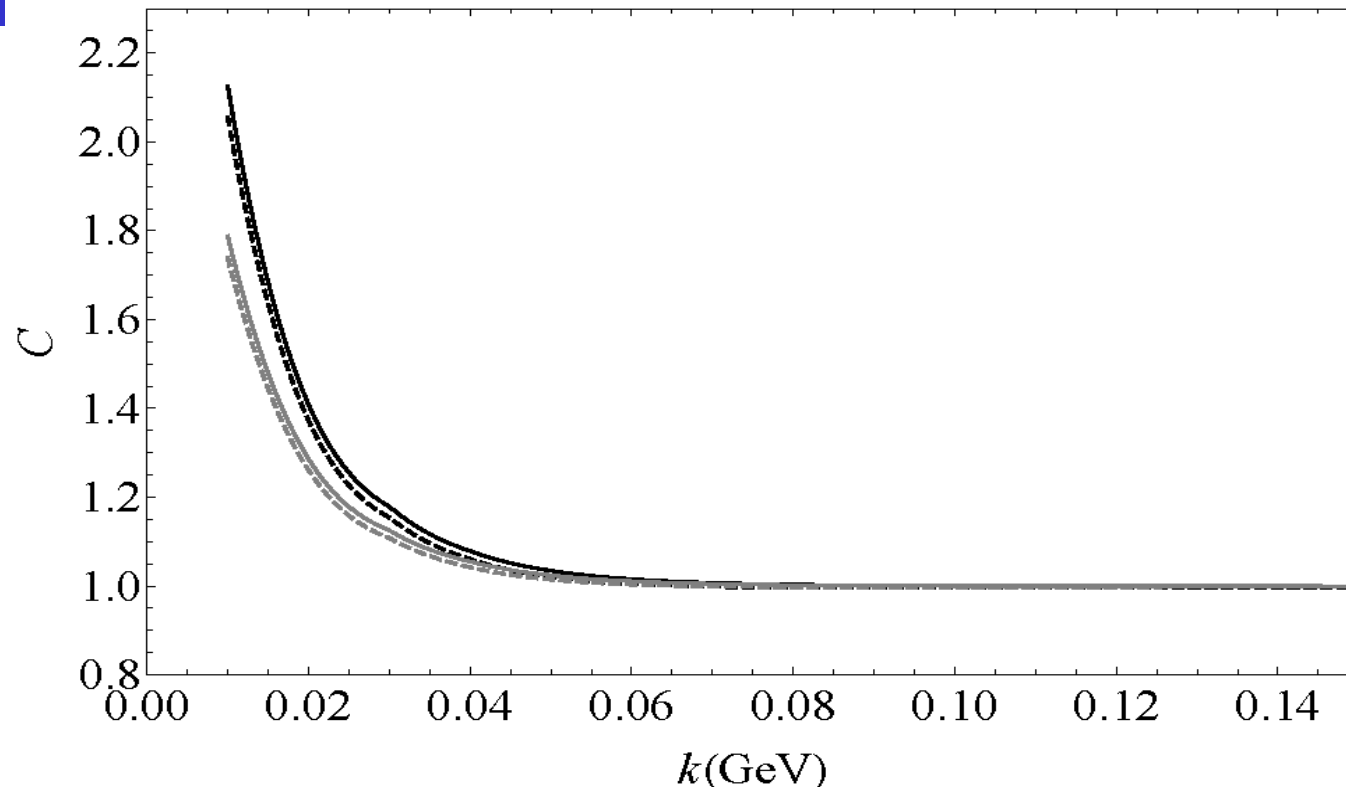
"Effective radius" d_0

are supposed to be as
for baryon-baryon
pairs

$p - \Lambda \oplus \bar{p} - \bar{\Lambda}$

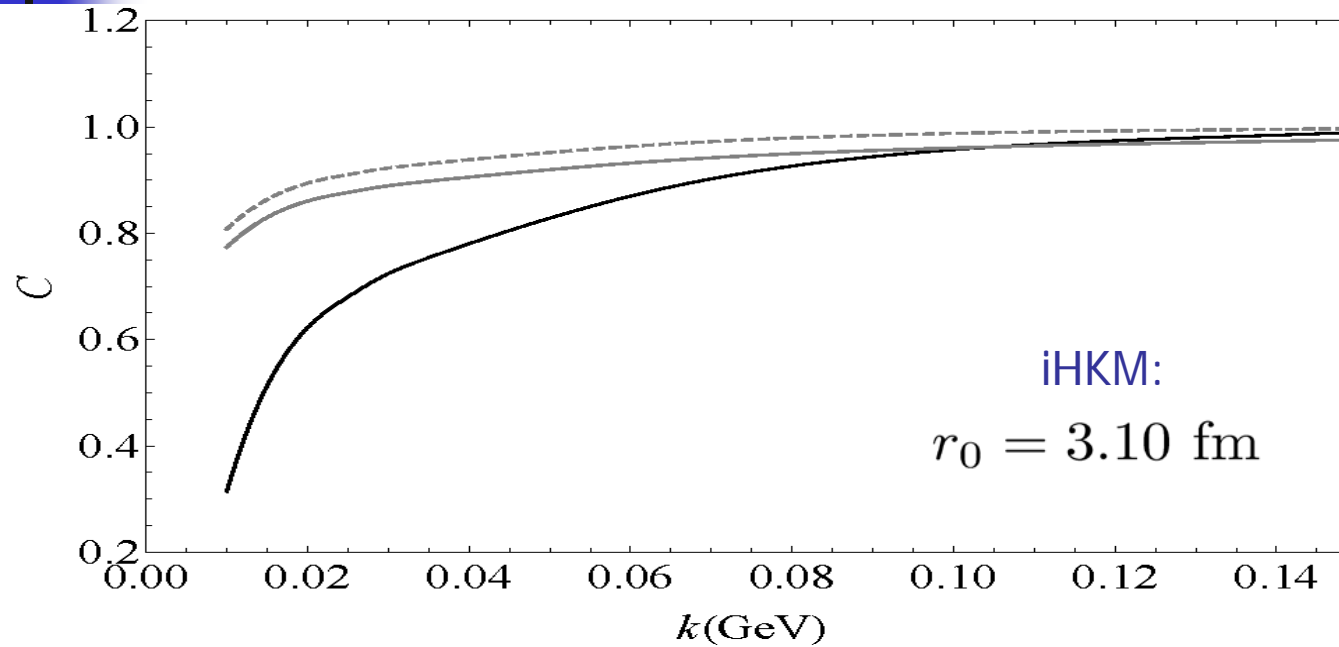
The pure (purity $\lambda = 1$) baryon-baryon correlation function between primary proton and cascade, $p\Xi^-$ obtained in iHKM simulations. The Gaussian radius in the source function distribution in iHKM is 3.1 fm. The scattering lengths for strong interactions are supposed to be the same as in $p - \Lambda$ systems. The solid line is related to utilizing the basic formula (1) for the Coulomb plus strong FSI at all distances r between the two baryons in the rest system of the pair. The dashed line corresponds to switching off all interactions at $r < 1$ fm. Therefore, the realistic results, accounting for the baryon kernel in short range interaction and electromagnetic form-factors of baryons is supposed to be between these two curves.

Measured $p - \Xi^- \oplus \bar{p} - \Xi^+$ correlation function (prediction for LHC)



The prediction for observed baryon-baryon correlation function proton and cascade, $p\Xi^-$

obtained in iHKM simulations. The purity that is result of long-lived resonance decays is found in iHKM as $\lambda_{res} = 0.28$. The gray lines account for purity connected with particle misidentification and some other detector aspects. We put this additional factor as 0.7, so that the gray lines are corresponding to $\lambda = 0.7\lambda_{res} = 0.196$.



The baryon-antibaryon correlation function for primary proton and anti-cascade, $p\Xi^+$, obtained in iHKM simulations. The Gaussian radius in the source function distribution in iHKM is 3.1 fm. The scattering lengths for strong interactions are supposed to be the same as for $\bar{\Lambda}$ extracted from STAR data in Ref. [4]. The solid line is related to purity = 1 in primary baryon-antibaryon system. The gray dashed line correspond to the purity that is result of long-lived resonance decays as it found in iHKM, $\lambda_{res} = 0.28$. Solid gray line corresponds account for the residual correlations among primary parents of p or/and Ξ^+ . The parameters of such a correction are taken from top RHIC energies [4] and adjust for LHC space scale by using iHKM.

Summary (spectra and meson correlations)

- The integrated hydrokinetic model (iHKM) of A+A collisions is developed and tuned to satisfactorily describe at different centralities the multiplicities, pion, kaon and antiproton spectra and elliptic flow of all charged particles.
- The results are not much sensitive to reasonable variations of shear viscosity, η/s relaxation time, τ_{rel} , time of thermalization τ_{th} , if their change is accompanied by re-scaling of the maximal initial energy density. But spectra are quite sensitive to the initial time τ_0 of (non-thermal) $T^{\mu\nu}(\tau_0)$ formation.
- The pion and kaon interferometry radii 1D and 3D are well described with above fixed parameters. The reasons for violation of m_T – scaling are intensive transverse flows and (mainly) re-scattering at the afterburner stage. The HKM prediction for pion-kaon k_T -scaling is conformed at RHIC and LHC experiments.
- It is found that effective emission time of kaon radiation is larger than the corresponding for pions. It is supposed to be conditioned by K^* with life-time 4-5 fm/c. Its decays in hadronic medium result in suppression of the possibility of K^* registration since the daughter particles are partially rescattered. The results for K^* (reduced 23% due to rescattering) and ϕ (increase 10% because of coalescence in UrQMD) as compared to their numbers at chemical freeze-out.

Summary (baryon correlations)

- Source functions extracted from iHKM, allows one to analyze the strong interaction in different baryon– (anti)baryon systems, including also multi-strange ones using FSI technique.
- The analysis of $p - \Lambda \oplus \bar{p} - \bar{\Lambda}$ and $p - \Lambda \oplus \bar{p} - \bar{\Lambda}$ correlations at RHIC with the residual correlations taken into account and source function taken from HKM, brings good description of the experimental results and allow to estimate the scattering lengths for $\bar{p} - \Lambda \oplus p - \bar{\Lambda}$
- The extracted at RHIC parameters, after the re-scaling for new energies, allow one to make prediction for the above hadron correlations at LHC within HKM.
- The first estimates for $\bar{p} - \Xi^- \oplus p - \Xi^+$ and $p - \Xi^- \oplus \bar{p} - \Xi^+$ within HKM model are presented.



Acknowledgement

Щиро дякую усіх присутніх за їх
присутність та увагу !

Thank you





Дякую за увагу!

Correlation femtoscopy of nucleus-nucleus collisions

- The femtoscopy analysis is used by Collaborations at SPS, RHIC and LHC. They provide the measurements of the space-time scales with accuracy 10^{-15} m та 10^{-23} c.

Some basic points

«Sinyukov-Makhlin formula”
for an estimate of the
duration of “Little Bang”

$$\tau \approx R_L \sqrt{\frac{m_T}{T_{f.o.}}}$$

Femto “homogeneity lengths”
general interpretation of the
femtoscopy measurements as
“homogeneity lengths”

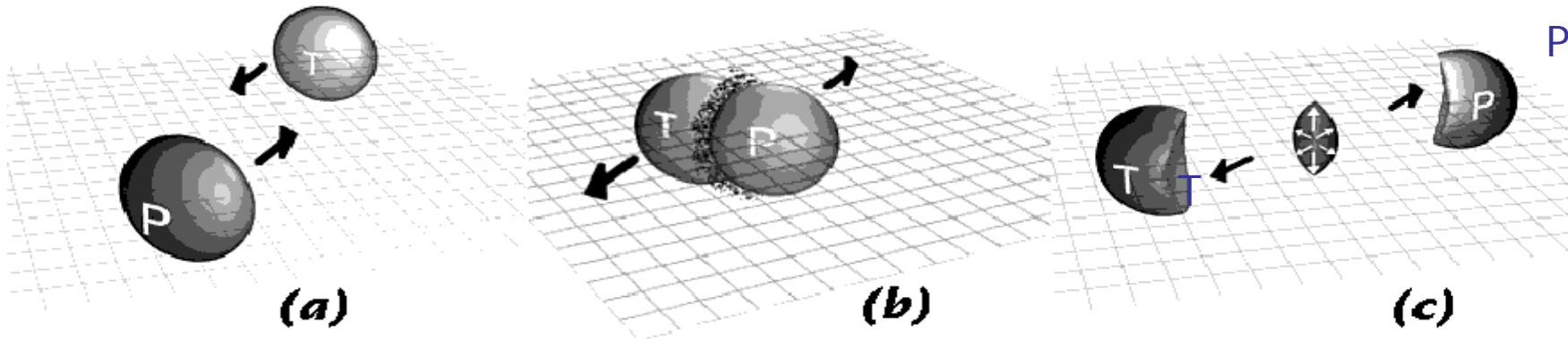
$$\lambda_i^2 = \frac{f(x_0, p)}{|f''_{x_i}(x_0, p)|}$$

“Bowler–Sinyukov
treatment”

extracts the femtoscopy
correlations from effects of
the Coulomb interactions and long-
lived resonances

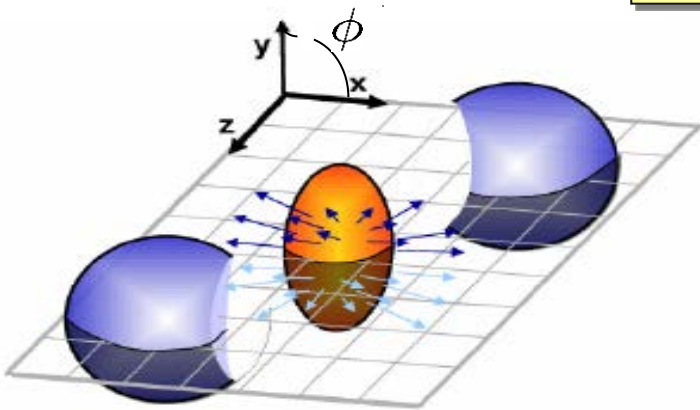
$$C(\mathbf{q}) = [(1-\lambda) + \lambda K(q_{inv})(1+G(\mathbf{q}))]$$

Collective transverse flows



Initial spatial anisotropy \rightarrow different pressure gradients \rightarrow momentum anisotropy v_2

$$\frac{dN}{d\phi} \sim [1 + 2v_1 \cos(\phi) + 2v_2 \cos(2\phi)]$$



Mid-rapidity data,
 ρ_t integrated

