Jet Quenching and Substructure Modifications in Heavy-Ion Collisions

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International School "Relativistic Heavy Ion Collisions, Cosmology and Dark Matter, Cancer Therapy"
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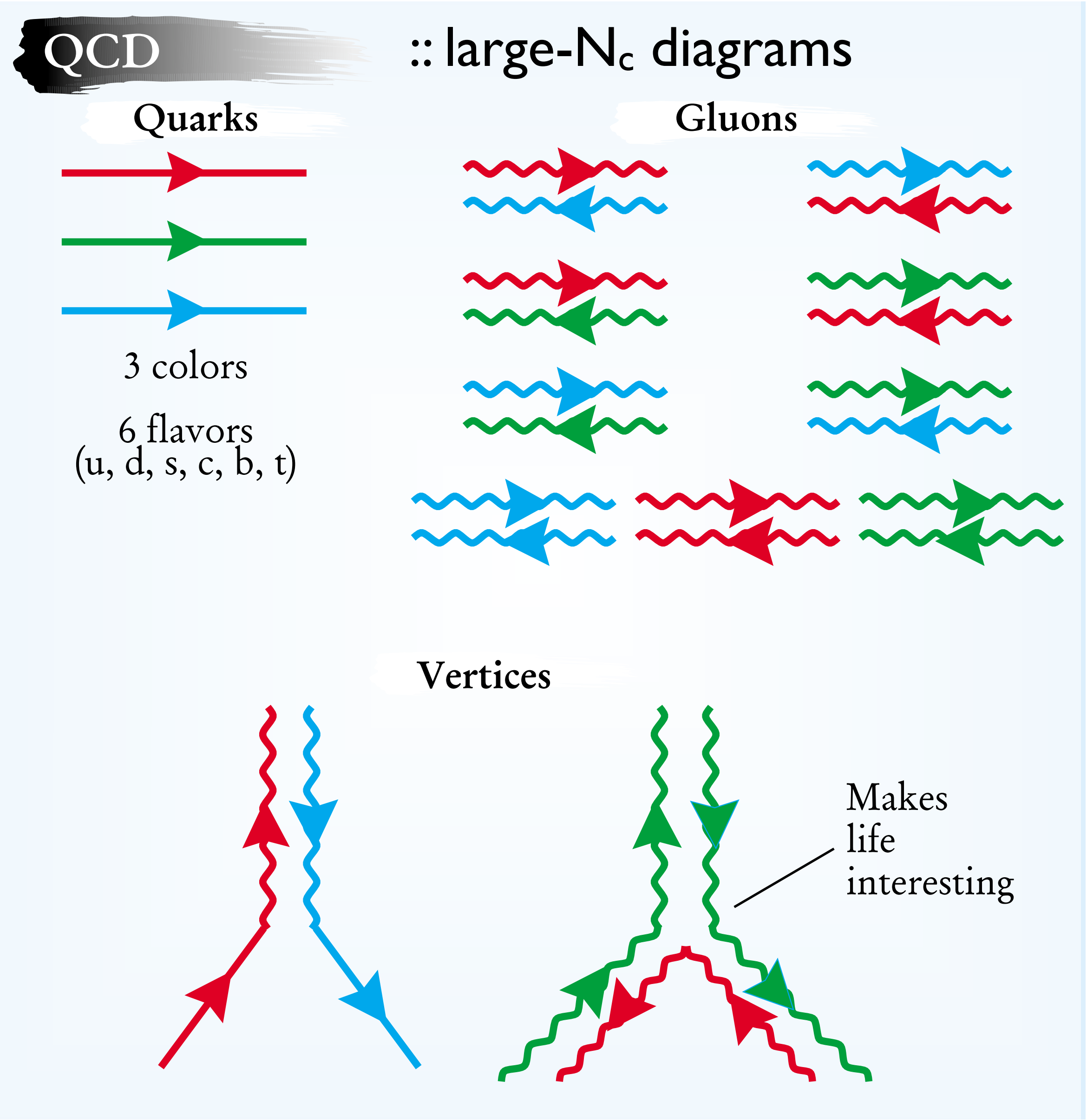
Lecture 1

QCD: QUARKS AND GLUONS



- keep in mind: we observe hadrons!
- quarks and gluons are DOF's in perturbation theory!

HARD PROBES

- there is a hard scale in the problem: **Q** ≫ Λ_{QCD}
  - separation of long and short-distance processes
  - uncertainty principle ($\Delta p \Delta x = 1$)

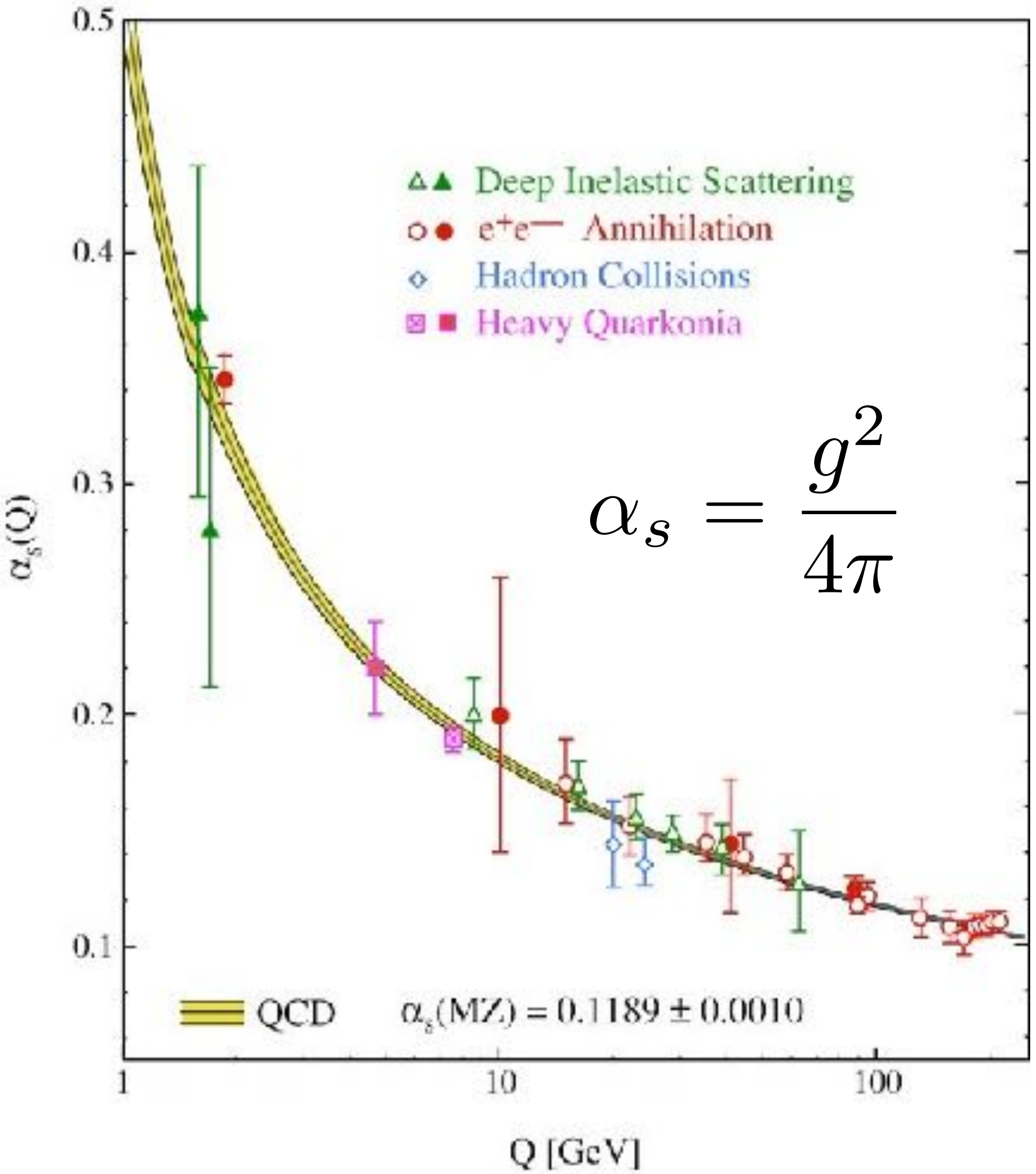
size ~ momentum^{-1}

Formation time: $$t_\mathrm{f} = \frac{1}{Q}\frac{E}{Q} = \frac{E}{Q^2}$$

life-time in particle rest frame (→ $\frac{1}{Q}$)

boost to lab frame (→ $\frac{E}{Q}$)

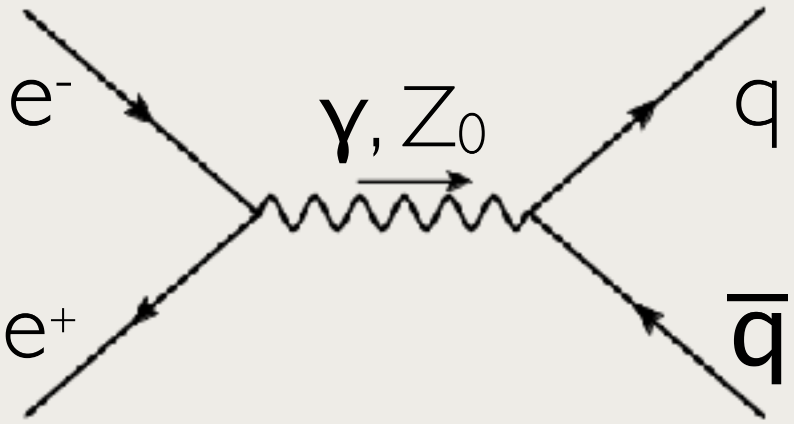
Coupling: Asymptotic Freedom



$$\alpha_s = \frac{g^2}{4\pi}$$

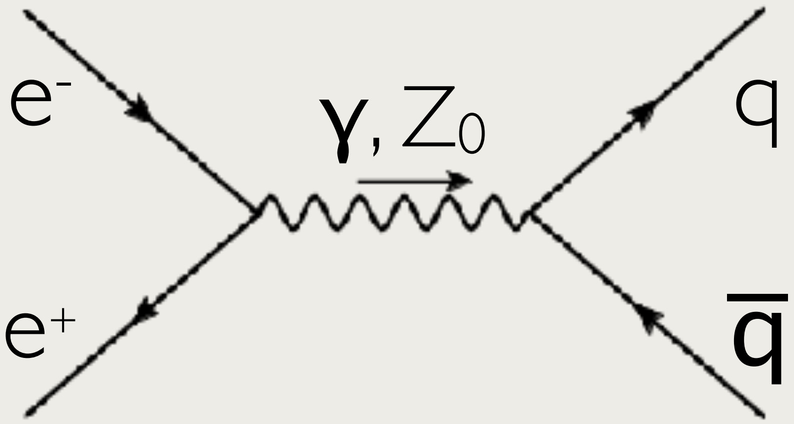
- QCD is weakly coupled at small distances — strongly coupled at large distances
  - "free" particles at short distances!
  - gluons & quarks
- can use perturbation theory when there is a large scale in the problem
- unfortunately, in many interesting situations this is not the case…

DIFFERENT COLLISION SYSTEMS



- jets (time-like branching): fragmentation functions
- resonance production
- clean environment for testing QCD
- soft physics: particles in between jets, interference phenomena

Different collision systems



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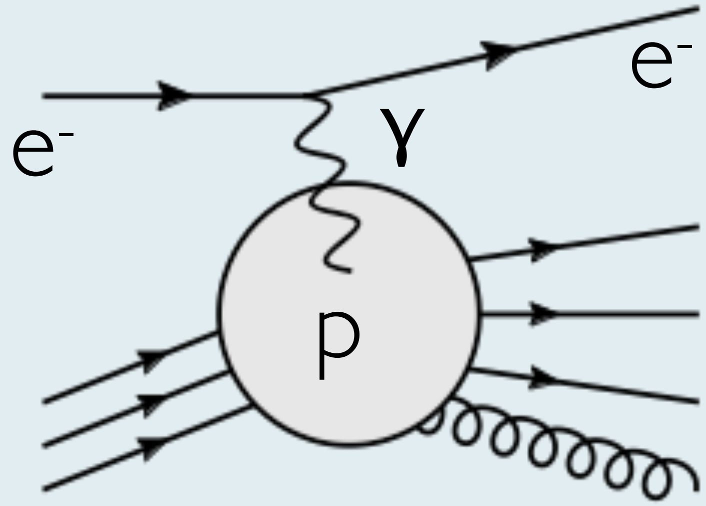

- deep inelastic scattering
- parton distribution functions (space-like branching): the structure of hadrons
- playground for QCD: could also involve non-linear phenomena at HE

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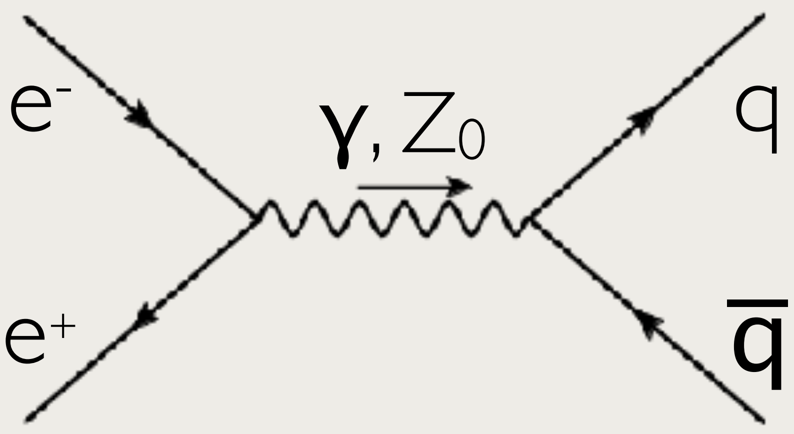

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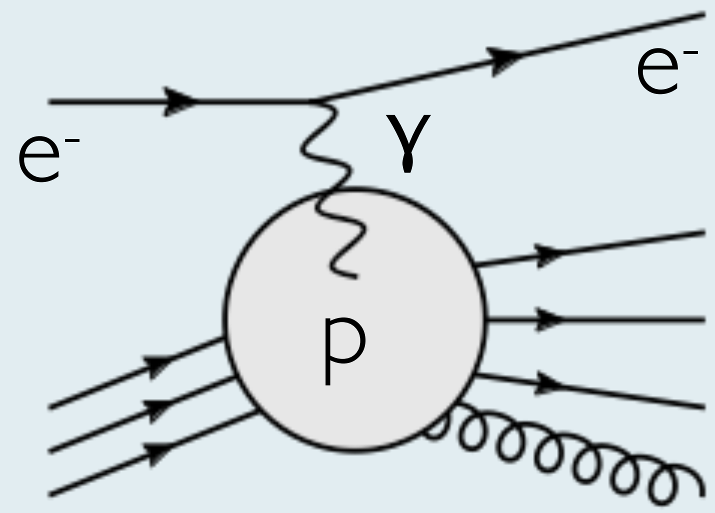

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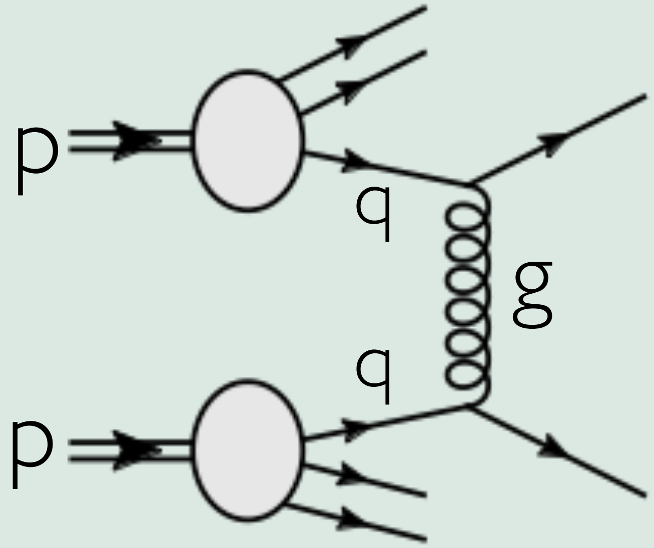
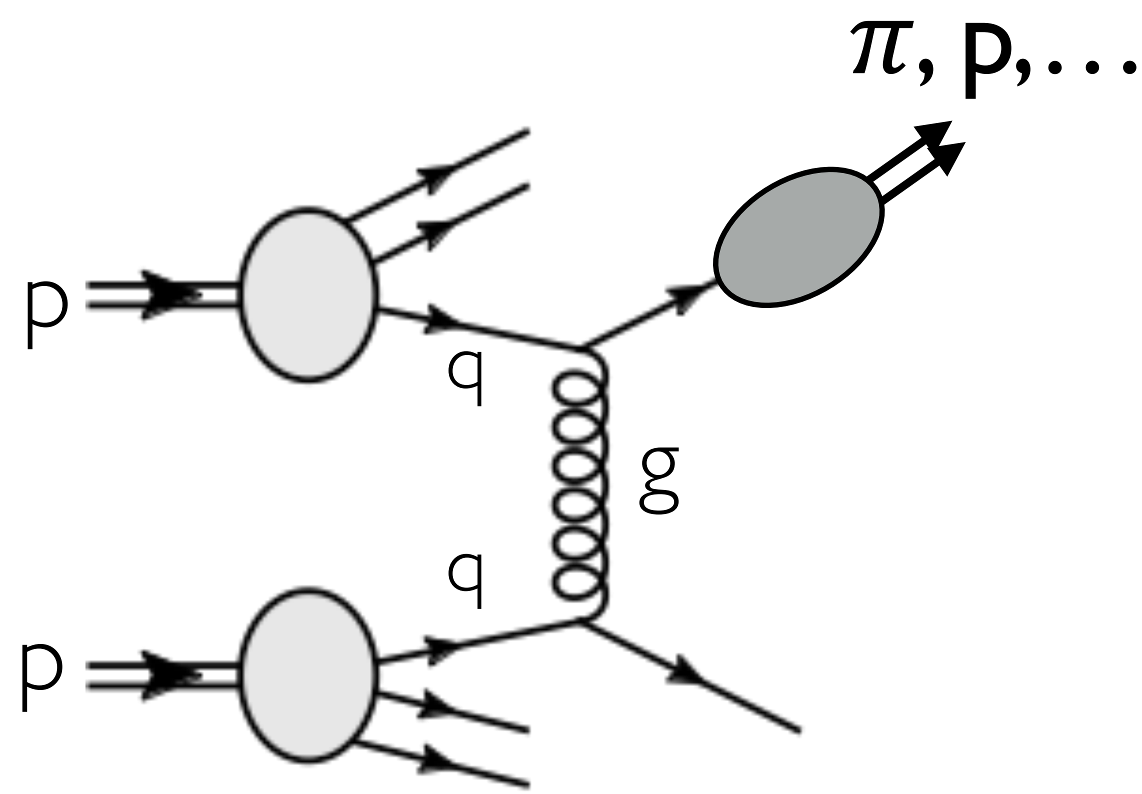


- factorisation theorems: separation of long- and short-distance processes
- hard physics: jets, heavy quarks, etc.
- soft physics: diffraction, underlying event
- collective phenomena?

QCD FACTORISATION



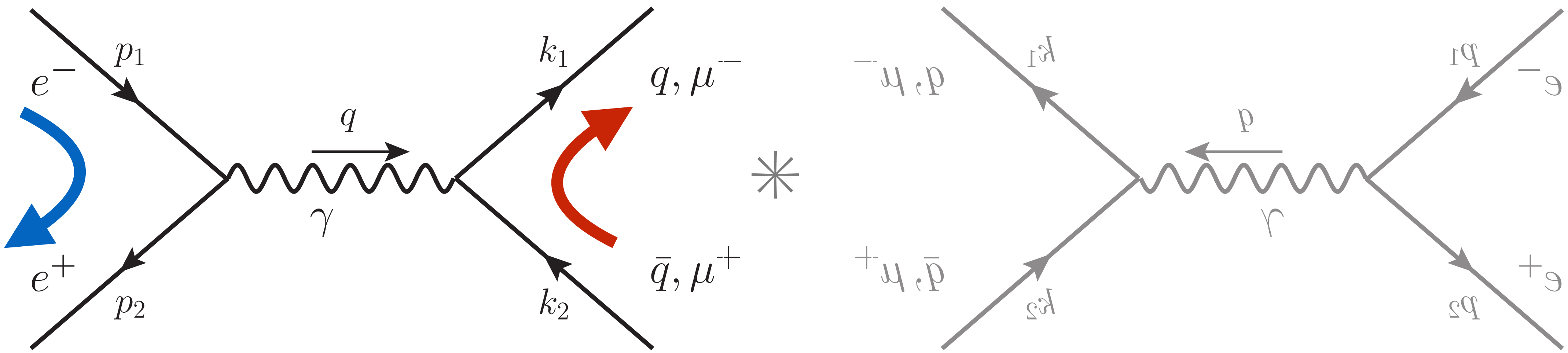
- hadron production
- separation of processes
  - short-distance (perturbative)
  - long-distance (non-perturbative)
  - **universal** distributions
- hard matrix element
- corrections suppressed $\sim 1/Q^2$

$$\sigma^{pp\to h} = f_p^i(x_1,Q^2) \otimes f_p^j(x_2,Q^2) \otimes \hat{\sigma}^{ij\to k} \otimes D^{k\to h}(z,Q^2)$$

parton distribution functions — fragmentation function

- we don't know how to compute PDF's/FF's!
- but we know how to evolve perturbatively!

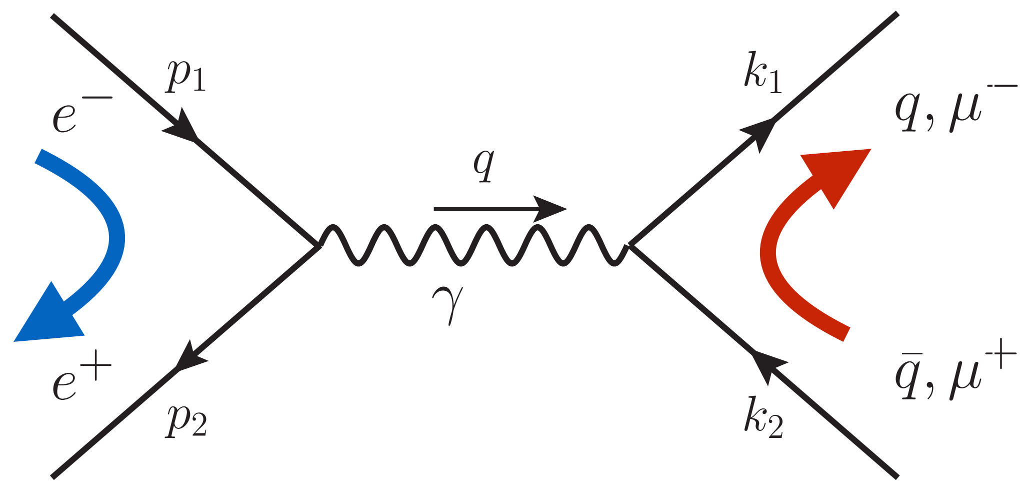
Electron-positron collisions



$$\frac{\mathrm{d}\sigma}{\mathrm{d}\cos\theta} = \frac{\pi\alpha_{\mathrm{em}}^2\, Q_f^2}{2s}(1+\cos^2\theta) \quad \Rightarrow \quad \sigma = \frac{4\pi\alpha_{\mathrm{em}}^2}{3\,s} Q_f^2$$

$$R = \frac{\sigma(e^+e^- \to \mathrm{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)} = \frac{\sum_f \sigma(e^+e^- \to f\bar{f})}{\sigma(e^+e^- \to \mu^+\mu^-)} = N_c \sum_f Q_f^2$$

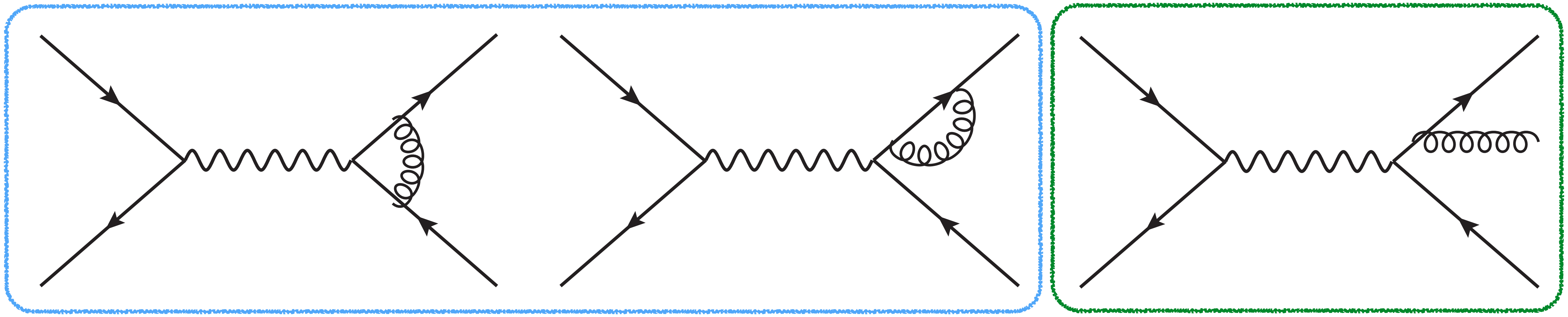
remarkably simple relation!



Q?? calculating quarks, but measure hadrons?

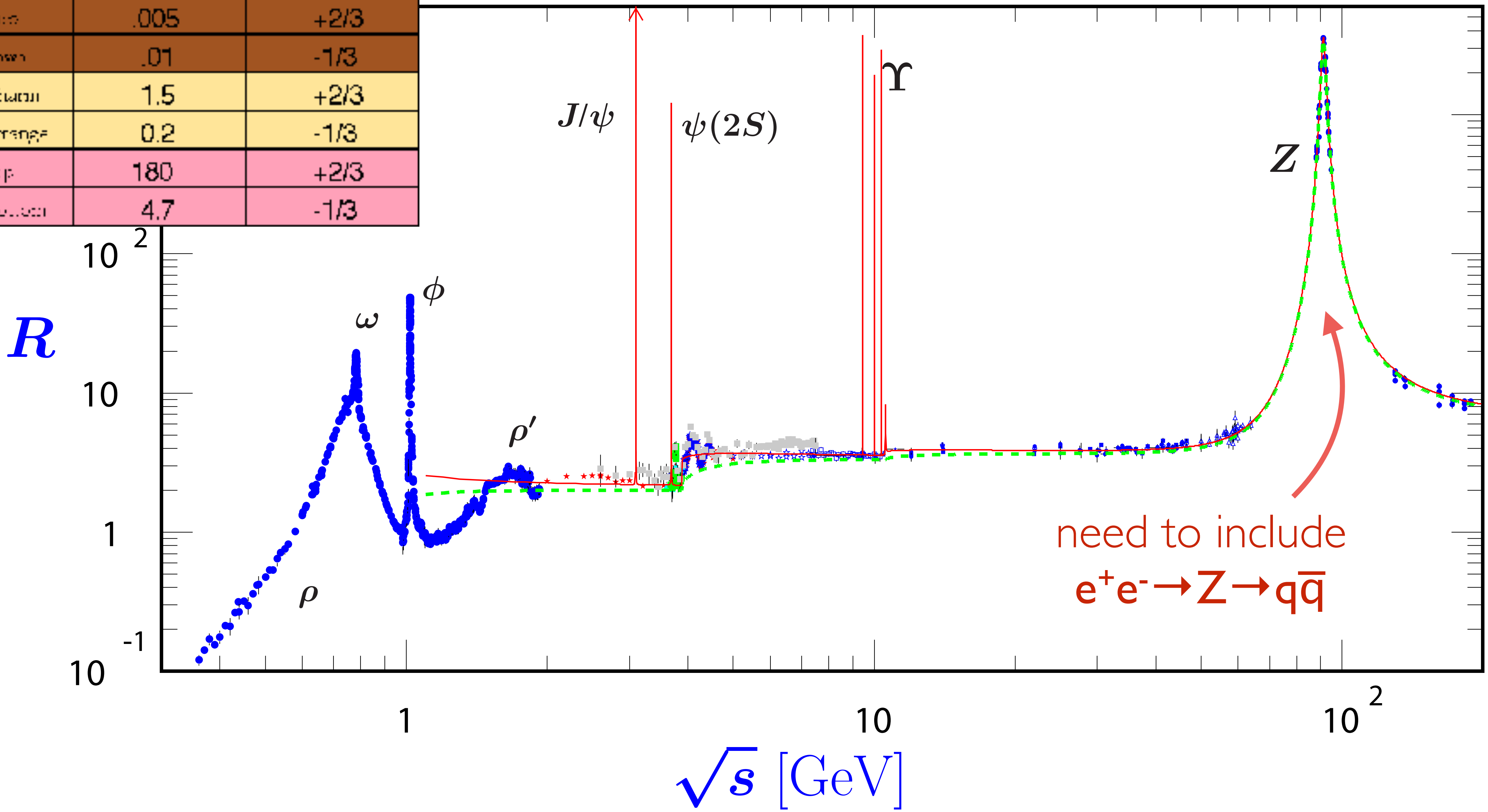
- short collision time $\quad t_{\rm coll} \sim 1/\sqrt{s}$
- hadronisation effects suppressed as $\mathcal{O}(m^2/s)$ $\quad t_{\rm hadr} \sim \sqrt{s}/m^2$

RADIATIVE CORRECTIONS



- next gluon emission — next order in α_s
- virtual and real emission contributions are separately IR divergent!
  - divergences cancel when summing the two
  - happens always for inclusive observables

Flavor	Mass(GeV/c)	Elect. Charge
u	.005	+2/3
d	.01	-1/3
c	1.5	+2/3
s	0.2	-1/3
t	180	+2/3
b	4.7	-1/3



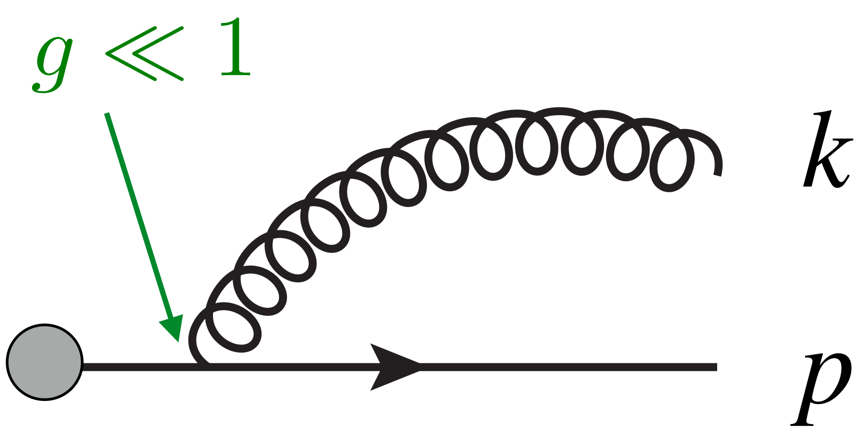
$$R = N_c \sum_f Q_f^2 \left[1 + \frac{\alpha_s}{\pi}\right]$$

$R_{(0)}$	2	if $Q \lesssim 3$ GeV
	3⅓	for $3\ \text{GeV} \lesssim Q \lesssim 10$ GeV
	3⅔	for $10\ \text{GeV} \lesssim Q$

Part 1) QCD radiation

- these lectures will deal with “real” emissions
  - in vacuum
  - in medium
  - how to deal with interference effects and re-sum multiple radiation
- aim: to know the fundamental splitting processes & establish a probabilistic picture
  - calculate jet spectra + other observables

Gluon emission



- apparently suppressed by two additional power of the coupling constant (in the cross section)

$$i\mathcal{M} = \bar{u}(p)\varepsilon^*_\mu(k)\big(-igt^a\gamma^\mu\big)\frac{i\left(\not{p}+\not{k}\right)}{(p+k)^2+i\epsilon}\, i\mathcal{M}_h$$

Work in light-cone coordinates: $p^\pm = \frac{1}{\sqrt{2}}\left(p^0 \pm p^3\right)$

$$\begin{aligned} p\cdot k &= p^+k^- + p^-k^+ - \boldsymbol{p}\cdot\boldsymbol{k} \\ &= \frac{1}{2}(p^0+p^z)(k^0-k^z) + \frac{1}{2}(p^0-p^z)(k^0+k^z) - p^xk^x - p^yk^y \\ &= p^0k^0 - p^xk^x - p^yk^y - p^zk^z \end{aligned}$$

High-energy approximation:
soft & collinear radiation

$$p^+ \gg k^+ \gg k_\perp$$

$$\bar{u}(p)\not{\varepsilon}\left(\not{p}+\not{k}\right) \simeq \bar{u}(p)\not{\varepsilon}\not{p} = 2\bar{u}(p)p\cdot\varepsilon - \bar{u}(p)\not{p}\not{\varepsilon}$$

$$\bar{u}(p)\not{\varepsilon}\,(\not{p}+\not{k}) \simeq \bar{u}(p)\not{\varepsilon}\not{p} = 2\bar{u}(p)p\cdot\varepsilon - \bar{u}(p)\not{p}\not{\varepsilon}$$

Anti-commutation relation:

$$\{\gamma^\mu,\gamma^\nu\} = 2g^{\mu\nu} \to \gamma^\mu\gamma^\nu = 2g^{\mu\nu} - \gamma^\nu\gamma^\mu$$

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Dirac equation: $\bar{u}(p)\not{p} = 0$

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On-shell condition: $(p+k)^2 = p^2 + k^2 + 2p\cdot k = 2p\cdot k$

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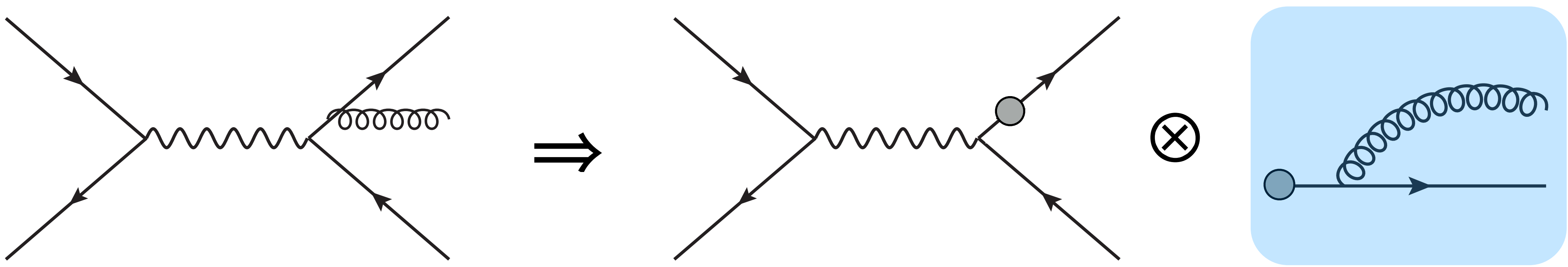
On-shell condition: $(p+k)^2 = p^2 + k^2 + 2p\cdot k = 2p\cdot k$

$$i\mathcal{M} = \bar{u}(p)i\mathcal{M}_h\,(gt^a)\,\frac{p\cdot\varepsilon(k)}{p\cdot k}$$



Calculating the cross section:

$$(2\pi)^3(2k^+)\frac{\mathrm{d}\sigma}{\mathrm{d}k^+\,\mathrm{d}^2k_\perp} = \sum |\mathcal{M}|^2 = |\mathcal{M}_h|^2\,(2\pi)^3(2k^+)\frac{\mathrm{d}N}{\mathrm{d}k^+\,\mathrm{d}^2k_\perp}$$

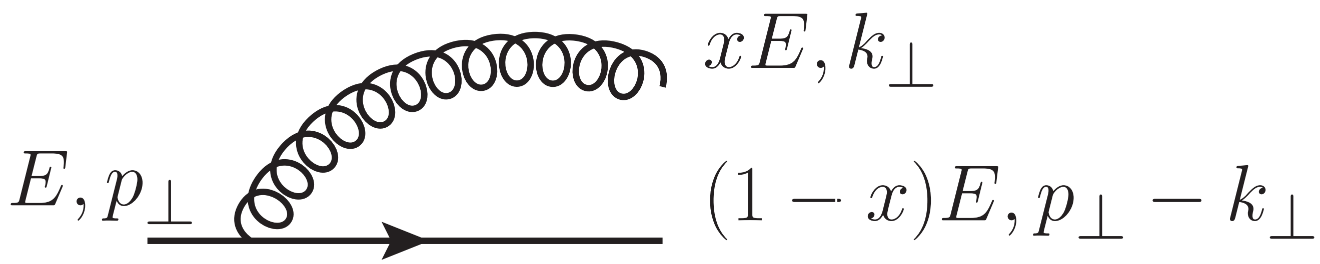
average over: incoming (quark) color
sum over: outgoing spin and polarisation

Exercise I) show that

$$\frac{\mathrm{d}N}{\mathrm{d}k^+\,\mathrm{d}^2\boldsymbol{k}} = \frac{\alpha_s}{\pi^2}C_F\frac{1}{k^+}\frac{1}{\boldsymbol{k}^2}$$

Exercise II) use the uncertainty principle to calculate the duration of the q→q+g splitting process
HINT: use three-momentum conservation!

Solution: Exercise II)



$$E = \sqrt{k_\perp^2 + (k^3)^2}$$

$$\begin{aligned}
\Delta E &= \sqrt{x^2 E^2 + \boldsymbol{k}^2} + \sqrt{(1-x)^2 E^2 + (\boldsymbol{k}+\boldsymbol{p})^2} - \sqrt{E^2 + \boldsymbol{p}^2} \\
&\approx xE + \frac{\boldsymbol{k}^2}{2xE} + (1-x)E + \frac{(\boldsymbol{k}+\boldsymbol{p})^2}{2(1-x)E} - E - \frac{\boldsymbol{p}^2}{2E} \\
&= \frac{(\boldsymbol{k}+x\boldsymbol{p})^2}{2x(1-x)E} \\
&= \frac{\boldsymbol{k}^2}{2\omega}
\end{aligned}$$

$\boldsymbol{p} = 0$

$$t_\mathrm{f} \sim \Delta t \sim \frac{1}{\Delta E} \sim \frac{2\omega}{\boldsymbol{k}^2}$$

$$t_\mathrm{f} \sim \frac{1}{\omega\theta^2}$$

New variables: $\theta = \frac{|\boldsymbol{k}|}{k^+}$ $x = \frac{k^+}{p^+}$

$$\mathrm{d}k^+ \mathrm{d}^2\boldsymbol{k} \to p^+ \mathrm{d}x \, (k^+)^2 \theta \, \mathrm{d}\theta \, \mathrm{d}\varphi$$

$$\frac{\mathrm{d}N}{\mathrm{d}x \, \mathrm{d}\theta} = \frac{2\alpha_s C_F}{\pi} \frac{1}{x} \frac{1}{\theta}$$

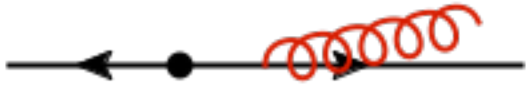
Small-angle approx:
relax soft condition,
replaced by Altarelli-Parisi
splitting function

$$\frac{2}{x} \to P(x)$$

Proportional to colour factor & coupling constant

Soft divergence: $x \to 0$

Collinear divergence: $\vartheta \to 0$



$$N = \frac{\alpha_s C_F}{\pi} 2 \int_{Q_0/E}^{1} \frac{\mathrm{d}x}{x} \int_{Q_0/(xE)}^{1} \frac{\mathrm{d}\theta}{\theta} = \frac{\alpha_s C_F}{\pi} \log^2 \frac{E}{Q_0}$$

- smallness of the coupling constant compensated by large phase space
  - double-logarithmic approximation
  - further improvements will include single-log contributions

Intra-jet processes:

$$k_\perp \ll k^+ \ll p^+$$

$$N \sim \frac{\alpha}{\pi} \log^2 E \gtrsim 1$$

log resummations (N…LL)

Inter-jet processes:

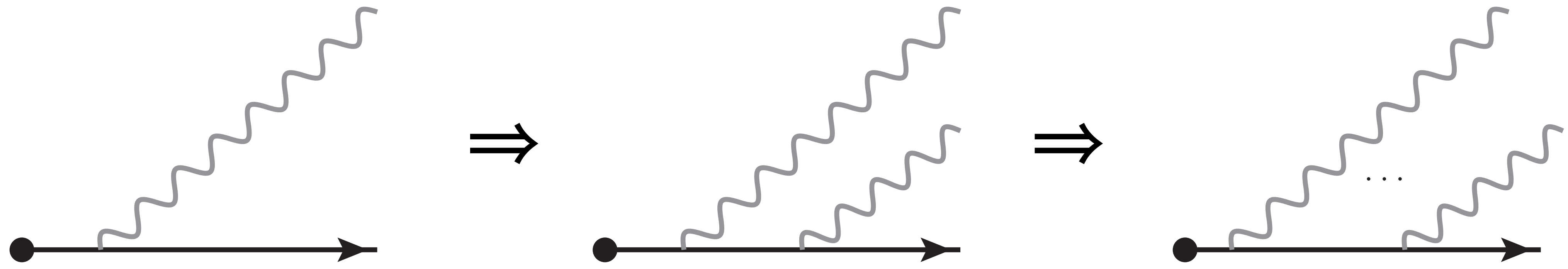
$$k_\perp \sim k^+ \sim p^+$$

$$N \sim \frac{\alpha}{\pi} \ll 1$$

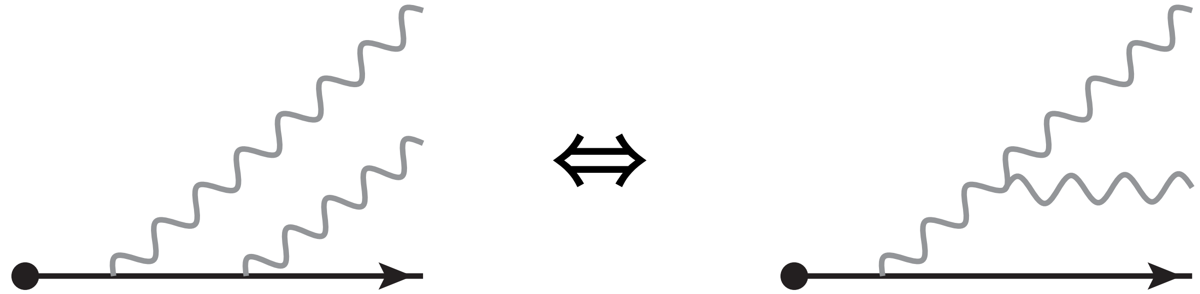
fixed-order (N…LO)

Part 2) interference effects in vacuum

MULTI-GLUON EMISSIONS

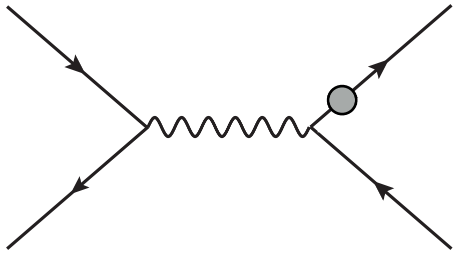


- soft & collinear emissions: need to consider emissions of multiple gluons
- can we simply reiterate single-emission formula?
  - for photons in QED: yes!
  - for gluons in QCD: not so fast!
  - there are interferences!



Current

$$i\mathcal{M} = \bar{u}(p) i\mathcal{M}_h \left(g t^a\right) \frac{p \cdot \varepsilon(k)}{p \cdot k}$$



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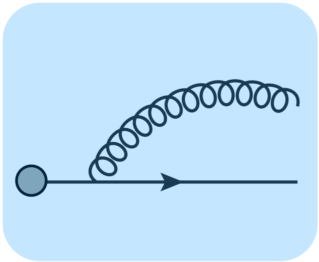


factorisation!

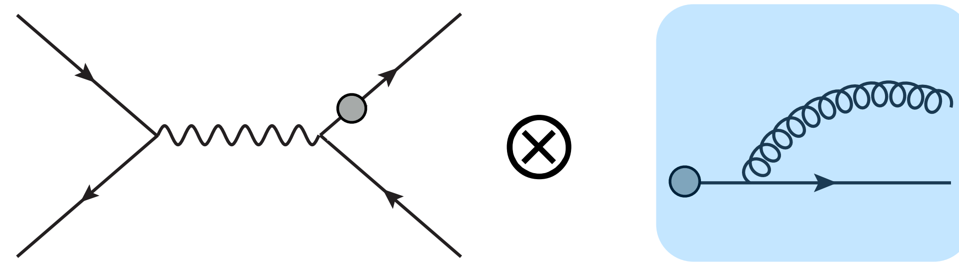
Defining a current:

proportional to the colour charge of the emitter

$$\mathcal{J}_i^{a,\mu}(k) = g \mathcal{Q}_i^a \frac{p_i^\mu}{p \cdot k}$$

Current

$$i\mathcal{M} = \bar{u}(p) i\mathcal{M}_h \left(g t^a\right) \frac{p \cdot \varepsilon(k)}{p \cdot k}$$



factorisation!

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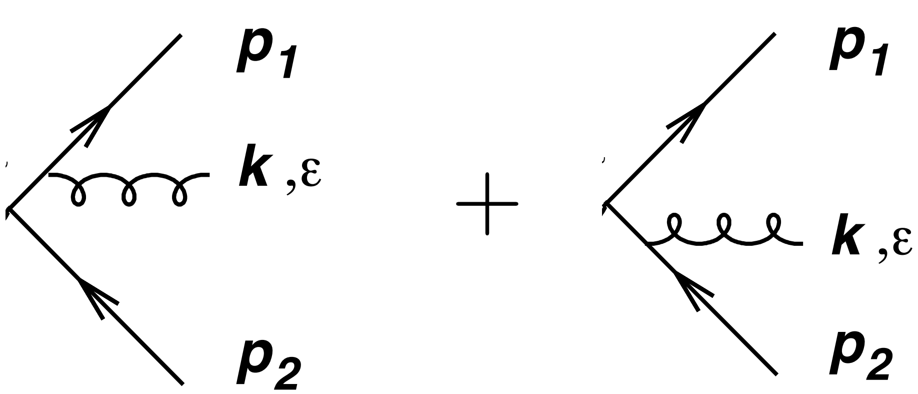
Emission off two quarks is simply a sum:



$$i\mathcal{M}_{q\bar{q}g} = i\mathcal{M}_{q\bar{q}} \, \mathcal{J}_{12}(k) \cdot \varepsilon(k)$$

$$\mathcal{J}_{12}^\mu(k) = g \mathcal{Q}_1^a \frac{p_1^\mu}{p_1 \cdot k} + g \mathcal{Q}_2^a \frac{p_2^\mu}{p_2 \cdot k}$$

Colour charge algebra

conservation of colour charge $\mathcal{Q}_1^a + \mathcal{Q}_2^a = \mathcal{Q}_3^a$

quark colour charge $\mathcal{Q}_q^2 = C_F$

gluon colour charge $\mathcal{Q}_g^2 = C_A$

2

$$\mathcal{Q}_1^2 + \mathcal{Q}_2^2 + 2\mathcal{Q}_1 \cdot \mathcal{Q}_2 = \mathcal{Q}_3^2 \quad \Rightarrow \quad \mathcal{Q}_1 \cdot \mathcal{Q}_2 = \frac{1}{2}\left(\mathcal{Q}_3^2 - \mathcal{Q}_1^2 - \mathcal{Q}_2^2\right)$$

$$\mathcal{Q}_1^2 = \mathcal{Q}_2^2 = C_F, \text{ and } \mathcal{Q}_3^2 = C_A \quad \text{for } g \to q + \bar{q}$$

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$$\mathcal{Q}_1^2 = \mathcal{Q}_3^2 = C_F, \text{ and } \mathcal{Q}_2^2 = C_A \quad \text{for } q \to q + g$$

$$\boldsymbol{v}_k \equiv \boldsymbol{k}/k^+$$

$$\boldsymbol{v}_k^2 = 1$$

The current is transverse: $\mathcal{J} \cdot \varepsilon = \mathcal{J}_\perp \cdot \boldsymbol{\varepsilon}$

(see Exercise I)

$$\mathcal{J}_{12,\perp} = \frac{2}{k^+}\left[\mathcal{Q}_1^a \frac{\boldsymbol{v}_k - \boldsymbol{v}_1}{(\boldsymbol{v}_k - \boldsymbol{v}_1)^2} + \mathcal{Q}_2^a \frac{\boldsymbol{v}_k - \boldsymbol{v}_2}{(\boldsymbol{v}_k - \boldsymbol{v}_2)^2}\right]$$

If $p_1 \to p_2$, or $\boldsymbol{\vartheta}_0 \to 0$, does the current vanish?

$$\mathcal{J}_{12,\perp}\big|_{\boldsymbol{v}_1 = \boldsymbol{v}_2} = \frac{2}{k^+}\frac{\boldsymbol{v}_k - \boldsymbol{v}_1}{(\boldsymbol{v}_k - \boldsymbol{v}_1)^2}\underbrace{[\mathcal{Q}_1^a + \mathcal{Q}_2^a]}_{\mathcal{Q}_3^a}$$

Not unless colour cancels!

$$
\begin{aligned}
\left|\mathcal{M}_{q\bar{q}g}\right|^2 &= \left|\mathcal{M}_{q\bar{q}}\right|^2 \sum_{\lambda} \mathcal{J}_{12,\perp} \cdot \boldsymbol{\varepsilon}_\lambda \left(\mathcal{J}_{12,\perp} \cdot \boldsymbol{\varepsilon}_\lambda\right)^* \\
&= \left|\mathcal{M}_{q\bar{q}}\right|^2 \left|\mathcal{J}_{12,\perp}\right|^2
\end{aligned}
$$

$$
\sum_{\lambda} \varepsilon^i_\lambda \varepsilon^j_\lambda = \delta^{ij}
$$

$$|\mathcal{M}_{q\bar{q}g}|^2 = |\mathcal{M}_{q\bar{q}}|^2 \sum_\lambda \mathcal{J}_{12,\perp} \cdot \boldsymbol{\varepsilon}_\lambda \left(\mathcal{J}_{12,\perp} \cdot \boldsymbol{\varepsilon}_\lambda\right)^* = |\mathcal{M}_{q\bar{q}}|^2 \, |\mathcal{J}_{12,\perp}|^2$$

$$\sum_\lambda \varepsilon^i_\lambda \varepsilon^j_\lambda = \delta^{ij}$$

$$|\mathcal{J}_{12,\perp}|^2 = \frac{4}{(k^+)^2}\left[\frac{\mathcal{Q}_1^2}{(\boldsymbol{v}_k - \boldsymbol{v}_1)^2} + \frac{\mathcal{Q}_2^2}{(\boldsymbol{v}_k - \boldsymbol{v}_2)^2} + 2\mathcal{Q}_1 \cdot \mathcal{Q}_2 \frac{(\boldsymbol{v}_k - \boldsymbol{v}_1)\cdot(\boldsymbol{v}_k - \boldsymbol{v}_2)}{(\boldsymbol{v}_k - \boldsymbol{v}_1)^2(\boldsymbol{v}_k - \boldsymbol{v}_2)^2}\right]$$

$$\mathcal{Q}_1 \cdot \mathcal{Q}_2 = \left(\mathcal{Q}_3^2 - \mathcal{Q}_1^2 - \mathcal{Q}_2^2\right)/2$$

$$= \frac{1}{(k^+)^2}\left[\mathcal{Q}_1^2 \mathcal{P}_1 + \mathcal{Q}_2^2 \mathcal{P}_2 + \mathcal{Q}_3^2 \mathcal{I}_{12}\right]$$

$$|\mathcal{M}_{q\bar{q}g}|^2 = |\mathcal{M}_{q\bar{q}}|^2 \sum_\lambda \mathcal{J}_{12,\perp} \cdot \boldsymbol{\varepsilon}_\lambda \left(\mathcal{J}_{12,\perp} \cdot \boldsymbol{\varepsilon}_\lambda\right)^* = |\mathcal{M}_{q\bar{q}}|^2 \, |\mathcal{J}_{12,\perp}|^2$$

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$$\mathcal{Q}_1 \cdot \mathcal{Q}_2 = \left(\mathcal{Q}_3^2 - \mathcal{Q}_1^2 - \mathcal{Q}_2^2\right)/2$$

$$= \frac{1}{(k^+)^2}\left[\mathcal{Q}_1^2 \mathcal{P}_1 + \mathcal{Q}_2^2 \mathcal{P}_2 + \mathcal{Q}_3^2 \mathcal{I}_{12}\right]$$

Coherent spectrum

$$\mathcal{P}_i = \mathcal{R}_i - \mathcal{I}_{12} = \frac{4}{(\boldsymbol{v}_k - \boldsymbol{v}_i)^2}\left[1 - \frac{(\boldsymbol{v}_k - \boldsymbol{v}_1)\cdot(\boldsymbol{v}_k - \boldsymbol{v}_2)}{(\boldsymbol{v}_k - \boldsymbol{v}_2)^2}\right]$$

$$|\mathcal{M}_{q\bar{q}g}|^2 = |\mathcal{M}_{q\bar{q}}|^2 \sum_\lambda \mathcal{J}_{12,\perp} \cdot \boldsymbol{\varepsilon}_\lambda \left(\mathcal{J}_{12,\perp} \cdot \boldsymbol{\varepsilon}_\lambda\right)^* = |\mathcal{M}_{q\bar{q}}|^2 \, |\mathcal{J}_{12,\perp}|^2$$

$$\sum_\lambda \varepsilon^i_\lambda \varepsilon^j_\lambda = \delta^{ij}$$

$$|\mathcal{J}_{12,\perp}|^2 = \frac{4}{(k^+)^2}\left[\frac{\mathcal{Q}_1^2}{(\boldsymbol{v}_k - \boldsymbol{v}_1)^2} + \frac{\mathcal{Q}_2^2}{(\boldsymbol{v}_k - \boldsymbol{v}_2)^2} + 2\mathcal{Q}_1 \cdot \mathcal{Q}_2 \frac{(\boldsymbol{v}_k - \boldsymbol{v}_1)\cdot(\boldsymbol{v}_k - \boldsymbol{v}_2)}{(\boldsymbol{v}_k - \boldsymbol{v}_1)^2(\boldsymbol{v}_k - \boldsymbol{v}_2)^2}\right]$$

$$\mathcal{Q}_1 \cdot \mathcal{Q}_2 = \left(\mathcal{Q}_3^2 - \mathcal{Q}_1^2 - \mathcal{Q}_2^2\right)/2$$

$$= \frac{1}{(k^+)^2}\left[\mathcal{Q}_1^2 \mathcal{P}_1 + \mathcal{Q}_2^2 \mathcal{P}_2 + \mathcal{Q}_3^2 \mathcal{I}_{12}\right]$$

Coherent spectrum

$$\mathcal{P}_i = \mathcal{R}_i - \mathcal{I}_{12} = \frac{4}{(\boldsymbol{v}_k - \boldsymbol{v}_i)^2}\left[1 - \frac{(\boldsymbol{v}_k - \boldsymbol{v}_1)\cdot(\boldsymbol{v}_k - \boldsymbol{v}_2)}{(\boldsymbol{v}_k - \boldsymbol{v}_2)^2}\right]$$

Independent & interference

$$\mathcal{R}_i = \frac{4}{(\boldsymbol{v}_k - \boldsymbol{v}_i)^2}$$

$$\mathcal{I}_{12} = 4\frac{(\boldsymbol{v}_k - \boldsymbol{v}_1)\cdot(\boldsymbol{v}_k - \boldsymbol{v}_2)}{(\boldsymbol{v}_k - \boldsymbol{v}_1)^2(\boldsymbol{v}_k - \boldsymbol{v}_2)^2}$$

Notation: angles

$$
\begin{aligned}
(\boldsymbol{v}_k - \boldsymbol{v}_i)^2 &= 2(1 - \boldsymbol{v}_k \cdot \boldsymbol{v}_i) \\
&= 2(1 - \cos\hat{\theta}_i) \\
&= 2a_i
\end{aligned}
$$

Notation: angles

$$
\begin{aligned}
(\boldsymbol{v}_k - \boldsymbol{v}_i)^2 &= 2(1 - \boldsymbol{v}_k \cdot \boldsymbol{v}_i) \\
&= 2(1 - \cos\hat{\theta}_i) \\
&= 2a_i
\end{aligned}
$$

Coherent spectrum: diverges only in the direction of quark 1

$$
\mathcal{P}_1 = \frac{1}{a_1}\left(1 - \frac{a_1 - a_{12}}{a_2}\right) \to \left\{ \begin{array}{cl} \infty & \text{for } a_1 \to 0 \, (a_{12} \to a_2) \\ 0 & \text{for } a_2 \to 0 \, (a_{12} \to a_1) \end{array} \right.
$$

Notation: angles

$$(\boldsymbol{v}_k - \boldsymbol{v}_i)^2 = 2(1 - \boldsymbol{v}_k \cdot \boldsymbol{v}_i) = 2(1 - \cos\hat{\theta}_i) = 2a_i$$

Coherent spectrum: diverges only in the direction of quark 1

$$\mathcal{P}_1 = \frac{1}{a_1}\left(1 - \frac{a_1 - a_{12}}{a_2}\right) \to \begin{cases} \infty & \text{for } a_1 \to 0\,(a_{12} \to a_2) \\ 0 & \text{for } a_2 \to 0\,(a_{12} \to a_1) \end{cases}$$

Exercise IV) put quark 1 on the z-axis and prove that

$$\int_0^{2\pi} \frac{\mathrm{d}\varphi}{2\pi}\mathcal{P}_1 = \frac{2}{1 - \cos\theta_1}\Theta(\theta_{12} - \theta_1)$$

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Independent

$$\frac{\mathrm{d}N_q}{\mathrm{d}x\,\mathrm{d}\theta} = \frac{2\alpha_s C_F}{\pi} \frac{1}{x} \frac{1}{\theta}$$

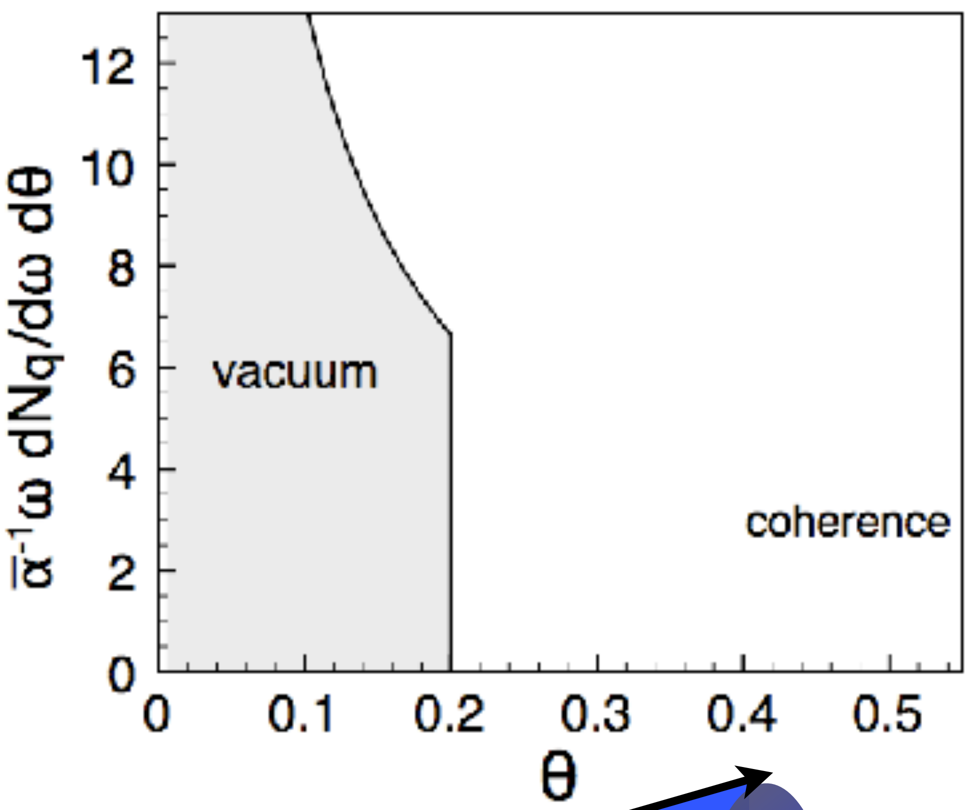
$\Rightarrow$

Coherent

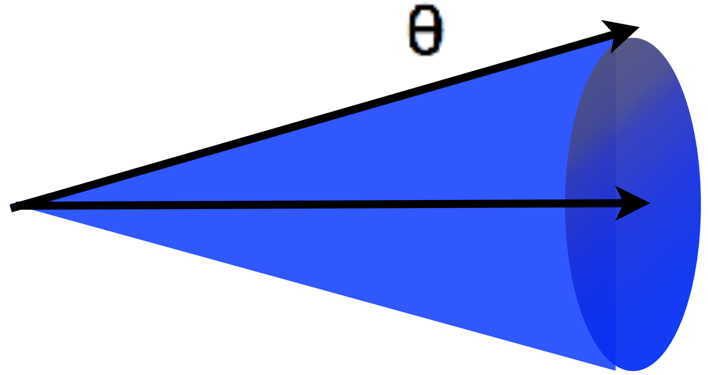
$$\frac{\mathrm{d}N_q}{\mathrm{d}x\,\mathrm{d}\theta} = \frac{2\alpha_s C_F}{\pi} \frac{1}{x} \frac{1}{\theta} \Theta(\theta_0 - \theta)$$

Angular ordering

$$\frac{dN_q}{dx\, d\theta} = \frac{2\alpha_s C_F}{\pi}\frac{1}{x}\frac{1}{\theta}\Theta(\theta_0 - \theta)$$



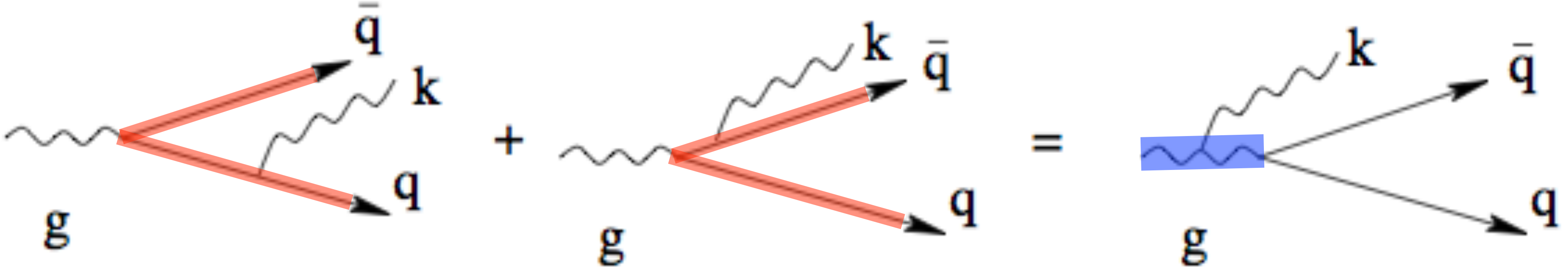
- interference effects = coherence limit phase space of emissions
- antenna grows during formation time
- if gluon is "too big" :: doesn't resolve the individual charges of the antenna, resolves total charge
- if gluon is "small" :: resolves the individual charges



$$\lambda_\perp \sim \frac{1}{k_\perp} = \frac{1}{\omega\theta} \quad r_\perp \sim \theta_0 t_f = \frac{\theta_0}{\omega\theta^2}$$

$$\lambda_\perp < r_\perp \;\rightarrow\; \theta < \theta_0$$

COLOUR CHARGED ANTENNA

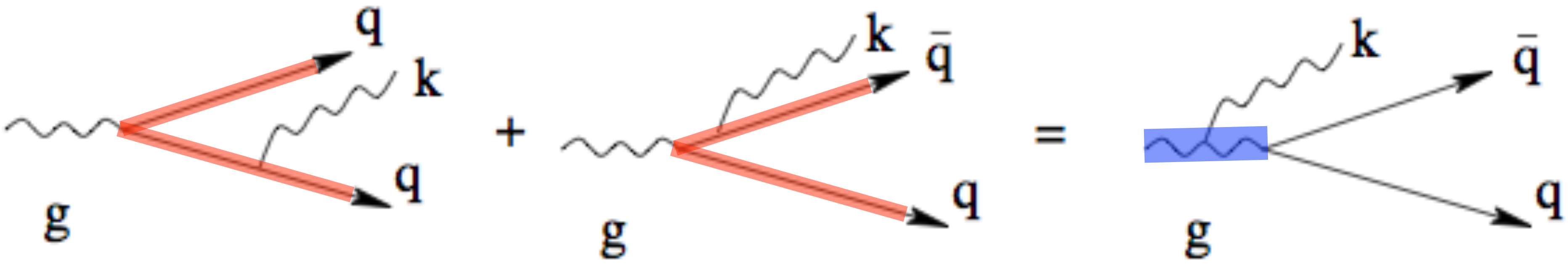


large-angle emissions are restored with the total charge!

$$\left|\mathcal{J}_{g\to q\bar{q},\perp}\right|^2 = \frac{1}{(k^+)^2}\left[C_F\mathcal{P}_1 + C_F\mathcal{P}_1 + C_A\mathcal{I}_{12}\right]$$

total charge = gluon charge!

COLOUR CHARGED ANTENNA



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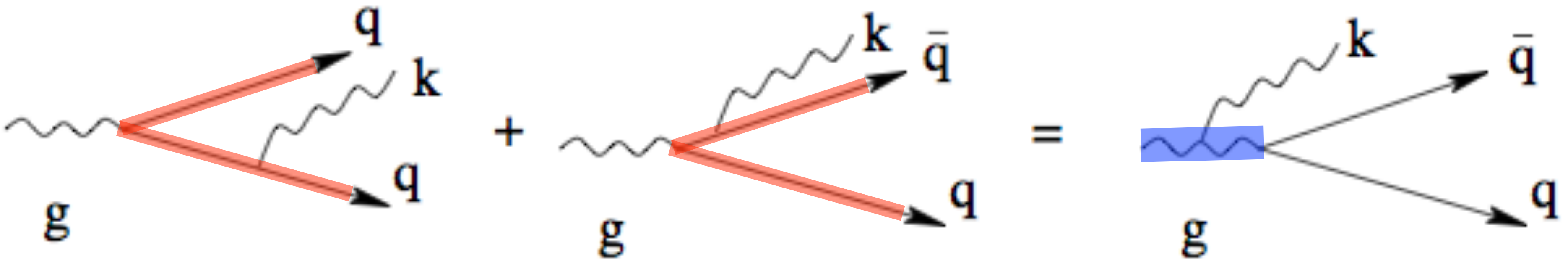
total charge = gluon charge!

Small angles: quarks

$$\omega\frac{dN_g}{d\omega d^2k_\perp} \propto \frac{\alpha_s C_F}{k_\perp^2} + (q \to \bar{q})$$

$$\theta \ll \theta_{q\bar{q}} \;(k_\perp \ll \omega\theta_{q\bar{q}})$$

COLOUR CHARGED ANTENNA



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Small angles: quarks

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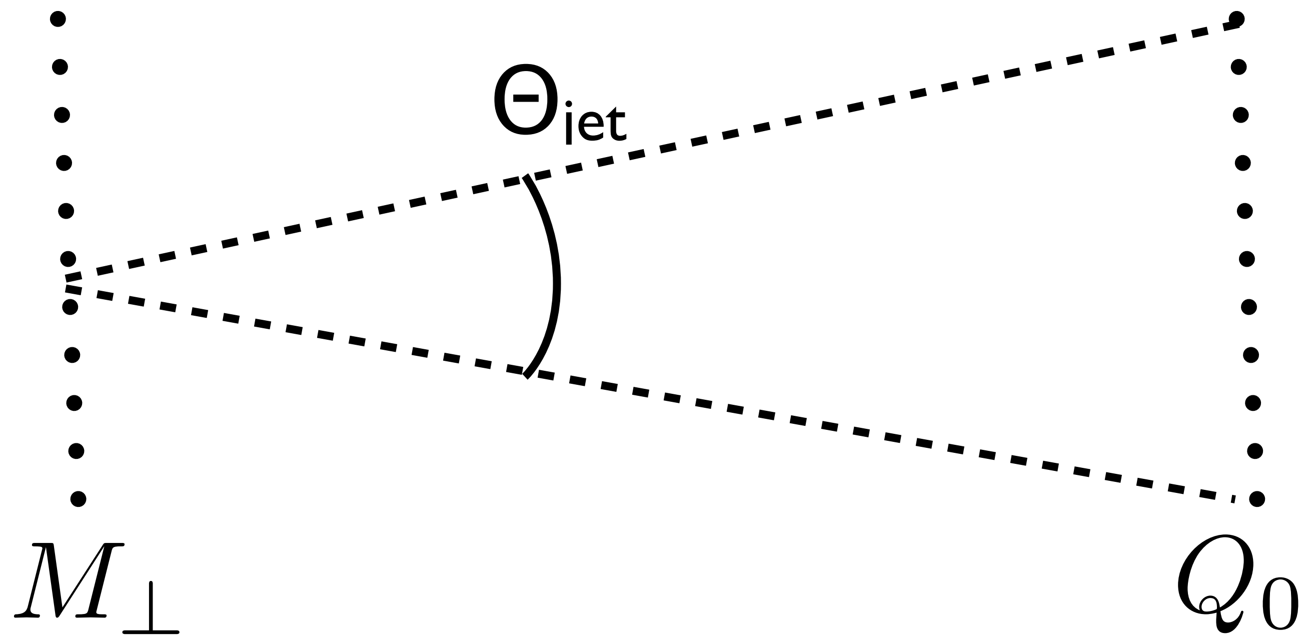
$$\theta \ll \theta_{q\bar{q}}\ (k_\perp \ll \omega\theta_{q\bar{q}})$$

Large angles: gluon

$$\omega\frac{dN_g}{d\omega d^2k_\perp} \propto \frac{\alpha_s C_A}{k_\perp^2}$$

$$\theta \gg \theta_{q\bar{q}}\ (k_\perp \gg \omega\theta_{q\bar{q}})$$

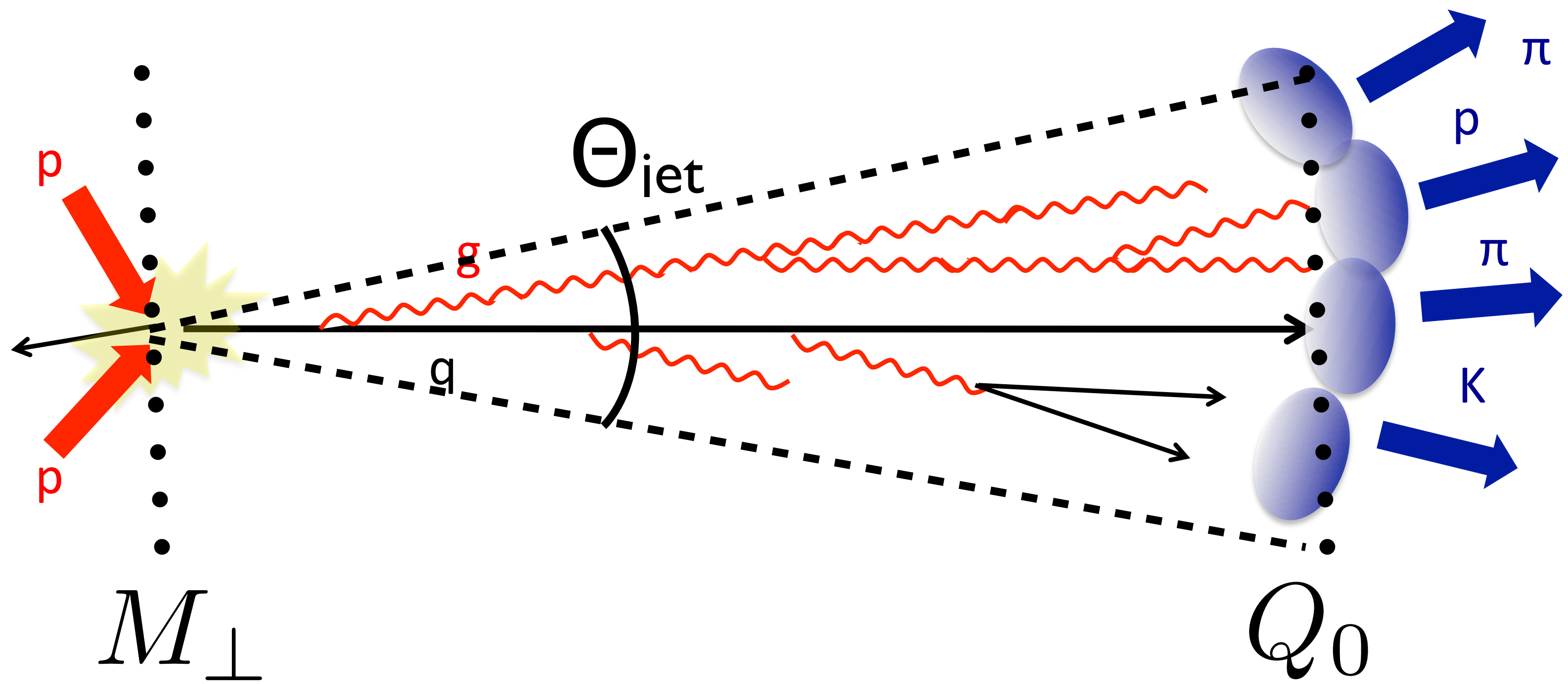
Part 3) Jet shower evolution equation



Global jet scales

$$M_\perp = E\Theta_{\rm jet}$$

$$Q_0 \sim \Lambda_{\rm QCD}$$



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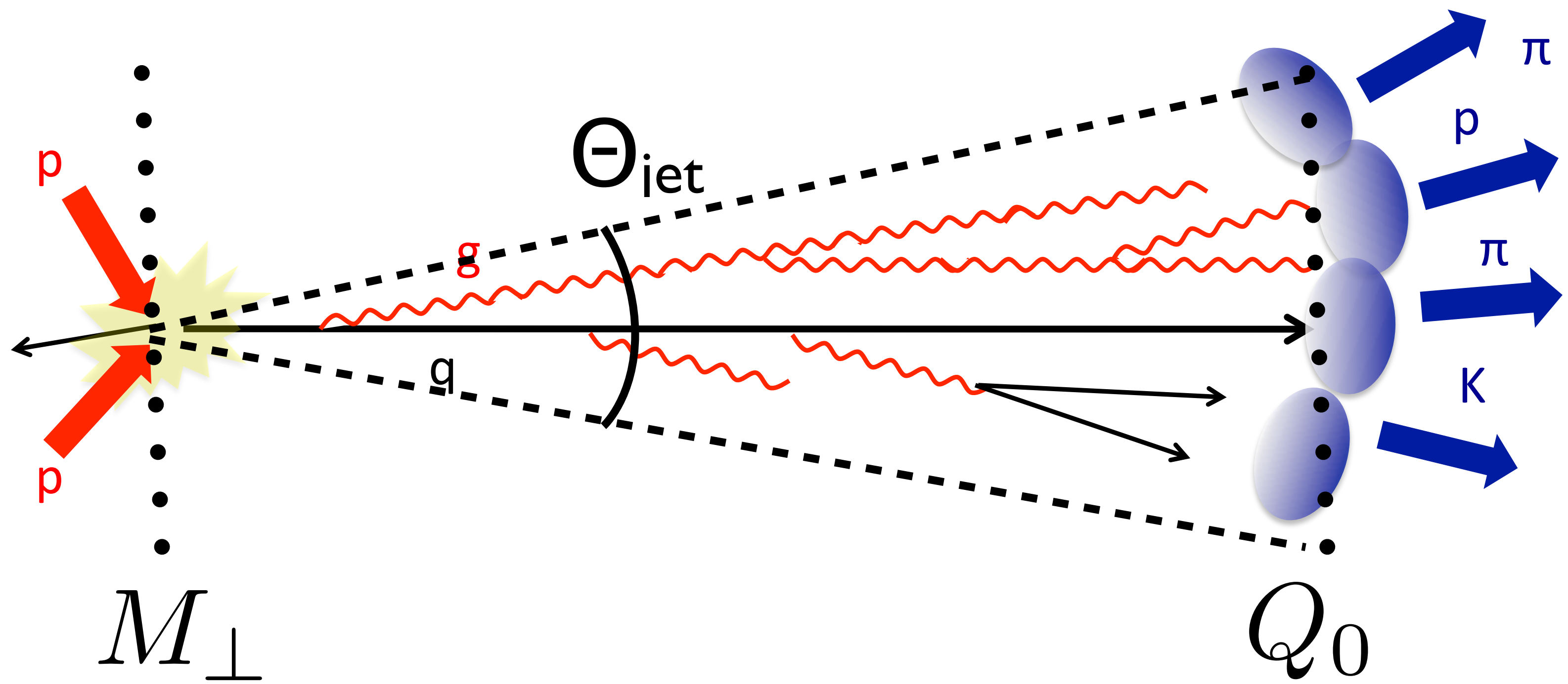


$$t_f \sim \frac{k_\parallel}{k_\perp^2} \sim \frac{1}{E}$$

$$t_{\rm had} \sim \frac{E}{\Lambda_{\rm QCD}^2}$$

$$t_{\rm had} \sim \frac{1-100}{(0.2)^2 \cdot 5}\ {\rm fm} = 5 - 500\ {\rm fm}$$

Global jet scales

$$M_\perp = E\Theta_{\rm jet}$$

$$Q_0 \sim \Lambda_{\rm QCD}$$

SPLITTING PROBABILITY

$$\frac{dN_q}{dx\, d\theta} = \frac{\alpha_s C_F}{\pi} \frac{1}{x} \frac{1}{\theta} \Theta(\theta_0 - \theta)$$

$$d\mathcal{P}_{\rm A}^{\rm BC} = \frac{\alpha_s}{\pi} P_{\rm A}^{\rm BC}(z) dz \frac{d\theta}{\theta} \Theta(\theta_0 - \theta)$$

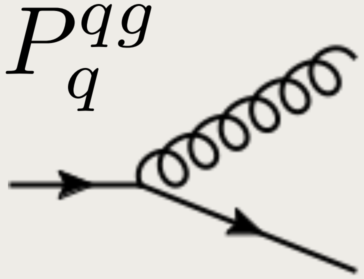
Sudakov form factor:
probability of no splitting

$$\Delta_{\rm A}(\theta_0, \theta) = \exp\left[-\int_\theta^{\theta_0} d\theta' \int_0^1 dz \sum_{\rm B,C} d\mathcal{P}_{\rm A}^{\rm BC}\right]$$

for now we will only consider gluon branching!

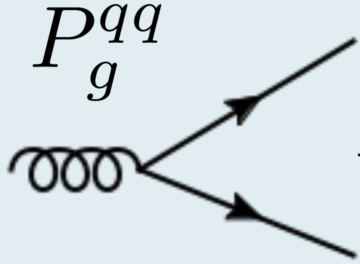
Altarelli-Parisi splitting functions (*z=1-x*)

P_q^{qg}



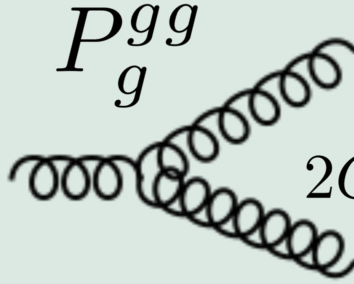
$$C_F \frac{1+z^2}{1-z}$$

$P_g^{q\bar{q}}$



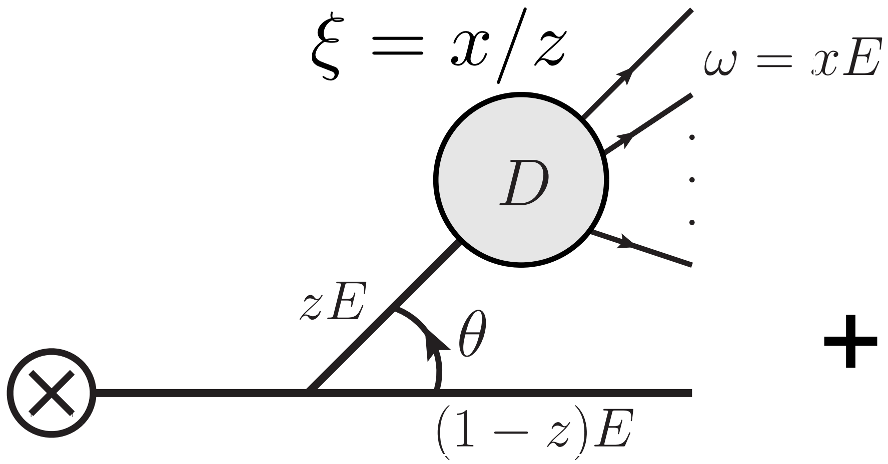
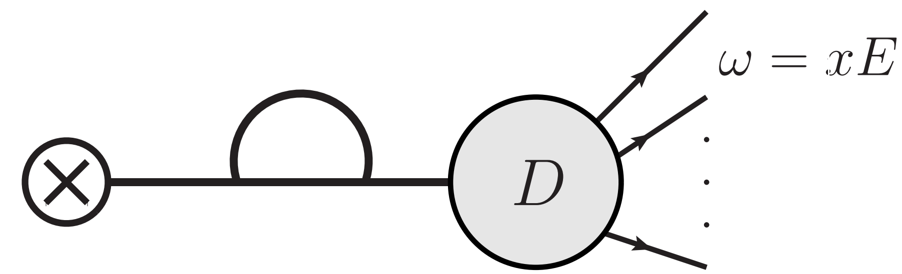
$$\frac{1}{2}(z^2 + (1-z)^2)$$

P_g^{gg}



$$2C_A \frac{(1 - z(1-z))^2}{z(1-z)}$$

GAIN & LOSS TERMS

$\delta D(x, M_\perp) =$  + 

Gain term :: particle formed within a sub-jet of energy $E' = zE$ and scale $k'_\perp = zE\theta$, whose distribution is probed at ξ

Loss term :: in course of a branching, the distribution of particles at x and zE is depleted by a splitting (virtual contribution)

$$\delta D_{\mathbb{G}} = \frac{\delta M_\perp}{M_\perp} \int_x^1 \mathrm{d}z \frac{\alpha}{2\pi} P(z) D\left(\frac{x}{z}, zM_\perp\right)$$

$$\delta D_{\mathbb{L}} = -\frac{\delta M_\perp}{M_\perp} D\left(x, M_\perp\right) \int_0^x \mathrm{d}z \frac{\alpha}{2\pi} P(z)$$

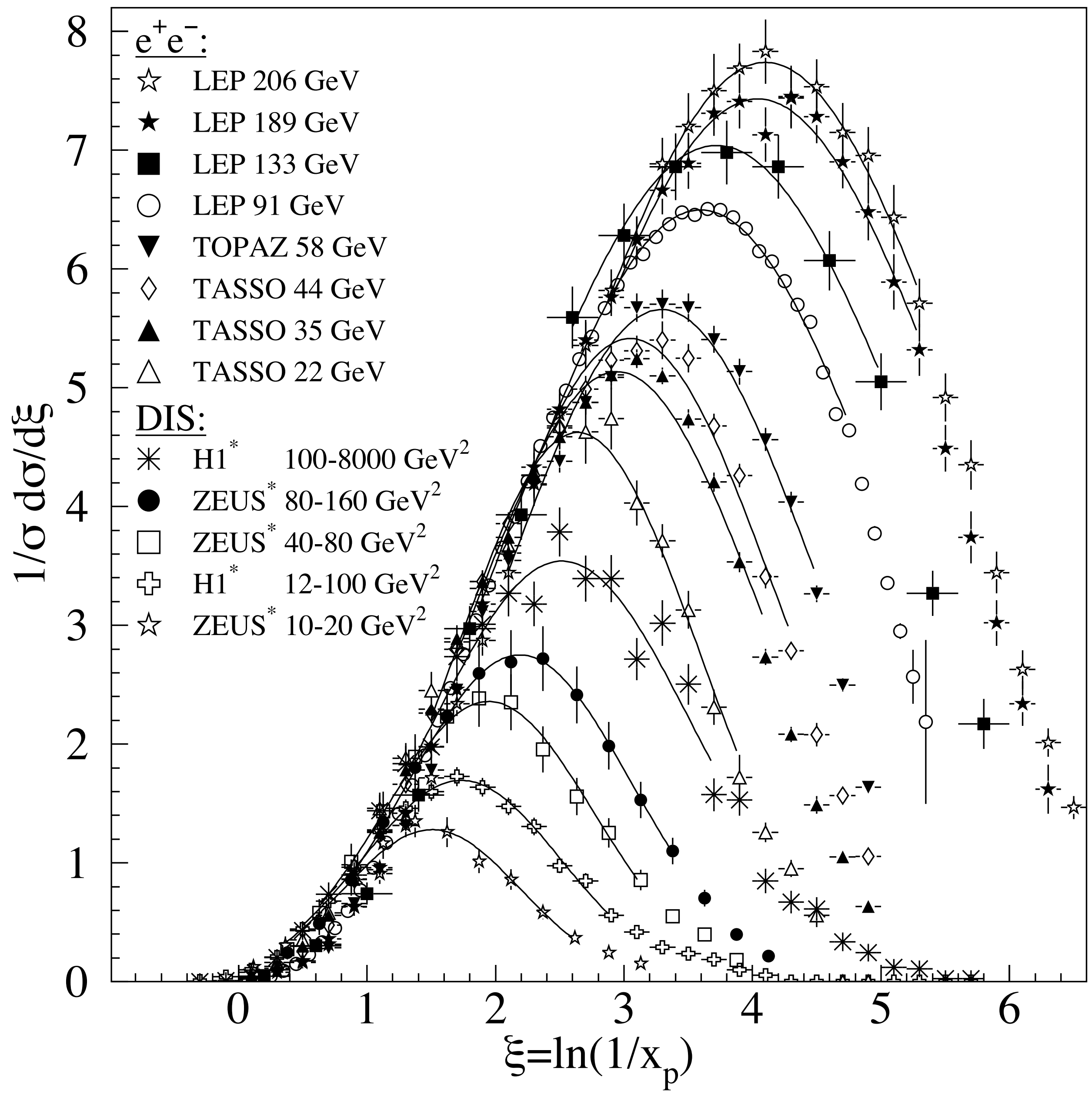
QCD EVOLUTION EQUATION

$$M_\perp \frac{\mathrm{d}}{\mathrm{d}M_\perp} D(x, M_\perp) = \int_x^1 \mathrm{d}z \frac{\alpha(k_\perp)}{2\pi} P(z) \left[D\left(\frac{x}{z}, zM_\perp\right) - \frac{1}{2} D(x, M_\perp) \right]$$

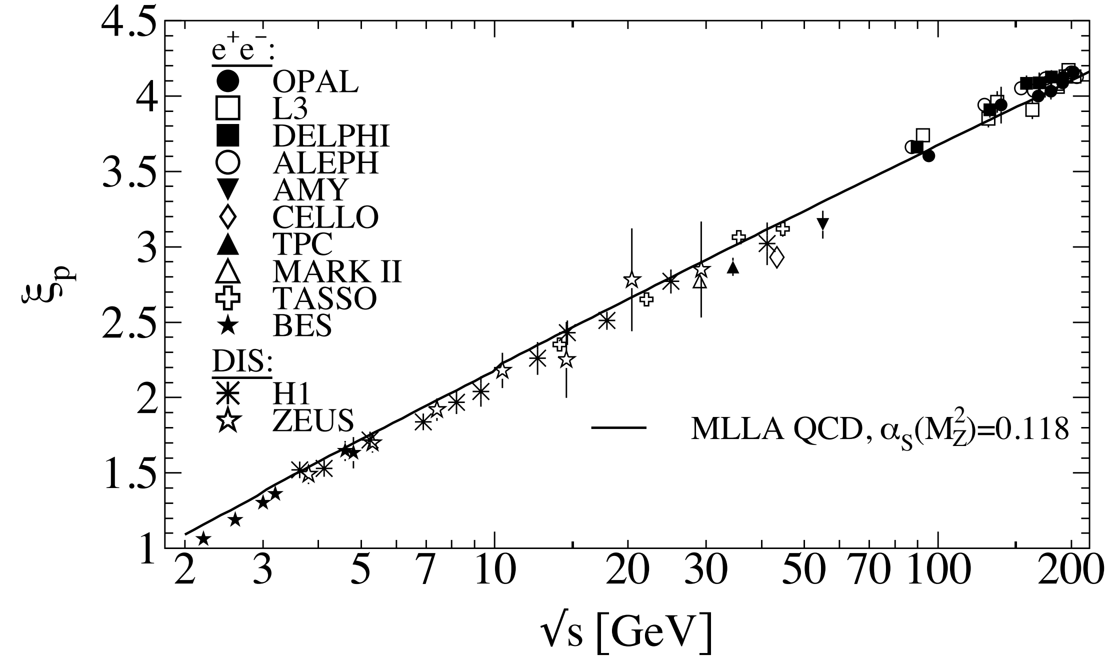
$$k_\perp = z(1-z)M_\perp$$

- coherent evolution: angular ordering
  - Double-Log Approximation
  - Modified Leading-Log Approximation
- resulting distribution has a maximum
  - suppression of the yield of soft particles
- similar to conventional DGLAP equation (which does not have angular ordering built in)

Interjet distribution: soft particles in the jet



hard ⟷ soft



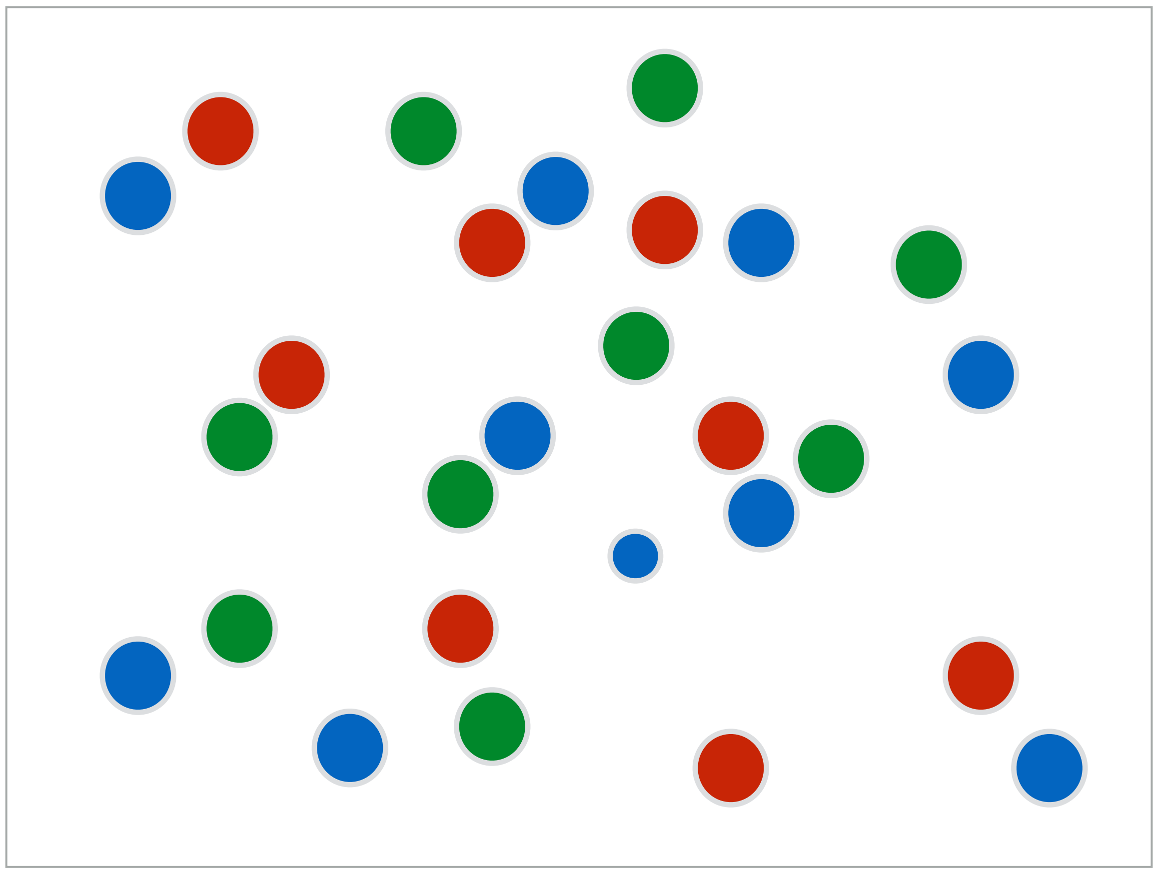
position of the peak

"Humpbacked" plateau

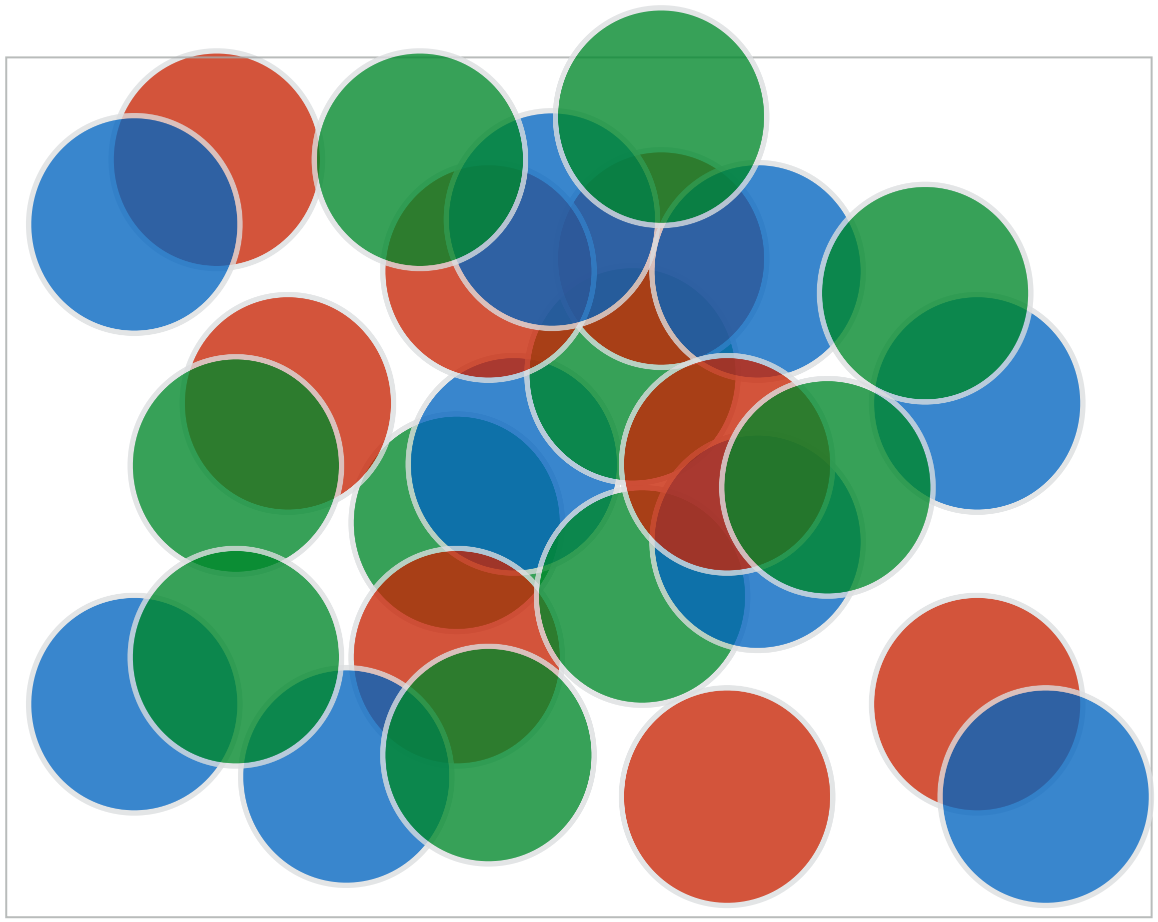
LECTURE 2

Part 1) Interactions with medium

A GLIMPSE OF THE QGP



weakly-coupled



strongly-coupled

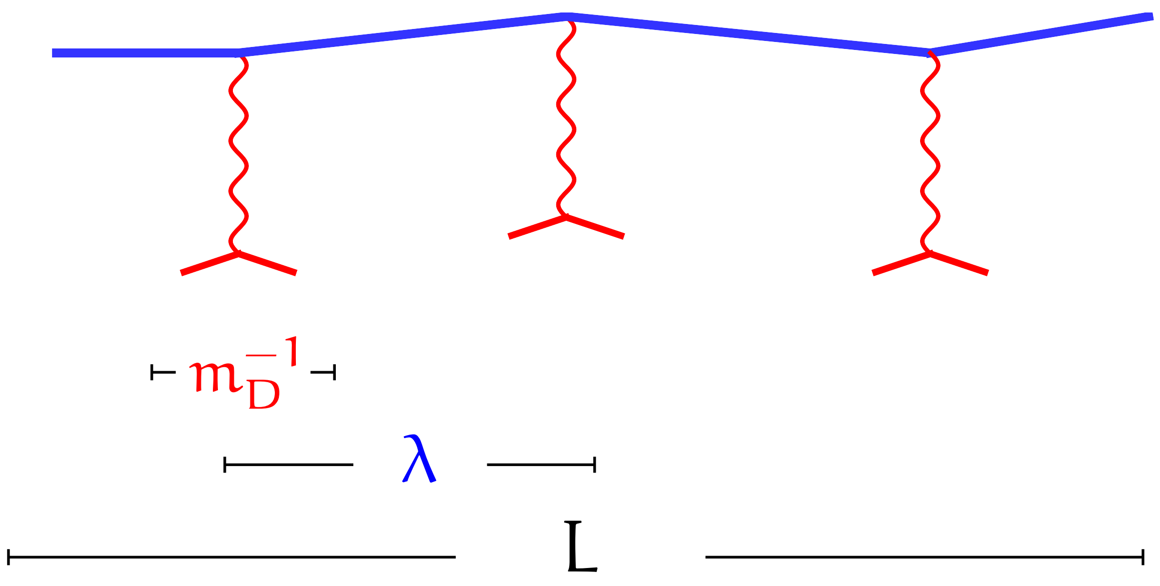
Simplest case $g \ll 1$ (*mostly* perturbative)

In the medium

1. Momentum broadening

$$\langle k_\perp^2 \rangle \sim \hat{q} t$$

$$m_D^{-1} \sim 1/(gT)$$
$$\lambda \sim 1/(g^2 T)$$



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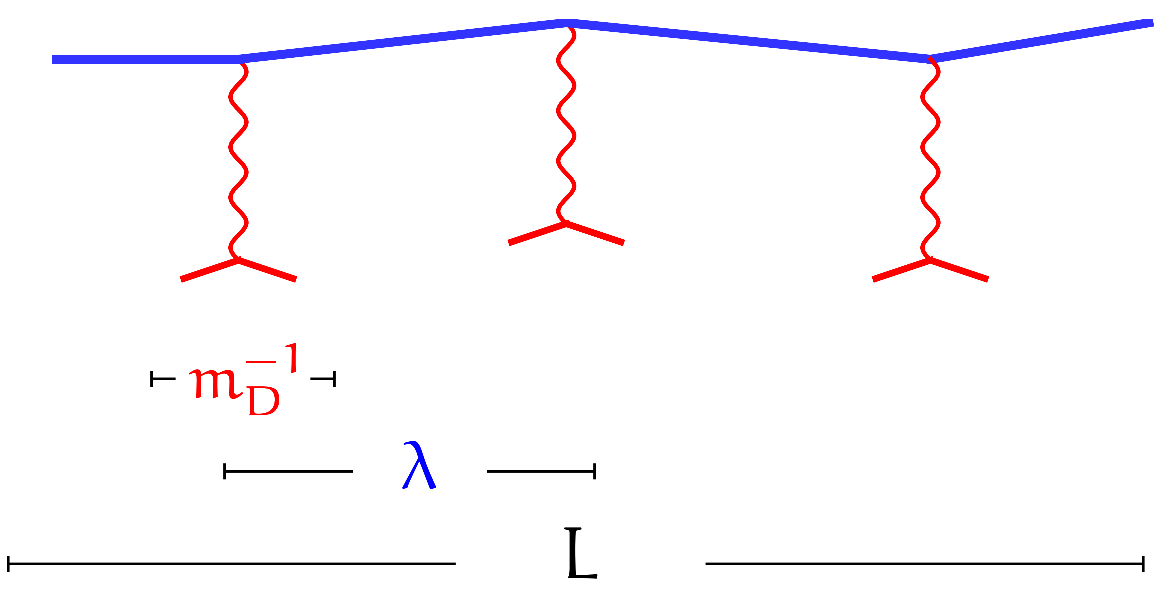
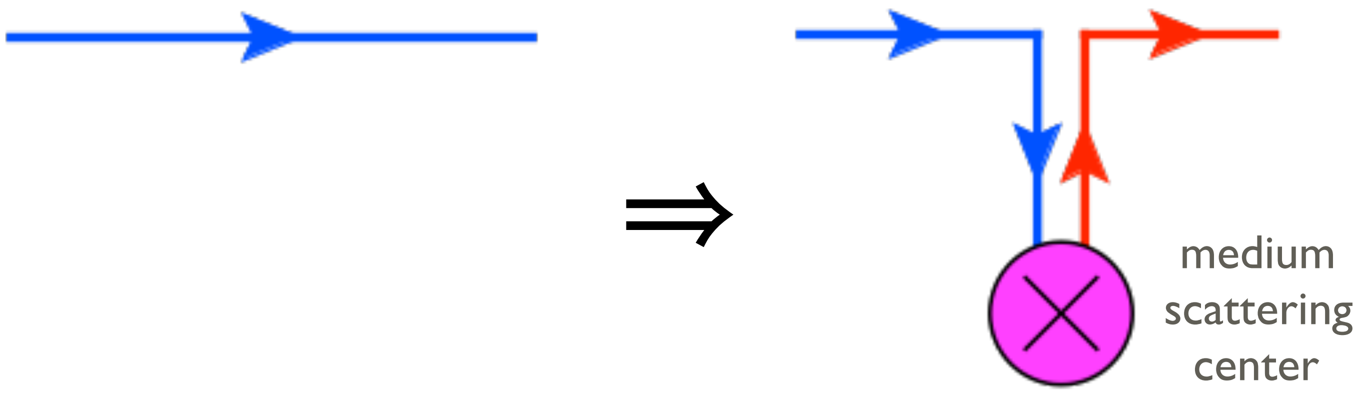


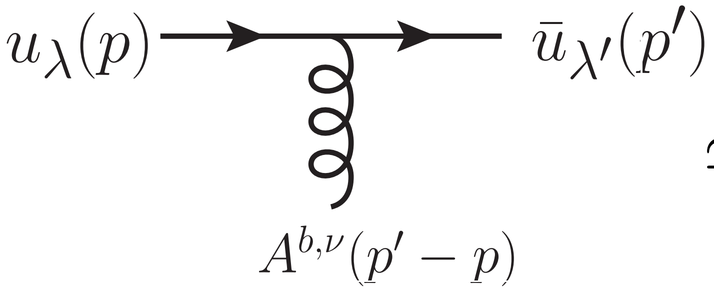
2. Color rotations

⇓

decoherence

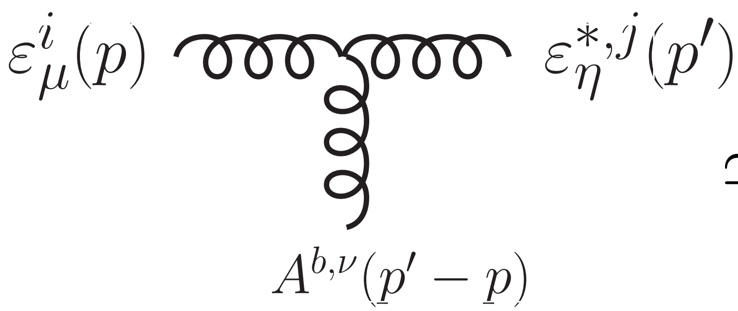


Eikonal interactions



spin

$$\simeq (ig_s)(2p^+)\delta^{\lambda,\lambda'}\, t^a \mathcal{A}^a(x^+, \boldsymbol{q})$$

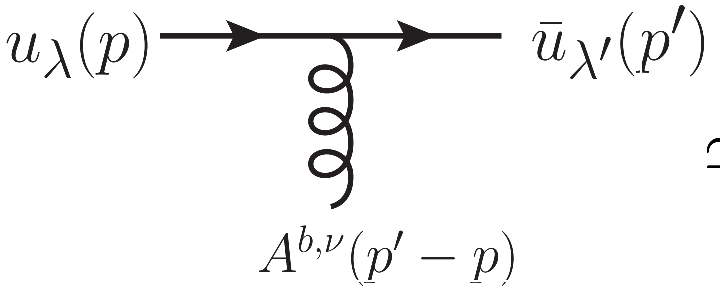


polarization

$$\simeq g_s(2p^+)\delta^{\lambda,\lambda'}\, f^{abc} \mathcal{A}^c(x^+, \boldsymbol{q})$$

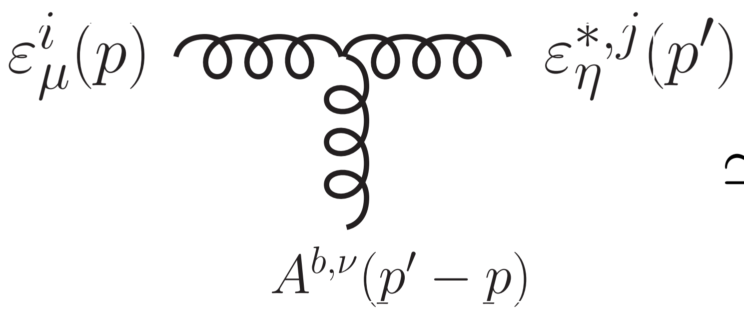
- conservation of energy during scattering
  - ***elastic energy loss can be neglected at high energies***
- no spin-flip or change of polarisation
- color precession

Eikonal interactions



spin

$$\simeq (ig_s)(2p^+)\delta^{\lambda,\lambda'}\, t^a \mathcal{A}^a(x^+,\boldsymbol{q})$$



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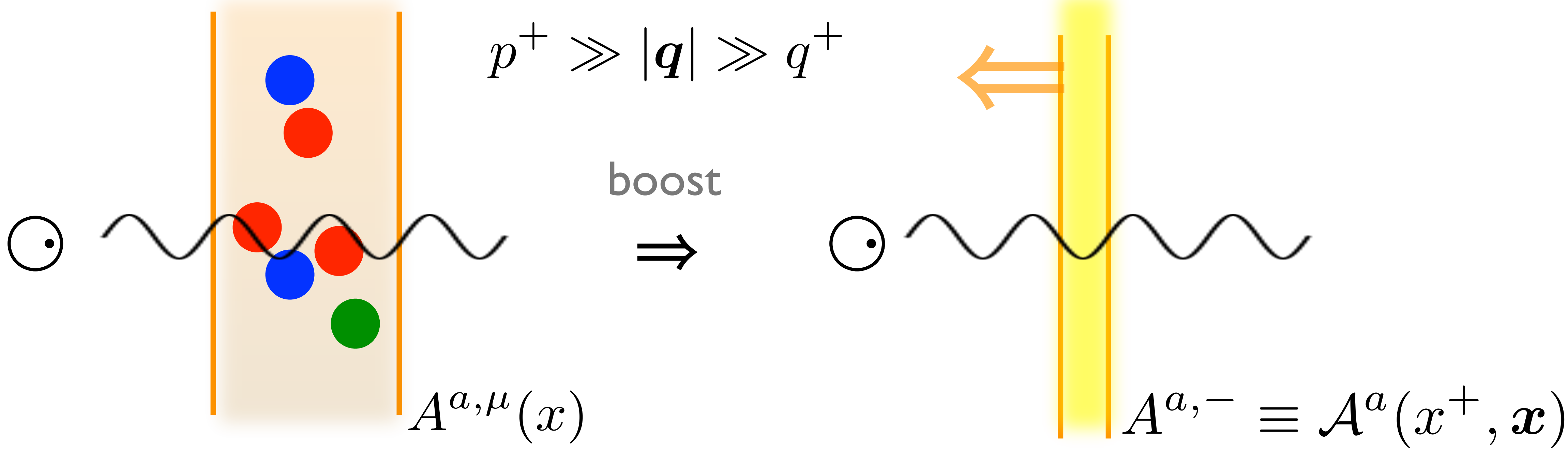
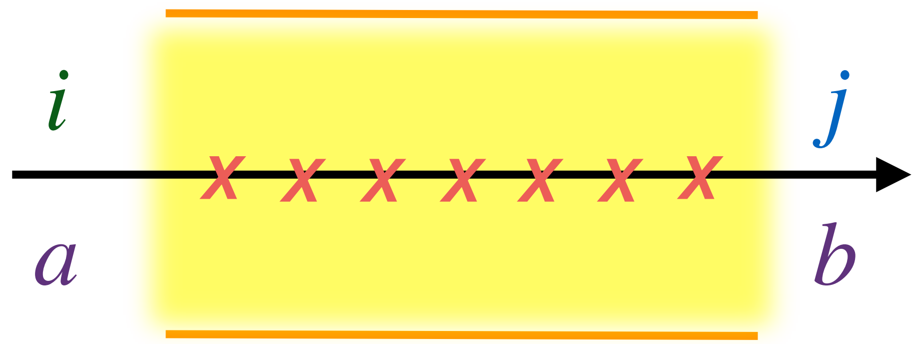


WILSON LINES



$$U(x_1^+, x_0^+; \boldsymbol{x}) = \mathcal{P} \exp\left[i g_s \int_{x_0^+}^{x_1^+} \mathrm{d}s\, T \cdot \mathcal{A}(s, \boldsymbol{x}(s)) \right]_{ij \atop ab}$$

$(T^a)_{ij} = t^a{}_{ij}$

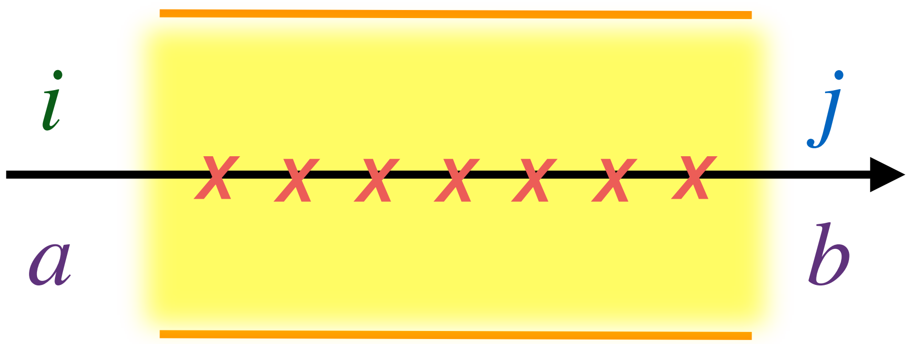
$(T^c)_{ab} = i\, f^{acb}$

colour matrix: describes colour rotation taking place from initial to final point

WILSON LINES

quark probe:

gluon probe:



$$U(x_1^+, x_0^+; \boldsymbol{x}) = \mathcal{P} \exp\left[i g_s \int_{x_0^+}^{x_1^+} \mathrm{d}s \, T \cdot \mathcal{A}(s, \boldsymbol{x}(s)) \right]_{ab}^{ij}$$

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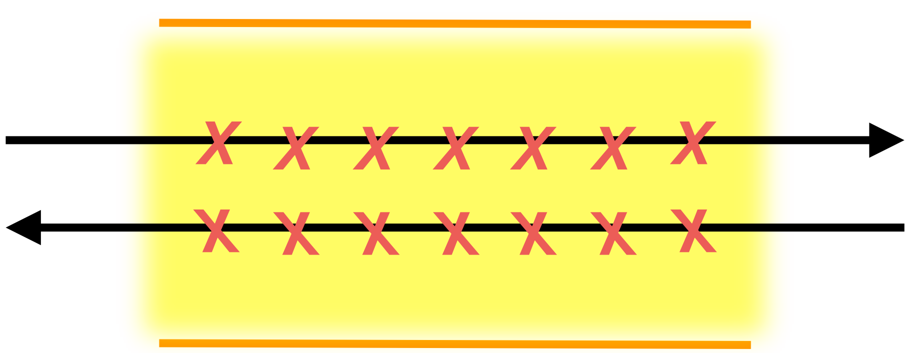
colour matrix: describes colour rotation taking place from initial to final point

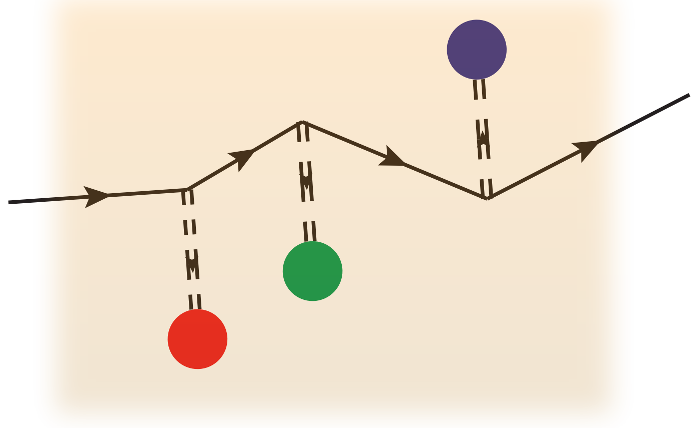


for physical processes: colour singlet

$$S(\boldsymbol{x} - \boldsymbol{y}) \sim U(\boldsymbol{x}) U^\dagger(\boldsymbol{y})$$

normalisation $\quad S(0) = 1$

BROADENING



- Green's function for propagation in the medium
  - EOM Schrödinger's equation in 2D
- solution in form of a path integral
  - accounts for fluctuations around the eikonal path

$$\left[i\frac{\partial}{\partial t} + \frac{\boldsymbol{\partial}^2}{2E} + g\mathcal{A}(t,\boldsymbol{x})\right]\mathcal{G}(\boldsymbol{x},t;\boldsymbol{x}_0,t_0) = i\delta(t-t_0)\delta(\boldsymbol{x}-\boldsymbol{x}_0)$$

$$\begin{aligned}\mathcal{G}(\boldsymbol{x},t;\boldsymbol{x}_0,t_0) &= \int_{\boldsymbol{r}(t_0)=\boldsymbol{x}_0}^{\boldsymbol{r}(t)=\boldsymbol{x}} \mathcal{D}\boldsymbol{r}\, \exp\left[i\frac{E}{2}\int_{t_0}^{t} \mathrm{d}s\, \dot{\boldsymbol{r}}^2(s)\right] U(t,t_0;[\boldsymbol{r}(s)]) \\ &= \int_{\boldsymbol{r}(t_0)=\boldsymbol{x}_0}^{\boldsymbol{r}(t)=\boldsymbol{x}} \mathcal{D}\boldsymbol{r}\, \exp\left\{\int_{t_0}^{t} \mathrm{d}s \left[i\frac{E}{2}\dot{\boldsymbol{r}}^2(s) + ig_s T\cdot\mathcal{A}(s,\boldsymbol{r}(s))\right]\right\}\end{aligned}$$

MEDIUM AVERAGES

medium average: $$\frac{1}{N_c^2-1}\mathrm{tr}\langle U(0)U^\dagger(\boldsymbol{x})\rangle \sim \exp\left[-\frac{1}{4}\int \mathrm{d}s\, \hat{q}(s)\boldsymbol{x}^2(x)\right]$$

transport coefficient (→ $\hat{q}(s)$)

MEDIUM AVERAGES

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$$\frac{1}{N_c^2 - 1}\mathrm{tr}\langle U(0)U^\dagger(\boldsymbol{x})\rangle \sim \exp\left[-\frac{1}{4}\int \mathrm{d}s\, \hat{q}(s)\boldsymbol{x}^2(x)\right]$$

transport coefficient

$$\langle \mathcal{A}^a(x^+;\boldsymbol{q})\mathcal{A}^{*\,b}(x'^+;\boldsymbol{q}')\rangle = \delta^{ab} m_D^2 n(x^+)\,\delta(x^+ - x'^+)\,(2\pi)^2\delta(\boldsymbol{q}-\boldsymbol{q}')\mathcal{V}(\boldsymbol{q})$$

Medium potential:

$$\mathcal{V}(\boldsymbol{q}) \sim \boldsymbol{q}^{-4}$$

Yukawa screening

$$\boldsymbol{q}^{-4} \to (\boldsymbol{q}^2 + m_\mathrm{D}^2)^{-2}$$

Hard-Thermal-Loop screening

$$\boldsymbol{q}^{-4} \to \boldsymbol{q}^{-2}(\boldsymbol{q}^2 + m_\mathrm{D}^2)^{-1}$$

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Hard-Thermal-Loop screening

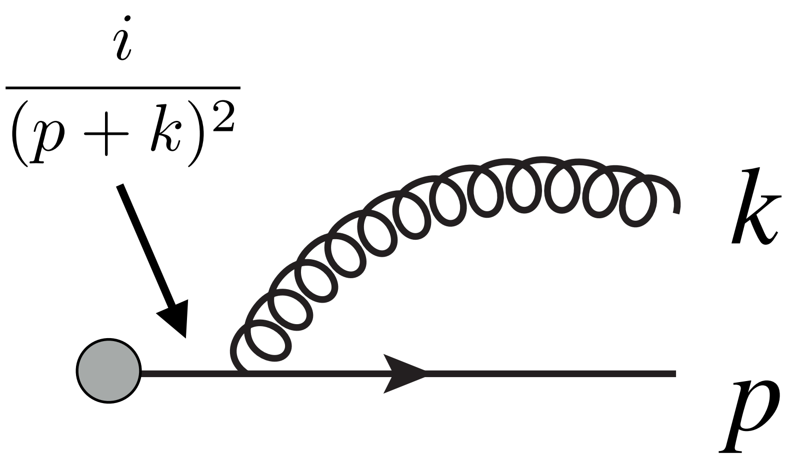
$$\boldsymbol{q}^{-4} \to \boldsymbol{q}^{-2}(\boldsymbol{q}^2+m_{\mathrm{D}}^2)^{-1}$$

Definition of $\hat{q}$

$$\hat{q} \sim n(x^+)\int \frac{\mathrm{d}^2\boldsymbol{q}}{(2\pi)^2}\boldsymbol{q}^2\mathcal{V}(\boldsymbol{q}) \sim \frac{m_{\mathrm{D}}^2}{\lambda}$$

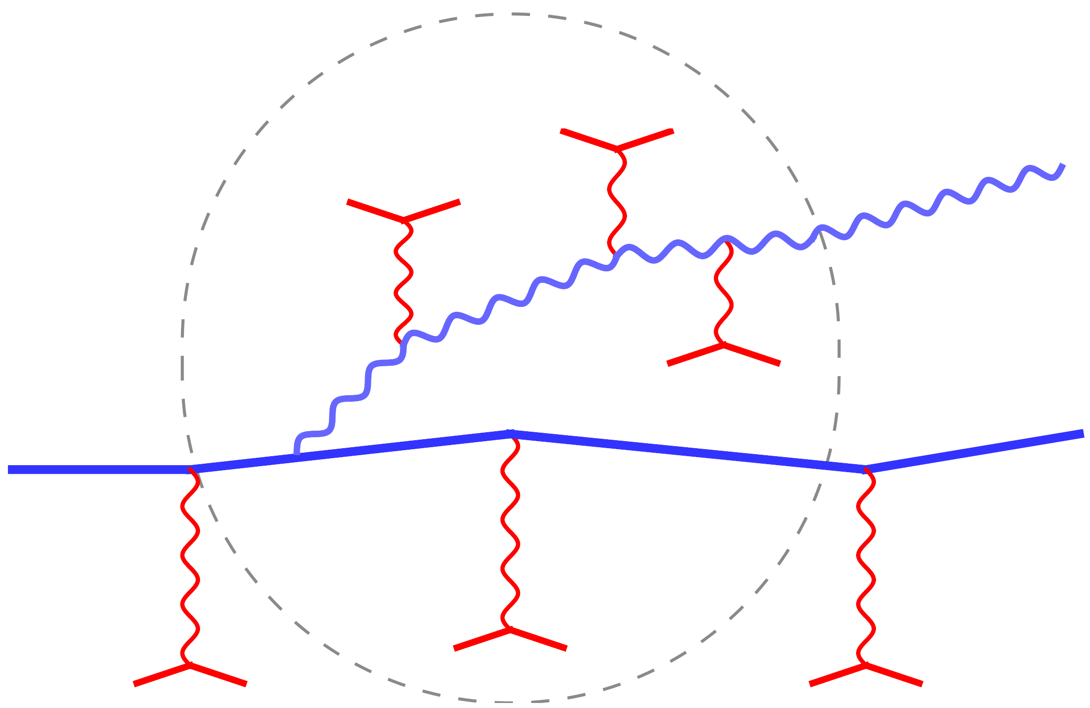
Part 2) Medium-induced radiation

Radiative processes in the medium



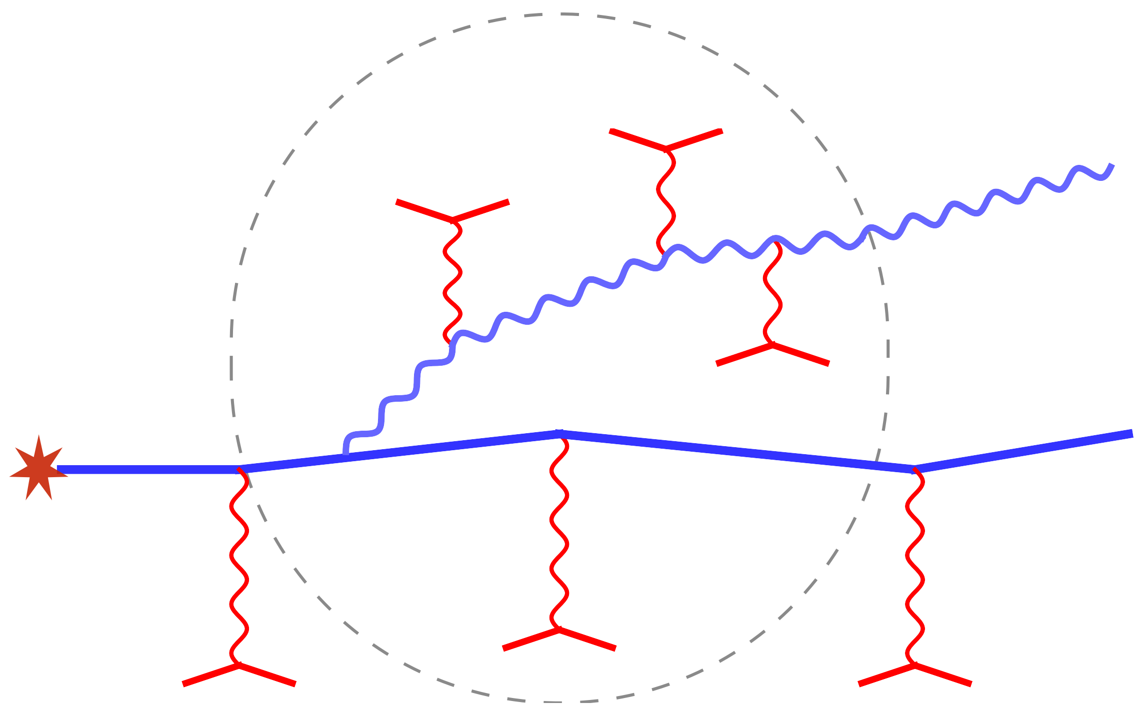
- additional radiation from interactions with the medium
- in vacuum: radiation due to off-shellness
  - hard process accelerates the particle to the speed of light
- in medium: an on-shell quark/gluon can radiate
  - transverse momentum of emitted gluon from accumulated kicks in the medium
- for jet quenching: accelerate a particle through a QGP!

GLUON RADIATION: AMPLITUDE



$$\mathcal{M}^{(a,i)}_{(\lambda,s)}(p,k) = \int_{\boldsymbol{k}',\boldsymbol{p}',\boldsymbol{p}_0} \int_0^\infty \mathrm{d}t\, \mathcal{G}(\boldsymbol{k},L;\boldsymbol{k}',t|zE)^{ab} \frac{1}{2E}$$
$$\times \left[\mathcal{G}(\boldsymbol{p},L;\boldsymbol{p}'-\boldsymbol{k}',t|(1-z)E)\, V^b_{\lambda,s,s'}(\boldsymbol{k}'-z\boldsymbol{p}',z)\, \mathcal{G}(\boldsymbol{p}',t;\boldsymbol{p}_0,0|E)\right]^{ij} \mathcal{M}^j_{s'}(p_0)$$

Gluon radiation: amplitude



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hard vertex

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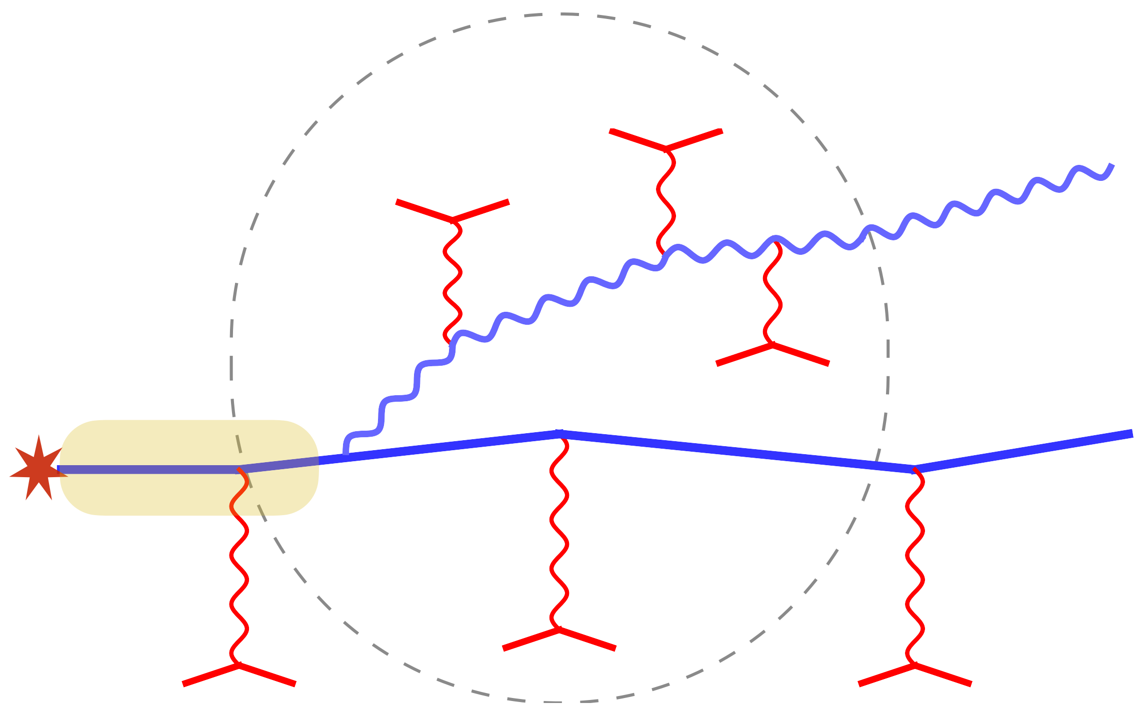

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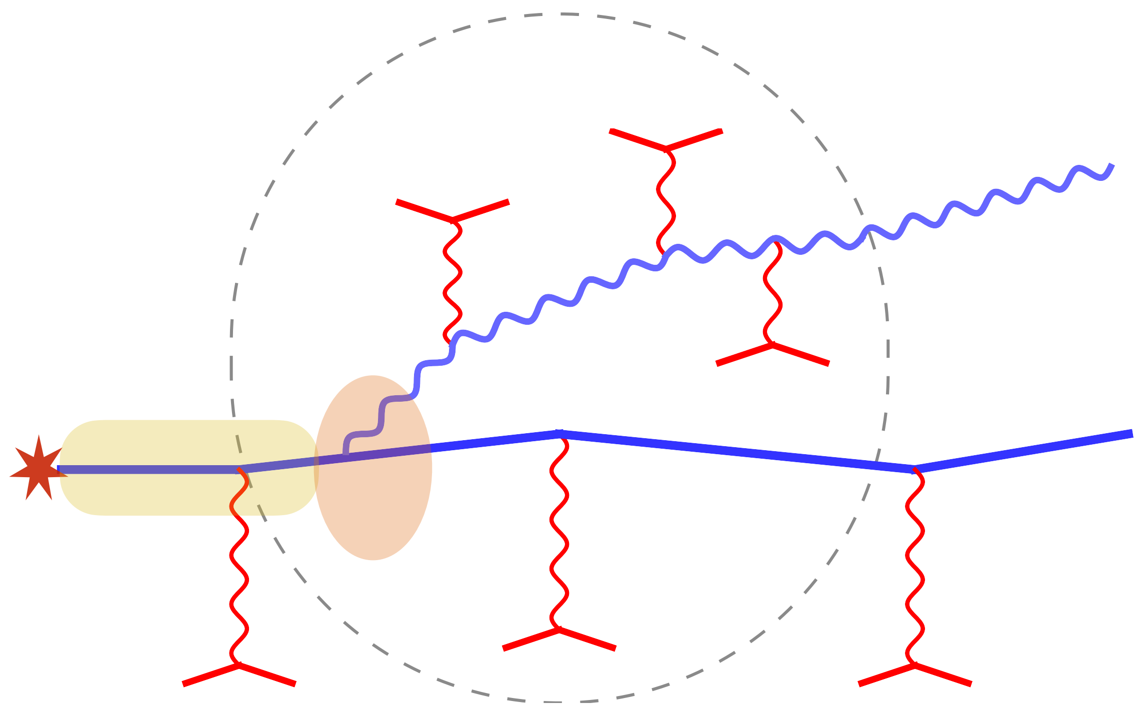
initial quark — hard vertex

GLUON RADIATION: AMPLITUDE



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radiation vertex — initial quark — hard vertex

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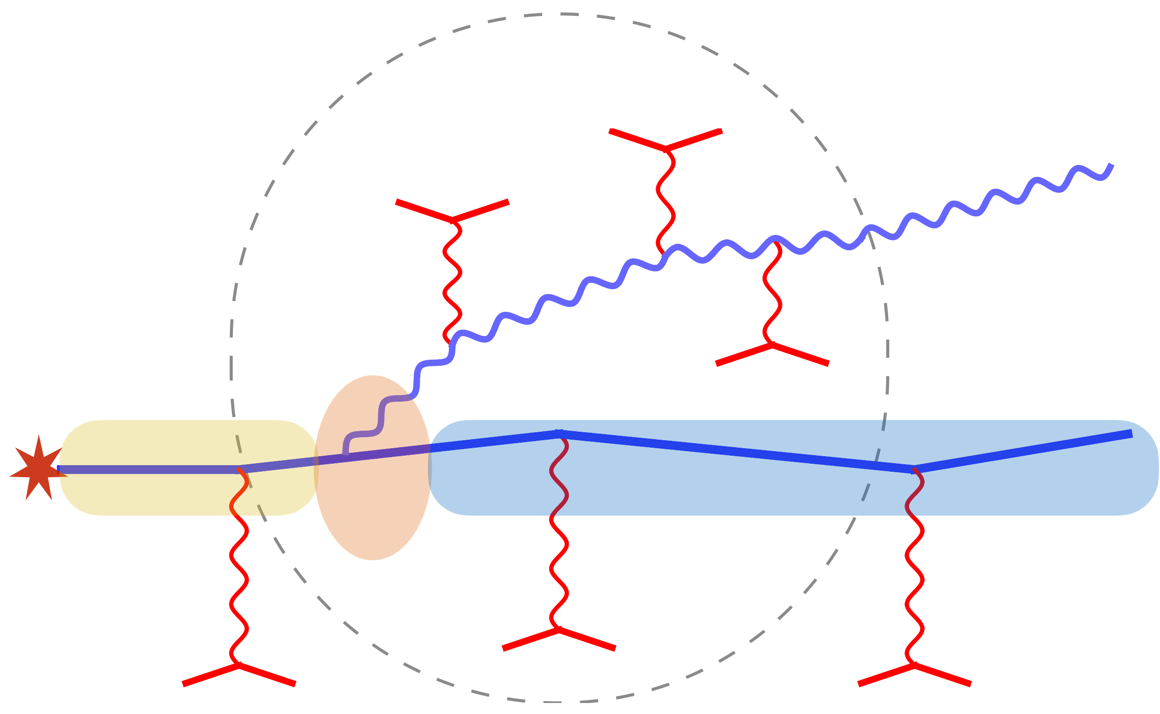

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final quark propagator — radiation vertex — initial quark — hard vertex

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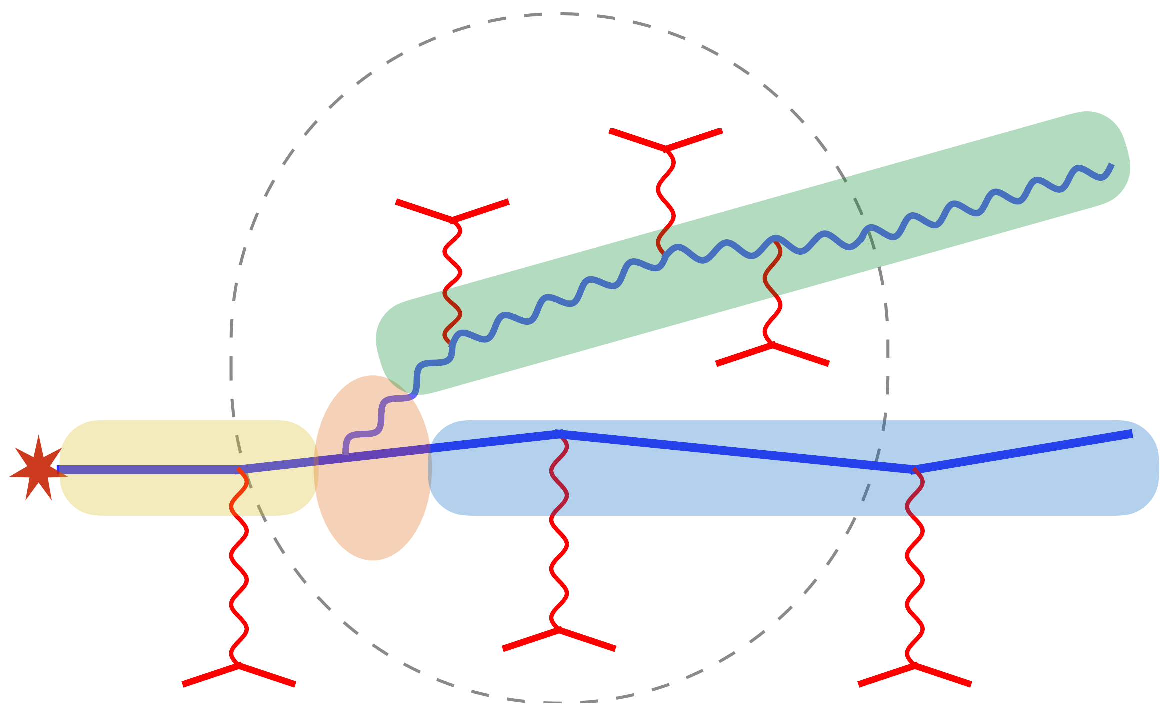

gluon propagator

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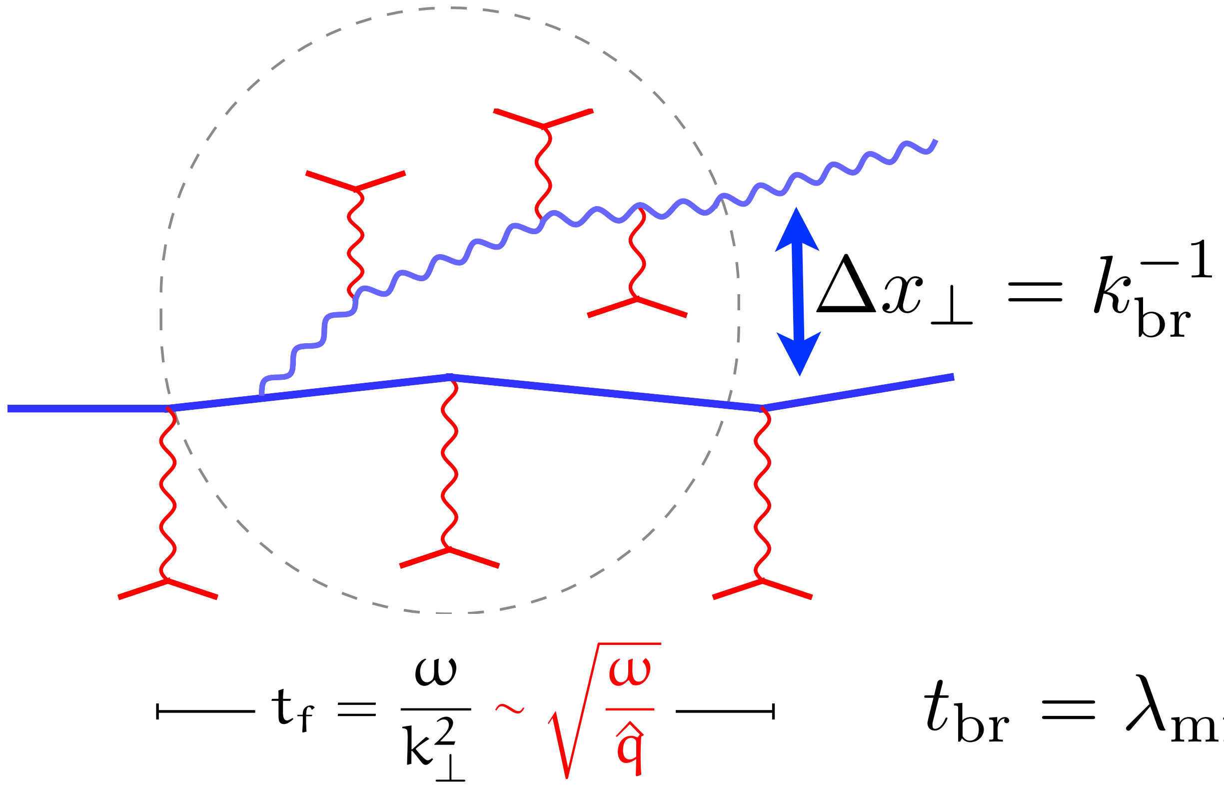
QUALITATIVE: MULTIPLE SCATTERINGS



Longitudinal coherence

induces a characteristic formation time larger than mean free path

$$t_{\rm br} = \lambda_{\rm mfp} N_{\rm coh}$$
$$k_{\rm br}^2 = \mu^2 N_{\rm coh}$$

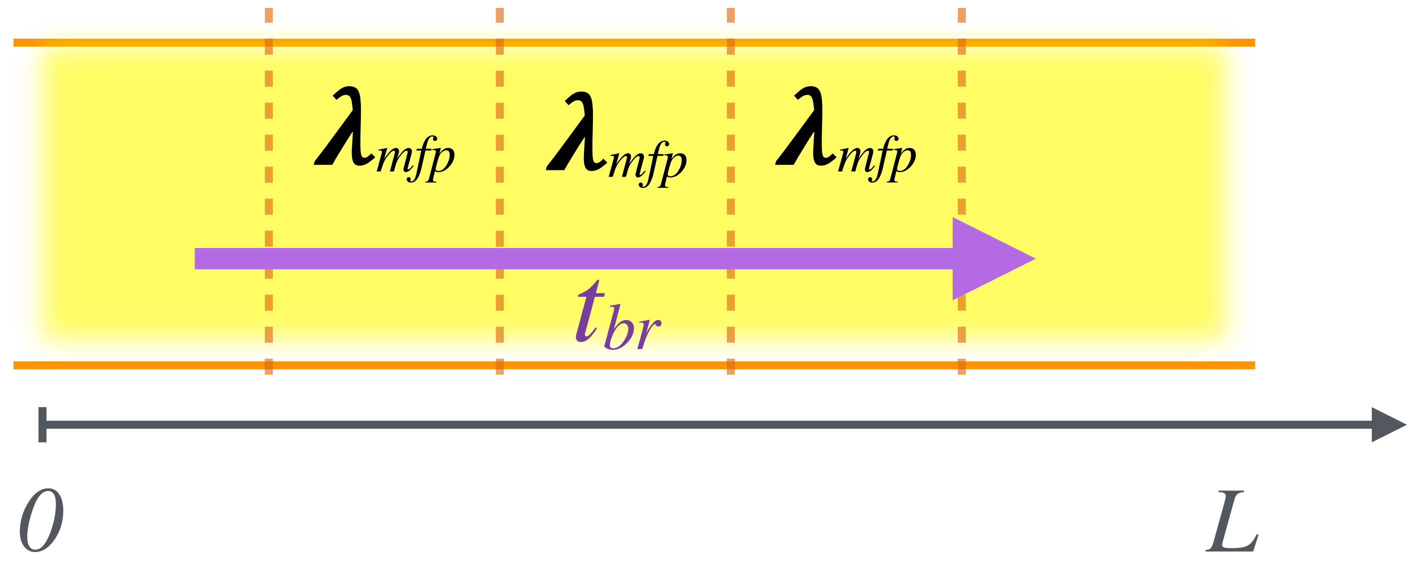
$$\Bigg\} \quad t_{\rm br} = \sqrt{\omega/\hat{q}}, \qquad k_{\rm br}^2 = \sqrt{\hat{q}\omega}$$

$$\hat{q} = \frac{\mu^2}{\lambda_{\rm mfp}}$$

Landau-Pomeranchuk-Migdal effect

- soft gluons are produced with very short times $t \sim \sqrt{\omega}$!
- opposite to vacuum (at finite angle) $t \sim 1/\omega\vartheta^2$

BDMPS-Z SPECTRUM



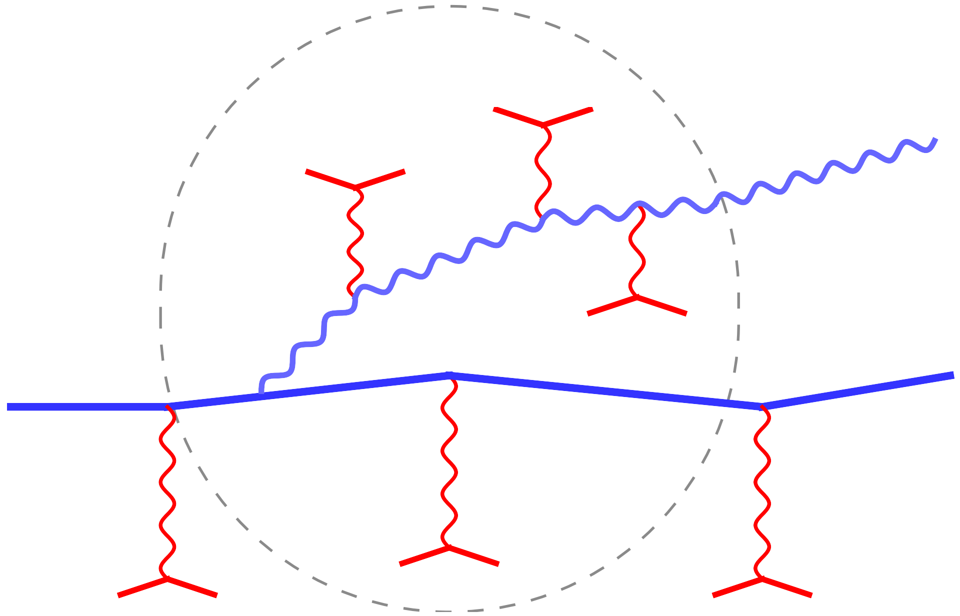
$$\omega \frac{\mathrm{d}N^{N=1}}{\mathrm{d}\omega\, \mathrm{d}L} \propto \frac{L}{\lambda_{\mathrm{mfp}}} \frac{\bar{\alpha}}{t_{\mathrm{coh}}} = \frac{L}{\lambda_{\mathrm{mfp}}} \frac{\bar{\alpha}\mu^2}{\omega} \quad \Rightarrow \quad \omega \frac{\mathrm{d}N^{LPM}}{\mathrm{d}\omega\, \mathrm{d}L} \propto \frac{\bar{\alpha}}{t_{\mathrm{br}}} = \bar{\alpha}\sqrt{\frac{\hat{q}}{\omega}}$$

Characteristic (maximal) gluon energy: $\omega_c = \frac{1}{2}\hat{q}L^2$

$\omega \gg \omega_c$ $$z\frac{dI^{\mathrm{ind}}}{dz} \simeq \frac{2\alpha_s C_R}{12\pi}\left(\frac{\omega_c}{\omega}\right)^2$$ spectrum strongly suppressed

Bethe-Heitler regime $t_{\mathrm{br}} \sim \lambda_{\mathrm{mfp}}$ $$\sqrt{\frac{\omega_{\mathrm{BH}}}{\hat{q}}} = \lambda \Rightarrow \omega_{\mathrm{BH}} = \lambda^2\hat{q} \sim \lambda m_D^2$$

MULTIPLICITY



$$t_f = \frac{\omega}{k_\perp^2} \sim \sqrt{\frac{\omega}{\hat{q}}}$$

$$\omega \frac{dI}{d\omega} = \bar{\alpha}\sqrt{\omega_c/\omega}$$

$$\omega_c = \hat{q}L^2$$

$$N(\omega) = \int_\omega^\infty \frac{dI}{d\omega} \sim \bar{\alpha}\sqrt{\frac{\omega_c}{\omega}}$$

multiplicity above a certain energy ω

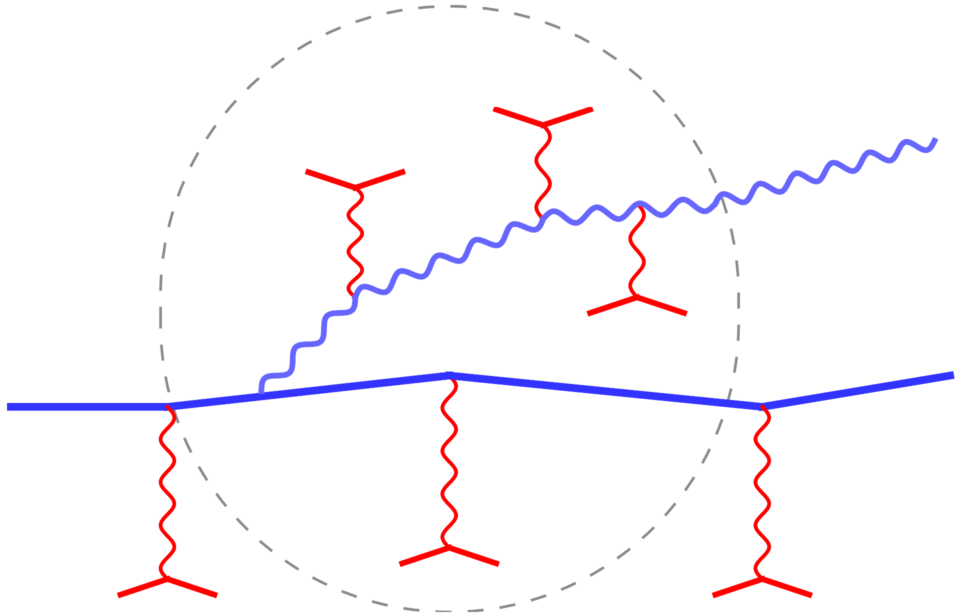
$N(\omega_c) \sim \mathcal{O}(\bar{\alpha})$ — rare emissions, hard BDMPS

$N(\omega_s) \sim \mathcal{O}(1)$ — copious production, need for resummation, large fluctuations

Baier, Dokshitzer, Mueller, Peigné, Schiff (1997-2000), Zakharov (1996), Wiedemann (2000), Gyulassy, Levai, Vitev (2000), Arnold, Moore, Yaffe (2001)

$$\omega_s = \bar{\alpha}^2\hat{q}L^2$$

MULTIPLICITY



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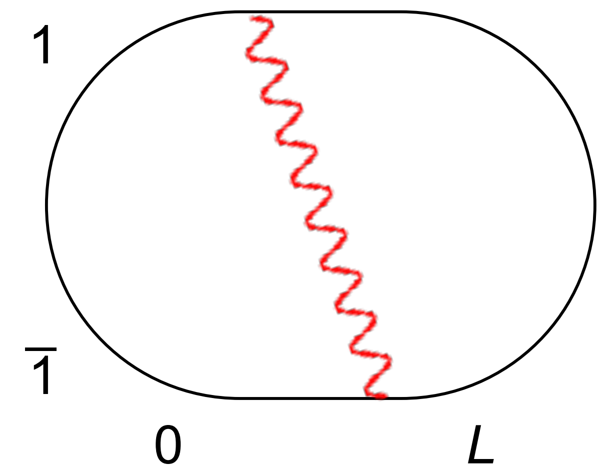
Colour flow diagram

Note: all lines are dressed propagators (Wilson lines)

⇒



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TWO REGIMES

$$t_{\rm br}(\omega) = \sqrt{\frac{\omega}{\hat{q}}}$$

$$t_{\rm br}(\omega_c) \sim \mathcal{O}(L)$$

takes a long time to form, emerge at *the end of the medium*

$$t_{\rm br}(\omega_s) \sim \bar{\alpha}\mathcal{O}(L)$$

produced rapidly, further branching highly probable

Blaizot, Mehtar-Tani, Iancu PRL (2013)

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$$\theta_{\rm br}(\omega) = \sqrt[4]{\frac{\hat{q}}{\omega^3}}$$

$$\theta_{\rm br}(\omega_c) \sim \sqrt{\frac{1}{\hat{q}L^3}} \equiv \theta_c$$

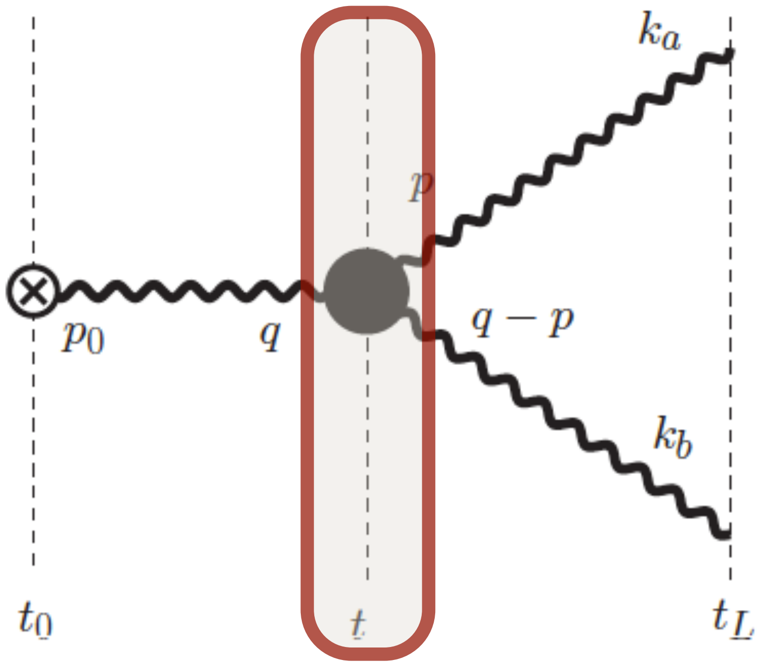
minimal angle!

$$\theta_{\rm br}(\omega_s) \sim \frac{1}{\bar{\alpha}^{3/2}}\theta_c$$

energy transported to parametrically large angles

Blaizot, Fister, Mehtar-Tani NPA (2015); Kurkela, Wiedemann PLB (2015); Iancu, Wu JHEP (2015);…

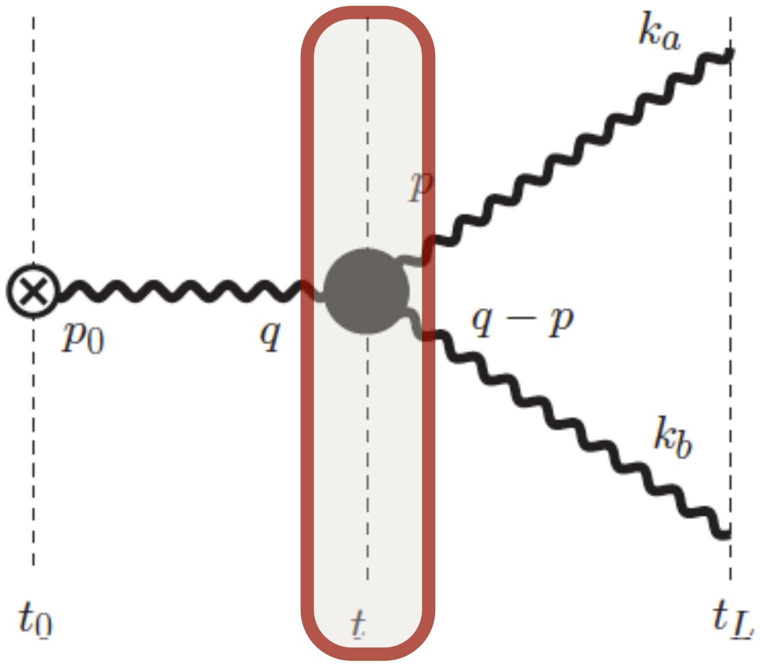
Factorisation



Emerging picture:

- for $t_{br} \ll L$ we can separate two processes
  - branching
  - broadening (we will neglect this at the moment since we are only interested in energy spectra)

FACTORISATION



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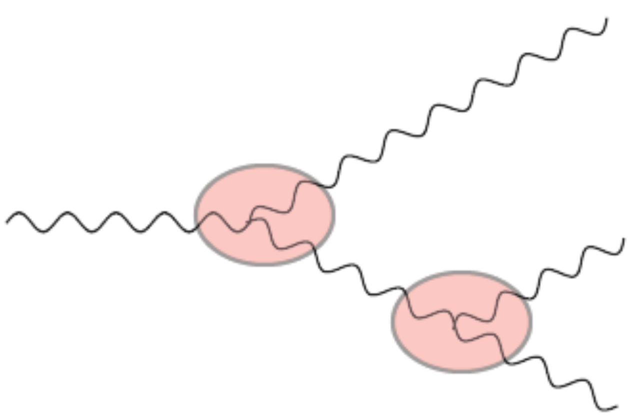
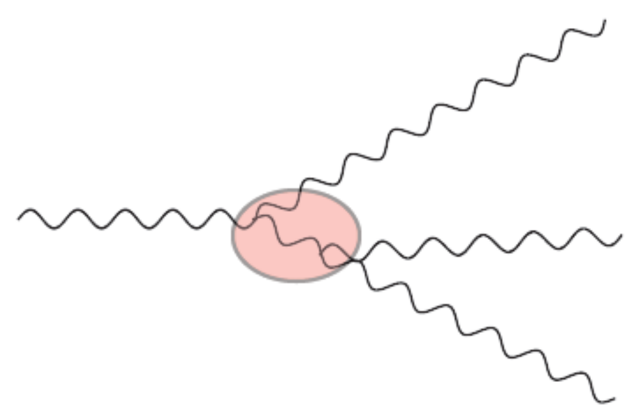


$$\sim \left(\alpha_s \frac{L}{t_f}\right)^2$$

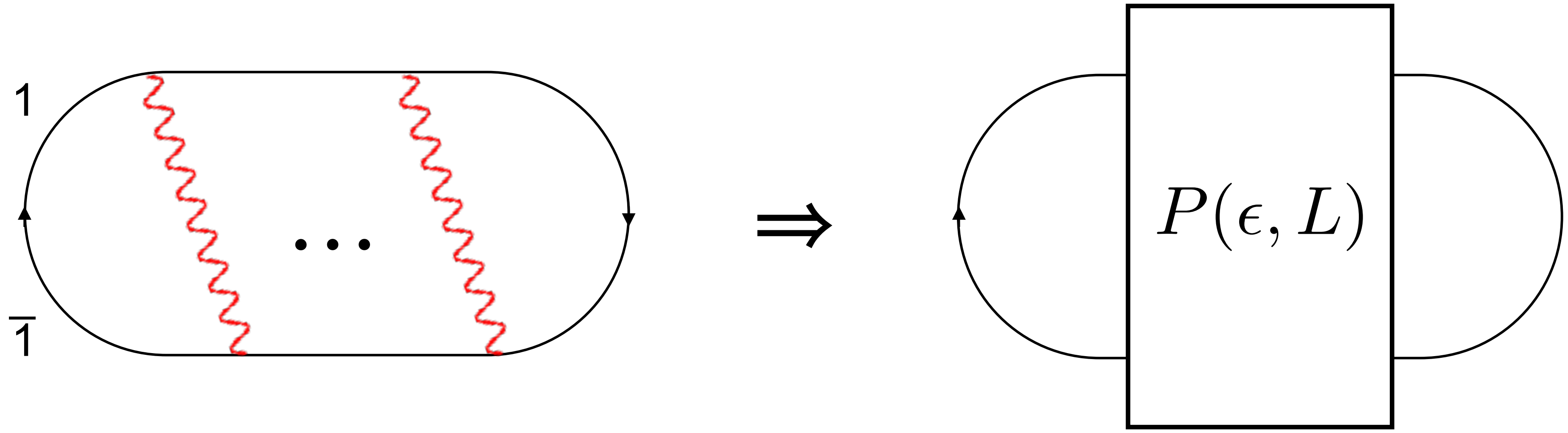


$$\sim \alpha_s^2 \frac{L}{t_f}$$

⇒ subsequent emissions are independent!

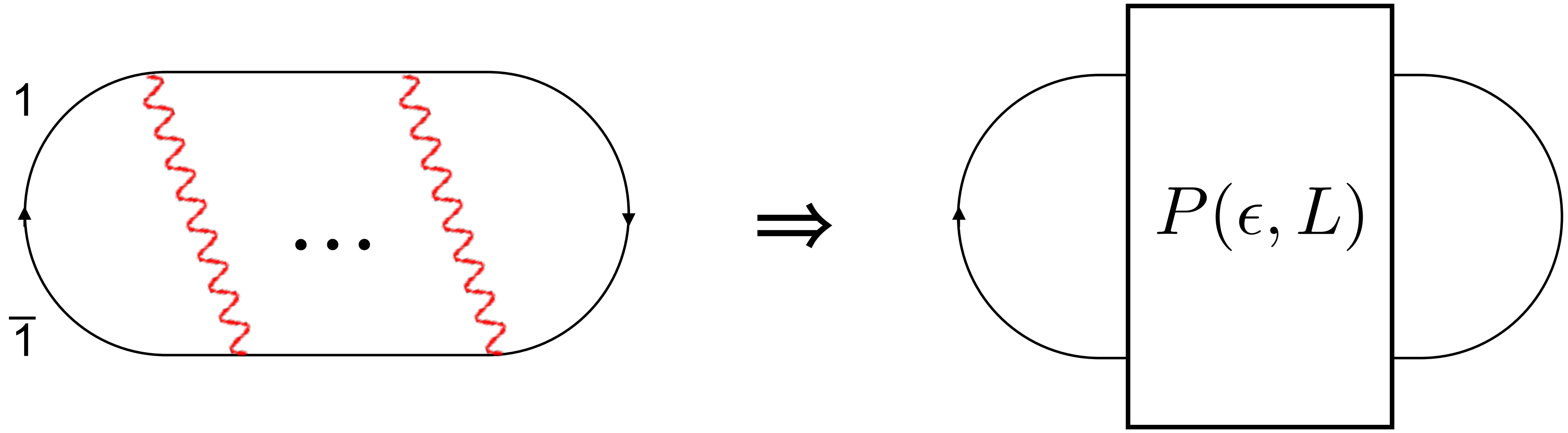
Part 3) Jet energy loss

ENERGY-LOSS PROBABILITY



We have to deal with primary emissions off the hard particle!

Energy-loss probability



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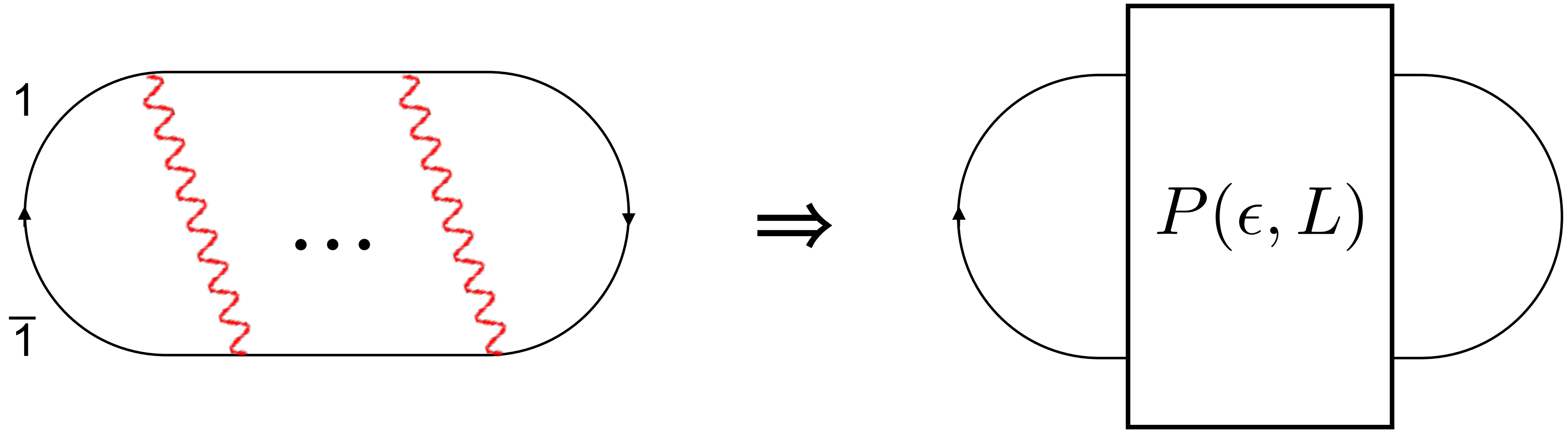
Resumming multiple emissions = solving evolution equation for the energy loss probability

$$\frac{\partial}{\partial t}P(\epsilon,t) = \int_0^\infty \mathrm{d}\omega \left[\frac{\mathrm{d}I}{\mathrm{d}\omega \mathrm{d}t} - \delta(\omega)\int_0^\infty \mathrm{d}\omega' \frac{\mathrm{d}I}{\mathrm{d}\omega' \mathrm{d}t}\right] P(\epsilon-\omega,t)$$

ENERGY-LOSS PROBABILITY



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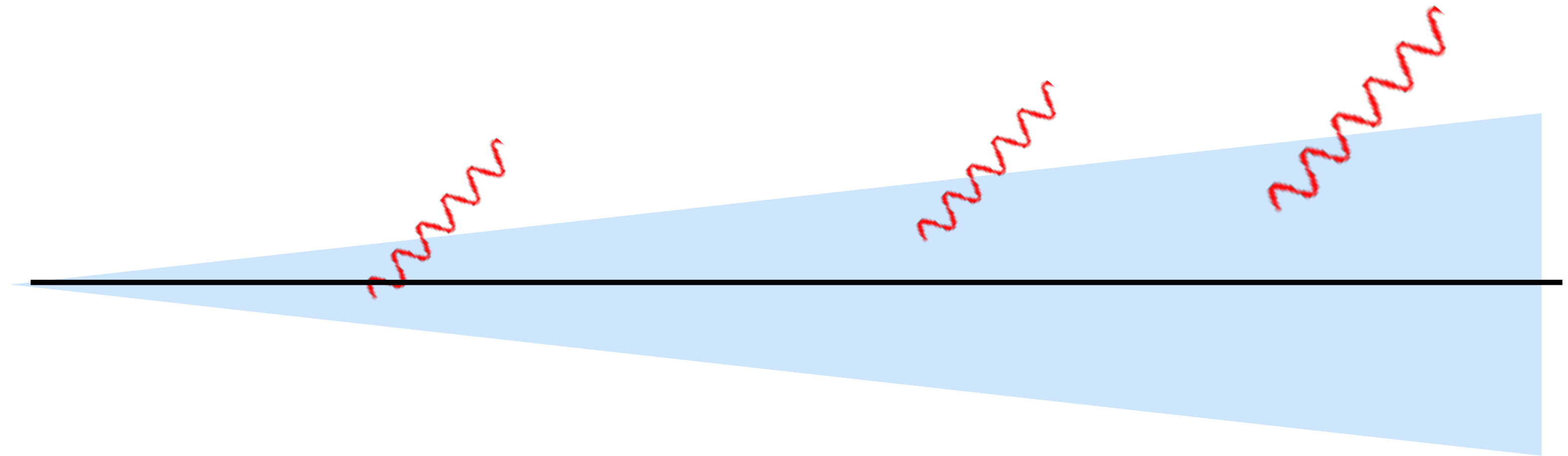
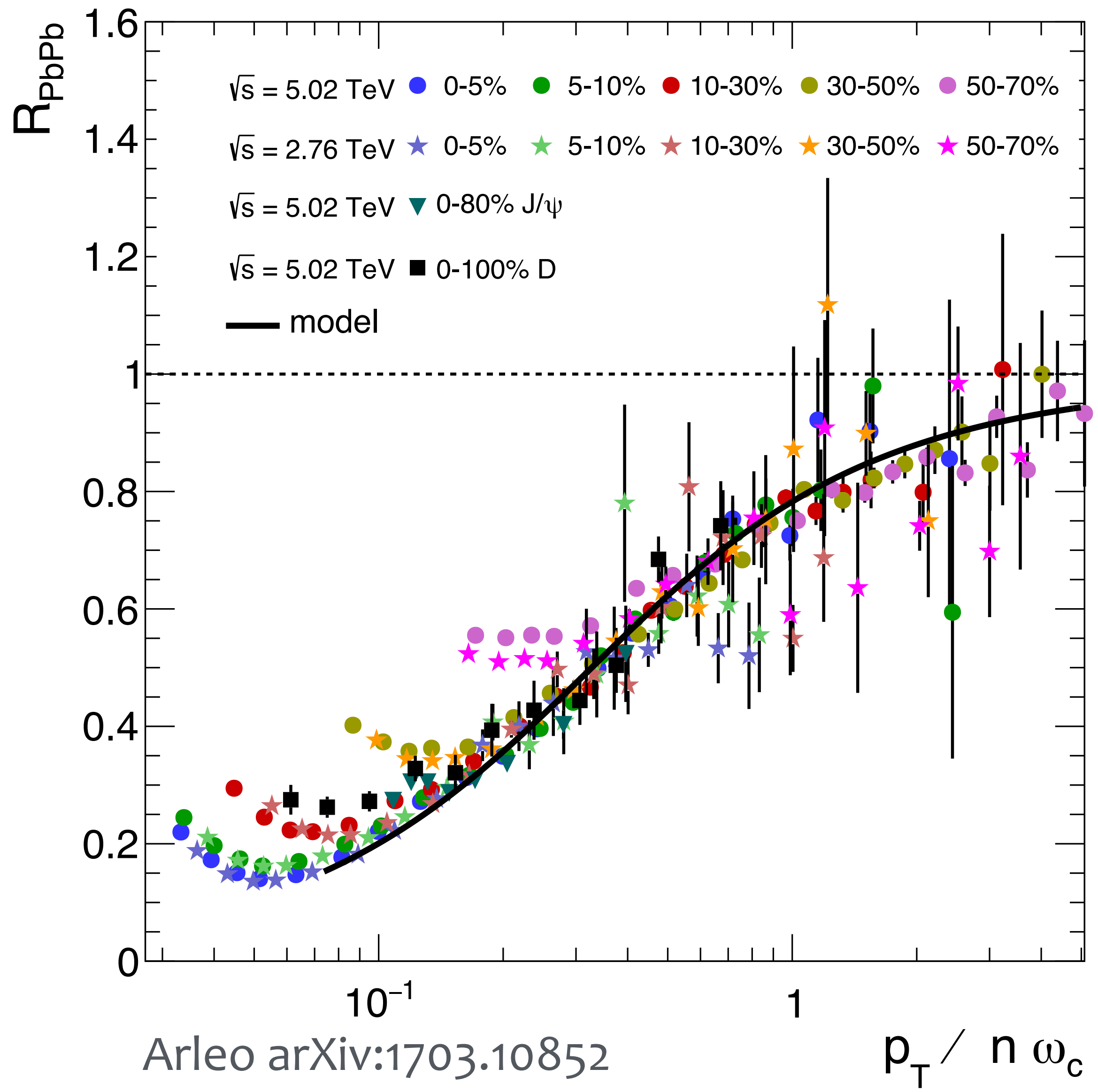
Energy loss dominated by typical gluon energy $\omega_s = \bar{\alpha}^2 \hat{q} L^2$

$$P(\epsilon, L) = \sqrt{\frac{\omega_s}{\epsilon^3}} \mathrm{e}^{-\frac{\pi \omega_s}{\epsilon}}$$

SINGLE-PARTICLE ENERGY LOSS





Arleo arXiv:1703.10852

Bias due to steeply falling spectrum (w/index n)

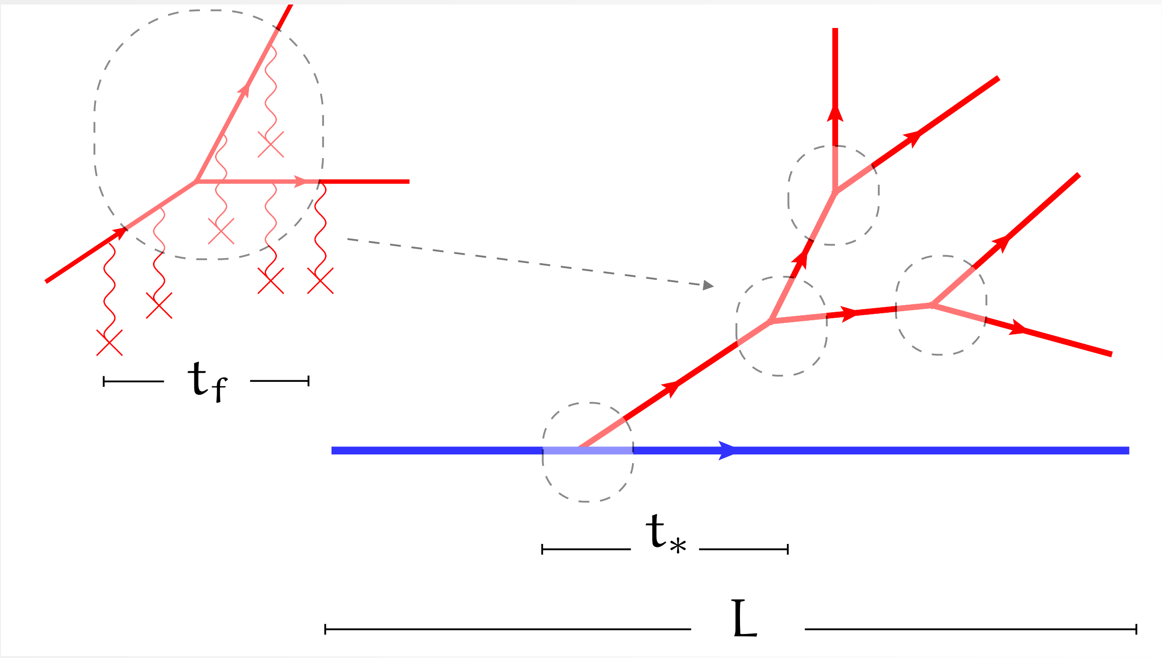
$$\frac{\mathrm{d}N}{\mathrm{d}E} = \int_0^\infty \mathrm{d}\epsilon \, P(\epsilon, L) \frac{\mathrm{d}N_0(E+\epsilon)}{\mathrm{d}E'}$$

Ratio: only two scales

$$Q(E) = \exp\left[- \int_0^L \mathrm{d}t \int_{E/(\pi n)}^\infty \frac{\mathrm{d}I}{\mathrm{d}\omega \mathrm{d}t} \right]$$

Baier, Dokshitzer, Mueller, Schiff (2001), Salgado, Wiedemann (2003)

TRACING THE SOFT EMISSIONS



multiple emission regime

$$t_{\rm f} \ll t_* = \frac{t_{\rm f}}{\alpha_s} \ll L$$

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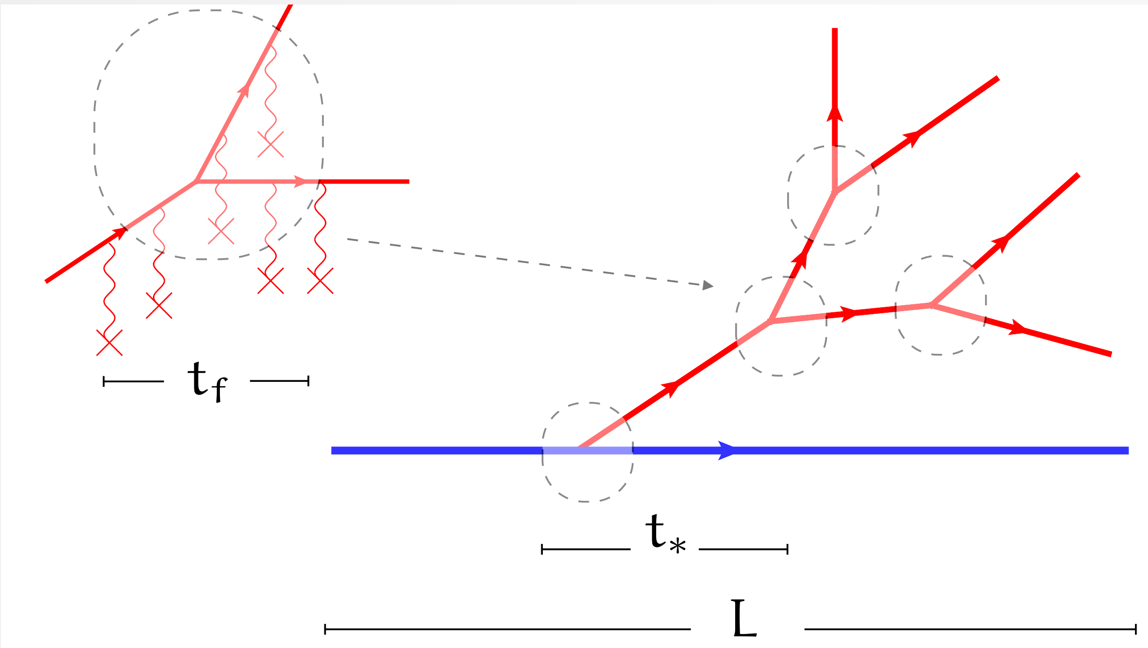

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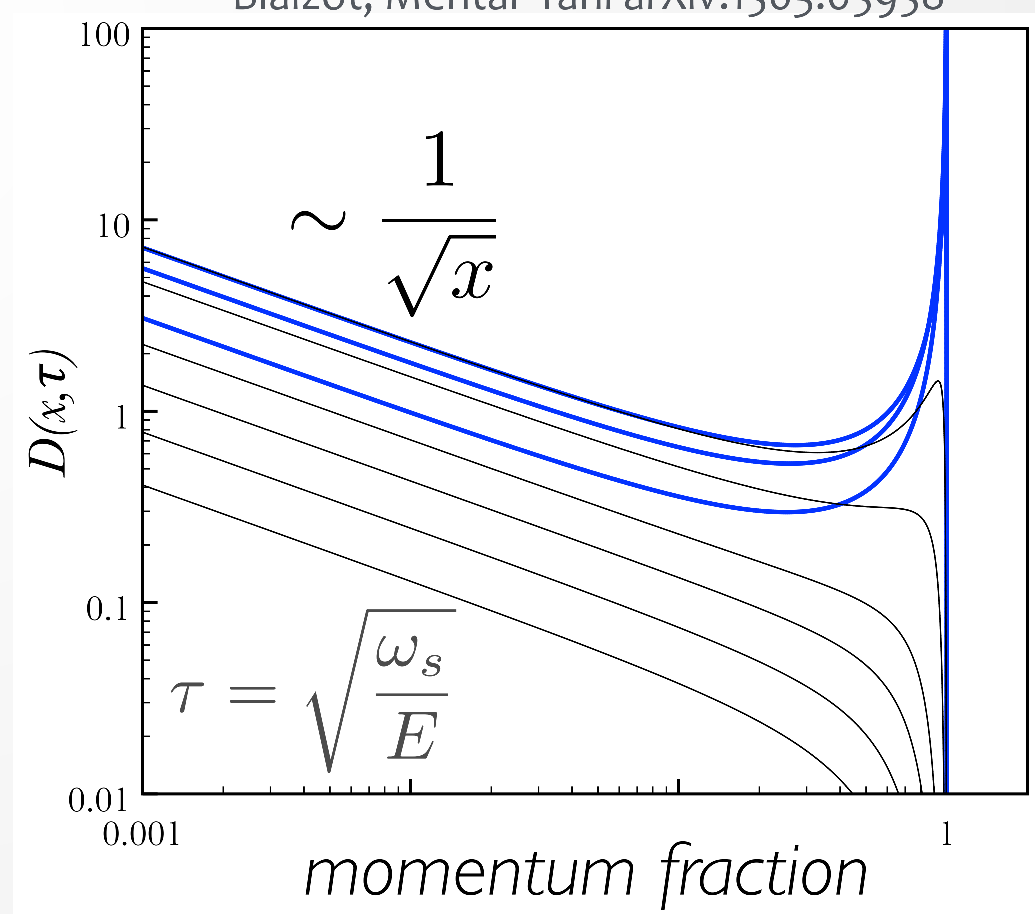
- probabilistic picture

  Blaizot, Dominguez, Iancu, Mehtar-Tani arXiv:1511:5823
- turbulent cascade: energy taken away from projectile into soft particles at large angles
- large fluctuations

  Escbedo, Iancu arXiv:1601.03629 ,1609.06104
- IR: thermalisation (bottom-up)

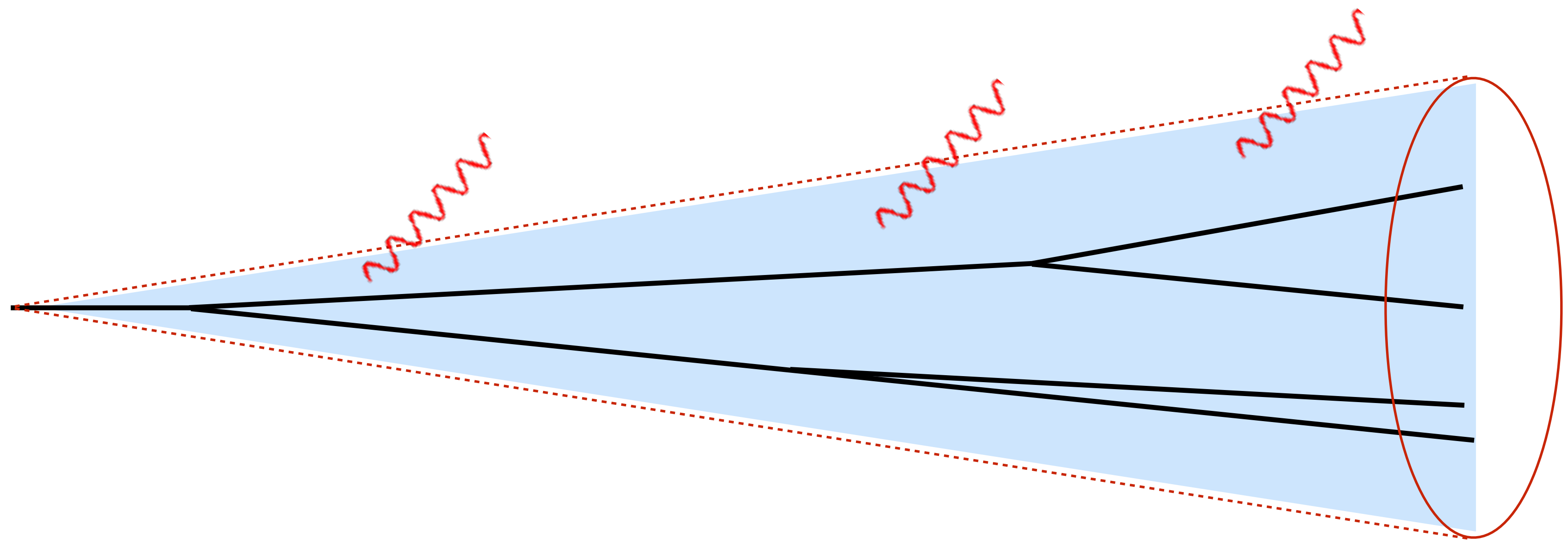
  Iancu, Wu arXiv:1506.07871; …

Blaizot, Mehtar-Tani arXiv:1503.05958



JET QUENCHING & FLUCTUATIONS

- how does an entire jet loose energy to the medium?
- need to account for fluctuations of energy loss due to fluctuations of the jet substructure!



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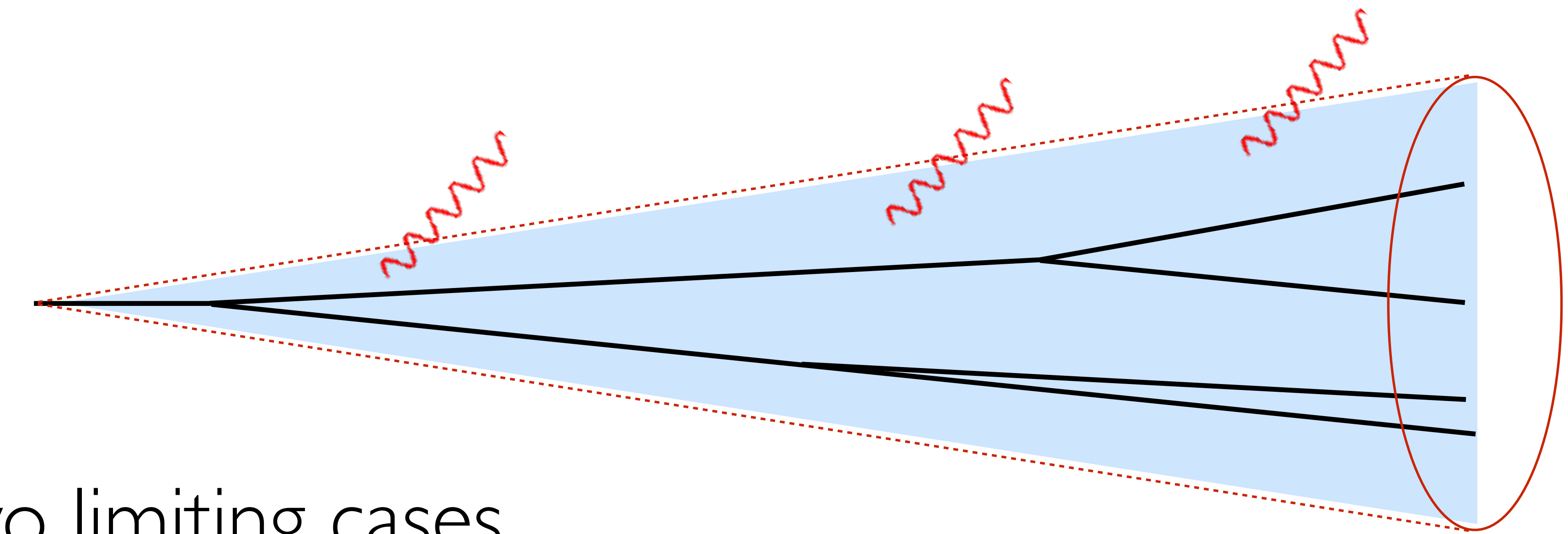

Two limiting cases

- coherently as a single colour charge (parton)
- incoherently as multiple charges

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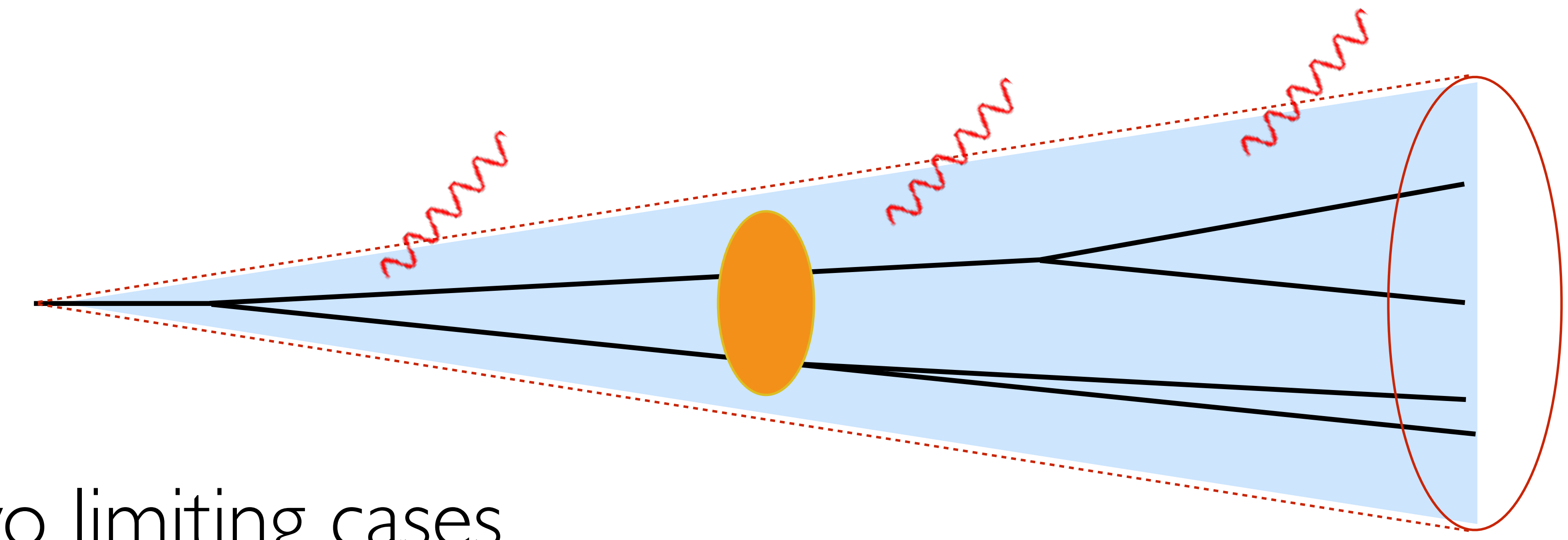

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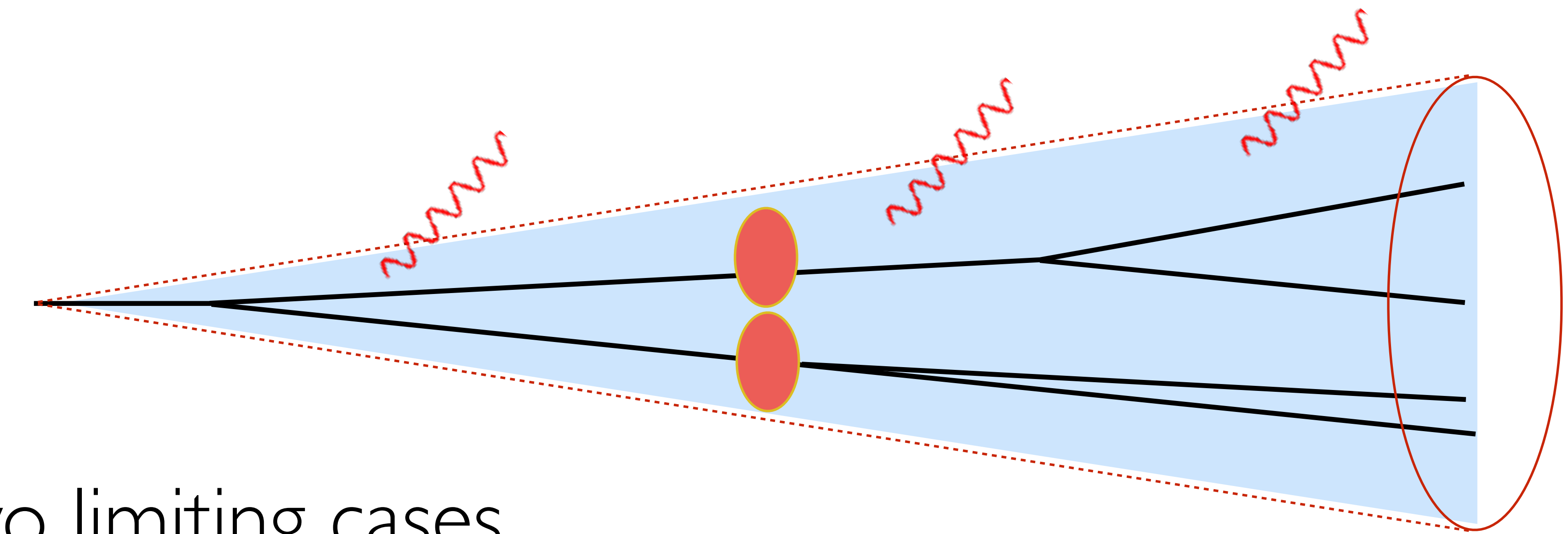

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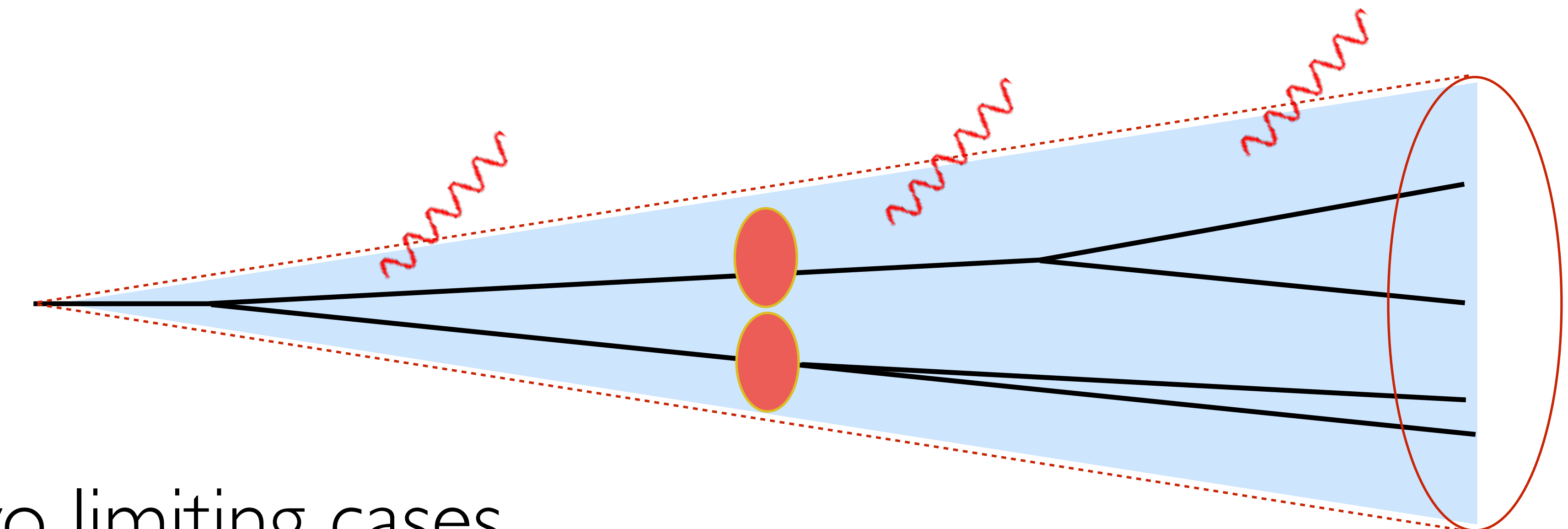

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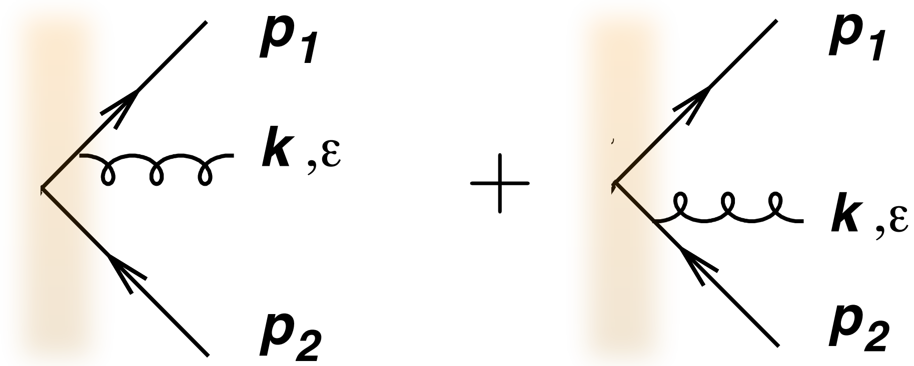
Formidable task: existing Monte-Carlo prescriptions

JEWEL: Zapp, Krauss, Wiedemann arXiv:1212.1599
MARTINI: Schenke, Gale, Jeon arXiv:0909.2037

INTERLUDE: IN-MEDIUM ANTENNA



for emissions
outside the medium

$$\mathcal{J}_i^{a,\mu}(k) = g\,Q_i^a \frac{p_i^\mu}{p\cdot k}$$

$$\Downarrow$$

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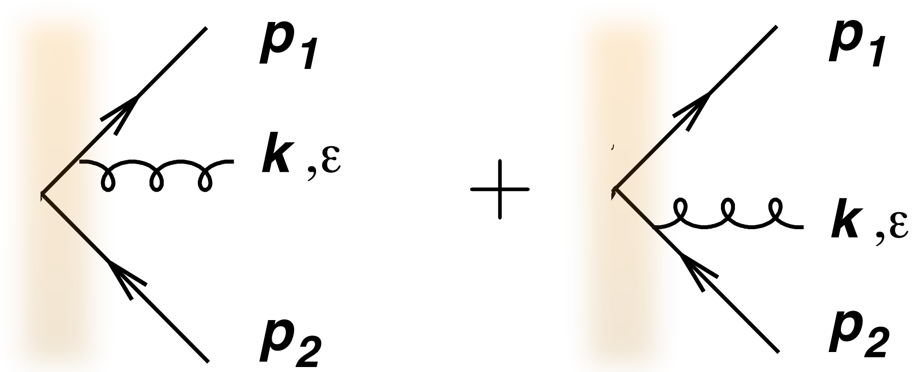

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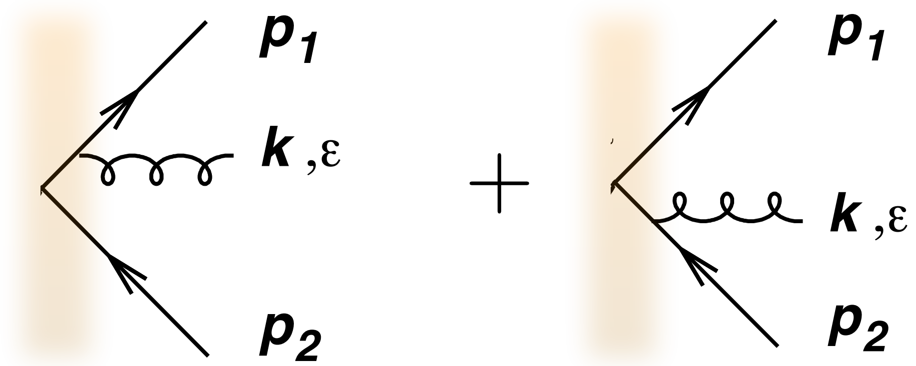

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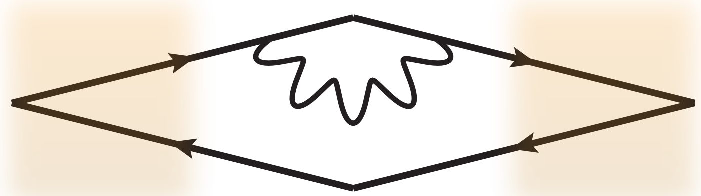
REMINDER:

$$|\mathcal{J}_{12,\perp}|^2 = \frac{1}{(k^+)^2}\left[\mathcal{Q}_1^2\mathcal{P}_1 + \mathcal{Q}_2^2\mathcal{P}_2 + \mathcal{Q}_3^2\mathcal{I}_{12}\right]$$

interference

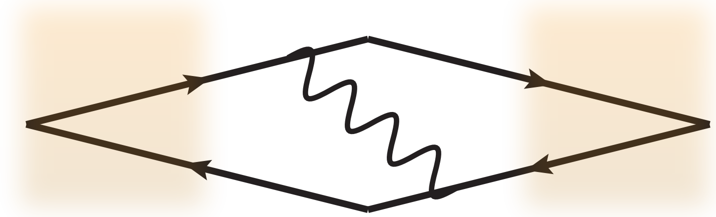
$$\mathcal{P}_i = \mathcal{R}_i - \mathcal{I}_{12}$$

DIRECT



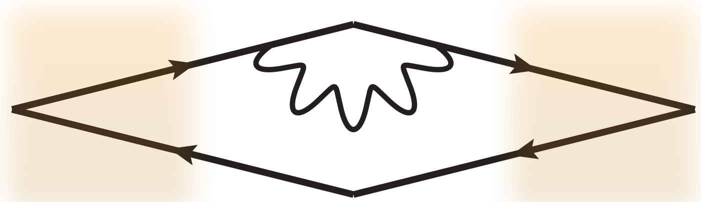
$$|\mathcal{J}_1|^2 \sim \frac{1}{\boldsymbol{w}_1^2} \mathrm{tr}\langle U(\boldsymbol{x}_1) \mathcal{Q}_1^a \mathcal{Q}_1^a U^\dagger(\boldsymbol{x}_1)\rangle \sim \frac{\mathcal{Q}_1^2}{\boldsymbol{v}_1^2}$$

INTERFERENCE



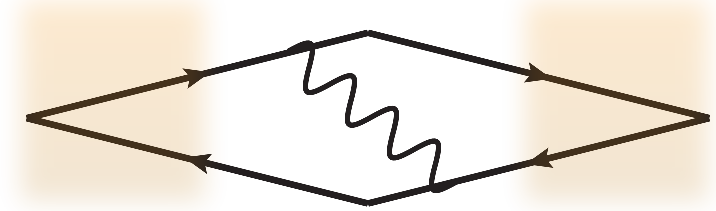
$$\langle \mathcal{J}_1 \mathcal{J}_2^* \rangle \sim \frac{\boldsymbol{w}_1 \cdot \boldsymbol{w}_2}{\boldsymbol{w}_1^2 \boldsymbol{w}_2^2} \mathrm{tr}\langle U(\boldsymbol{x}_1) \mathcal{Q}_1^a \mathcal{Q}_2^a U^\dagger(\boldsymbol{x}_2)\rangle$$

DIRECT



$$|\mathcal{J}_1|^2 \sim \frac{1}{\boldsymbol{w}_1^2} \mathrm{tr}\langle U(\boldsymbol{x}_1) \mathcal{Q}_1^a \mathcal{Q}_1^a U^\dagger(\boldsymbol{x}_1)\rangle \sim \frac{\mathcal{Q}_1^2}{\boldsymbol{v}_1^2}$$

INTERFERENCE



$$\langle \mathcal{J}_1 \mathcal{J}_2^* \rangle \sim \frac{\boldsymbol{w}_1 \cdot \boldsymbol{w}_2}{\boldsymbol{w}_1^2 \boldsymbol{w}_2^2} \mathrm{tr}\langle U(\boldsymbol{x}_1) \mathcal{Q}_1^a \mathcal{Q}_2^a U^\dagger(\boldsymbol{x}_2)\rangle$$

Resulting spectrum: modification of interferences

$$|\mathcal{J}_{12,\perp}|^2 = \frac{1}{(k^+)^2} \left[\mathcal{Q}_1^2 \mathcal{P}_1 + \mathcal{Q}_2^2 \mathcal{P}_2 + \mathcal{Q}_3^2 (1 - \Delta_{\mathrm{med}}) \mathcal{I}_{12}\right]$$

$$\mathcal{P}_i = \mathcal{R}_i - (1 - \Delta_{\mathrm{med}}) \mathcal{I}_{12}$$

Decoherence parameter:

$$1 - \Delta_{\mathrm{med}} = \frac{1}{N_c^2 - 1} \mathrm{tr}\langle U(\boldsymbol{x}_1) U^\dagger(\boldsymbol{x}_2)\rangle = \exp\left[-\frac{1}{4}\int_0^L \mathrm{d}s\, \hat{q}\, (\boldsymbol{x}_1 - \boldsymbol{x}_2)^2\right] = \exp\left[-\frac{1}{12}\hat{q}\theta_0^2 L^3\right]$$

$$(\boldsymbol{x}_1 - \boldsymbol{x}_2)^2 = (\theta_0\, s)^2$$

Decoherence parameter:

survival probability

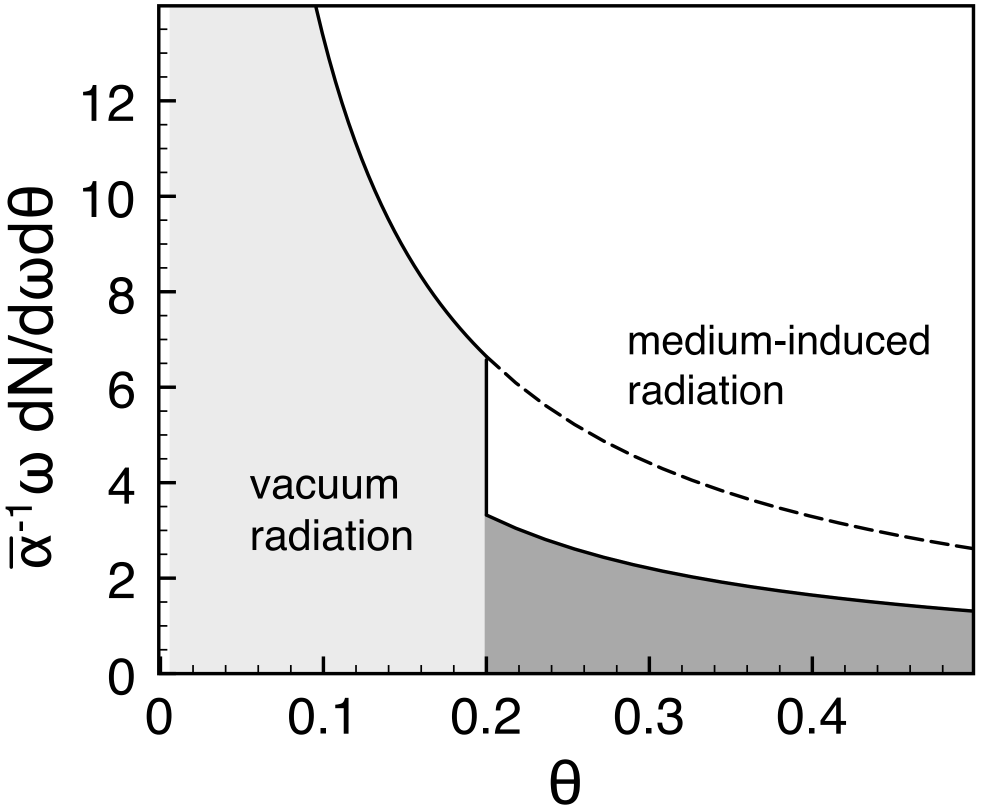
$$\Delta_{\text{med}} = 1 - \exp\left[-\frac{1}{12} Q_s^2 r_\perp^2\right]$$

New scales:

$$Q_s = \sqrt{\hat{q}\, L}$$

$$r_\perp^{-1} = 1/(\theta_0\, L)$$

$$\frac{\mathrm{d}N_q}{\mathrm{d}x\,\mathrm{d}\theta} = \frac{2\alpha_s C_F}{\pi}\frac{1}{x}\frac{1}{\theta}\Big[\Theta\big(\theta_0 - \theta\big) + \Delta_{\text{med}}\Theta\big(\theta - \theta_0\big)\Big]$$



$\Delta_{\text{med}} \to 0$ Coherence

antenna is not resolved = survives

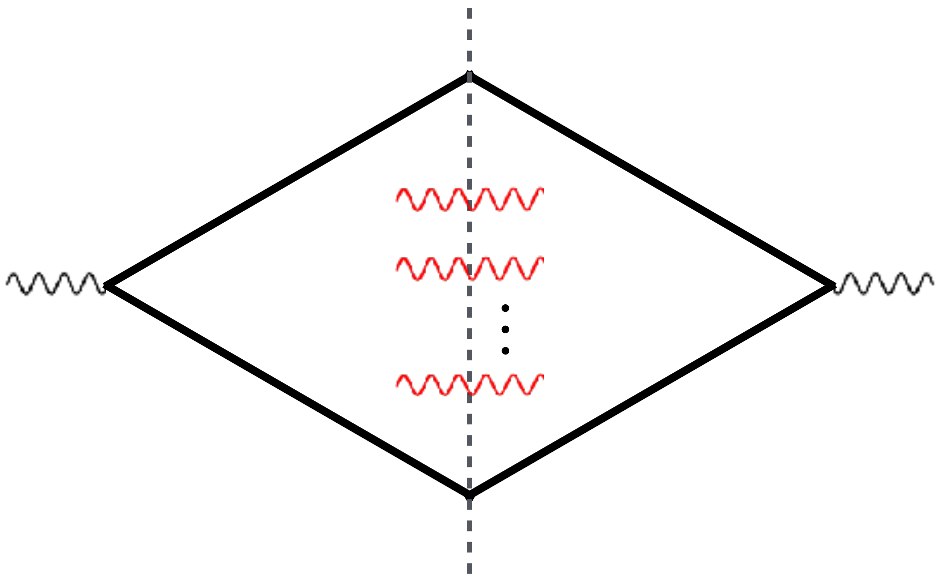
vacuum spectrum = angular ordering

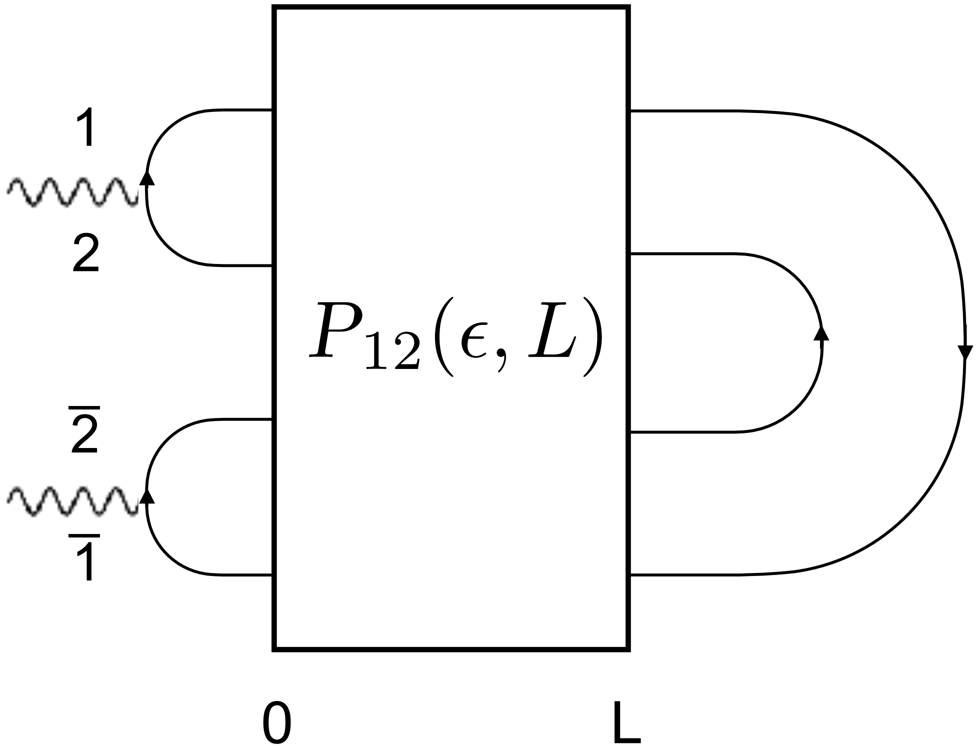
$\Delta_{\text{med}} \to 1$ Decoherence

antenna is resolved = destroyed

incoherent spectrum = no angular ordering

TWO-PRONG ENERGY LOSS





- **how do two colour-connected charges lose energy?**
  - tagging two hard sub-jets within a jet cone
  - fixed opening angle
- depends on direct emissions + interference
- pair gradually decoheres: interpolates between
  - small angle: no eloss (photon)
  - large angle: independent eloss

SOLUTION

$$P_{12}(\epsilon, L) = \int \mathrm{d}\epsilon_1 \int \mathrm{d}\epsilon_2 \, \delta(\epsilon - \epsilon_1 - \epsilon_2) \, P(\epsilon_1, L) P(\epsilon_2, L)$$
$$+ \int_0^L \mathrm{d}t \int \mathrm{d}\epsilon_1 \int \mathrm{d}\epsilon_2 \int \mathrm{d}\omega \, \delta(\epsilon - \epsilon_1 - \epsilon_2 - \omega)$$
$$\times P(\epsilon_1, L - t) P(\epsilon_2, L - t) \left[1 - \Delta_{\mathrm{med}}\right] \left(\frac{\mathrm{d}I_{\mathrm{int}}}{\mathrm{d}\omega \mathrm{d}t} - \mathrm{virt.} \right)$$

- quantum decoherence (instantaneous)
  - hard emissions can resolve the internal colour structure
  - corresponds to collinear emissions in vacuum…
- colour decoherence (accumulative)
  - the pair gradually becomes disconnected in colour & behave independently
- probabilistic formulation

SOLUTION

$$P_{12}(\epsilon, L) = \int d\epsilon_1 \int d\epsilon_2\, \delta(\epsilon - \epsilon_1 - \epsilon_2) \boxed{P(\epsilon_1, L) P(\epsilon_2, L)}$$

$$+ \int_0^L dt \int d\epsilon_1 \int d\epsilon_2 \int d\omega\, \delta(\epsilon - \epsilon_1 - \epsilon_2 - \omega)$$

$$\times P(\epsilon_1, L - t) P(\epsilon_2, L - t) \left[1 - \Delta_{\rm med}\right] \left(\frac{dI_{\rm int}}{d\omega dt} - \text{virt.} \right)$$

incoherent energy loss

- quantum decoherence (instantaneous)
  - hard emissions can resolve the internal colour structure
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SOLUTION

$$P_{12}(\epsilon, L) = \int \mathrm{d}\epsilon_1 \int \mathrm{d}\epsilon_2\, \delta(\epsilon - \epsilon_1 - \epsilon_2)\, P(\epsilon_1, L) P(\epsilon_2, L)$$

$$+ \int_0^L \mathrm{d}t \int \mathrm{d}\epsilon_1 \int \mathrm{d}\epsilon_2 \int \mathrm{d}\omega\, \delta(\epsilon - \epsilon_1 - \epsilon_2 - \omega)$$

$$\times P(\epsilon_1, L - t) P(\epsilon_2, L - t) [1 - \Delta_{\mathrm{med}}] \left(\frac{\mathrm{d}I_{\mathrm{int}}}{\mathrm{d}\omega \mathrm{d}t} + \mathrm{virt.} \right)$$

incoherent energy loss

interferences!

- quantum decoherence (instantaneous)
  - hard emissions can resolve the internal colour structure
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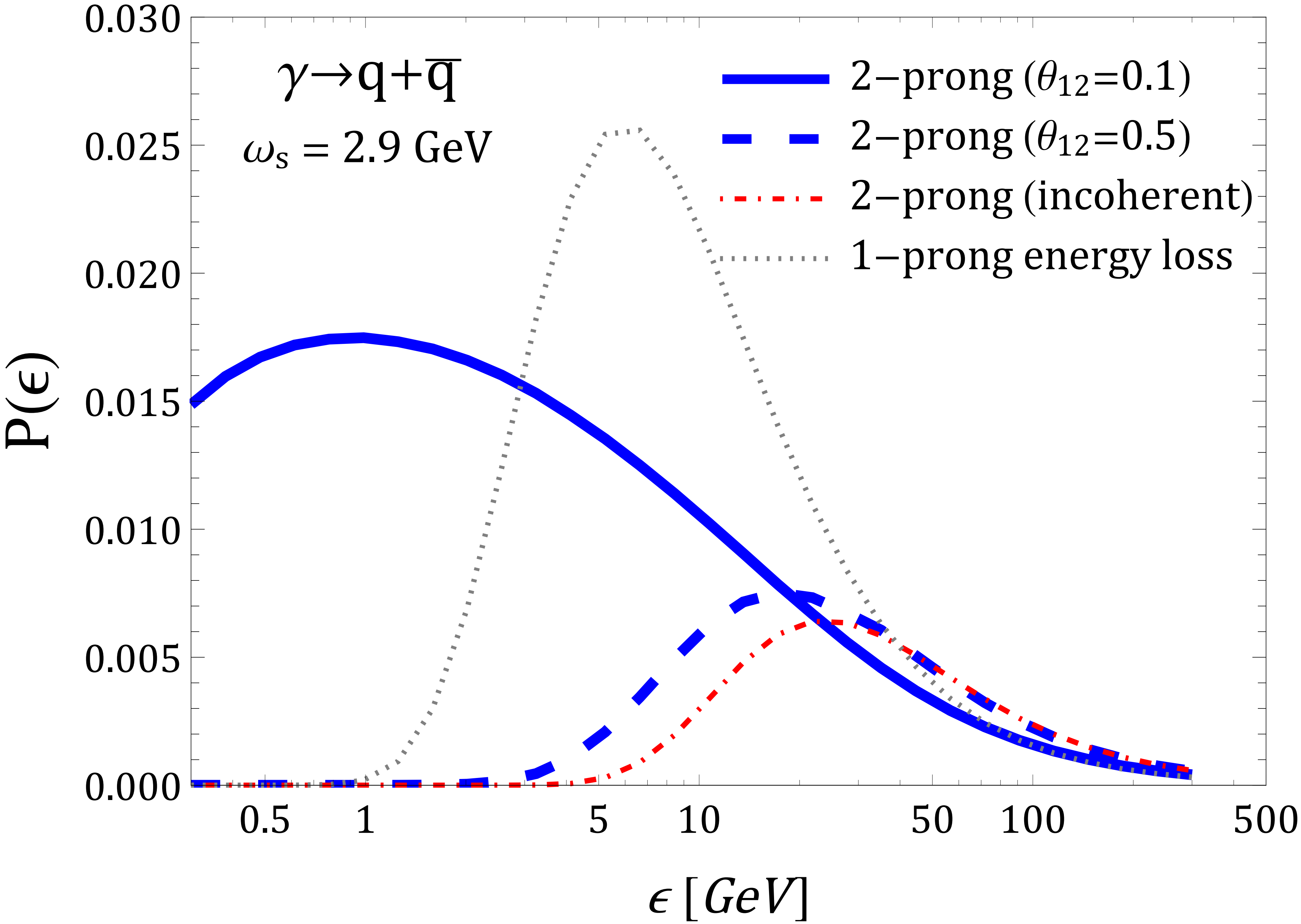
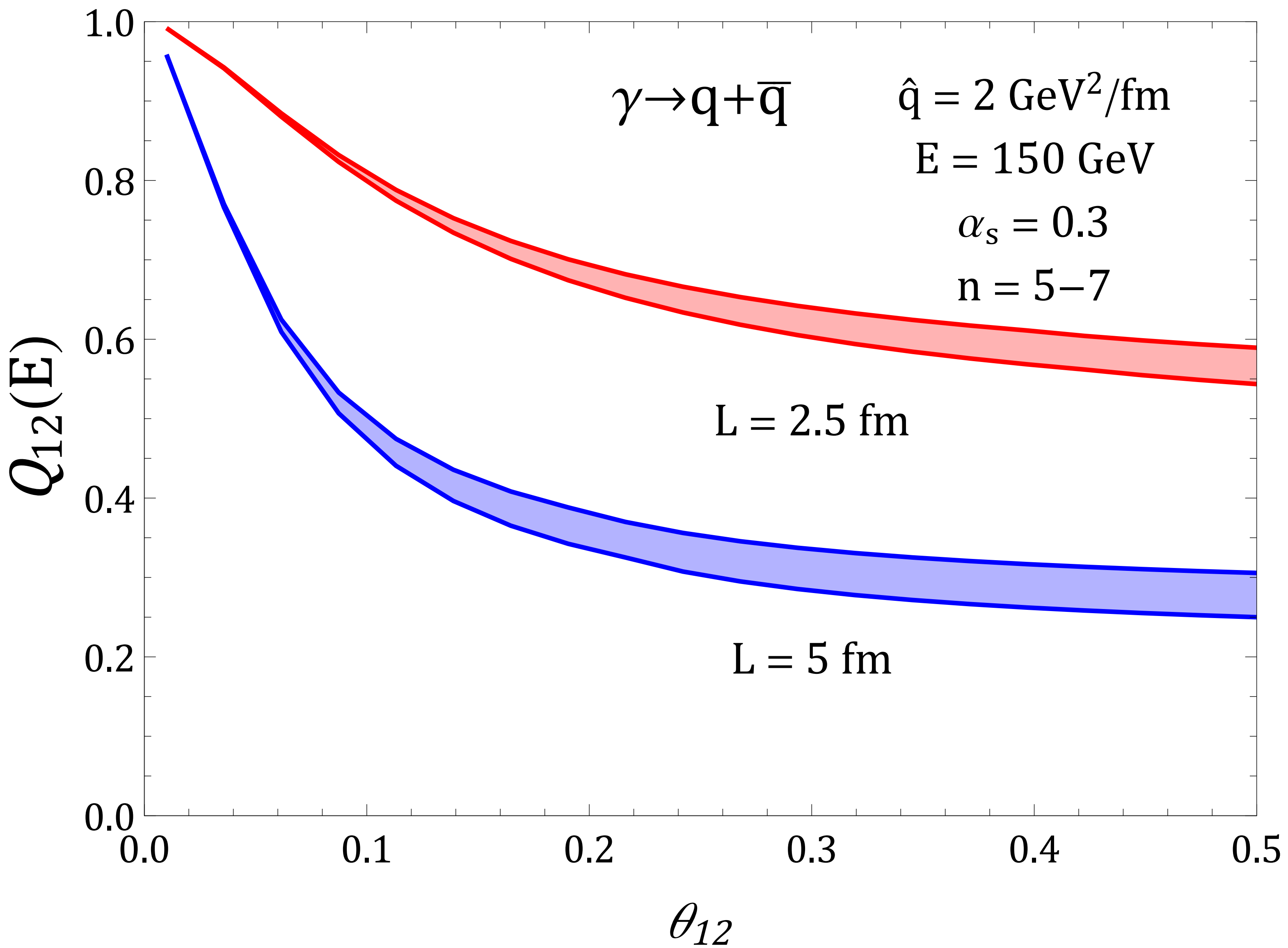
NEW QUENCHING WEIGHT



A NEW OBSERVABLE

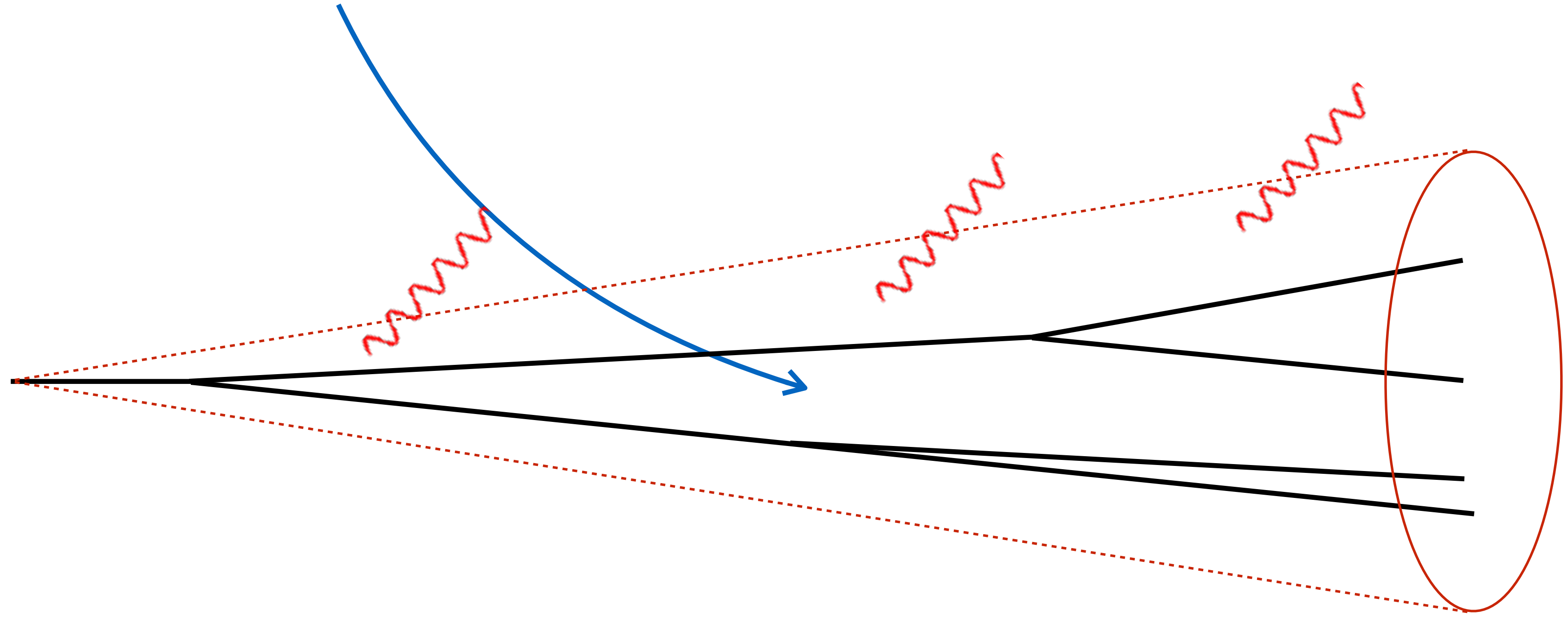
- quenching depends on the opening angle!
- large-angle structures within jets are strongly suppressed



$$\frac{\mathrm{d}N_{2j}}{\mathrm{d}z\mathrm{d}E\,\mathrm{d}\theta} = \int_0^\infty \mathrm{d}\epsilon\, P_{12}\left(\epsilon, L; \theta_{12}\right) \frac{\alpha_s}{\pi} \frac{P(z)}{\theta} \frac{\mathrm{d}N_0(E+\epsilon)}{\mathrm{d}E'}$$

SUMMARY OF THE LECTURES

vacuum
soft & collinear divergences
colour coherence (angular ordering)
multi-gluon emissions (MLLA)



Summary of the lectures

vacuum
soft & collinear divergences
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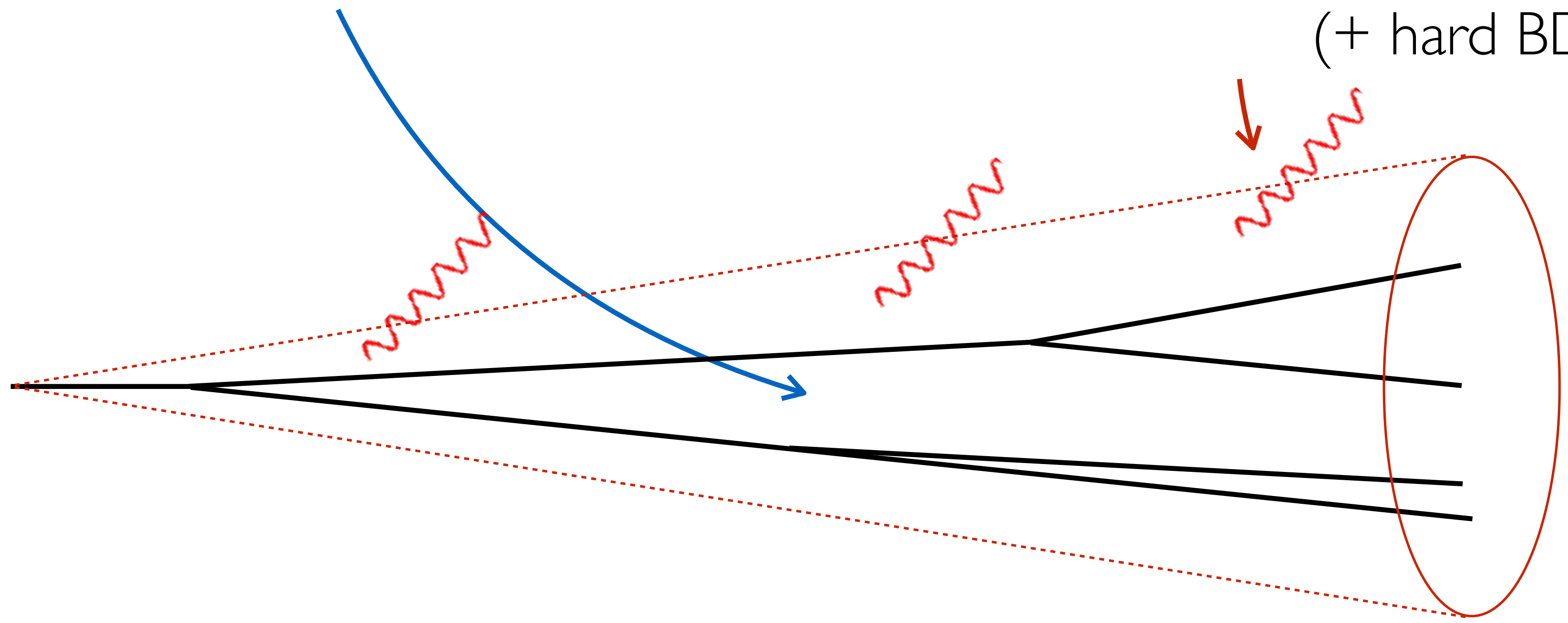
medium
collinear finite & soft enhanced spectrum
gradual breaking of colour coherence
multi-gluon emissions lead to energy loss
(+ hard BDMPS radiation)



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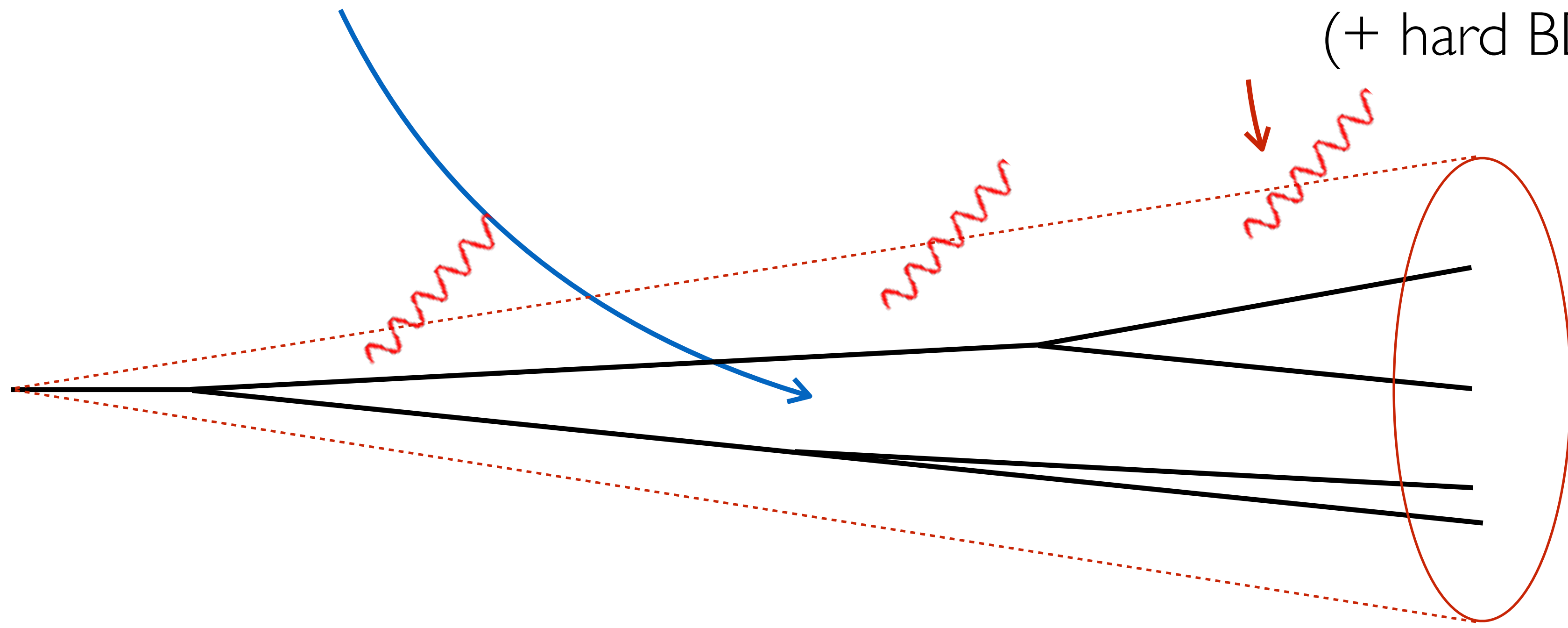

outlook
theoretical progress prompted by exciting experimental results
new aspects of QCD are studied (jet perspective, medium perspective)
toward building a full understanding of hard probes @ LHC

LIST OF VALUABLE RESOURCES

- Peskin, Schroeder "An introduction to QFT" (Addison-Wesley Publishing)
- Sterman "An introduction to QFT" (Cambridge University Press)
- Ellis, Stirling, Webber "QCD and collider physics" (Cambridge University Press)
- Dokshitzer, Khoze, Mueller, Troyan "Basics of perturbative QCD" (Editions Frontieres)
  - online on: www.lpthe.jussieu.fr/~yuri/BPQCD/BPQCD.pdf
- Khoze, Ochs "Perturbative-QCD approach to multiparticle production" IJMPA 12 (1997) 2949
- Mangano "Introduction to QCD", http://cern.ch/~mlm/talks/cern98.ps.gz
- Seymour "Quantum ChromoDynamics", arXiv:1010.2330
- Salam "Elements of QCD for hadron colliders", arXiv:1011.5131
  - more things on: https://gsalam.web.cern.ch/gsalam/teaching/PhD-courses.html
- Mikko Laine "Basics of thermal field theory" http://www.laine.itp.unibe.ch/basics.pdf
- Kapusta & Gale "Finite-temperature Field Theory: Principles and Applications"
- Salgado & Casalderrey-Solana "Introductory lectures on jet quenching in heavy ion collisions" arXiv:0712.3443
- Mehtar-Tani, Milhano, Tywoniuk "Jet physics in heavy-ion collisions" arXiv:1302.2579
- Blaizot, Mehtar-Tani "Jet structure in heat ion collisions" arXiv:1503:05958