UiO : Fysisk institutt
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# Introduction to Supersymmetry

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# Overview

- What is SUSY?
- The Minimal Supersymmetric Standard Model
- Phenomenology

# What is SUSY?

# Symmetries in physics

- We know Einstein's (Poincaré's) symmetry well. Lengths of **external** four-vectors are invariant:

$$x^\mu \rightarrow x'^\mu = \Lambda^\mu_\nu x^\nu + a^\mu, \quad (x'-y')^2 = (x-y)^2$$

- We also have **internal** gauge symmetries:

$$SU(3)_c \times SU(2)_L \times U(1)_Y$$

- Can the Poincaré symmetry be extended? Yes
- Can we unify internal and external symmetries?
Yes, but not really the way we would have liked...

# Symmetries in physics

- Symmetries are described by a group and its algebra (relations of generators of the group).
- The Poincaré algebra:

$$[P_\mu, P_\nu] = 0$$

$$[M_{\mu\nu}, M_{\rho\sigma}] = -i(g_{\mu\rho}M_{\nu\sigma} - g_{\mu\sigma}M_{\nu\rho} - g_{\nu\rho}M_{\mu\sigma} + g_{\nu\sigma}M_{\mu\rho}).$$

$$[M_{\mu\nu}, P_\rho] = -i(g_{\mu\rho}P_\nu - g_{\nu\rho}P_\mu)$$

- *No-go theorem*, Coleman & Mandula (1967).
- Haag, Lopuszanski and Sohnius (1975): allow for anti-commutators in algebra.

# Supersymmtry

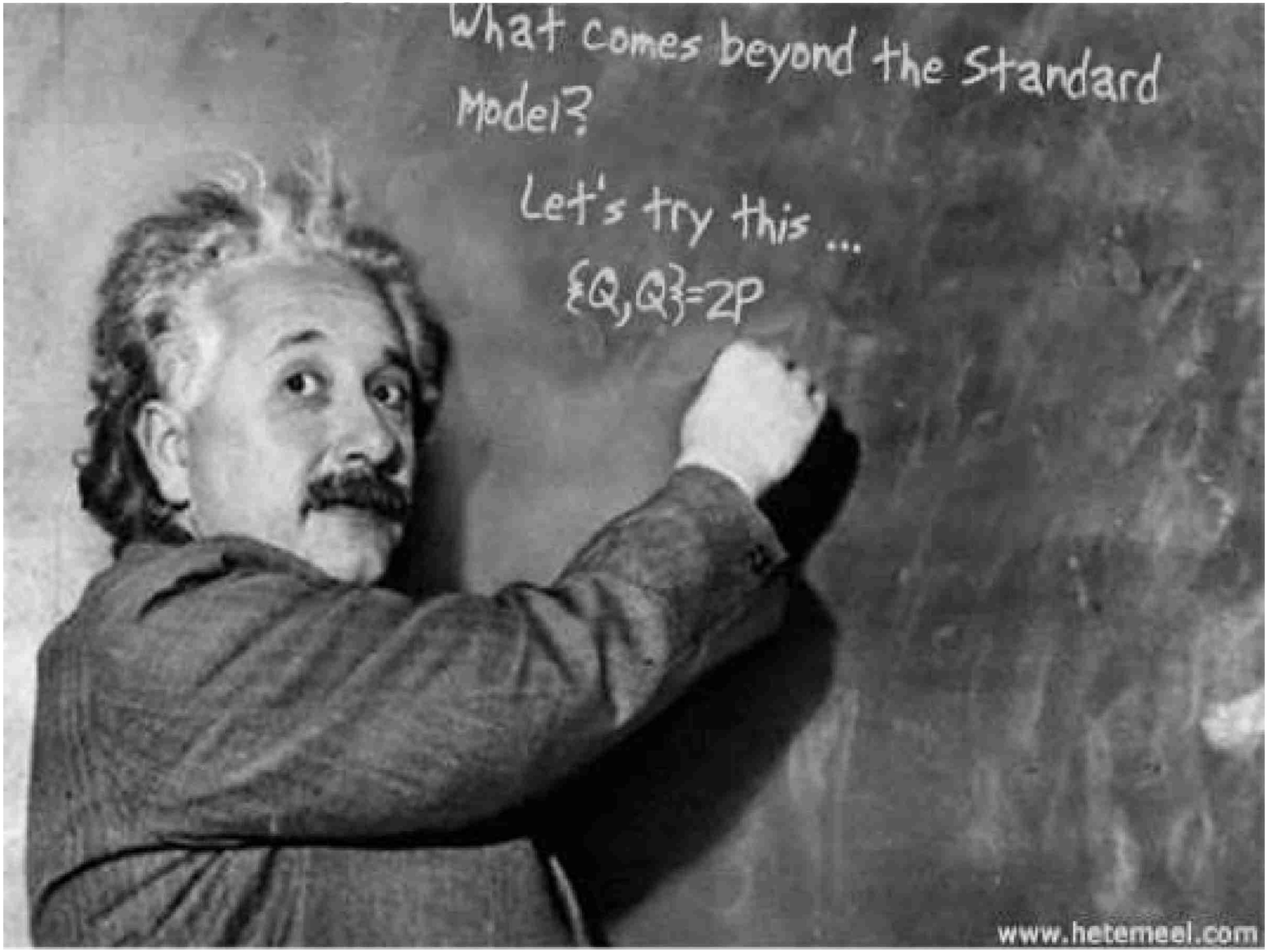
- Introduce new generator $Q_a$ (a Majorana spinor) mapping fermion states to bosons and back.
- The (N=1) super-Poincaré algebra:

$$[P_\mu, P_\nu] = 0$$

$$[M_{\mu\nu}, M_{\rho\sigma}] = -i(g_{\mu\rho}M_{\nu\sigma} - g_{\mu\sigma}M_{\nu\rho} - g_{\nu\rho}M_{\mu\sigma} + g_{\nu\sigma}M_{\mu\rho}).$$

$$[M_{\mu\nu}, P_\rho] = -i(g_{\mu\rho}P_\nu - g_{\nu\rho}P_\mu)$$

$$\begin{aligned} [Q_a, P_\mu] &= 0 \\ [Q_a, M_{\mu\nu}] &= (\sigma_{\mu\nu}Q)_a \\ \{Q_a, \bar{Q}_b\} &= 2\not{P}_{ab} \end{aligned}$$


What comes beyond the Standard Model?
Let's try this ...
{Q,Q}=2P
www.hetemeel.com

# Supersymmtry

- Some immediate consequences:
  - Equal number of fermion and boson states (not particles!)
  - Partners inherit mass (and other couplings)
  - Proof directly from algebra.
- These properties are vital to the solution of the Higgs hierarchy problem.
- But where have all the bosons gone?

# Spontaneous SUSY breaking

- Just as in the Higgs mechanism we can use the scalar potential of the theory to break SUSY.
- However, we are limited by the supertrace relation

$$\mathrm{STr}\ M^2 = \sum_s (-1)^{2s} (2s+1)\ \mathrm{Tr}\ M_s^2 = 0$$

- All new scalars can not be heavier than all the fermions!
- Solution: put breaking at high scale with extra fermions.

# Spontaneous SUSY breaking

- We parametrize our ignorance of the exact mechanism by adding SUSY breaking terms

$$\mathcal{L}_{\text{soft}} = -\frac{1}{2}M\lambda^A\lambda_A - (\frac{1}{6}a_{ijk}A_iA_jA_k + \frac{1}{2}b_{ij}A_iA_j + t_iA_i + \frac{1}{2}c_{ijk}A_i^*A_jA_k + c.c.) -m_{ij}^2A_i^*A_j$$

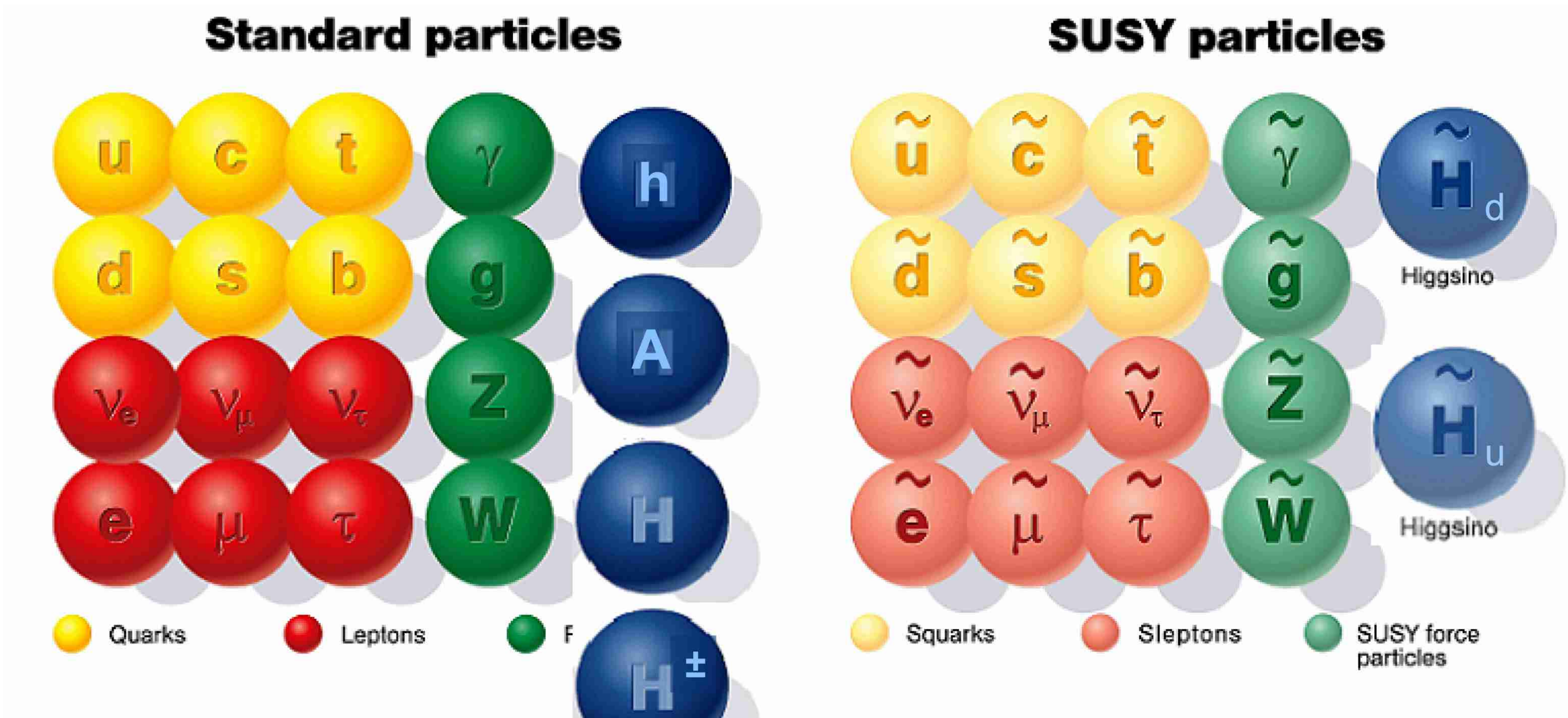
- These are the **soft breaking terms** (do not reintroduce the hierarchy problem).
- Dramatic phenomenological effect!

# Minimal Supersymmetric Standard Model

# MSSM

- The Minimal Supersymmetric Standard Model (MSSM) is the smallest model in terms of fields that contains all SM particles.
- In addition to the partners of all SM particles it is necessary to introduce two Higgs doublets.
  - Anomaly cancellation.
  - Give mass to both up- and down-type quarks

# MSSM




$$\widetilde{\chi}_i^0 = N_{i1}\widetilde{B}^0 + N_{i1}\widetilde{W}^0 + N_{i3}\widetilde{H}_u^0 + N_{i4}\widetilde{H}_d^0$$

$$\widetilde{\chi}_i^+ = C_{i1}\widetilde{W}^+ + C_{i1}\widetilde{H}_u^+$$

# MSSM

- The extra Higgs doublet adds only one new parameter μ coupling the two Higgs doublets.
- For historical reasons no neutrino mass. (Or right handed neutrinos.)
- There are 104 new soft breaking parameters!

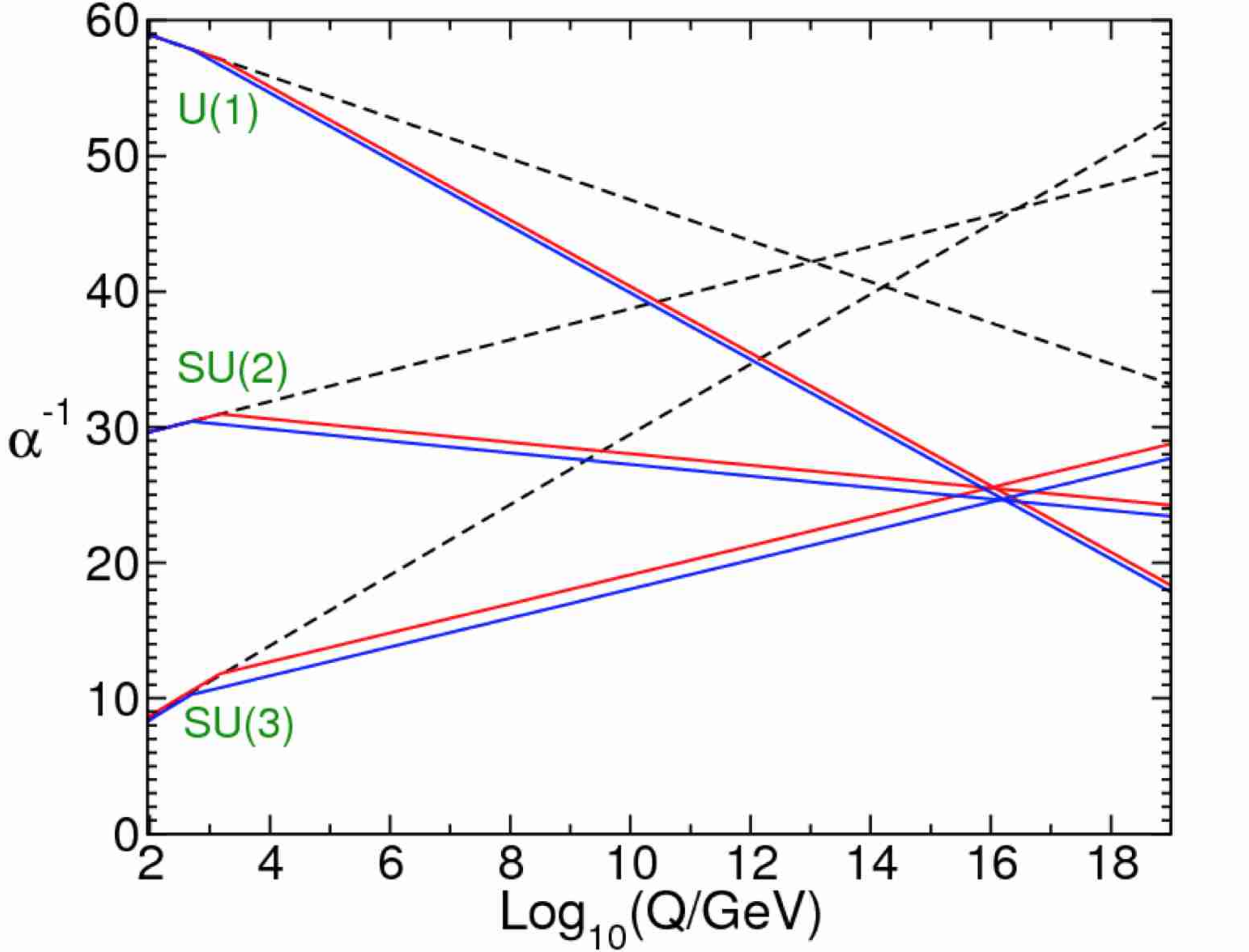
  (In addition to 19 free parameters in the SM)
- Does this ruin predictability?

# R-parity

- To remove lepton & baryon number violating interactions we introduce a new multiplicative quantum number R-parity

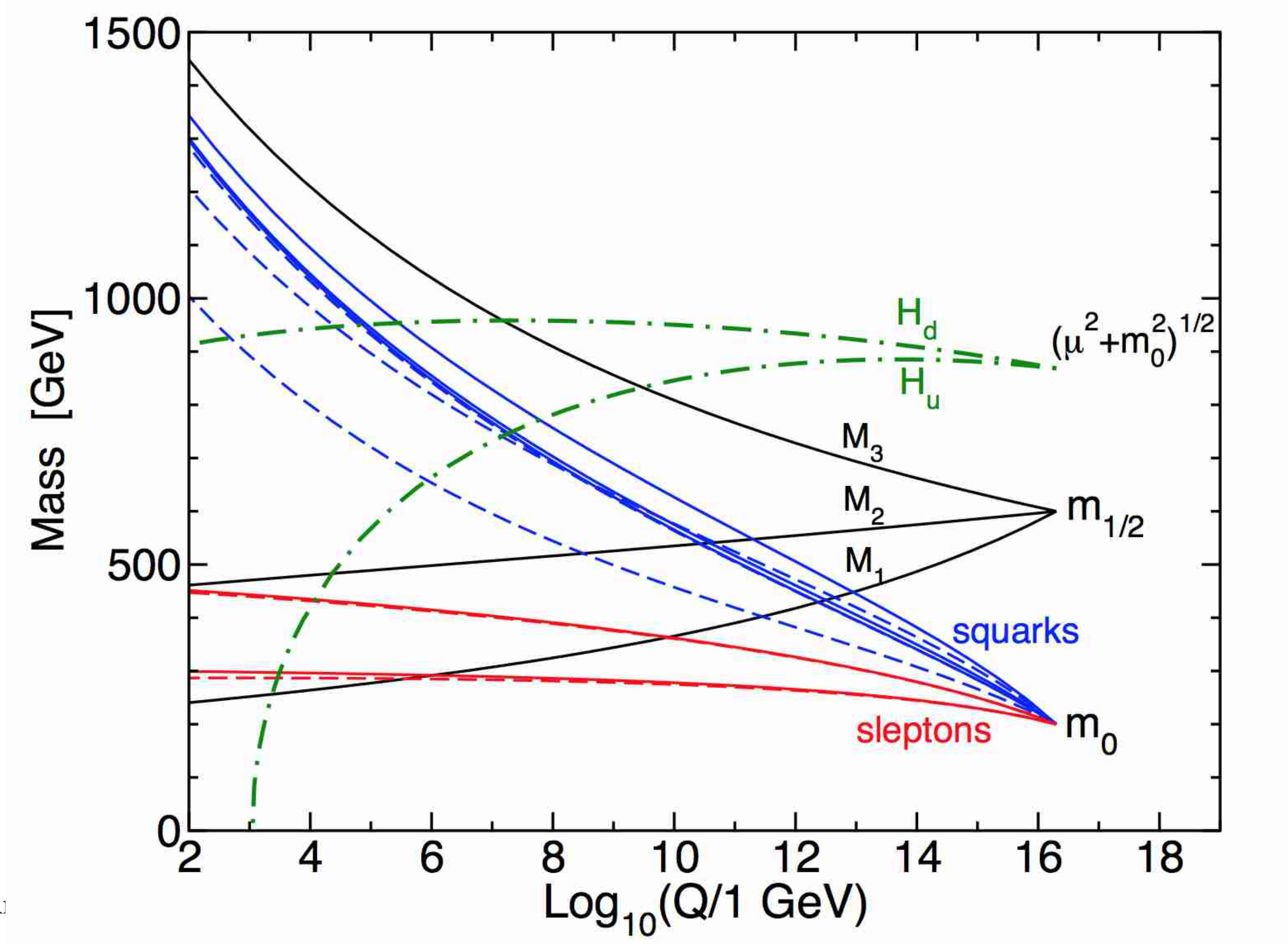
$$R = (-1)^{3B+L+2s}$$

  - All interactions have an even number of sparticles.
  - Sparticles can only be pair-produced.
  - The lightest sparticle (LSP) is absolutely stable. (Usually the lightest neutralino.)

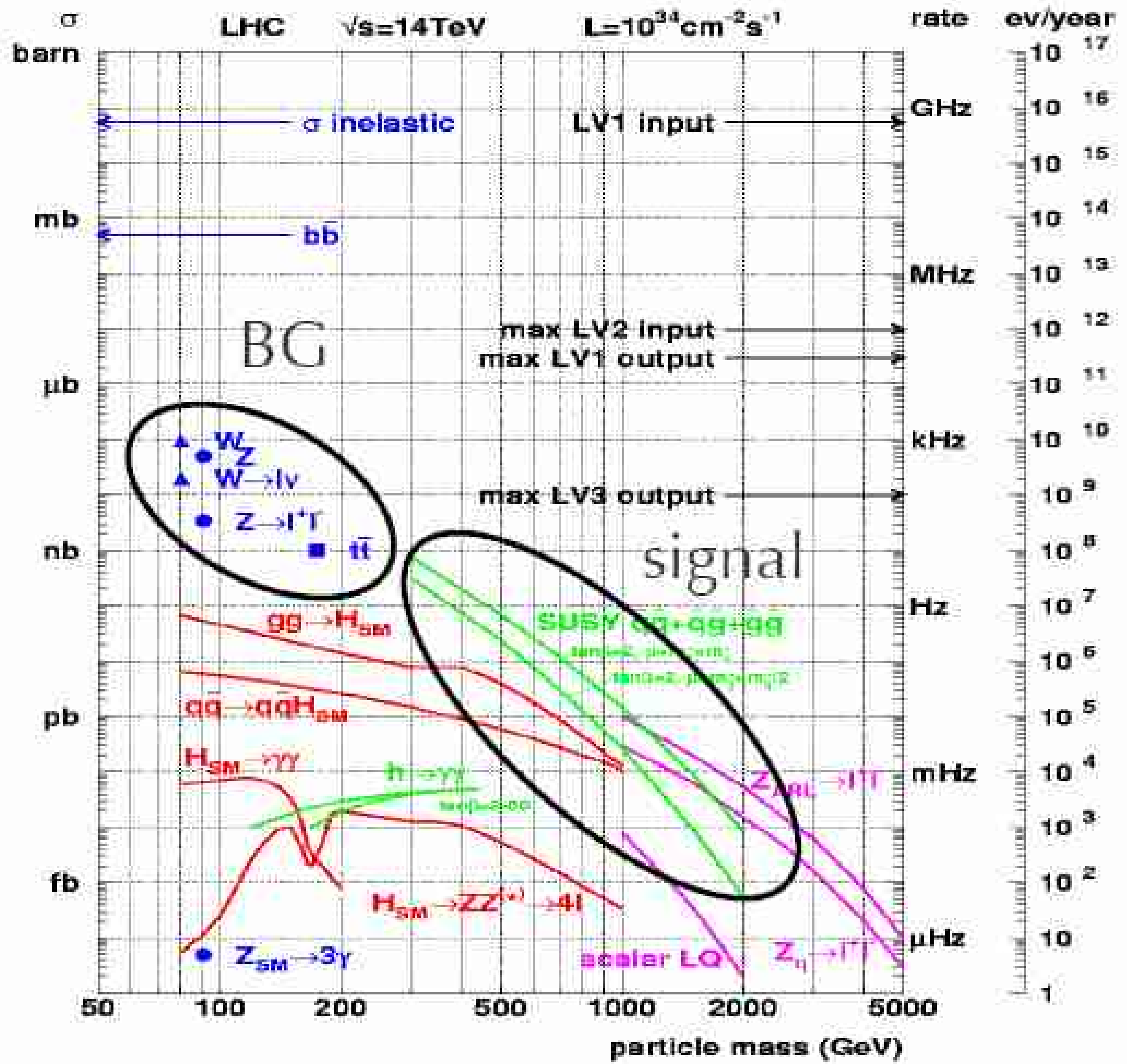


U(1)
SU(2)
SU(3)
α^-1
Log10(Q/GeV)
0
10
20
30
40
50
60
2
4
6
8
10
12
14
16
18

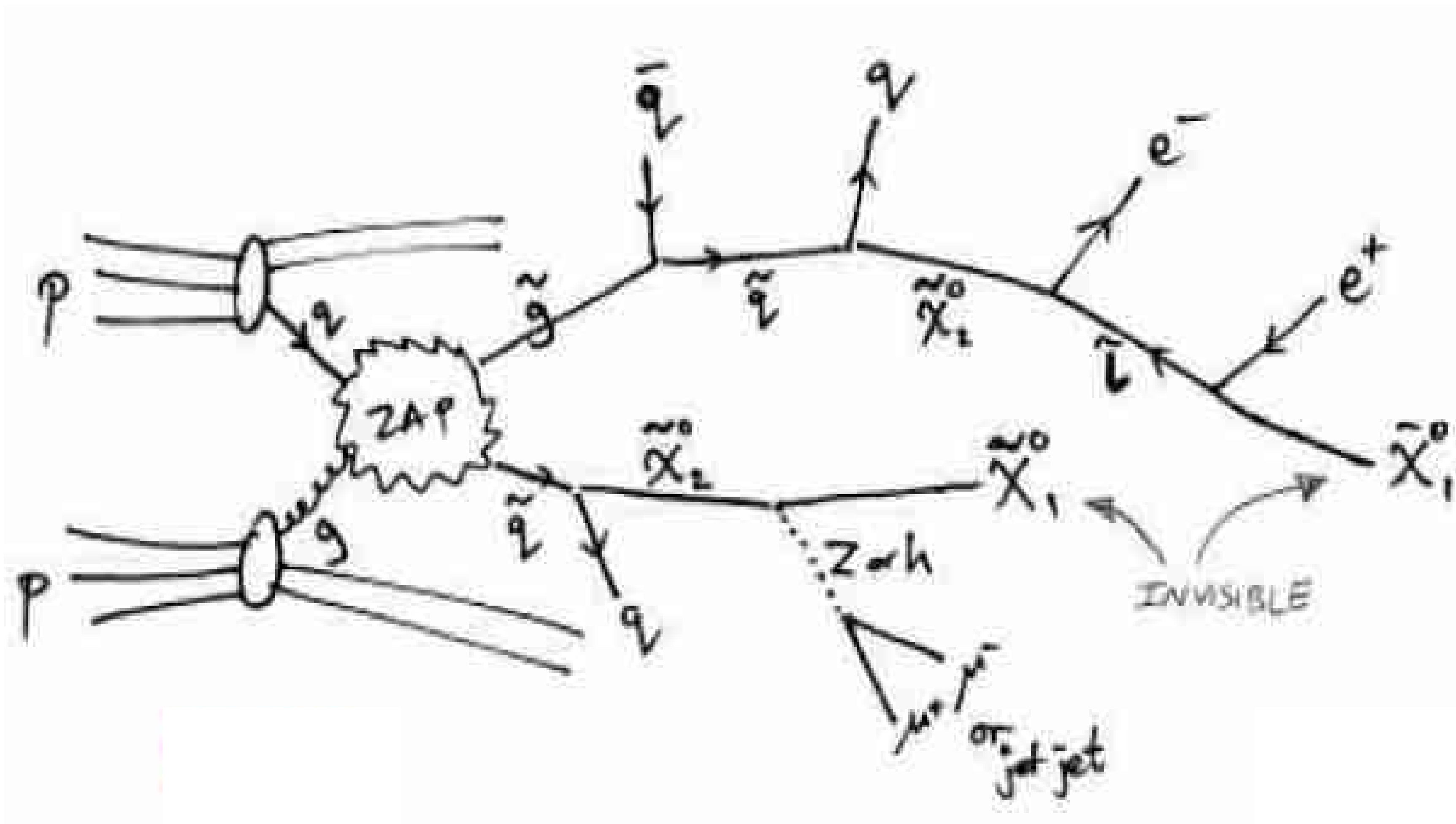
# GUT-motivated models



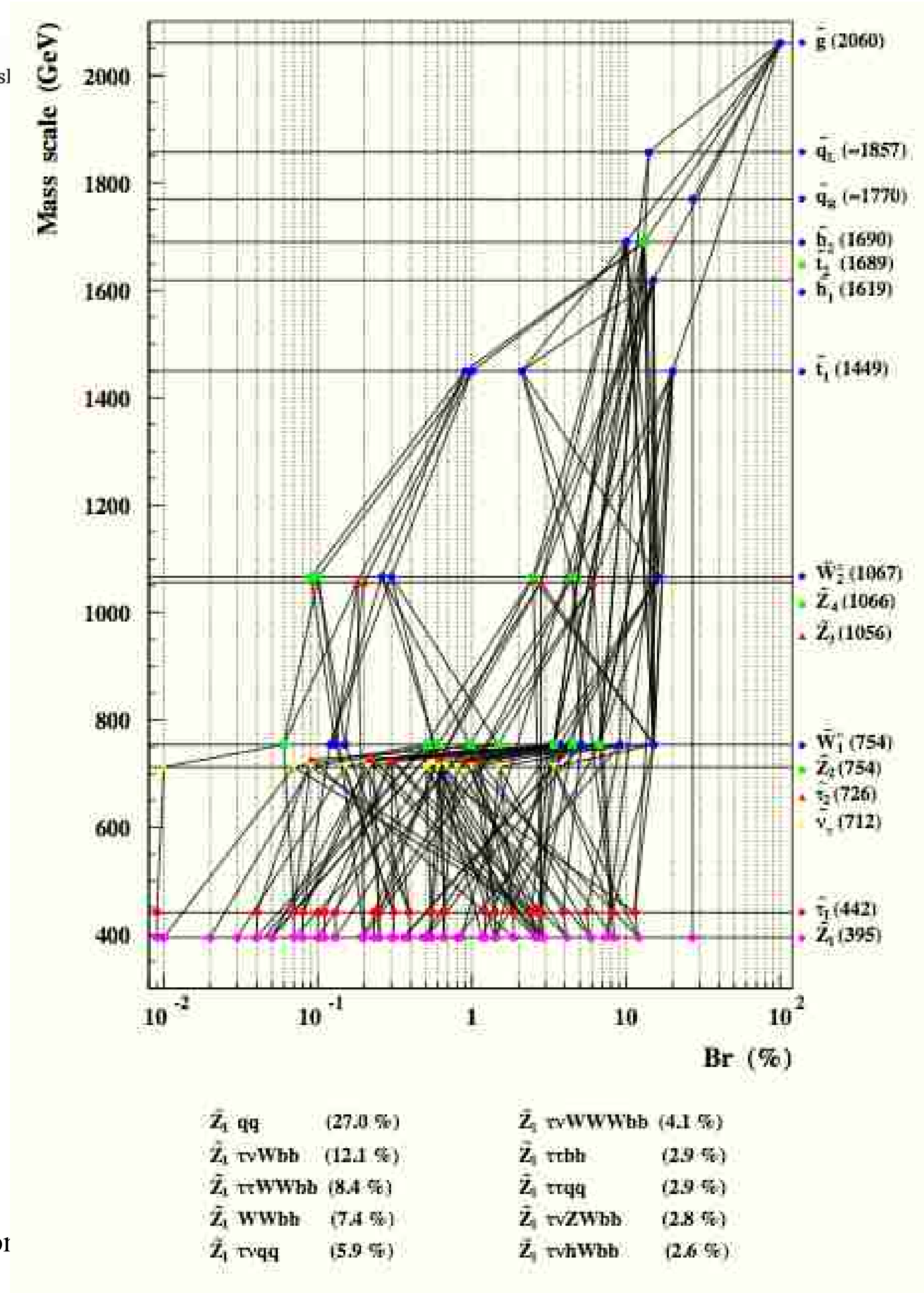
# Phenomenology


σ
barn
LHC √s=14TeV L=10^34 cm^-2 s^-1
rate
ev/year
σ inelastic
LV1 input
GHz
mb
bb̄
MHz
BG
max LV2 input
max LV1 output
μb
W
Z
W→lν
kHz
max LV3 output
Z→l⁺l⁻
tt̄
nb
signal
Hz
gg→H_SM
SUSY q̃q̃+q̃g̃+g̃g̃
qq→qqH_SM
pb
H_SM→γγ
h→γγ
mHz
fb
H_SM→ZZ(*)→4l
Z_SM→3γ
scalar LQ
μHz
50
100
200
500
1000
2000
5000
particle mass (GeV)
10^17
10^16
10^15
10^14
10^13
10^12
10^11
10^10
10^9
10^8
10^7
10^6
10^5
10^4
10^3
10^2
10
1

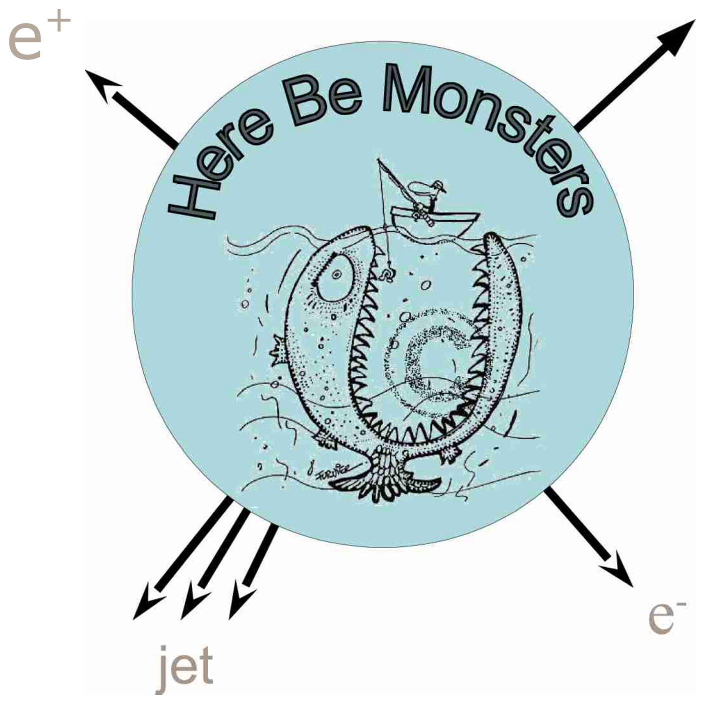
# SUSY cascades



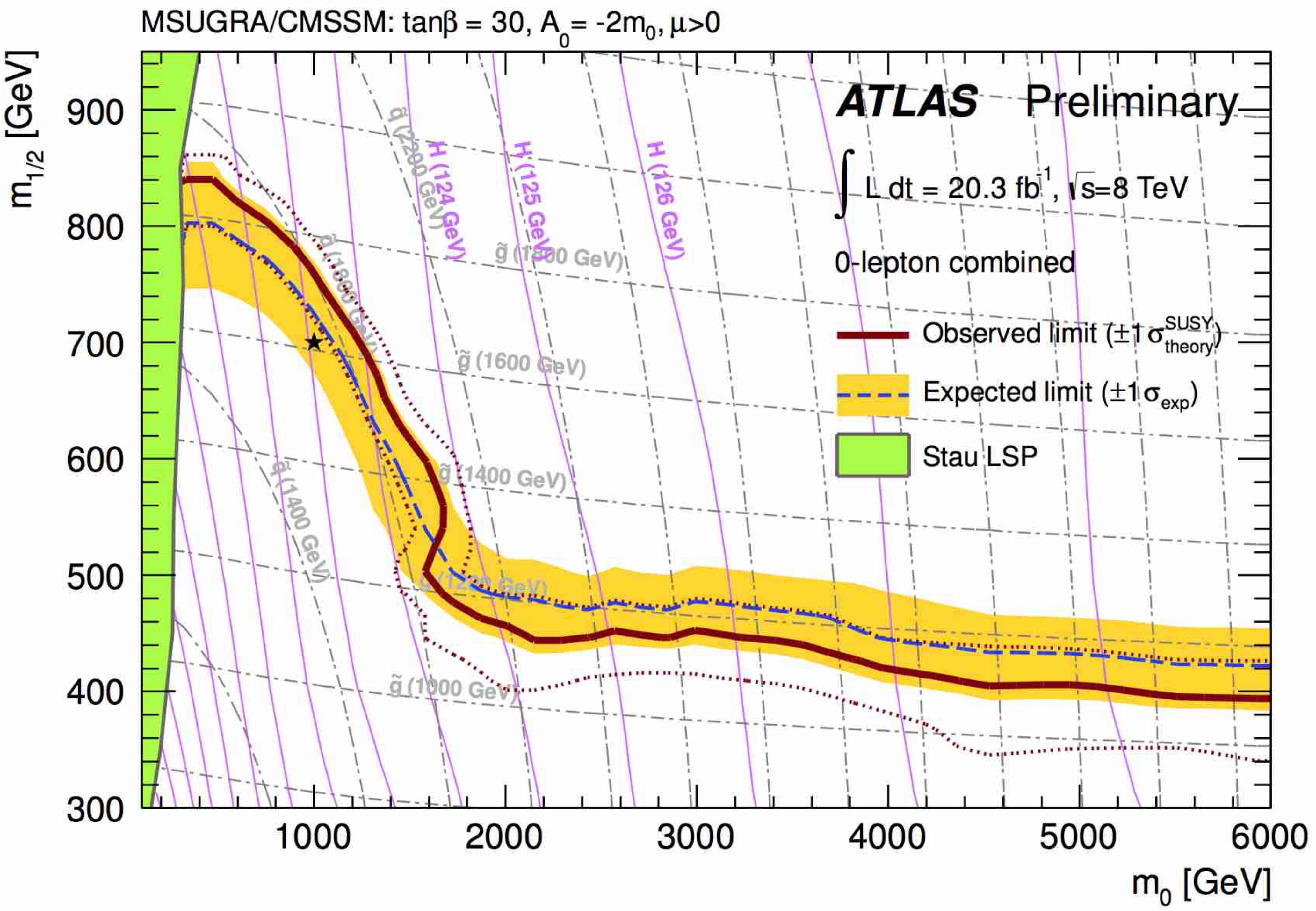

[Drawing by Chris Lester]

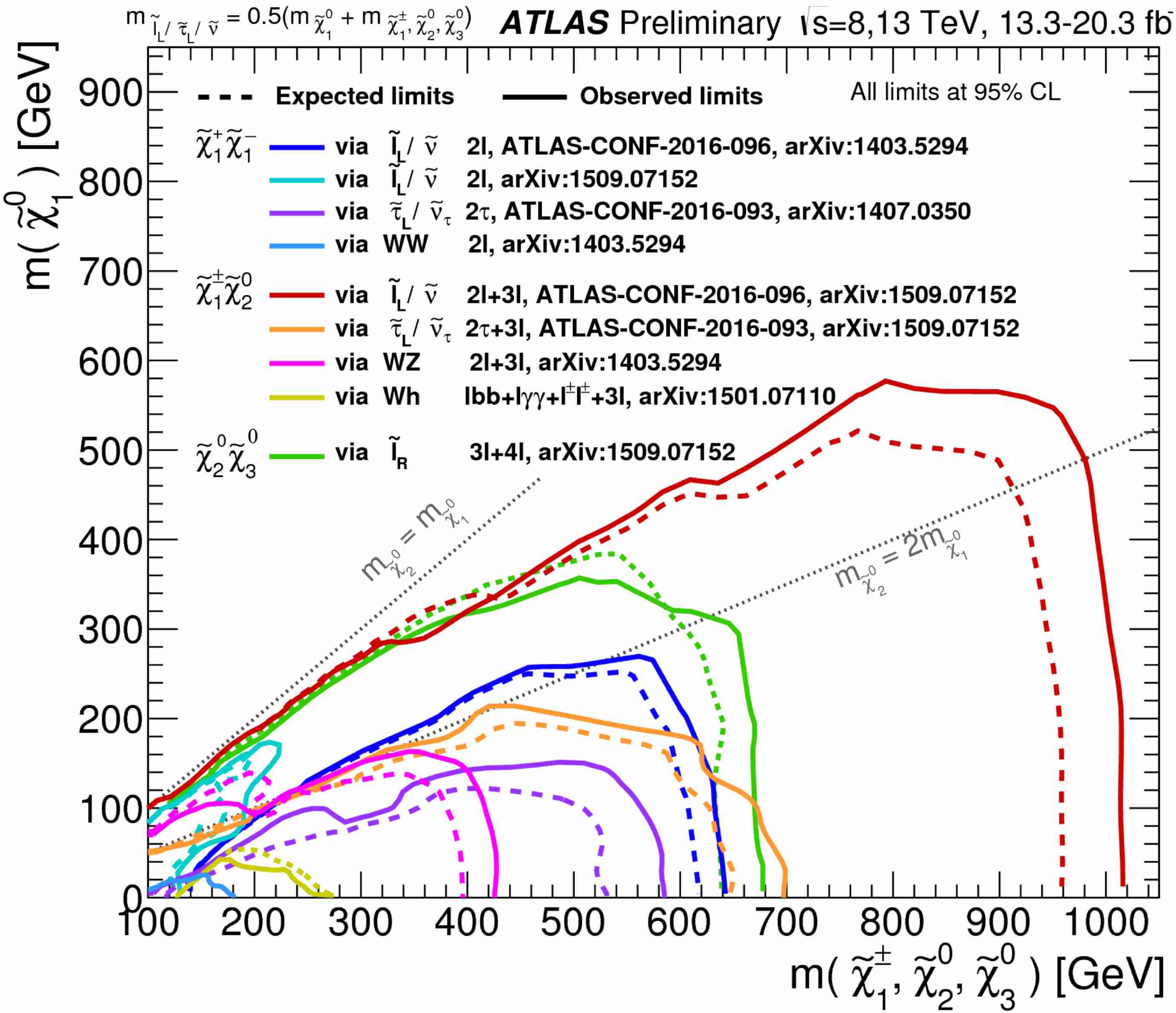

Mass scale (GeV)
2000
1800
1600
1400
1200
1000
800
600
400
10^-2
10^-1
1
10
10^2
Br (%)
g̃ (2060)
q̃_L (≈1857)
q̃_R (≈1770)
b̃_2 (1690)
t̃_2 (1689)
b̃_1 (1619)
t̃_1 (1449)
W̃_2^± (1067)
Z̃_4 (1066)
Z̃_3 (1056)
W̃_1^± (754)
Z̃_2 (754)
τ̃_2 (726)
ν̃_τ (712)
τ̃_1 (442)
Z̃_1 (395)
Z̃_1 qq (27.0 %)
Z̃_1 τνWbb (12.1 %)
Z̃_1 ττWWbb (8.4 %)
Z̃_1 WWbb (7.4 %)
Z̃_1 τνqq (5.9 %)
Z̃_1 τνWWWbb (4.1 %)
Z̃_1 ττbb (2.9 %)
Z̃_1 ττqq (2.9 %)
Z̃_1 τνZWbb (2.8 %)
Z̃_1 τνhWbb (2.6 %)

# Realistic detector




Missing transverse energy


MSUGRA/CMSSM: tanβ = 30, A0= -2m0, μ>0
ATLAS Preliminary
∫ L dt = 20.3 fb-1, √s=8 TeV
0-lepton combined
Observed limit (±1 σ SUSY theory)
Expected limit (±1 σexp)
Stau LSP
H (124 GeV)
H (125 GeV)
H (126 GeV)
g̃ (2200 GeV)
g̃ (1800 GeV)
g̃ (1600 GeV)
g̃ (1400 GeV)
g̃ (1200 GeV)
g̃ (1000 GeV)
m1/2 [GeV]
m0 [GeV]
300
400
500
600
700
800
900
1000
2000
3000
4000
5000
6000



$m_{\tilde{l}_L/\tilde{\tau}_L/\tilde{\nu}} = 0.5(m_{\tilde{\chi}_1^0} + m_{\tilde{\chi}_1^\pm,\tilde{\chi}_2^0,\tilde{\chi}_3^0})$
ATLAS Preliminary $\sqrt{s}$=8,13 TeV, 13.3-20.3 fb$^{-1}$
Expected limits
Observed limits
All limits at 95% CL
$\tilde{\chi}_1^+\tilde{\chi}_1^-$ via $\tilde{l}_L/\tilde{\nu}$ 2l, ATLAS-CONF-2016-096, arXiv:1403.5294
via $\tilde{l}_L/\tilde{\nu}$ 2l, arXiv:1509.07152
via $\tilde{\tau}_L/\tilde{\nu}_\tau$ 2τ, ATLAS-CONF-2016-093, arXiv:1407.0350
via WW 2l, arXiv:1403.5294
$\tilde{\chi}_1^\pm\tilde{\chi}_2^0$ via $\tilde{l}_L/\tilde{\nu}$ 2l+3l, ATLAS-CONF-2016-096, arXiv:1509.07152
via $\tilde{\tau}_L/\tilde{\nu}_\tau$ 2τ+3l, ATLAS-CONF-2016-093, arXiv:1509.07152
via WZ 2l+3l, arXiv:1403.5294
via Wh lbb+lγγ+l$^\pm$l$^\pm$+3l, arXiv:1501.07110
$\tilde{\chi}_2^0\tilde{\chi}_3^0$ via $\tilde{l}_R$ 3l+4l, arXiv:1509.07152
$m_{\tilde{\chi}_2^0} = m_{\tilde{\chi}_1^0}$
$m_{\tilde{\chi}_2^0} = 2m_{\tilde{\chi}_1^0}$
$m(\tilde{\chi}_1^0)$ [GeV]
0
100
200
300
400
500
600
700
800
900
1000
$m(\tilde{\chi}_1^\pm, \tilde{\chi}_2^0, \tilde{\chi}_3^0)$ [GeV]

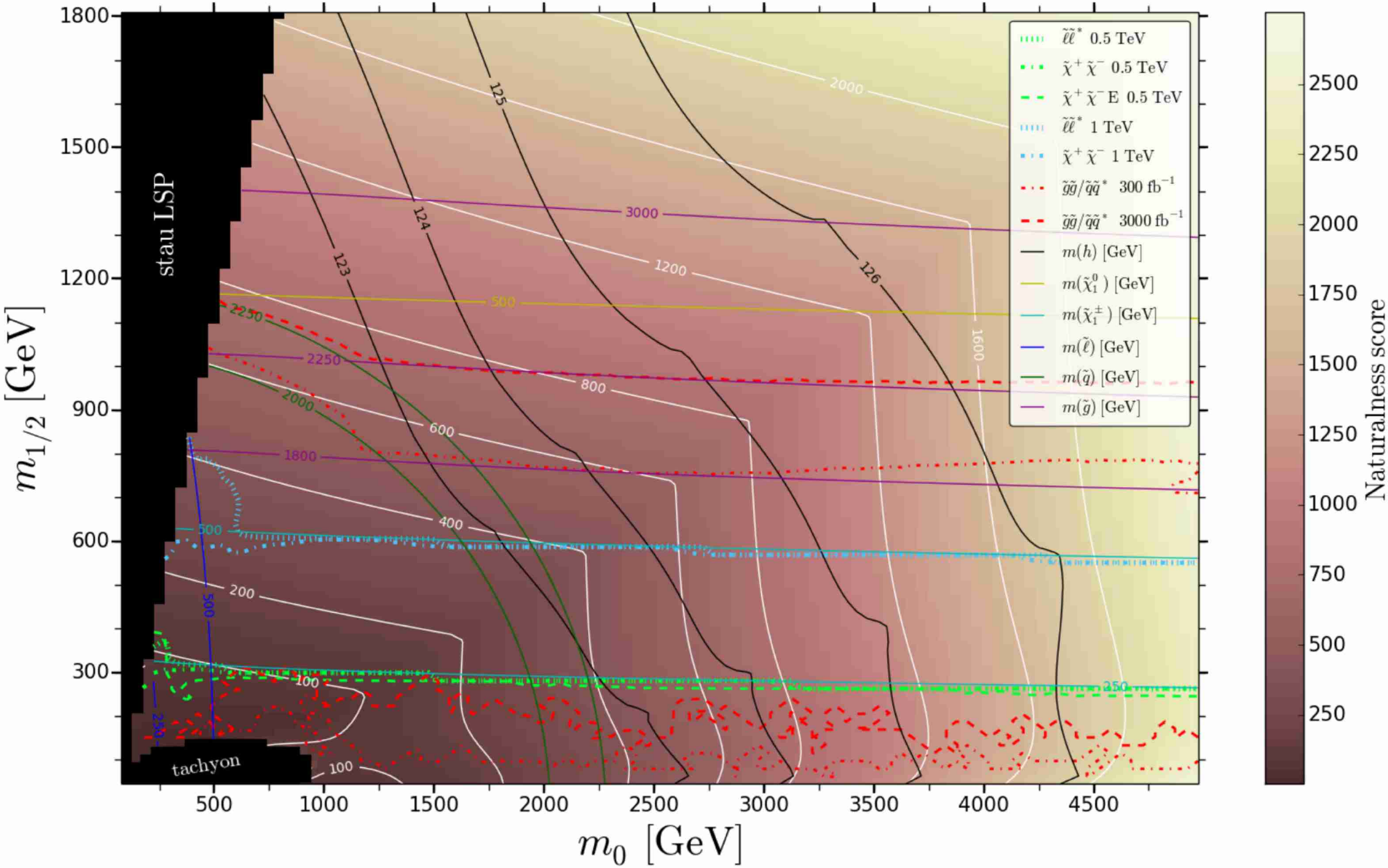
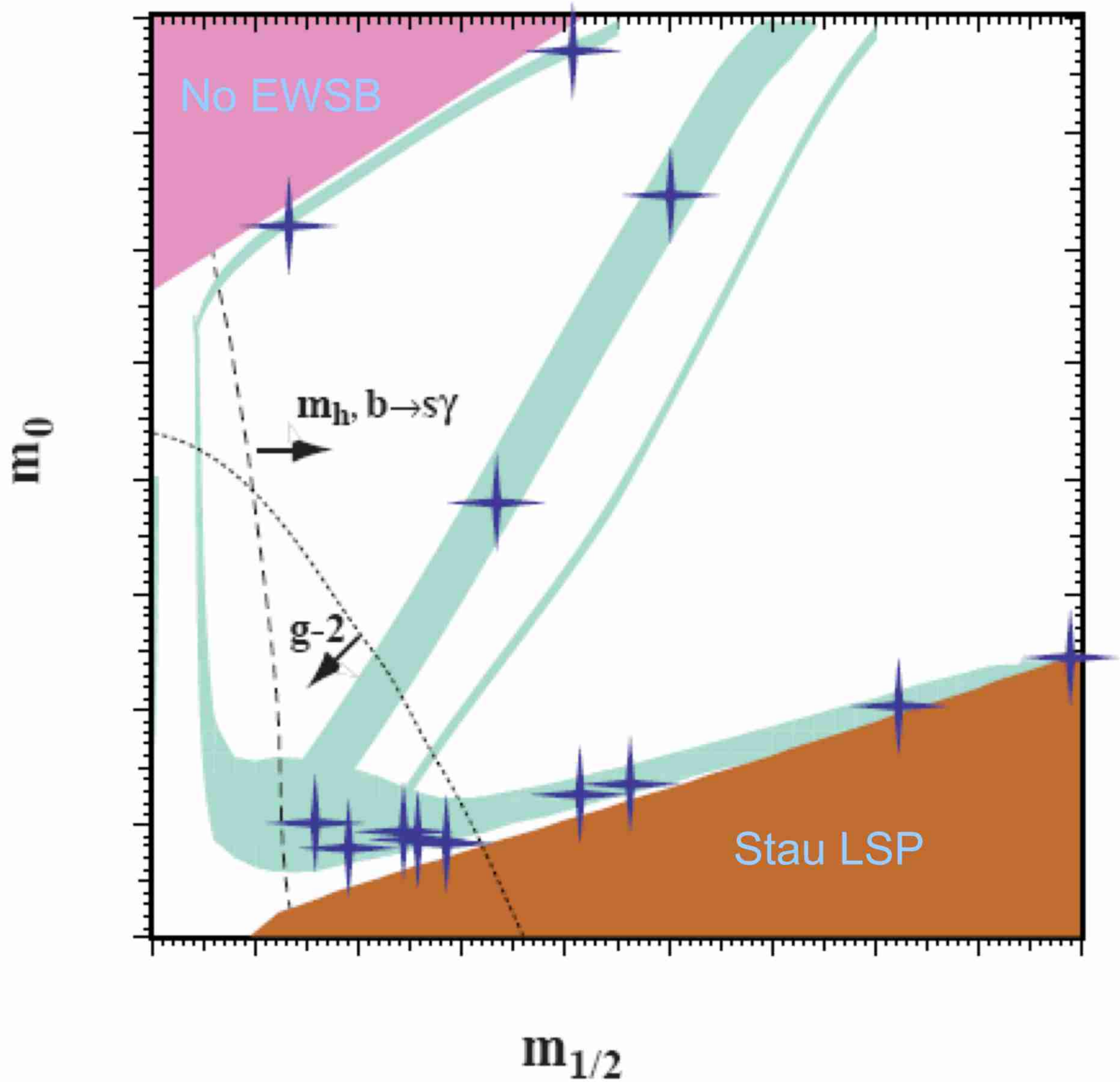



**Figure 6.4:** The mSUGRA30 scenario with $A_0 = -2m_0$, $\tan\beta = 30$, $\text{sgn}(\mu) = +$. The white lines are contours for the naturalness score.

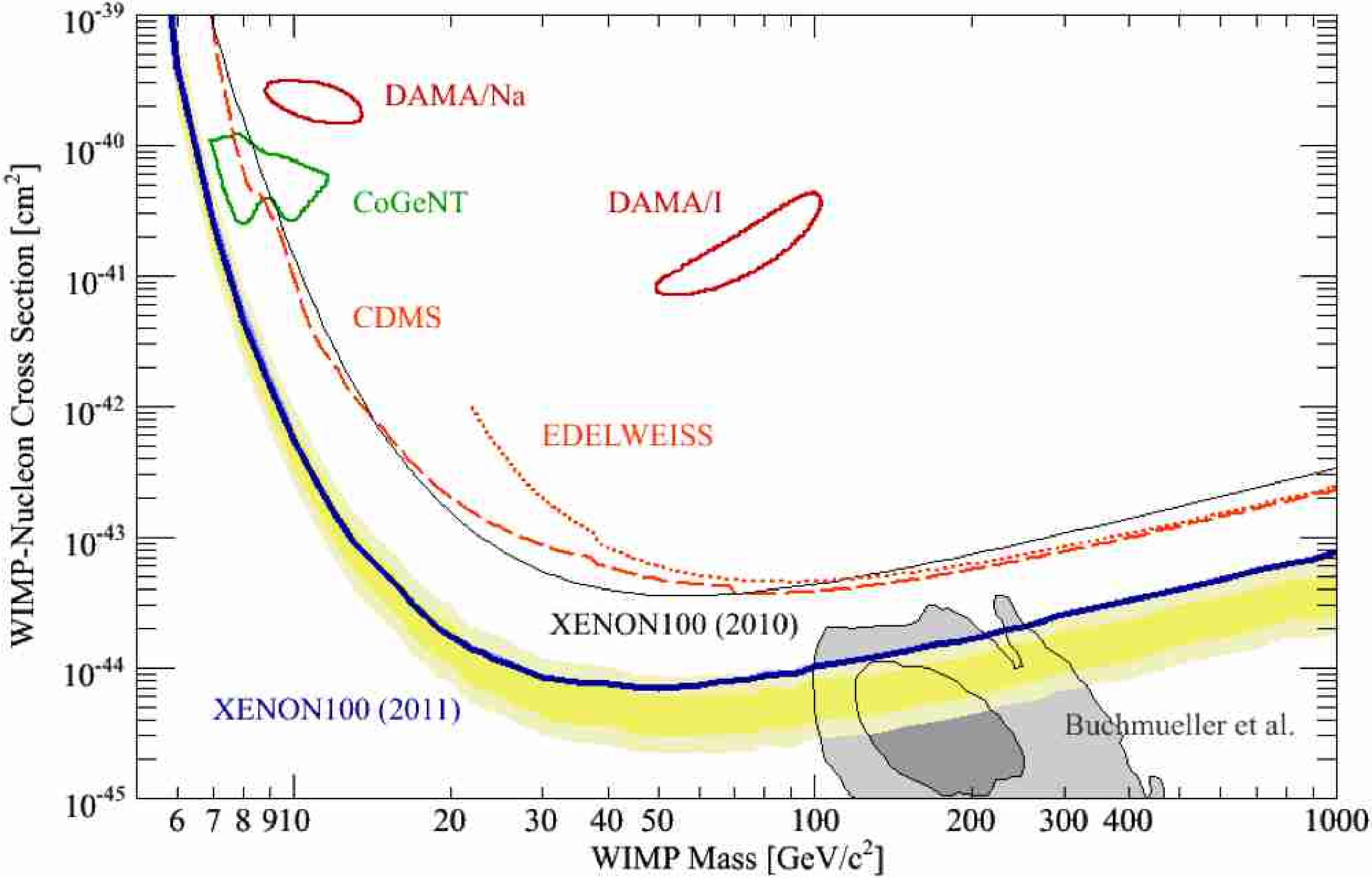
[M. Røysheim, MSc. thesis]

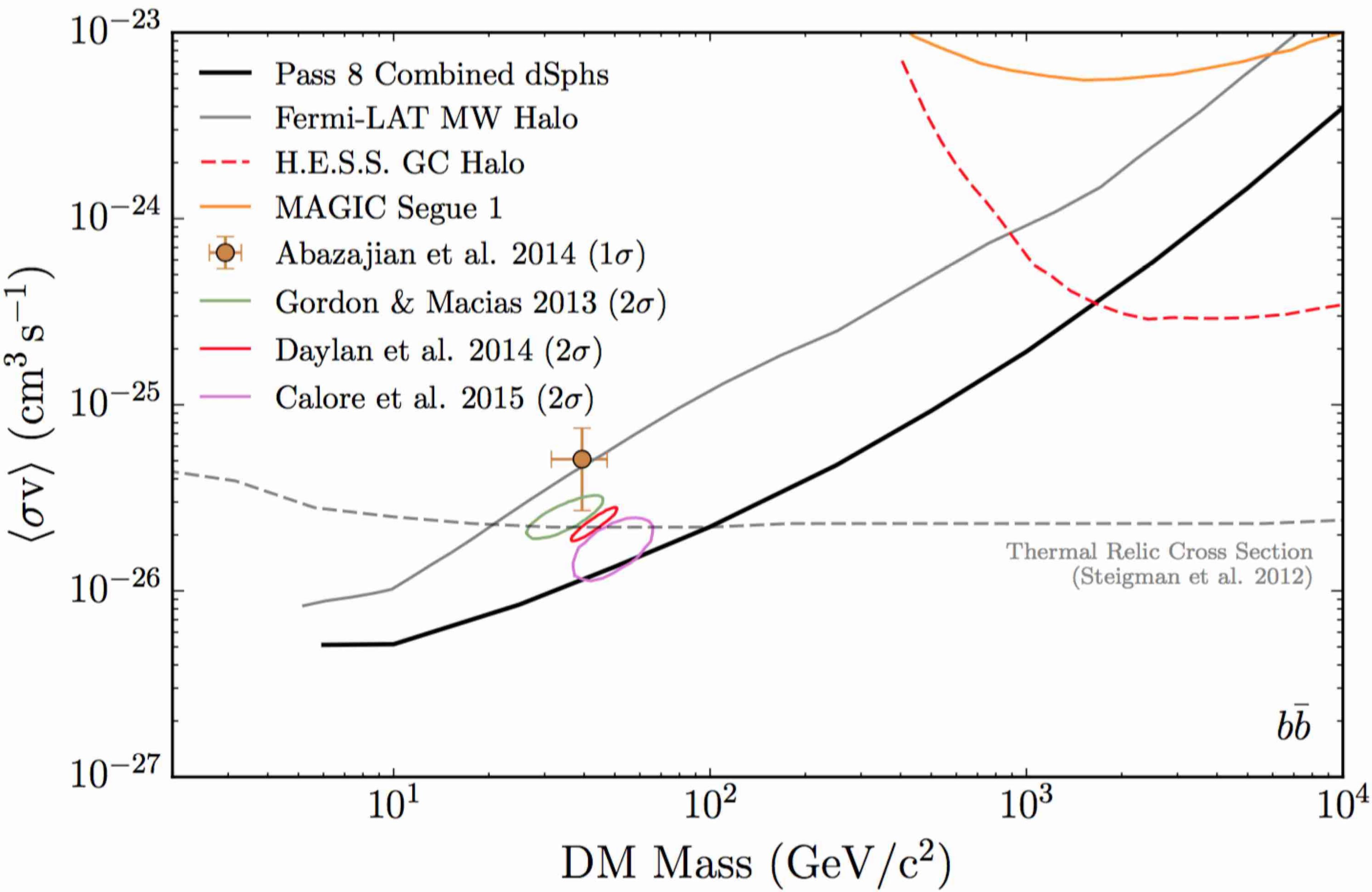
# Neutralino Dark Matter

- A neutralino LSP is a good Dark Matter candidate:
  - It is stable (R-parity).
  - It has no charge (electric or strong).
  - It is weakly interacting $\Rightarrow$ WIMP candidate.

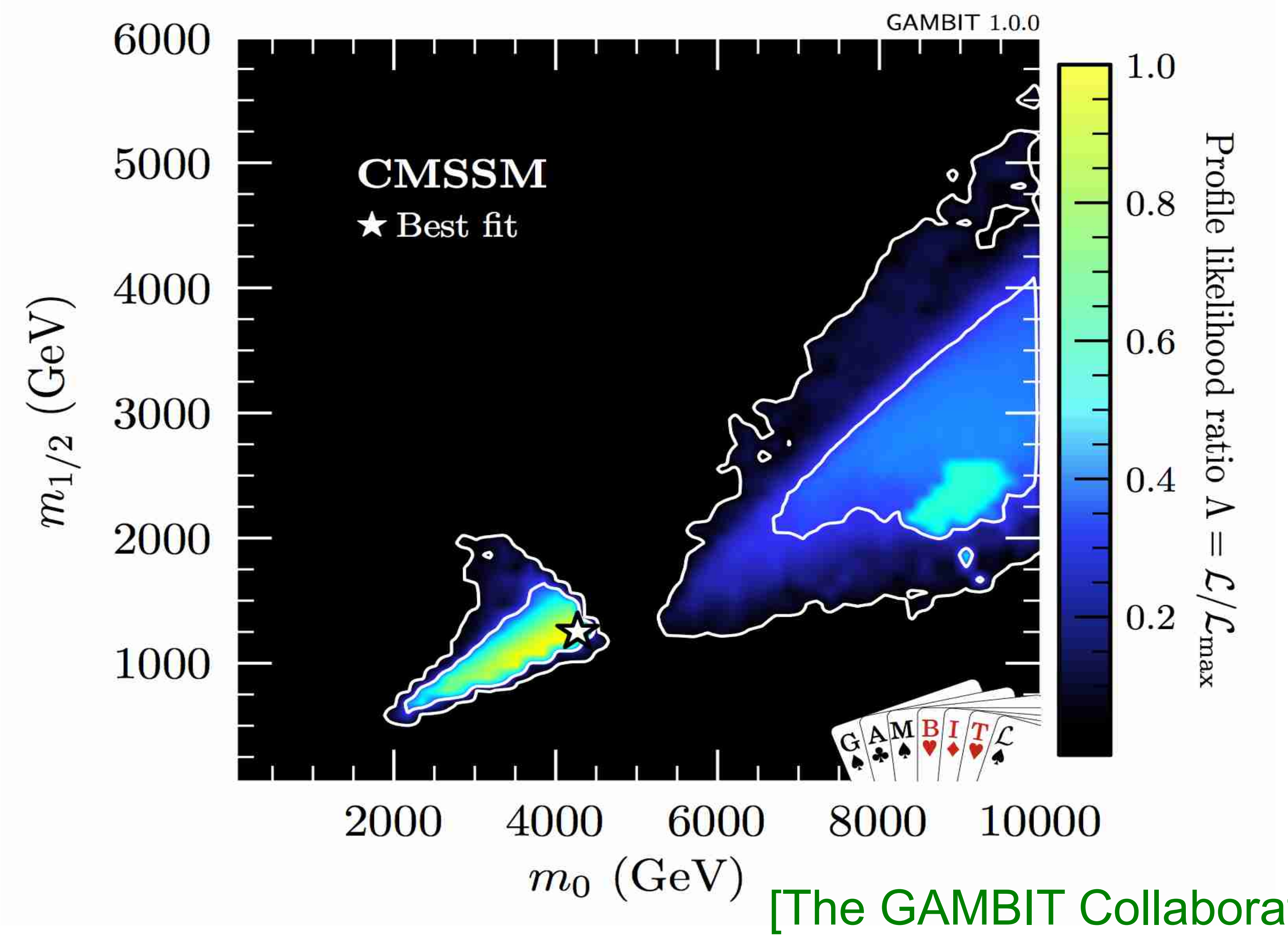



[Battaglia *et al.*, hep-ph/0106204]


DAMA/Na
CoGeNT
DAMA/I
CDMS
EDELWEISS
XENON100 (2010)
XENON100 (2011)
Buchmueller et al.
WIMP-Nucleon Cross Section [cm$^2$]
WIMP Mass [GeV/c$^2$]
10$^{-39}$
10$^{-40$}
10$^{-41}$
10$^{-42}$
10$^{-43}$
10$^{-44}$
10$^{-45}$
6 7 8 910 20 30 40 50 100 200 300 400 1000


Pass 8 Combined dSphs
Fermi-LAT MW Halo
H.E.S.S. GC Halo
MAGIC Segue 1
Abazajian et al. 2014 (1σ)
Gordon & Macias 2013 (2σ)
Daylan et al. 2014 (2σ)
Calore et al. 2015 (2σ)
Thermal Relic Cross Section
(Steigman et al. 2012)
$b\bar{b}$
$\langle\sigma v\rangle$ (cm$^3$ s$^{-1}$)
DM Mass (GeV/c$^2$)
$10^{-23}$
$10^{-24}$
$10^{-25}$
$10^{-26}$
$10^{-27}$
$10^1$
$10^2$
$10^3$
$10^4$

# Final slide (almost)




[The GAMBIT Collaboration, unpublished]

# Final slide (really!)

- Supersymmetry is well motivated:
  - It solves the Higgs hierarchy problem.
  - It has (multiple) good Dark Matter candidates.
  - It can provide GUT-models.
- Searches have turned up nothing:
  - Performed on very simplified models.
  - Mostly assuming R-parity.
  - Collider searches blind to degeneracies.
  - The Higgs mass tells us that SUSY should be heavy.

# Bonus material

# The full superalgebra

$$
\begin{aligned}
\{Q_a^\alpha, Q_b^\beta\} &= \{\bar{Q}_a^\alpha, \bar{Q}_b^\beta\} = 0 \\
\{Q_a^\alpha, \bar{Q}_b^\beta\} &= 2\delta^{\alpha\beta}\gamma^\mu_{ab}P_\mu \\
[Q_a^\alpha, P_\mu] &= [\bar{Q}_a^\alpha, P_\mu] = 0 \\
[Q_a^\alpha, M^{\mu\nu}] &= \sigma^{\mu\nu}_{ab} Q_b^\alpha \\
[Q_a^\alpha, B_l] &= iS_l^{\alpha\beta} Q_a^\beta \\
[B_k, B_l] &= ic_{klm} B_m \\
\{Q_a^\alpha, Q_b^\beta\} &= \epsilon_{ab} Z^{\alpha\beta} \\
Z^{\alpha\beta} &= -Z^{\beta\alpha} \\
[Z^{\alpha\beta}, B_l] &= 0
\end{aligned}
$$

[Haag, Lopuszanski and Sohnius, '75]

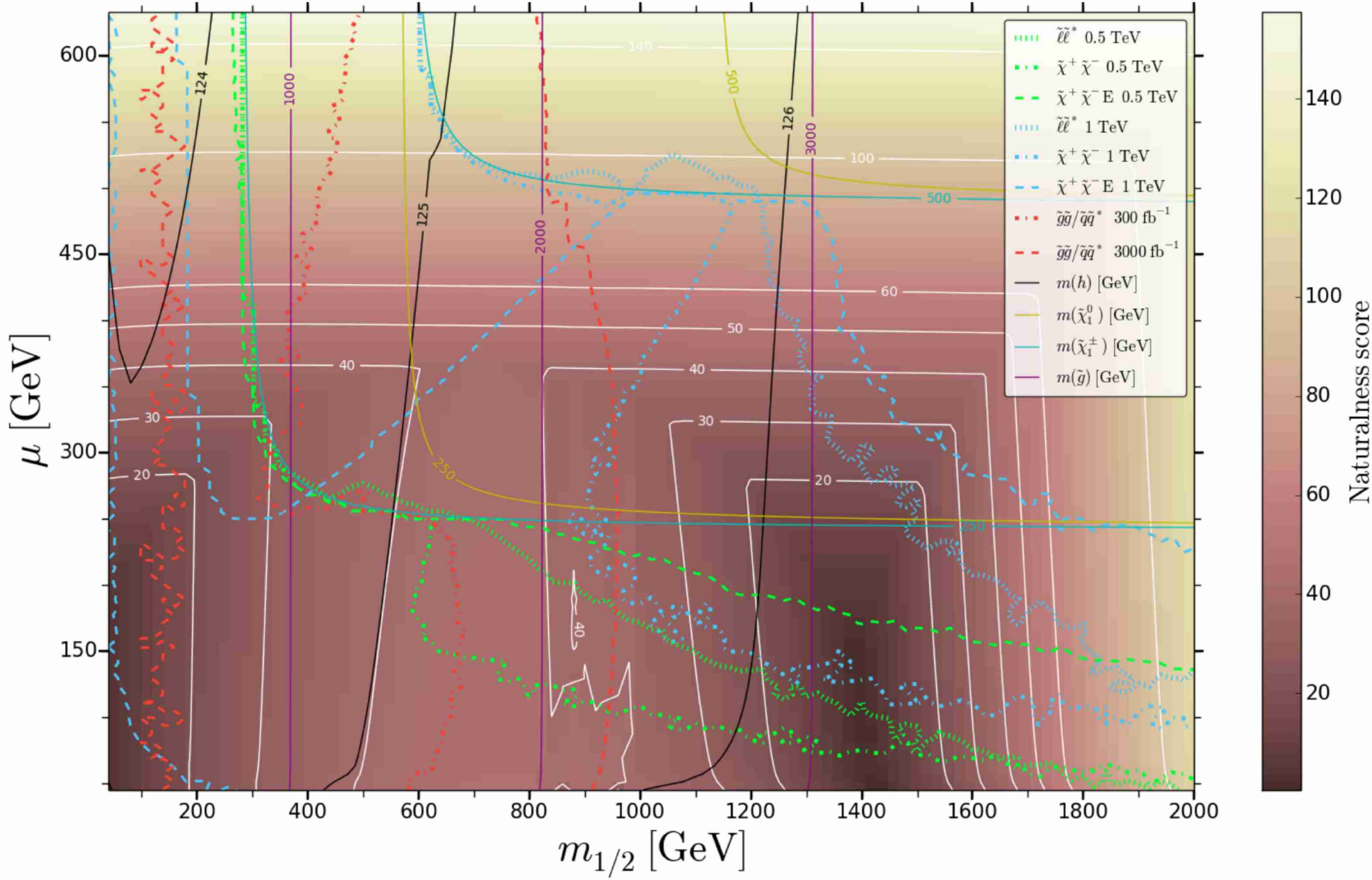
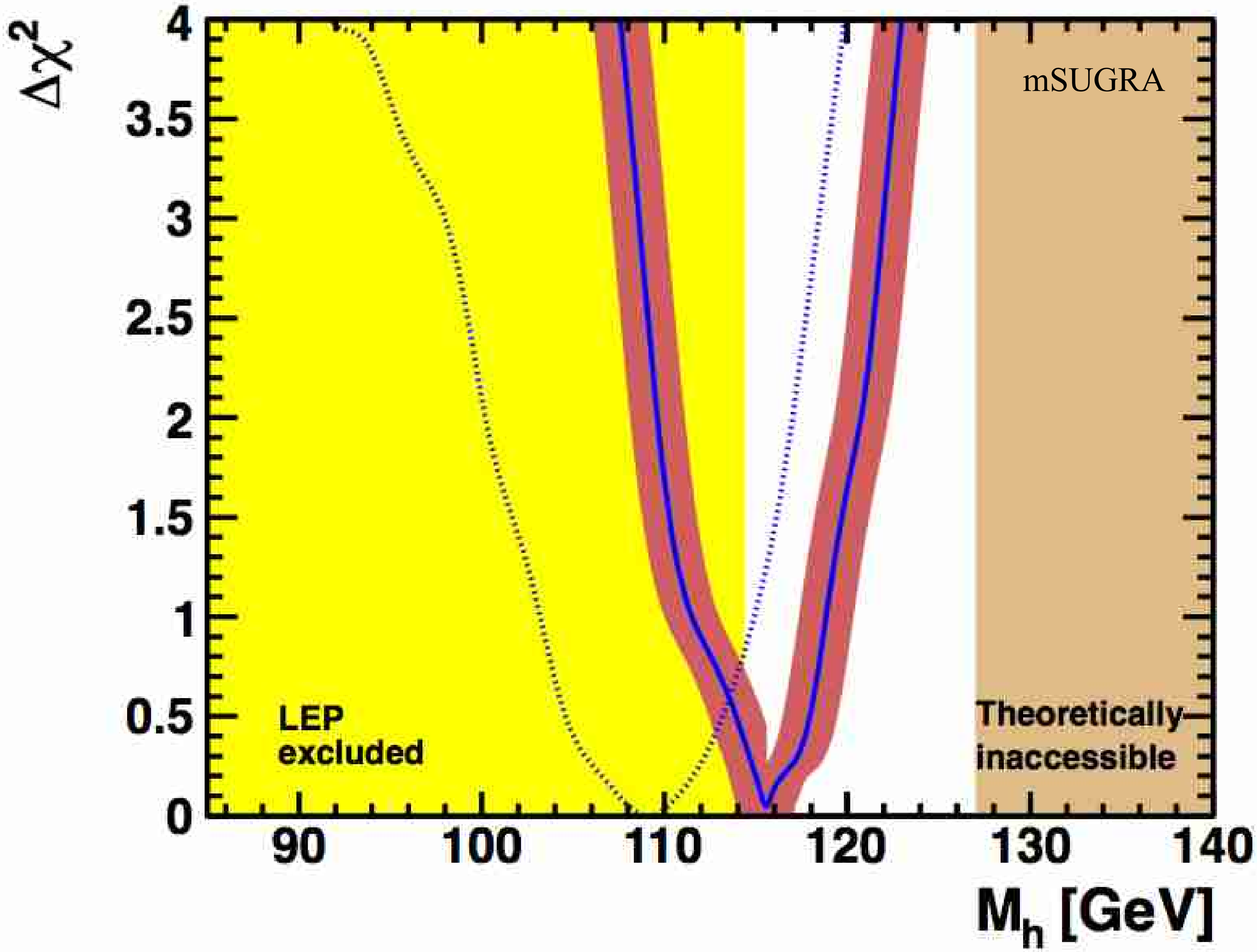



**Figure 6.6:** The NUHM2 scenario with $m_0 = 4$ TeV, $\tan\beta = 15$, $A_0 = -1.6m_0$, $m_A = 1$ TeV. The white lines are contours for the naturalness score.



[M. Røysheim, MSc. thesis]


$\Delta\chi^2$
4
3.5
3
2.5
2
1.5
1
0.5
0
90
100
110
120
130
140
$M_h$ [GeV]
mSUGRA
LEP excluded
Theoretically inaccessible

# $(g\text{-}2)_\mu$

$$a_\mu(\tilde{\chi}^-) = \frac{1}{8\pi^2}\frac{m_\mu}{m_{\tilde{\nu}_\mu}}\sum_{j=1}^{2}\left\{\left(\left|g_L^{\tilde{\chi}_j^-\mu\tilde{\nu}_\mu}\right|^2 + \left|g_R^{\tilde{\chi}_j^-\mu\tilde{\nu}_\mu}\right|^2\right)\frac{m_\mu}{m_{\tilde{\nu}_\mu}}G_1\left(\frac{m^2_{\tilde{\chi}_j^-}}{m^2_{\tilde{\nu}_\mu}}\right)\right.$$
$$\left. + \mathrm{Re}\left[\left(g_R^{\tilde{\chi}_j^-\mu\tilde{\nu}_\mu}\right)^* g_L^{\tilde{\chi}_j^-\mu\tilde{\nu}_\mu}\right]\frac{m_{\tilde{\chi}_j^-}}{m_{\tilde{\nu}_\mu}}G_3\left(\frac{m^2_{\tilde{\chi}_j^-}}{m^2_{\tilde{\nu}_\mu}}\right)\right\}$$

$$a_\mu(\tilde{\chi}^0) = -\frac{1}{8\pi^2}\sum_{i=1}^{2}\frac{m_\mu}{m_{\tilde{\mu}_i}}\sum_{j=1}^{4}\left\{\left(\left|g_L^{\tilde{\chi}_j^0\mu\tilde{\mu}_i}\right|^2 + \left|g_R^{\tilde{\chi}_j^0\mu\tilde{\mu}_i}\right|^2\right)\frac{m_\mu}{m_{\tilde{\mu}_i}}G_2\left(\frac{m^2_{\tilde{\chi}_j^0}}{m^2_{\tilde{\mu}_i}}\right)\right.$$
$$\left. + \mathrm{Re}\left[\left(g_R^{\tilde{\chi}_j^0\mu\tilde{\mu}_i}\right)^* g_L^{\tilde{\chi}_j^0\mu\tilde{\mu}_i}\right]\frac{m_{\tilde{\chi}_j^0}}{m_{\tilde{\mu}_i}}G_4\left(\frac{m^2_{\tilde{\chi}_j^0}}{m^2_{\tilde{\mu}_i}}\right)\right\}$$