Event-by-Event Fluctuations in Relativistic Nucleus-Nucleus Collisions

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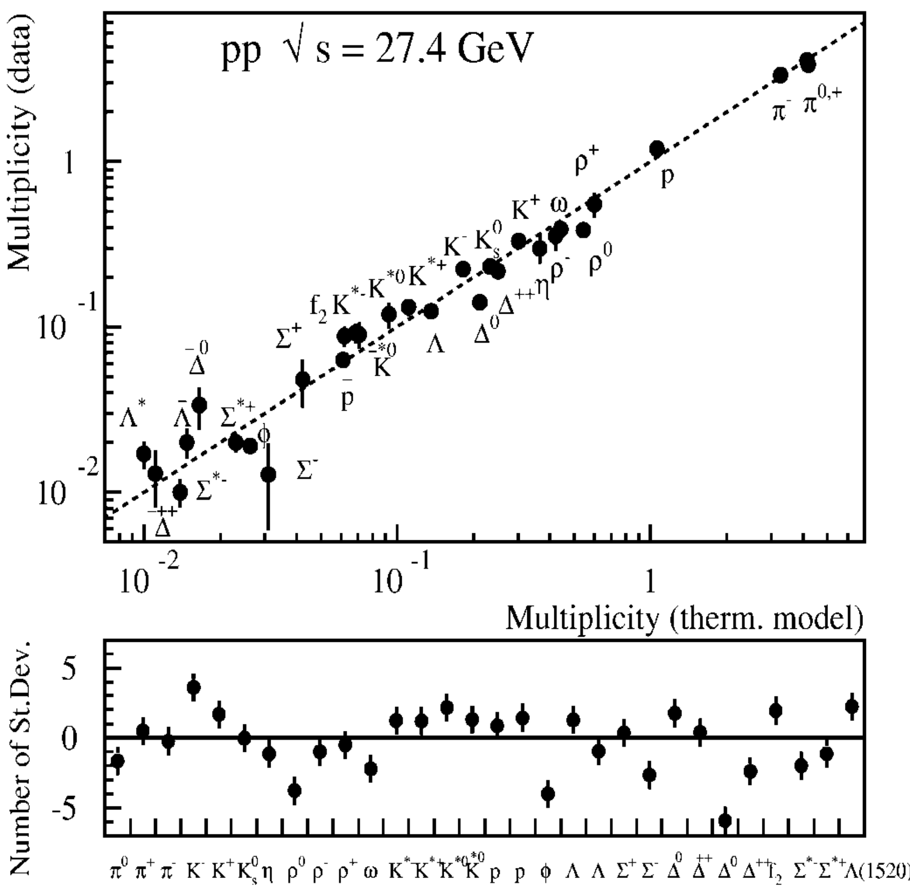
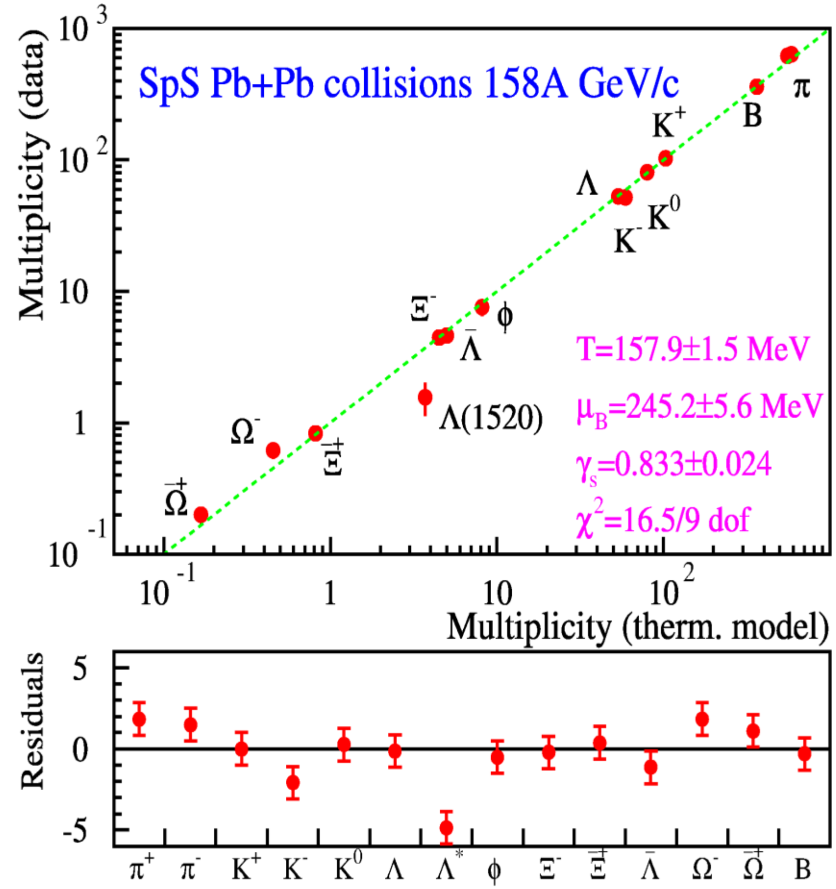
I. Introduction: Mean Multiplicities



Becattini, Heinz, Z. Phys. (1997) CE in p+p



Becattini, Manninen, Gazdzicki, Phys. Rev. C (2006) GCE in A+A

$$\mathbf{\Xi} = \exp\left\{\mathbf{V}\sum_{\mathbf{j}} \eta_{\mathbf{j}}\mathbf{d}_{\mathbf{j}} \int \frac{\mathbf{d^3p}}{(\mathbf{2}\pi)^{\mathbf{3}}} \ln\left[\mathbf{1} + \lambda_{\mathbf{j}}\eta_{\mathbf{j}} \exp\left(-\sqrt{p^2 + m_j^2}/\mathbf{T}\right)\right]\right\}$$

$$\mu_j = b_j \mu_B + s_j \mu_S + q_j \mu_Q, \qquad S = 0, \qquad \frac{Q}{B} = 0.4 \div 0.5$$

$$\overline{\mathbf{A}} = \frac{\mathbf{1}}{\mathbf{\Xi}} \lambda_{\mathbf{A}} \frac{\partial}{\partial \lambda_{\mathbf{A}}} \mathbf{\Xi} = \mathbf{V}\mathbf{n}_{\mathbf{A}}$$

$$\lambda_{\mathbf{j}} = \exp\left(\frac{\mu_{\mathbf{j}}}{\mathbf{T}}\right)$$

$$\eta_{\mathbf{j}} = \pm 1$$

$$n_A = \frac{d_A}{2\pi^2} \int_0^\infty k^2 dk \; \frac{1}{\exp\left[\dfrac{\sqrt{k^2 + m_A^2} - \mu_A}{T}\right] \pm 1}$$

$$\mathbf{n}_{\mathbf{A}} = \overline{\mathbf{A}}/\mathbf{V}$$

$$< A > \;=\; \overline{A} + \sum_i \overline{R_i}\, b\left(R_i \to A\right)$$

Momentum Spectra

Becattini, Passaleva,

Eur. Phys. J. (2002),

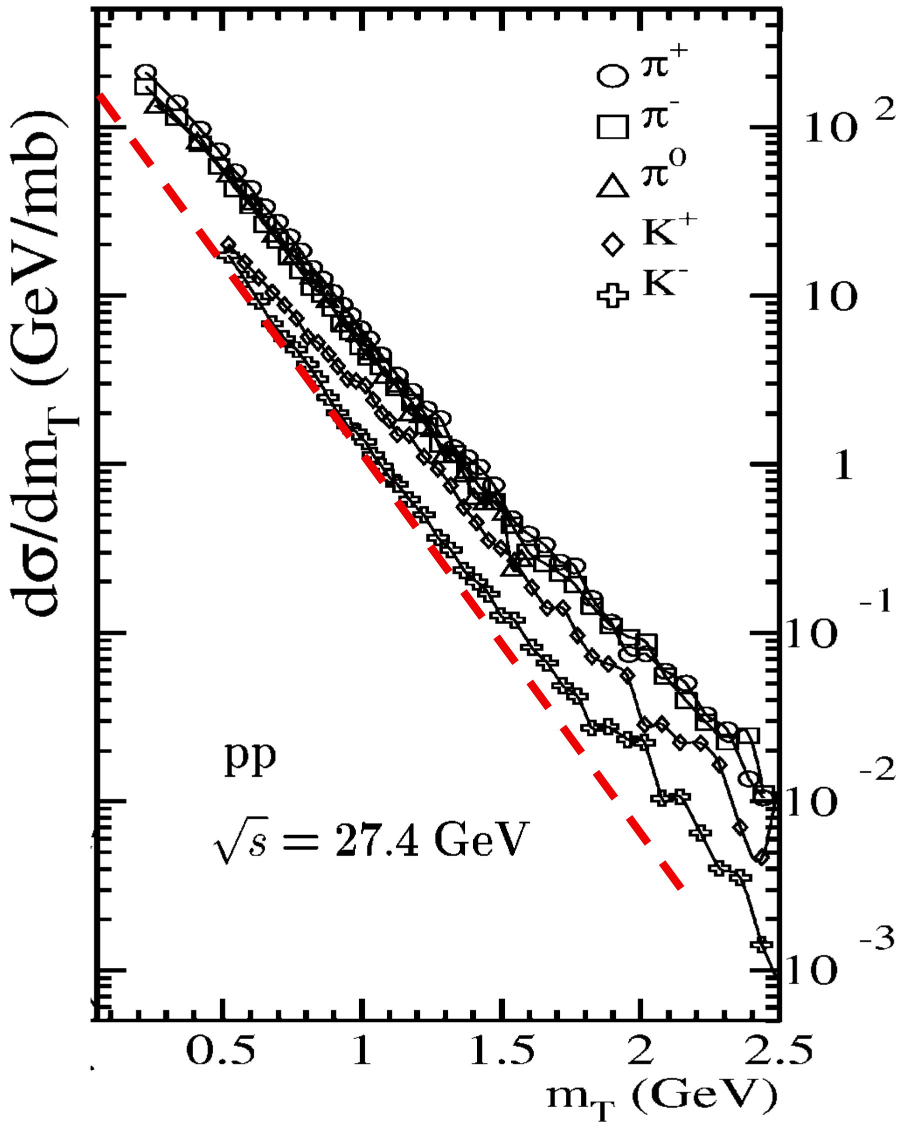
data from

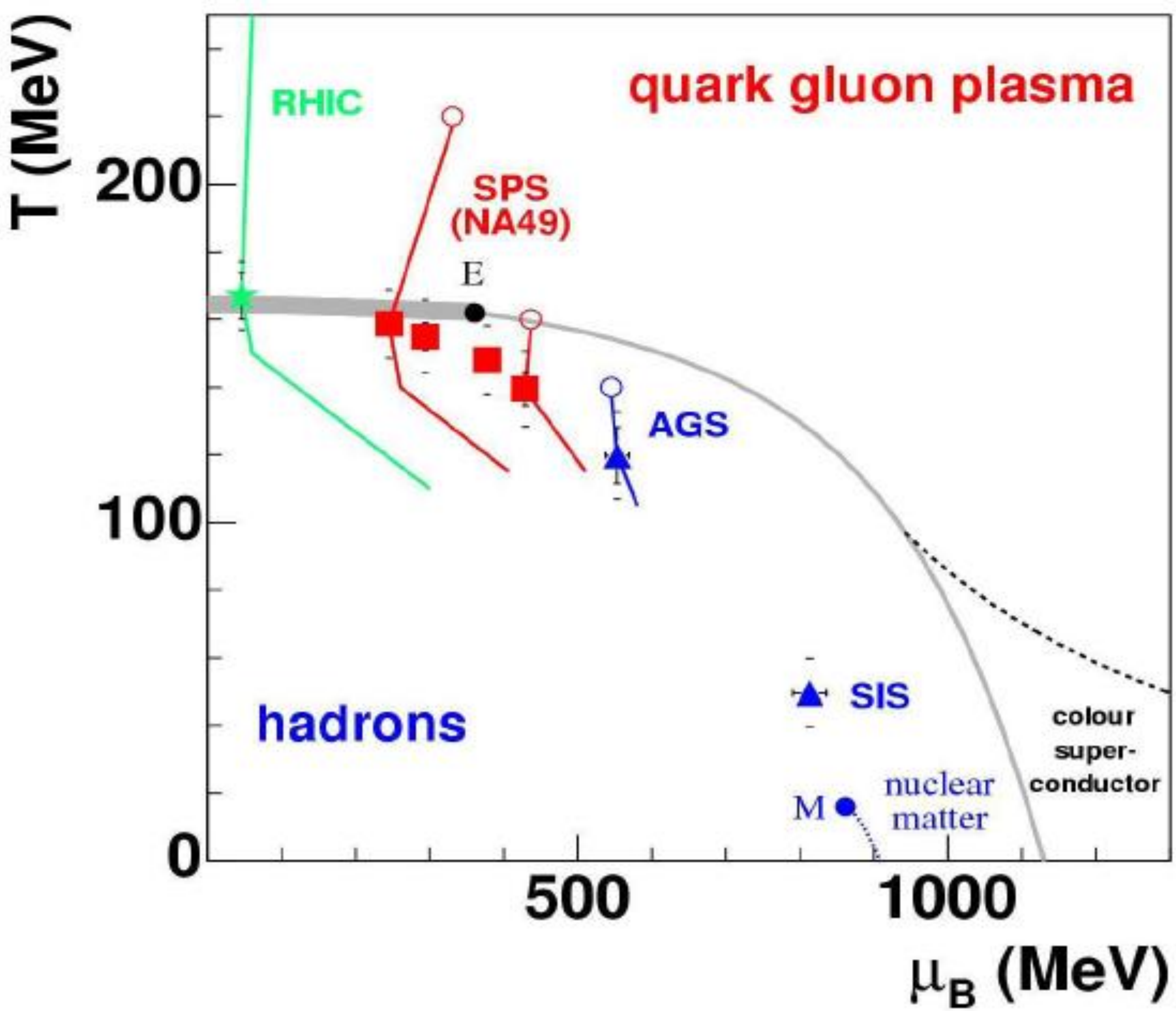
Aguilar-Benitez et al.,

Z. Phys. C (1990)

$$\boxed{\exp\left(-\frac{\mathrm{m_T}}{\mathrm{T}}\right)}$$

$$\mathrm{m_T} = \sqrt{p_T^2 + m^2}$$




T (MeV)
200
100
0
500
1000
μ_B (MeV)
RHIC
quark gluon plasma
SPS
(NA49)
E
AGS
SIS
hadrons
colour
super-
conductor
nuclear
matter
M

I. Global Conservation Laws. Statistical Ensembles

$E \Longleftrightarrow T$ $V \Longleftrightarrow p$ $Q \Longleftrightarrow \mu_Q$

$2^3 = 8$

E, V, Q MCE

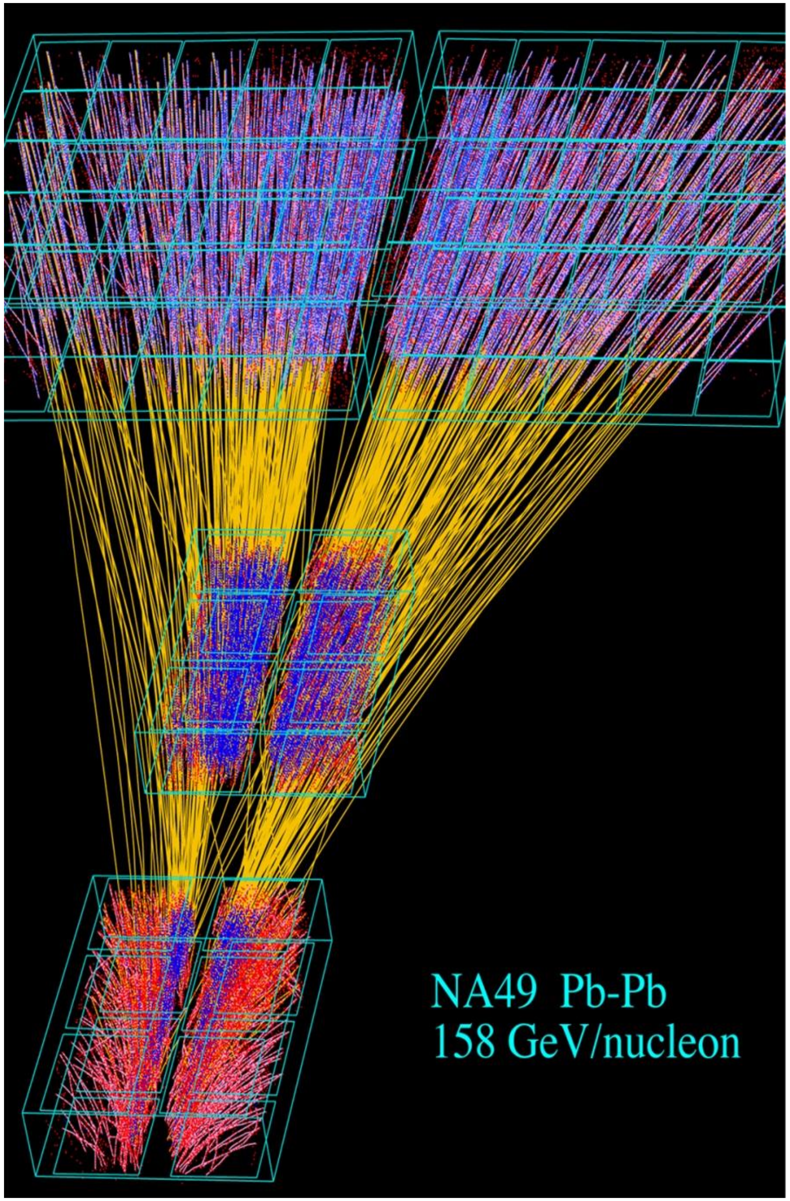
T, V, Q CE

T, V, μ_Q GCE

E, V, μ_Q MGCE

$\left.\begin{array}{ll} E, p, Q & T, p, Q \\ T, p, \mu_Q & E, p, \mu_Q \end{array}\right\}$ Pressure Ensembles

M.I.G. J. Phys. G (2008)



$$N = 10^2 \div 10^4$$

$$P(N)\ , \qquad \langle N^k \rangle = \sum_N N^k\, P(N)$$

$$Var(N) = \langle N^2 \rangle - \langle N \rangle^2$$
$$= \langle (N - \langle N \rangle)^2 \rangle = \langle (\Delta N)^2 \rangle$$

$$\omega = \frac{Var(N)}{\langle N \rangle}$$

Scaled Variances are not equal to each other in different SE

GCE and CE with Q=0

Rafelski, Danos, Phys. Lett. B (1980)
Redlich, Turko, Z. Phys. C (1980)

$$Z_{gce} = \sum_{N_+,N_-=0}^{\infty} \frac{z^{N_-}}{N_-!} \frac{z^{N_+}}{N_+!} = \exp(2z)$$

$$\mu = 0 \;\rightarrow\; \langle Q \rangle = 0 \qquad\qquad z = \frac{V}{2\pi^2} T m^2 \; K_2(m/T)$$

$$Z_{ce} = \sum_{N_+,N_-=0}^{\infty} \frac{z^{N_-}}{N_-!} \frac{z^{N_+}}{N_+!} \delta(N_+ - N_-) = I_0(2z)$$

$$\omega^- = \frac{\langle N_-^2 \rangle - \langle N_- \rangle^2}{\langle N_- \rangle}, \qquad \langle N_- \rangle_{gce} = z, \qquad \omega^-_{gce} = 1$$

$$\langle N_- \rangle_{ce} = z \frac{I_1(2z)}{I_0(2z)}, \qquad \omega^-_{ce} = 1 - z \left[\frac{I_1(2z)}{I_0(2z)} - \frac{I_2(2z)}{I_1(2z)} \right]$$

M.I.G., Gazdzicki, Greiner,
Phys. Lett. B (2000) **CE** for antibaryons in p+A

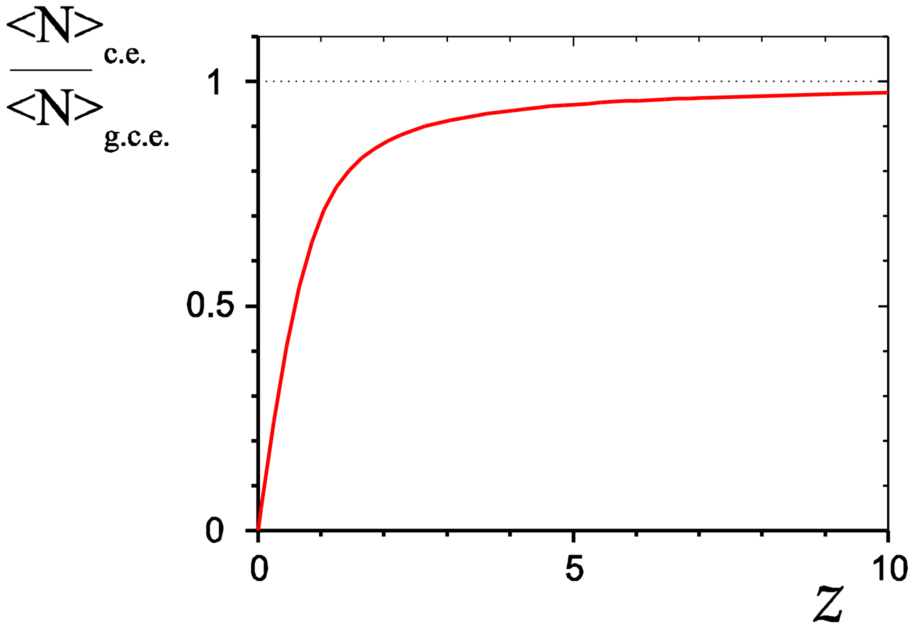
M.I.G., Kostyuk, Stoecker, Greiner,
Phys. Lett. B (2001) **CE** for charmed hadrons

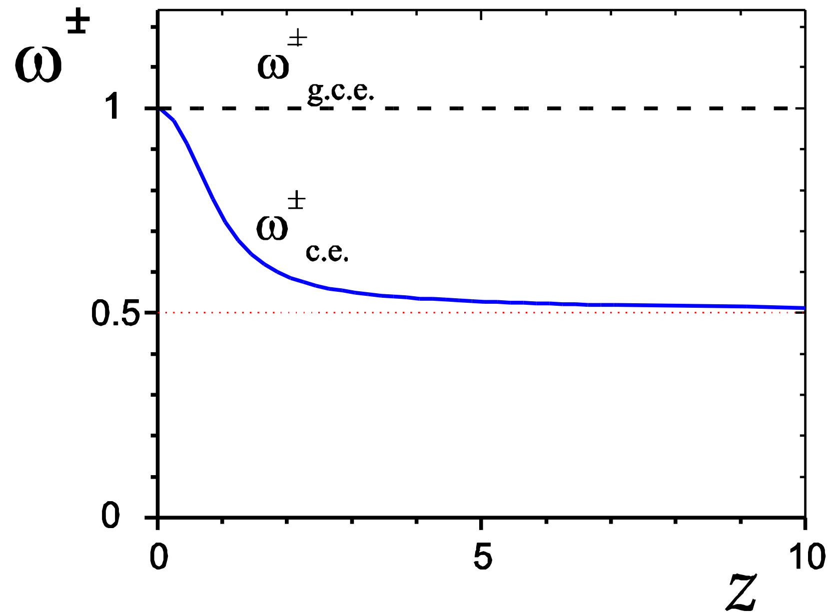
Begun, Gazdzicki, M.I.G., Zozulya,
Phys. Rev. C (2004)

$$Q=0$$

GCE: $\mu=0 \rightarrow \;<N_+> \;=\; <N_->$,

CE: $\delta(N_+ - N_-) \rightarrow N_+ = N_-$



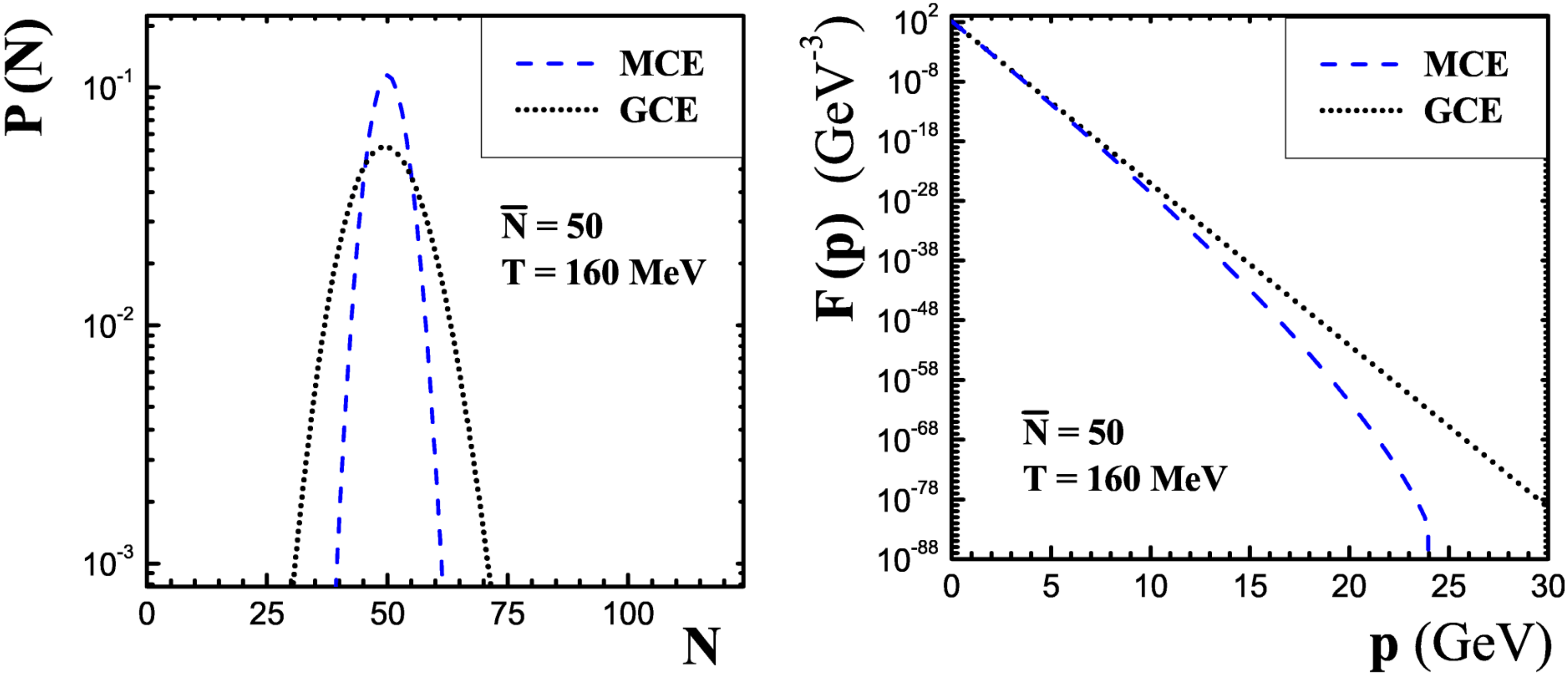


$$z = \frac{V}{2\pi^2} T m^2 K_2\left(m/T\right) = <N_\pm>_{\text{gce}} \;; \qquad \omega[N] = \frac{<N^2> - <N>^2}{<N>} ; \qquad \omega_{\text{gce}}[N_\pm] = 1$$

$$\frac{<N_\pm>_{\text{ce}}}{<N_\pm>_{\text{gce}}} = \frac{I_1(2z)}{I_0(2z)} \rightarrow 1 \qquad \omega_{\text{ce}}[N_\pm] = 1 - z\left[\frac{I_1(2z)}{I_0(2z)} - \frac{I_2(2z)}{I_1(2z)}\right] \rightarrow \frac{1}{2}$$

$$I_n(2z) \cong \frac{\exp(2z)}{\sqrt{4\pi z}}\left[1 - \frac{4n^2-1}{16z}\right], \qquad z \rightarrow \infty \;; \qquad I_n(2z) \cong \frac{z^n}{n!}, \qquad z \ll 1$$

Micro Canonical Ensemble of massless neutral particles

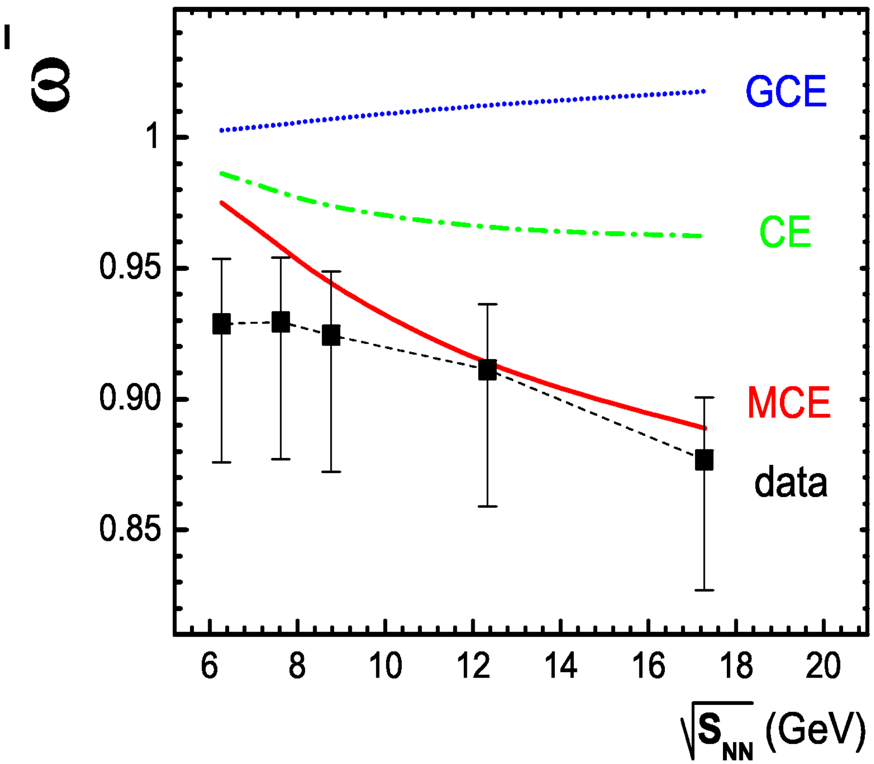


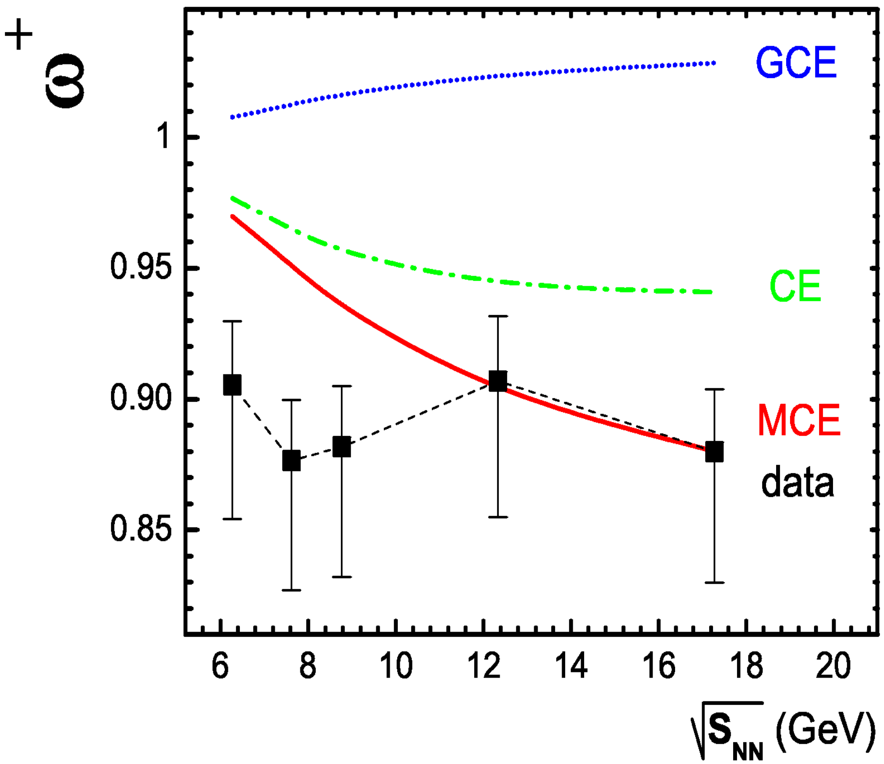
$$\omega_{gce} = 1, \qquad \omega_{mce} = \frac{1}{4}$$

Begun, M.I.G., Kostyuk, Zozulya, Phys. Rev. C (2005)

Comparison with the NA49 data

Pb+Pb 1% most central events



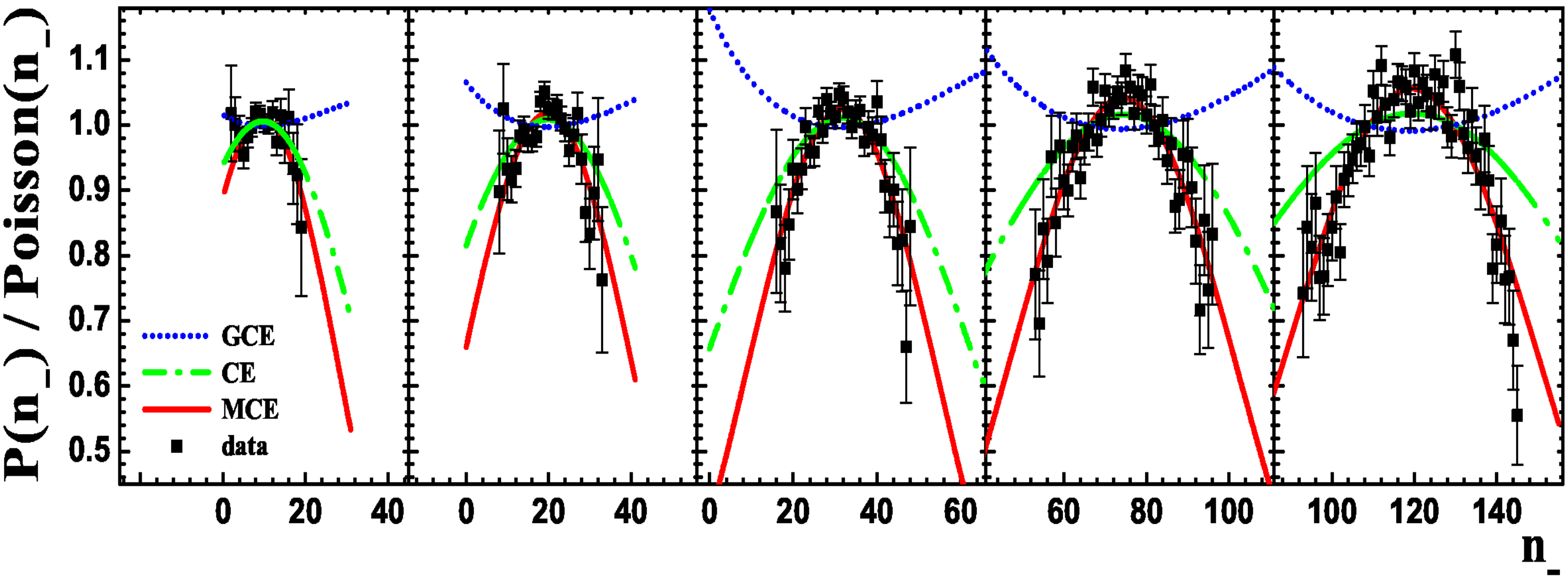


Begun, Gazdzicki, M.I.G., Hauer, Konchakovski, Lungwitz, Phys. Rev. C (2007)

Measured distribution / Poisson distribution

NA49, Pb+Pb, < 1% of most central events

20A GeV 30A GeV 40A GeV 80A GeV 158A GeV



Begun, Gazdzicki, M.I.G., Hauer, Konchakovski, Lungwitz, Phys. Rev. C (2006)

$$\vec{A} = (E, V, Q_1, \ldots, Q_k)$$

Alpha-Enesmble

$$P_\alpha(X) = \int d\vec{A}\; P_\alpha(\vec{A})\; P_{mce}(X; \vec{A})$$

M.I.G. and Hauer, Phys. Rev. C (2008)

Theory (standard statistical ensembles):

Begun, M.I.G., Kostyuk, Zozulya, Phys. Rev. C (2005), J.Phys. G
Begun, M.I.G., Zozulya, Phys. Rev. C (2005)
Becattini, Ferroni, Eur. Phys. J. C(2005) (2007)
Mekjian, Nucl. Phys. A (2005)
Keranen, Becattini, Begun, M.I.G., Zozulya, J. Phys. G (2005)
Cleymans, Redlich, Turko, Phys. Rev. C (2005), J. Phys. G (2005)
Becattini, Keranen, Ferroni, Gabbriellini, Phys. Rev. C (2005)
Torrieri, Jeon, Rafelski, Phys. Rev. C (2006), Nucleonics (2006)
Hauer, Phys. Rev. C (2008); Hauer, Wheaton, Phys. Rev. C (2009)
Turko, Int. J. Mod. Phys. E (2007), Phys. Part. Nucl. (2008)
Becattini, Ferroni, Eur. Phys. J. C (2007)
Torrieri, J. Phys. G (2006, 2008), Eur. Phys. J. C (2007) , (2012),
Yang, Wang, Phys. Rev. C (2011, 2012)
Torrieri, Bellweid, Market, Westfall (2012)

Theory (generalized statistical ensembles):

M.I.G., Hauer, Phys. Rev. C (2008);
Begun, Gazdzicki, M.I.G., Phys. Rev. C (2008, 2009)
Wilk, Wlodarczyk, Physica (2011)
Biro, Barnafoldi, Van, Eur. Phys. J. A (2013), Physica (2014)

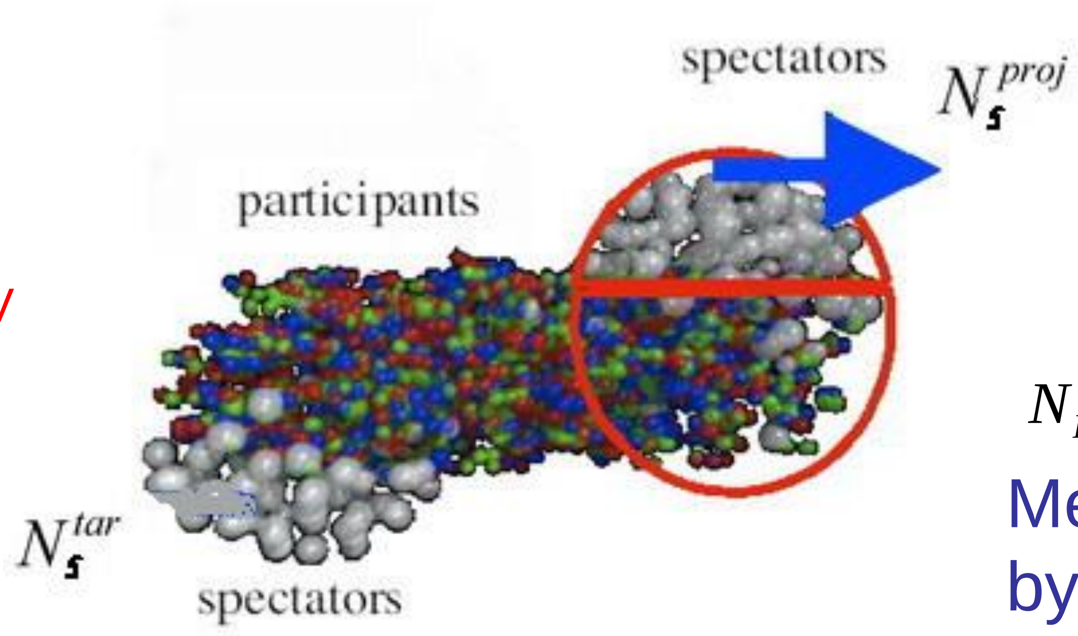
Experiment:

Rybczynski et. al, NA49 Collaboration, J. Phys. Conf. Ser. (2005)
Lungwitz (NA49 Collaboration) (2006, 2007)
Alt, et.al. NA49 Collaboration, Phys. Rev C (2007) (2008)
Center, et. al, NA61 Collaboration, Phys. Atom. Nucl. (2012)

II. Strongly Intensive Measures of Fluctuations

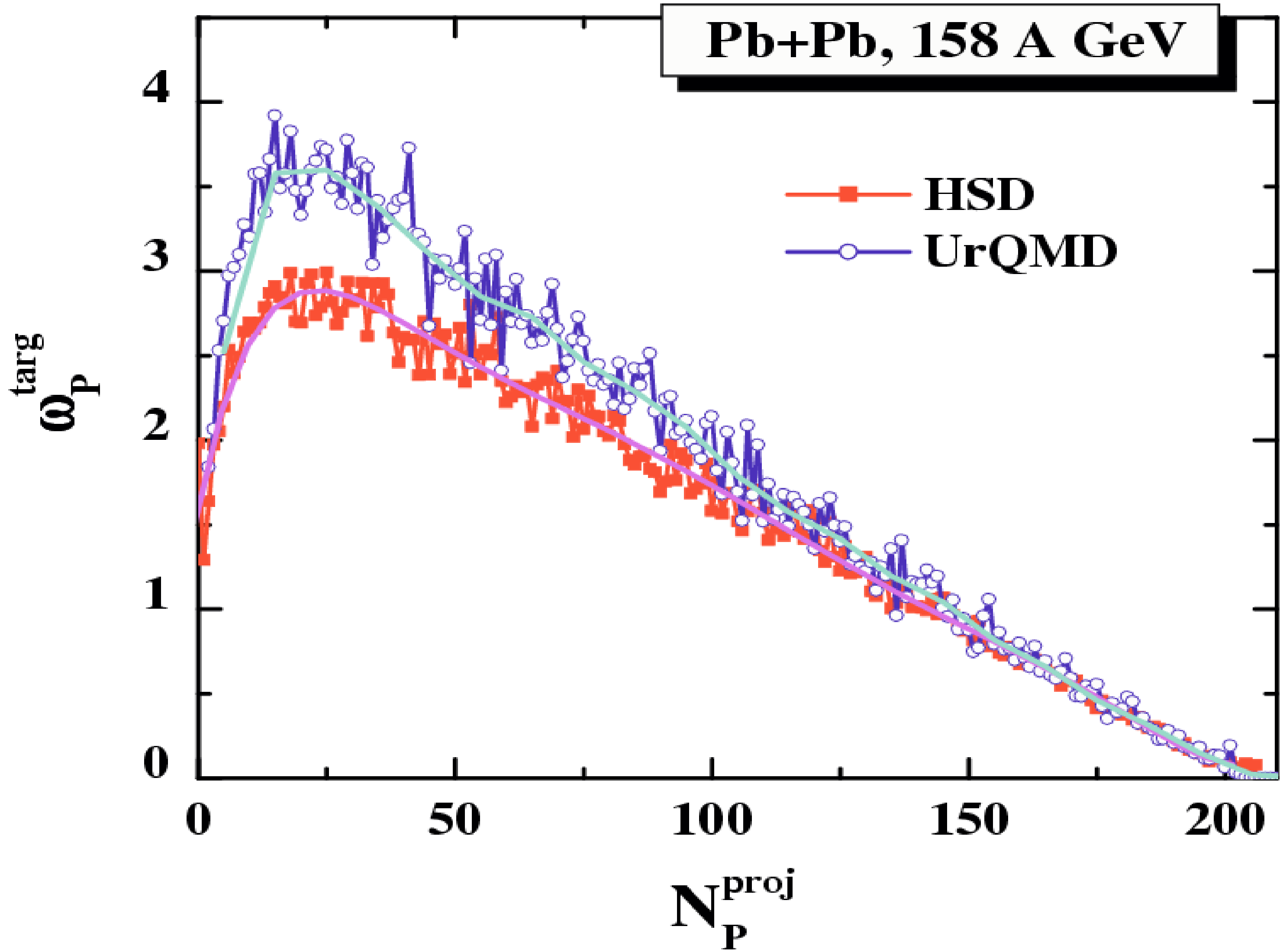
Nucleons: participants and spectators

Pb-Pb
158 AGeV



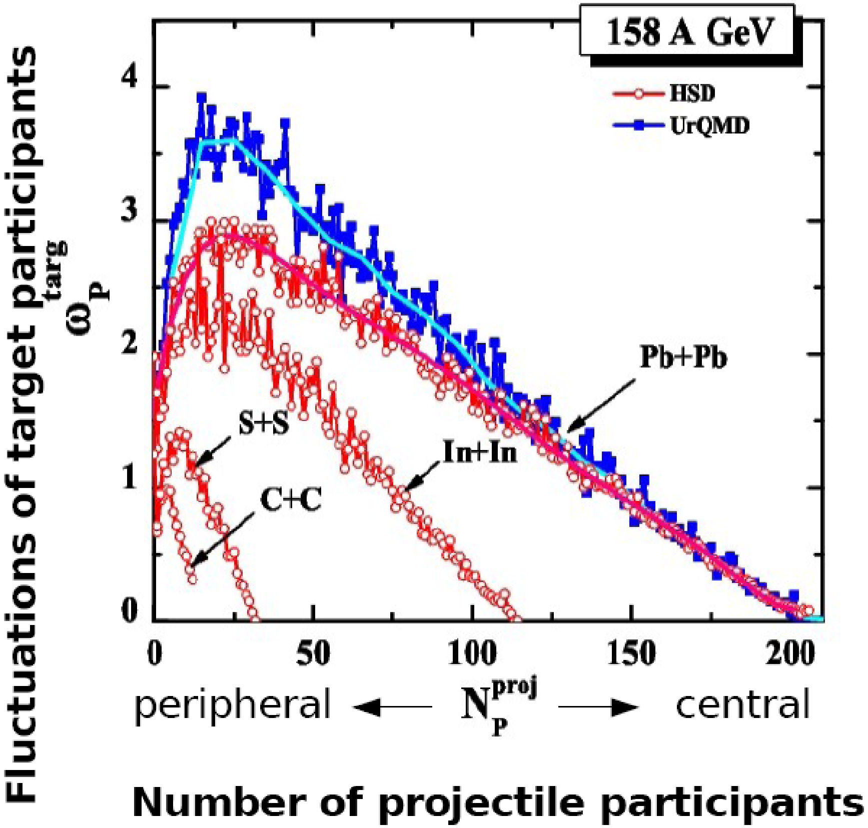
$$N_P^{proj} = A^{proj} - N_S^{proj}$$

Measured
by ZDC



Konchakovski, Hausler, M.I.G., Bratkovskaya, Bleicher, Stoecker,
Phys. Rev. C (2006)

Central collisions of light and medium size nuclei are required for the proposed fluctuation studies



Event-by-event fluctuations in the number of interacting (participant) nucleons are the main source of the background in the fluctuation studies

The fluctuations of the number of projectile participants are suppressed by selecting collisions with fixed number of projectile spectators (in NA49-future measured by PSD)

The fluctuations of the number of target participants can be suppressed only by selection of very central collisions

Konchakovski, M.I.G., et al, Phys. Rev C (2006)

Volume Fluctuations

$F(V)$ event-by-event volume distribution

$A \sim V, \quad B \sim V$ Extensive Quantities

$\Delta[A, B], \Sigma[A, B]$ are independent of the average volume and of volume fluctuations

M.I.G., Gazdzicki, Phys Rev. C (2011)

Examples:

$$A = P_T = |\mathrm{p}_T^{(1)}| + \ldots + |p_T^{(N)}|$$

$$B = N$$

$$A = N_1,$$

$$B = N_2$$

$$\mathbf{\Xi} = \exp\left\{\mathbf{V}\sum_{\mathbf{j}} \eta_{\mathbf{j}} \mathbf{d}_{\mathbf{j}} \int \frac{\mathbf{d^3p}}{(\mathbf{2}\pi)^{\mathbf{3}}} \ln\left[\mathbf{1} + \lambda_{\mathbf{j}}\eta_{\mathbf{j}} \exp\left(-\sqrt{p^2 + m_j^2}/\mathbf{T}\right)\right]\right\}$$

$$\mu_j = b_j \mu_B + s_j \mu_S + q_j \mu_Q, \qquad S = 0, \qquad \frac{Q}{B} = 0.4 \div 0.5$$

$$\overline{\mathbf{A}} = \frac{\mathbf{1}}{\mathbf{\Xi}} \lambda_{\mathbf{A}} \frac{\partial}{\partial \lambda_{\mathbf{A}}} \mathbf{\Xi} = \mathbf{V} \mathbf{n}_{\mathbf{A}}$$

$$\lambda_{\mathbf{j}} = \exp\left(\frac{\mu_{\mathbf{j}}}{\mathbf{T}}\right)$$

$$\eta_{\mathbf{j}} = \pm 1$$

$$\overline{\mathbf{A^2}} = \frac{\mathbf{1}}{\mathbf{\Xi}} \left(\lambda_{\mathbf{A}} \frac{\partial}{\partial \lambda_{\mathbf{A}}}\right)^{\mathbf{2}} \mathbf{\Xi} = \mathbf{V^2 n_A^2}$$

$$\mathbf{n_A} = \overline{\mathbf{A}}/\mathbf{V}$$

$$+\mathbf{V} \int \frac{\mathbf{d^3p}}{(\mathbf{2}\pi)^{\mathbf{3}}} \frac{\mathbf{d_A} \lambda_{\mathbf{A}}^{-\mathbf{1}} \exp\left(\sqrt{p^2 + m_j^2}/\mathbf{T}\right)}{\left[\lambda_{\mathbf{A}}^{-\mathbf{1}} \exp\left(\sqrt{p^2 + m_j^2}/\mathbf{T}\right) + \eta_{\mathbf{A}}\right]^{\mathbf{2}}}$$

$$\overline{\mathbf{AB}} = \frac{\mathbf{1}}{\mathbf{\Xi}} \lambda_{\mathbf{A}} \frac{\partial}{\partial \lambda_{\mathbf{A}}} \lambda_{\mathbf{B}} \frac{\partial}{\partial \lambda_{\mathbf{B}}} \mathbf{\Xi} = \mathbf{V^2 n_A n_B}$$

$$\omega^*_{\mathbf{A}} = \frac{\overline{\mathbf{A}^2} - \overline{\mathbf{A}}^2}{\overline{\mathbf{A}}} = \mathbf{n}_{\mathbf{A}}^{-1} \int \frac{\mathbf{d^3p}}{(2\pi)^3} \frac{\mathbf{d_A} \lambda_{\mathbf{A}}^{-1} \exp\left(\sqrt{p^2 + m_j^2}/\mathbf{T}\right)}{\left[\lambda_{\mathbf{A}}^{-1} \exp\left(\sqrt{p^2 + m_j^2}/\mathbf{T}\right) + \eta_{\mathbf{A}}\right]^2}$$

$$\int \mathbf{dV} \ \ldots \mathbf{F(V)} = \langle \ldots \rangle$$

$$\langle \mathbf{A} \rangle \ = \ \langle \mathbf{V} \rangle \ \mathbf{n_A} \qquad\qquad \langle \mathbf{A^2} \rangle \ = \ \langle \mathbf{V^2} \rangle \ \mathbf{n_A^2} \ + \ \langle \mathbf{V} \rangle \ \mathbf{n_A} \ \omega^*_{\mathbf{A}}$$

$$\langle \mathbf{AB} \rangle \ = \ \langle \mathbf{V^2} \rangle \ \mathbf{n_A n_B}$$

$$\omega_{\mathbf{A}} \ \equiv \ \frac{\langle \mathbf{A^2} \rangle \ - \ \langle \mathbf{A} \rangle^2}{\langle \mathbf{A} \rangle} \ = \ \omega^*_{\mathbf{A}} \ + \ \mathbf{n_A} \ \frac{\langle \mathbf{V^2} \rangle - \langle \mathbf{V} \rangle^2}{\langle \mathbf{V} \rangle}$$

$$< AB > \ - \ < A > \ < B > \ = \ n_A n_B \ \left(< V^2 > \ - \ < V >^2 \right)$$

$$\Delta\left[A, B\right] = \frac{1}{C_\Delta}\left[< B > \omega[A] - < A > \omega[B]\right]$$

$$\Sigma\left[A, B\right] = \frac{1}{C_\Sigma}[< B > \omega[A] + < A > \omega[B]$$

$$- 2\left(< AB > - < A > < B >\right)]$$

$$< C_\Delta >, < C_\Sigma > \sim < V >$$

These combinations of second moments $< A^2 >, < B^2 >, < AB >$
are independent of $< V >$ and $\omega\left[V\right]$

Normalization. For the Independent Particle Model: $\Delta[A, B] = 1$

IB-GCE ; Mixed Event Model $\Sigma[A, B] = 1$

$$C_{\Delta} = C_{\Sigma} = \omega[p_T] < N > \qquad [A = P_T, \; B = N]$$

$$C_{\Delta} = < N_1 > - < N_2 >$$

$$C_{\Sigma} = < N_1 > + < N_2 >$$

$$[A = N_1, \; B = N_2]$$

Gazdzicki, M.I.G., Mackowiak-Pawlowska, Phys. Rev. C (2013)

Effects of Quantum Statistics
M.I.G., Rybczynski, Phys. Lett. B (2014),

$$\Delta[P_T, N], \ \Sigma[P_T, N]$$

The strongest effect is
in $\Delta[P_T, N]$ of pions

p+p at 158 GeV/c : There is a correlation between the inverse slope of m_T -spectrum and hadron multiplicity *N*

$$\Delta[P_T, N] \cong 0.82, \qquad \Sigma[P_T, N] \cong 1.01$$

M.I.G., Grebieszkow, Phys. Rev. C (2014)

in a good agreement with (preliminary) NA61/SHINE data in p+p reactions

Experimental results for the Δ and Σ measures:

Rustamov for the NA61/SHINE and NA49 Collaborations, arXiv:1303.5671
Anticic, et.al Phys. Rev. C (2014)
Mackowiak-Pawlowska, PoS CPOD2013 (2013), J. Phys. Conf. Ser. (2014)
Rybczynski, NA61/SHINE Collaboration, PoS EPS-HEP2013, arXiv:1301.3360
Mackowiak-Pawlowska, Wilczek for the NA61 Collaboration (2013)
Grebieszkow, Acta Phys. Pol. (2013), PoS CPOD2013 (2013)
Seyboth, arXiv:1402.4619 (2014)
Stefanek, NA61/SHINE and NA49, arXiv:1411.2396

Resonance Decays

$$R \to \pi^+\pi^-, \qquad\qquad \omega\left[\pi^+\right] \cong \omega\left[\pi^-\right] \cong \omega[R] \cong 1,$$

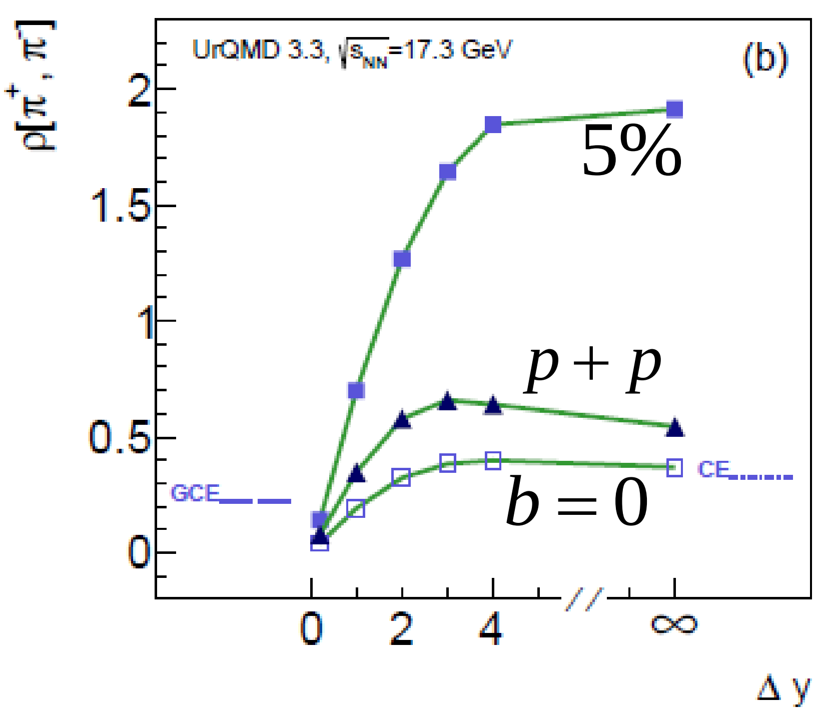
$$\frac{<R>}{<\pi^+> + <\pi^->} \cong \rho[\pi^+,\pi^-] \equiv \frac{<\pi^+\pi^-> - <\pi^+><\pi^->}{<\pi^+> + <\pi^->}$$

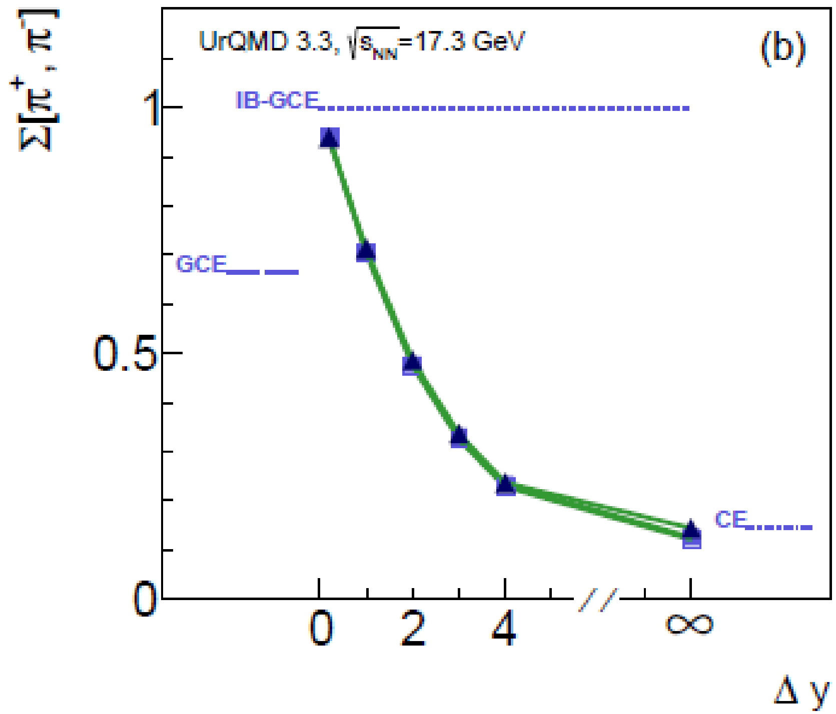
Jeon, Koch, Phys. Rev. Lett. (1999)

$$\frac{<R>}{<\pi^+> + <\pi^->} \cong \frac{1 - \Sigma[\pi^+,\pi^-]}{2}$$

UrQMD

Begun, M.I.G., Grebieszkow, arXiv:1409.3032





■ Pb+Pb, 5% most central

□ Pb+Pb, b=0

▲ p+p

$$-\frac{1}{2}\Delta y < y < \frac{1}{2}\Delta y$$

rapidity interval of pion acceptance in the c.m.s.

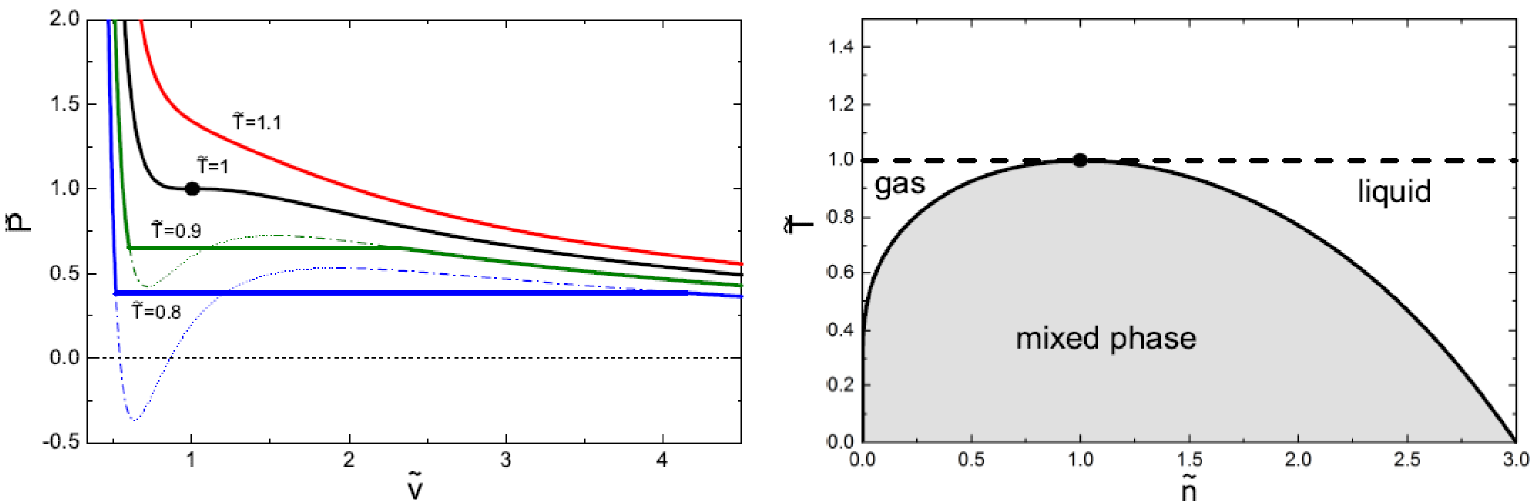
III. Van der Waals Equation of State in the GCE and Particle Number Fluctuations

Vovchenko, Anchishkin, M.I.G.
arXiv (2015)

$$p(V,T,N) = \frac{NT}{V-bN} - a\frac{N^2}{V^2} = \frac{nT}{1-bn} - an^2, \quad \text{CE}$$

$$T_c = \frac{8a}{27b}, \quad n_c = \frac{1}{3b}, \quad p_c = \frac{a}{27b^2}$$

$$\tilde{p} = \frac{8\tilde{T}\tilde{n}}{3-\tilde{n}} - 3\tilde{n}^2, \qquad \tilde{n} = n/n_c, \quad \tilde{p} = p/p_c, \quad \tilde{T} = T/T_c,$$

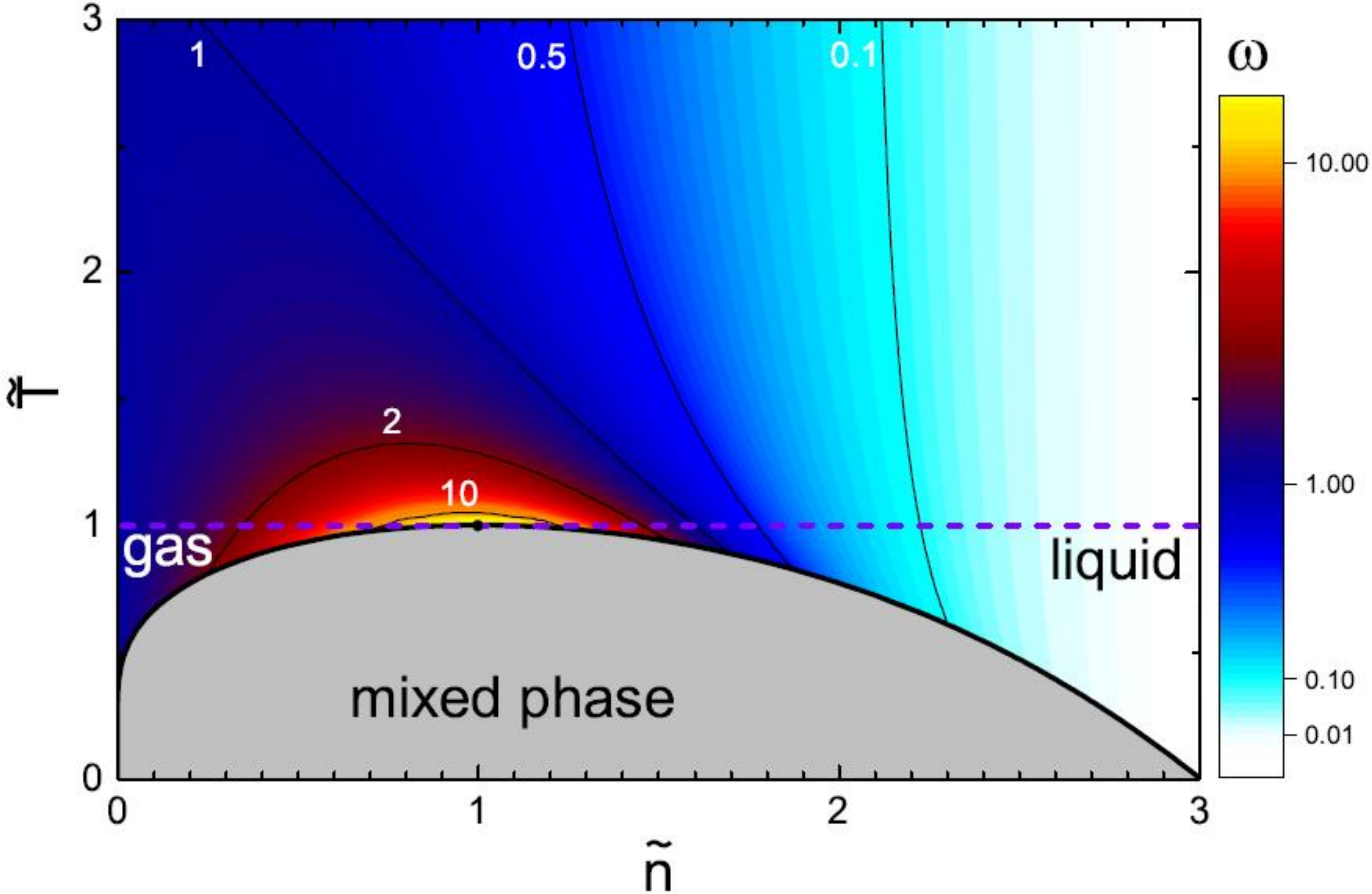


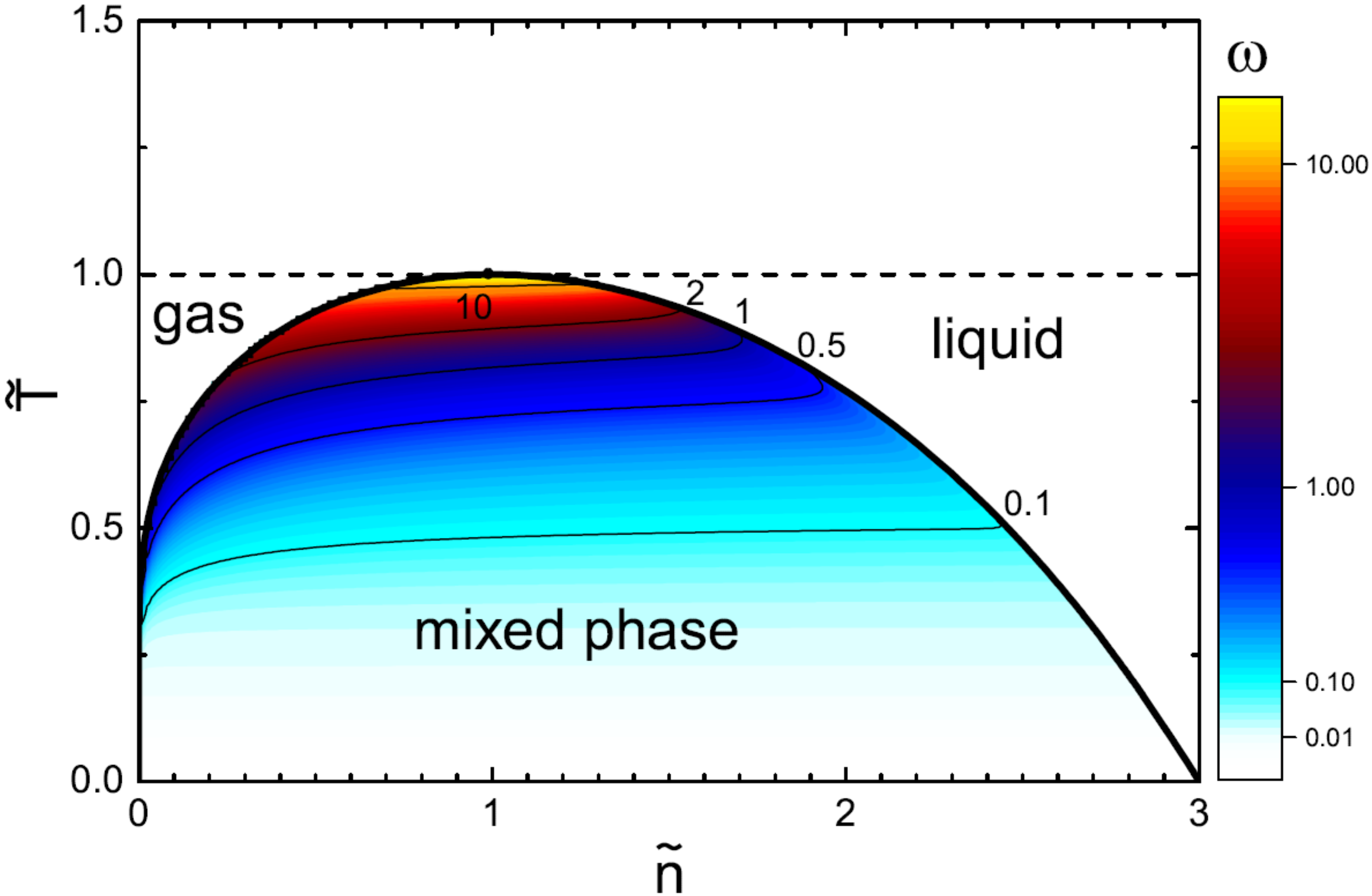
$$p(V,T,N) = \frac{nT}{1-bn} - an^2, \quad \text{CE}$$

$$n(T,\mu) = \frac{n_{\rm id}(T,\mu^*)}{1+bn_{\rm id}(T,\mu^*)}, \qquad \mu^* = \mu - b\frac{nT}{1-bn} + 2an, \qquad \text{GCE}$$

$$\omega[N] \equiv \frac{<N^2> - <N>^2}{<N>} = \left[\frac{1}{(1-bn)^2} - \frac{2an}{T}\right]^{-1}$$

$$\omega[N] = \frac{1}{9}\left[\frac{1}{(3-\tilde{n})^2} - \frac{\tilde{n}}{4\tilde{T}}\right]^{-1}$$





1.5
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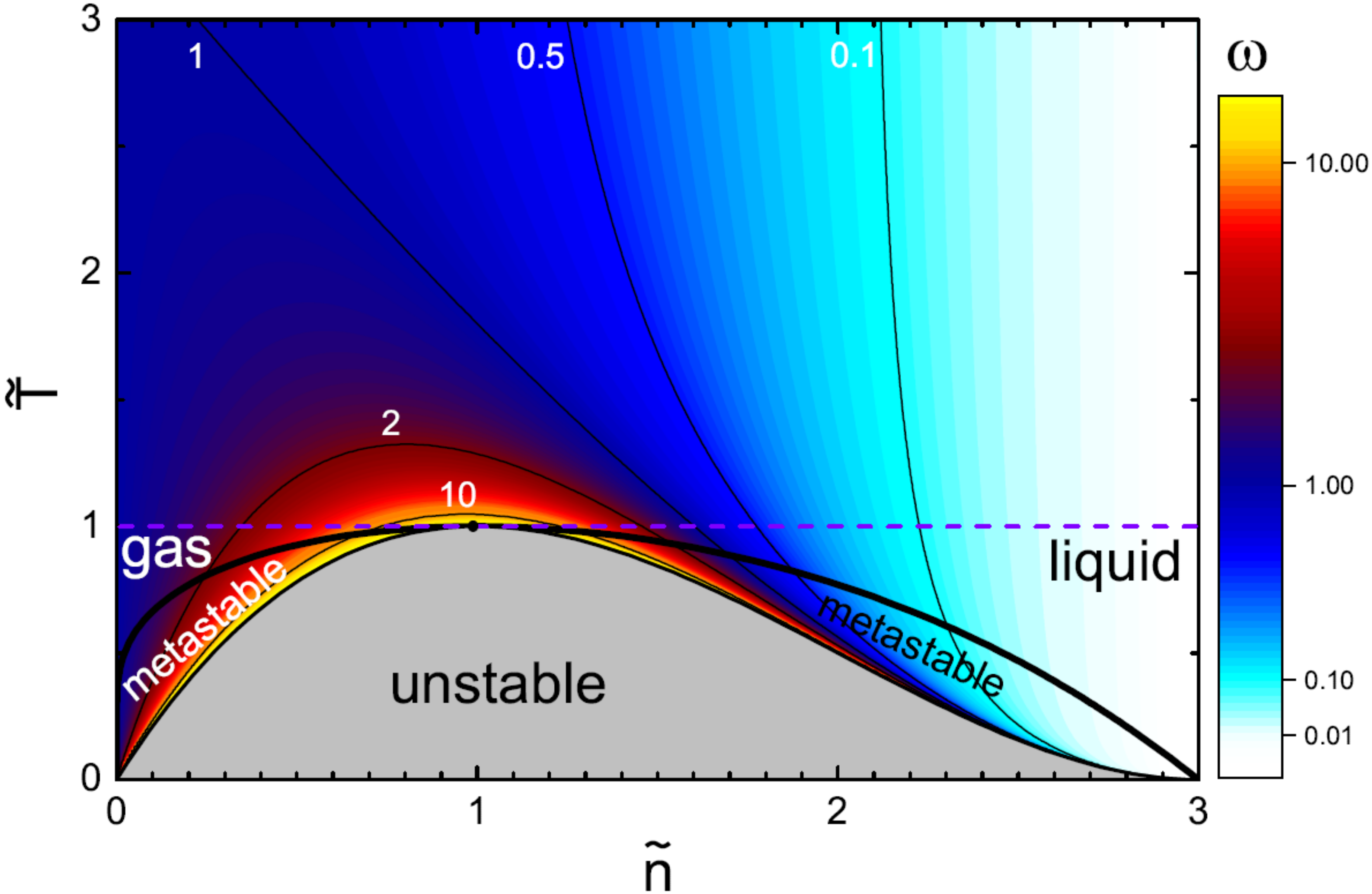
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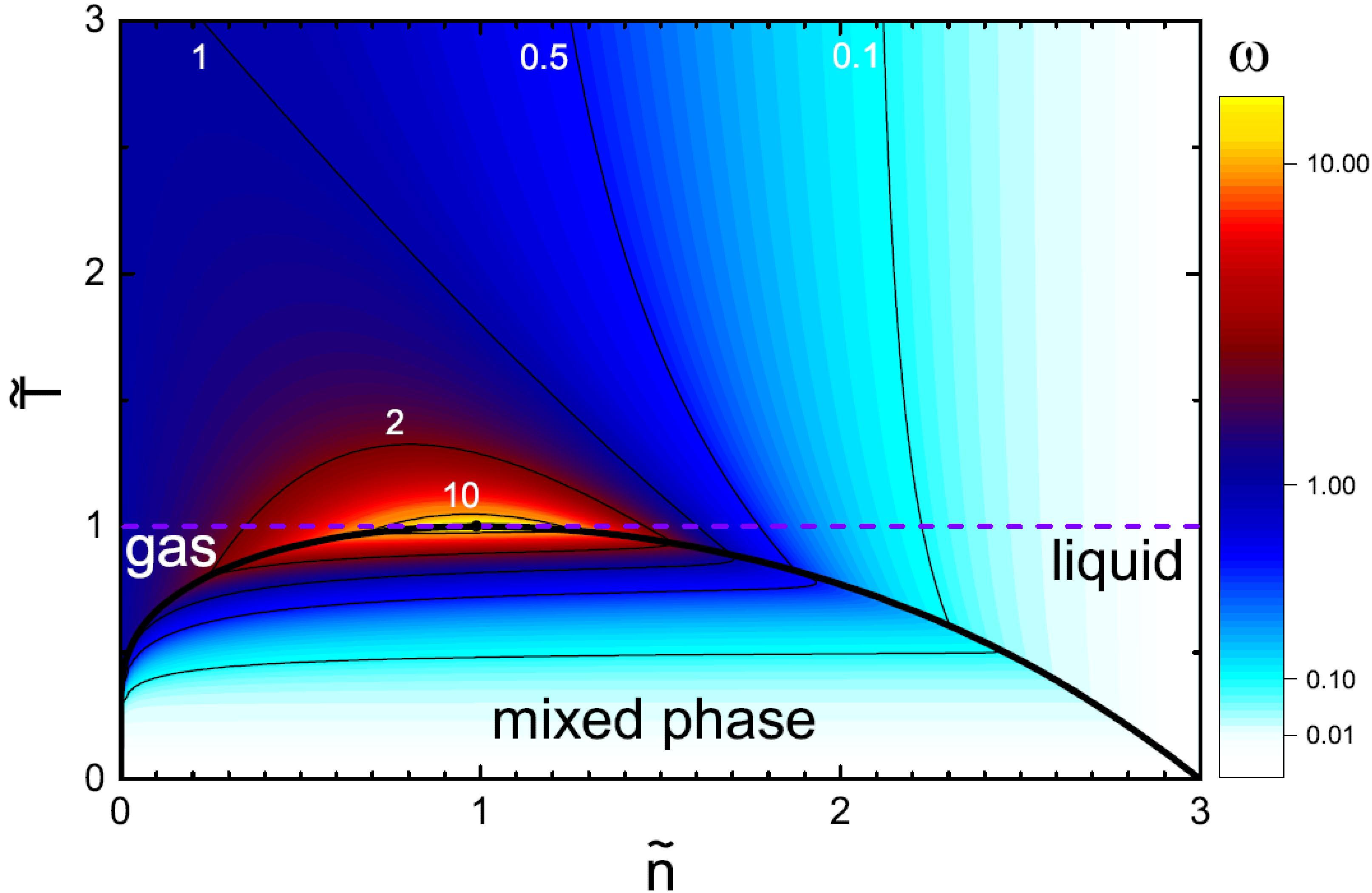

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gas
liquid
mixed phase
$\tilde{T}$
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ω
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0
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3

Summary

1. Global Conservation Laws

   suppress particle number fluctuations and introduce interparticle correlations. This is valid also in the thermodynamic limit $V \to \infty$.

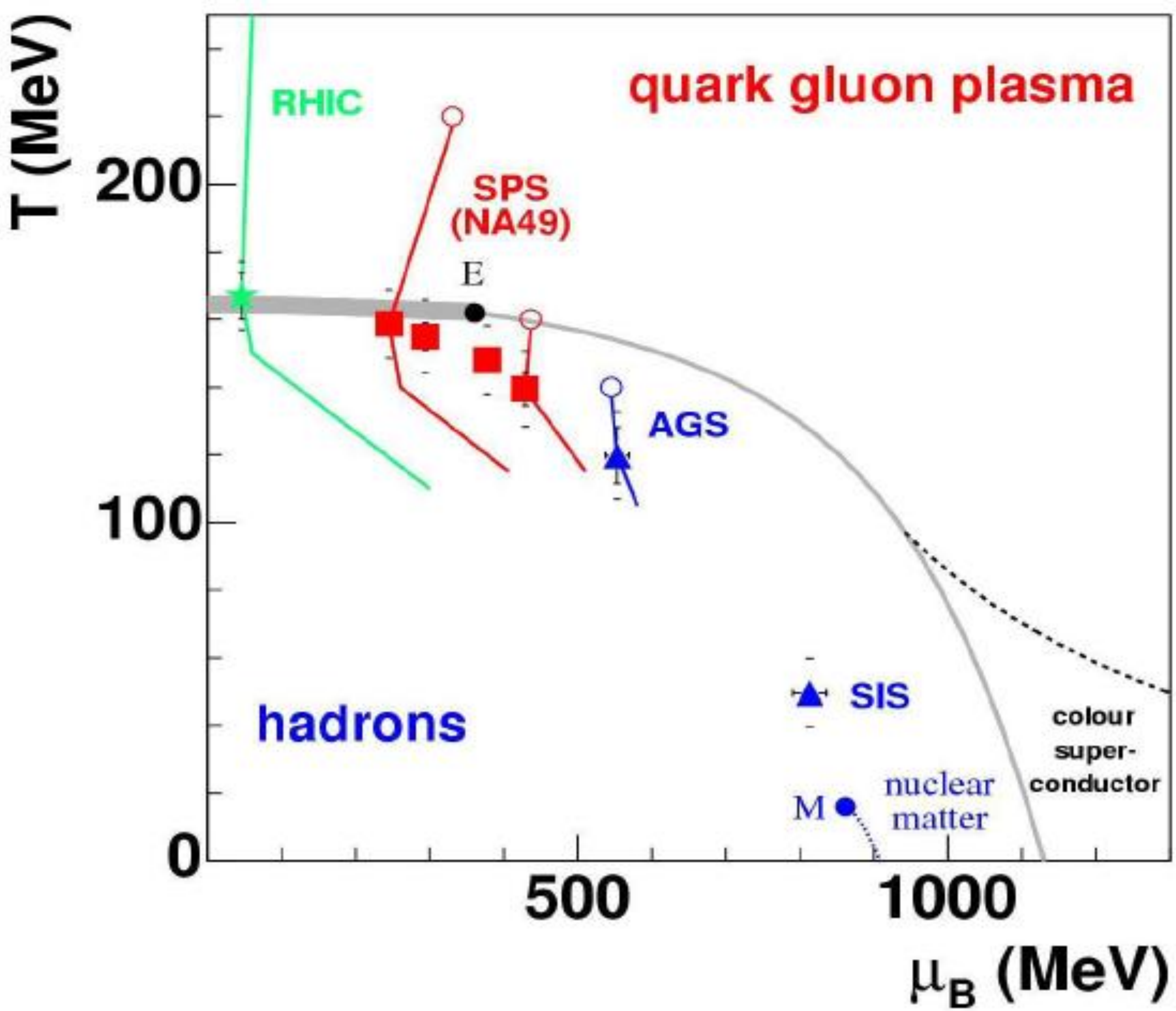
2. Strongly Intensive Measures $\Delta[A, B]$ and $\Sigma[A, B]$

   are independent of the average size of the system and of the fluctuations of the size.
   Examples: GCE, Model of Independent Sources, Mixed Events,….

3. Van der Waals Equation of State and Particle Number Fluctuations

   provides an analytical example of the fluctuations in systems with 1st order liquid-gas phase transition and critical point.

Thank you!


T (MeV)
200
100
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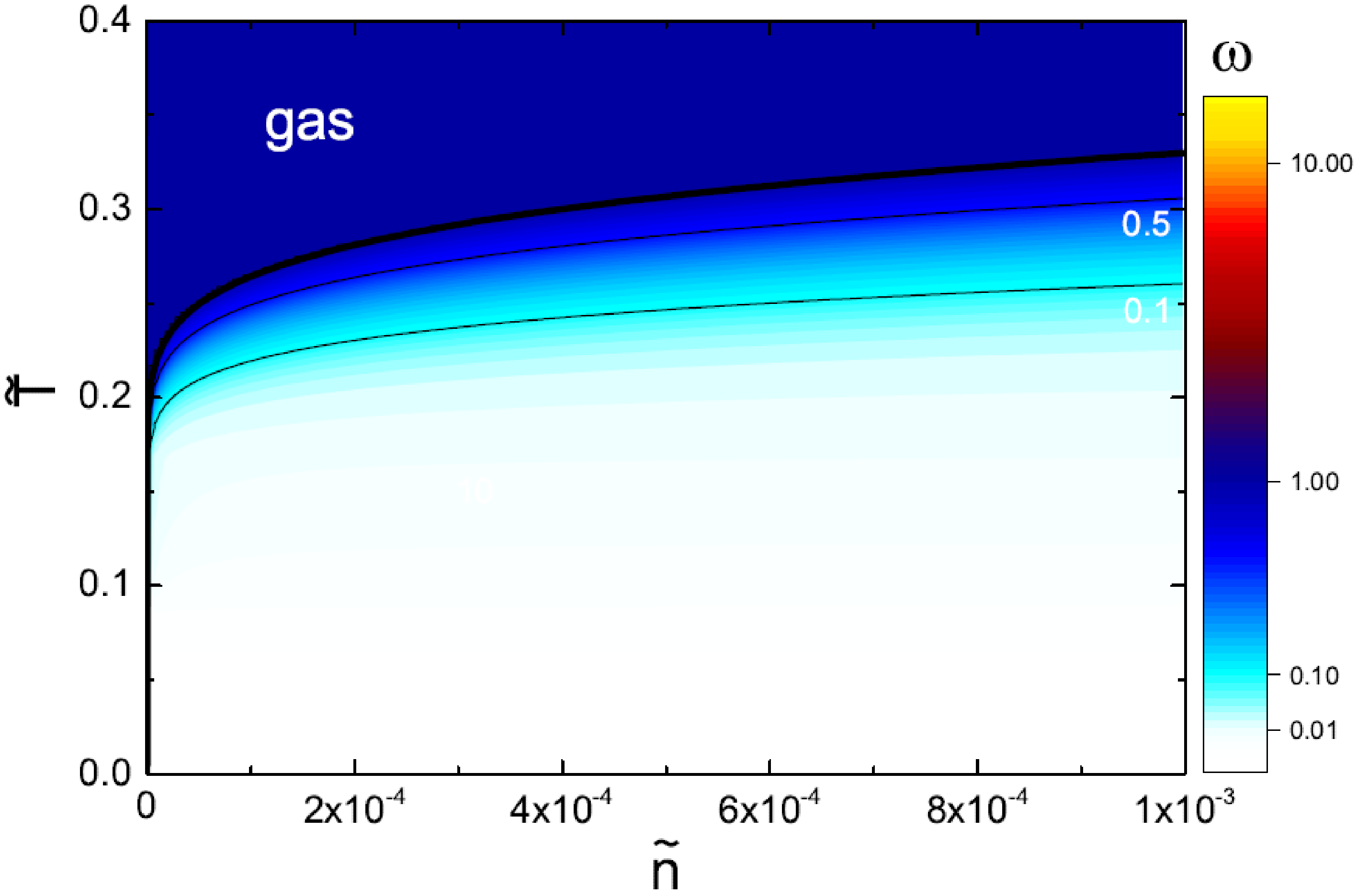
μB (MeV)
quark gluon plasma
RHIC
SPS
(NA49)
E
AGS
SIS
hadrons
colour
super-
conductor
M
nuclear
matter



0.4
0.3
0.2
0.1
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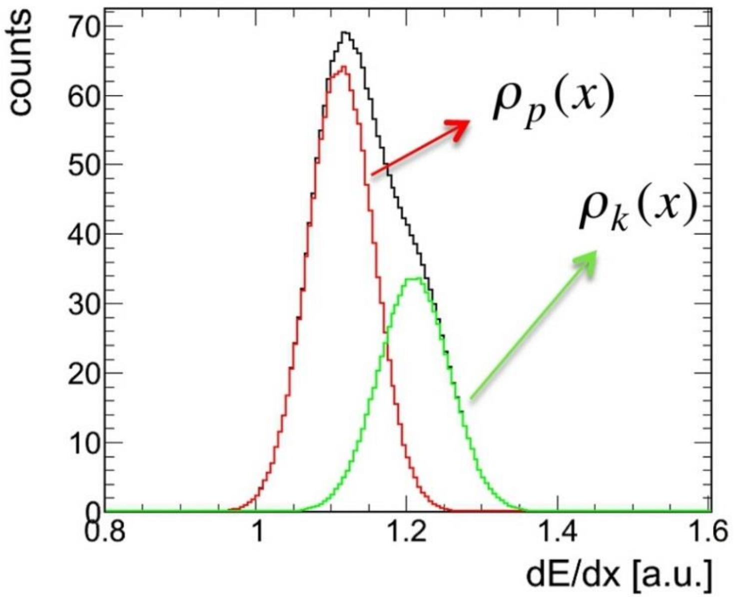
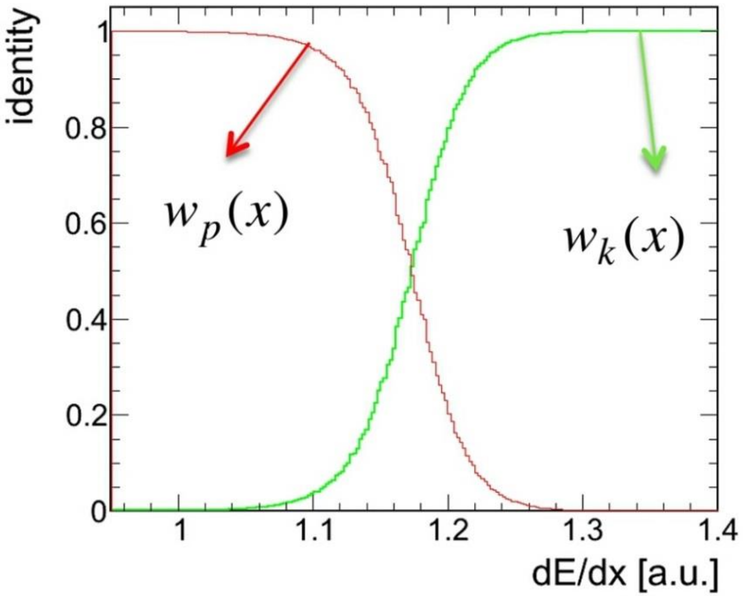
$\tilde{T}$
gas
0.5
0.1
0
2x10-4
4x10-4
6x10-4
8x10-4
1x10-3
$\tilde{n}$
ω
10.00
1.00
0.10
0.01

III. Fluctuations with Incomplete Particle Identifications





$$\int dm \rho_j(m) = < N_j >,$$

$$\rho(m) = \sum_{i=1}^{k} \rho_i(m)$$

identity variable $$w_j(m) = \frac{\rho_j(m)}{\rho(m)} \subset [0,1]$$

Complete identification:

$\rho_j(m)$ do not overlap

Gazdzicki, Grebieszkow, Mackowiak, Mrowczynski, Phys. Rev. C (2011)
M.I.G., Phys. Rev. C (2011); Rustamov, M.I.G., Phys. Rev. C (2012)

$$W_j = \sum_{i=1}^{N(n)} w_j(m_i), \qquad W_j^2 = [\sum_{i=1}^{N(n)} w_j(m_i)]^2$$

$$W_p W_q = [\sum_{i=1}^{N(n)} w_p(m_i)][\sum_{i=1}^{N(n)} w_q(m_i)]$$

$N(n) = N_1(n) + ... + N_k(n)$ total multiplicity in n-th event

$$< W_j^2 >= \frac{1}{N_{ev}} \sum_{n=1}^{N_{ev}} W_j^2, \qquad < W_p W_q >= \frac{1}{N_{ev}} \sum_{n=1}^{N_{ev}} W_p W_q$$

N_{ev} is the number of events

In the case of complete identifications: $W_j = N_j$,

$$< W_j^2 > \ = \ < N_j^2 >, \qquad < W_p W_q > \ = \ < N_p N_q >$$

$$\sum_{i=1}^{k} < N_i^2 > u_{ji}^2 + 2 \sum_{1 \le i < l \le k} < N_i N_l > u_{ji} u_{jl} = b_j$$

$$\sum_{i=1}^{k} < N_i^2 > u_{pi} u_{qi} + \sum_{1 \le i < l \le k} < N_i N_l > (u_{pi} u_{ql} + u_{pl} u_{qi}) = b_{pq}$$

$$u_{ji}^s \equiv \frac{1}{< \mathrm{N}_i >} \int dm \; w_j^s, \quad u_{pqi} \equiv \frac{1}{< \mathrm{N}_i >} \int dm \; w_p \; w_q \; \rho_i(m)$$

$$b_j \equiv < W_j^2 > - \sum_{i=1}^{k} < N_i > [u_{ji}^2 - (u_{ij})^2],$$

$$b_{pq} \equiv < W_p W_q > - \sum_{i=1}^{k} < N_i > [u_{pqi} - u_{pi} u_{qi}]$$

$$< N_j^2 >, \quad j = 1, \dots, k \qquad < N_p N_q >, \quad 1 \le p < q \le k$$

The system of k+k(k-1)/2 linear equations

$$< N_1^2 > \; = \; \frac{b_1 u_{22}^2 + b_2 u_{12}^2 - 2 b_{12} u_{12} u_{22}}{(u_{11} u_{22} - u_{12} u_{21})^2}$$

$$< N_2^2 > \; = \; \frac{b_2 u_{11}^2 + b_1 u_{21}^2 - 2 b_{12} u_{21} u_{11}}{(u_{11} u_{22} - u_{12} u_{21})^2}$$

$$< N_1 N_2 > \; = \; \frac{b_{12}(u_{11} u_{22} + u_{12} u_{21}) - b_1 u_{22} u_{21} - b_2 u_{11} u_{22}}{(u_{11} u_{22} - u_{12} u_{21})^2}$$

Experimental results for fluctuations with the identity method:

Rustamov for the NA61/SHINE and NA49, J. Phys. Ser. and arXiv:1303.5671
Anticic, et.al, Phys. Rev. C (2013, 2014)
Mackowiak-Pawlowska, PoS CPOD2013 (2013), J. Phys. Conf. Ser. (2014)
Grebieszkow, Acta Phys. Pol. (2013), PoS CPOD2013 (2013)
Seyboth, arXiv:1402.4619
Stefanek, NA61/SHINE and NA49, arXiv:1411.2396

Summary

1. Global Conservation Laws

   suppress particle number fluctuations and introduce interparticle correlations. This is valid also in the thermodynamic limit $V \to \infty$.

2. Strongly Intensive Measures $\Delta[A, B]$ and $\Sigma[A, B]$

   are independent of the average size of the system and of the fluctuations of the size.
   Examples: GCE, Model of Independent Sources, Mixed Events,….

3. Identity Method

   provides the values of all the second and higher moments of identified particle number distributions in a model independent way for the case of incomplete particle identifications.

Thank you!

Additional Slides

Ideal Botzmann gas in the GCE with $F(V)$ volume distribution

$$A = a_1 + a_2 + \ldots + a_N \,, \qquad B = b_1 + b_2 + \ldots + b_N$$

$$< A > = \bar{a} < N > = \bar{a}\; n < V >, \qquad < B > = \bar{b} < N > = \bar{b}\; n < V >$$

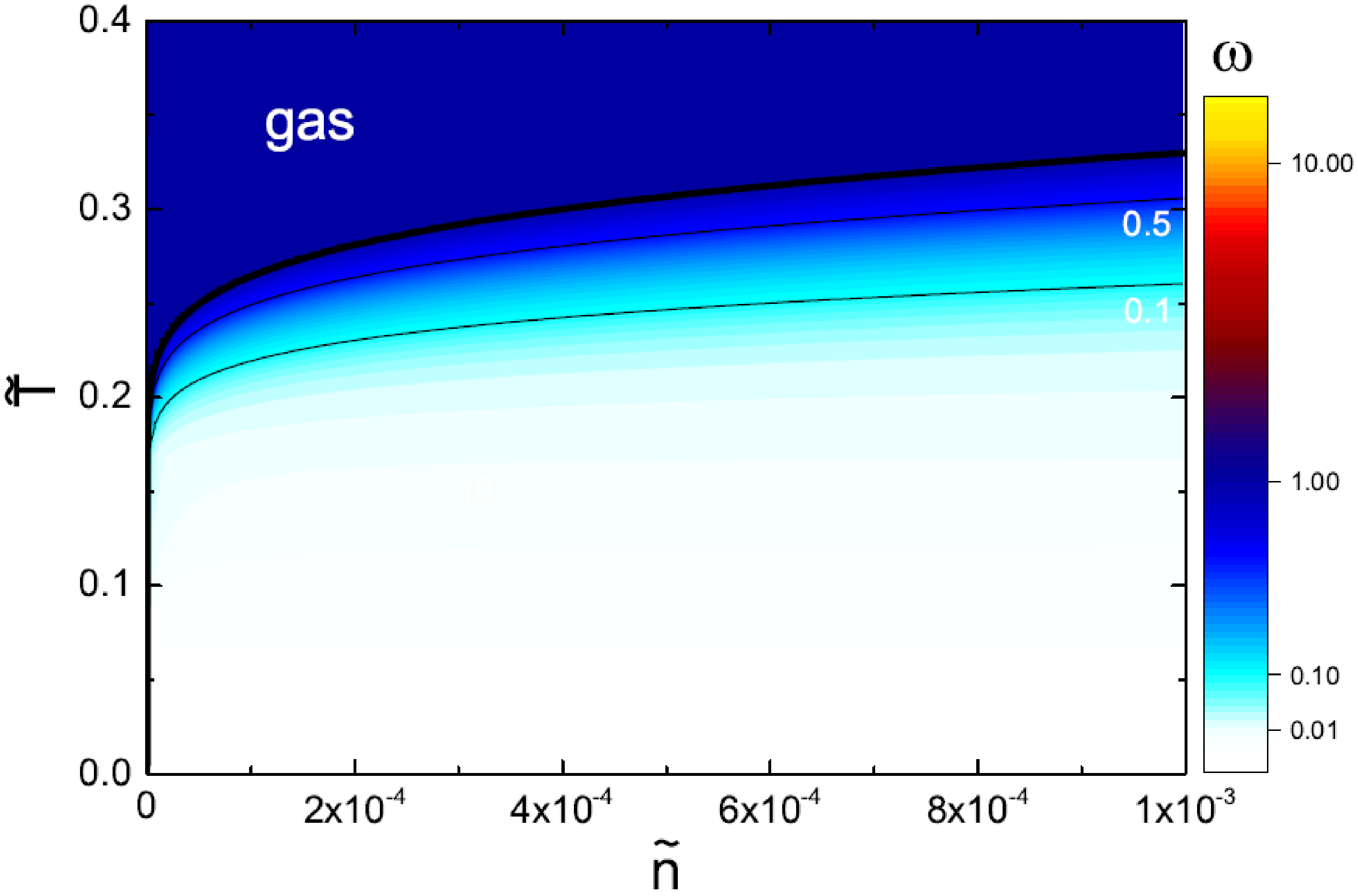
$$< A^2 > \;=\; \overline{a^2}\; n < V > \;+\; \bar{a}^2 \left[n^2 < V^2 > \;-\; n < V > \right]$$

$$< B^2 > \;=\; \overline{b^2}\; n < V > \;+\; \bar{b}^2 \left[n^2 < V^2 > \;-\; n < V > \right]$$

$$< AB > \;=\; \overline{ab}\; n < V > \;+\; \bar{a}\; \bar{b} \left[n^2 < V^2 > \;-\; n < V > \right]$$

$$\omega\,[A] \;\equiv \frac{< A^2 > - < A >^2}{< A >} = \frac{\overline{a^2} - \bar{a}^2}{\bar{a}} + \bar{a}\; n\; \frac{< V^2 > - < V >^2}{< V >}$$

$$\equiv \omega[a] + \bar{a}\; n\; \omega[V]$$



0.4
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0.2
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gas
0.5
0.1
$\tilde{T}$
0
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4x10^{-4}
6x10^{-4}
8x10^{-4}
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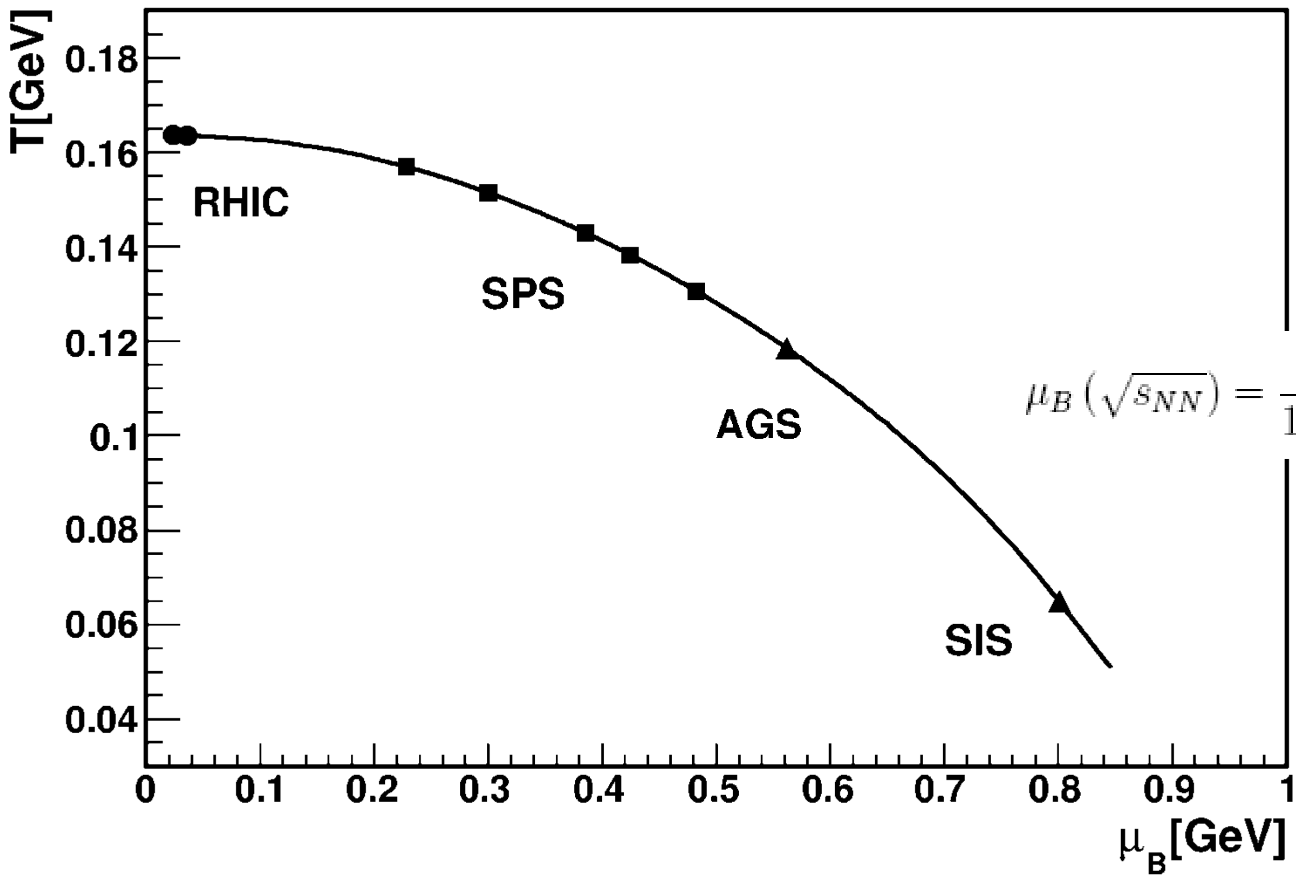
$\tilde{n}$
ω
10.00
1.00
0.10
0.01

Line of the chemical freeze-out

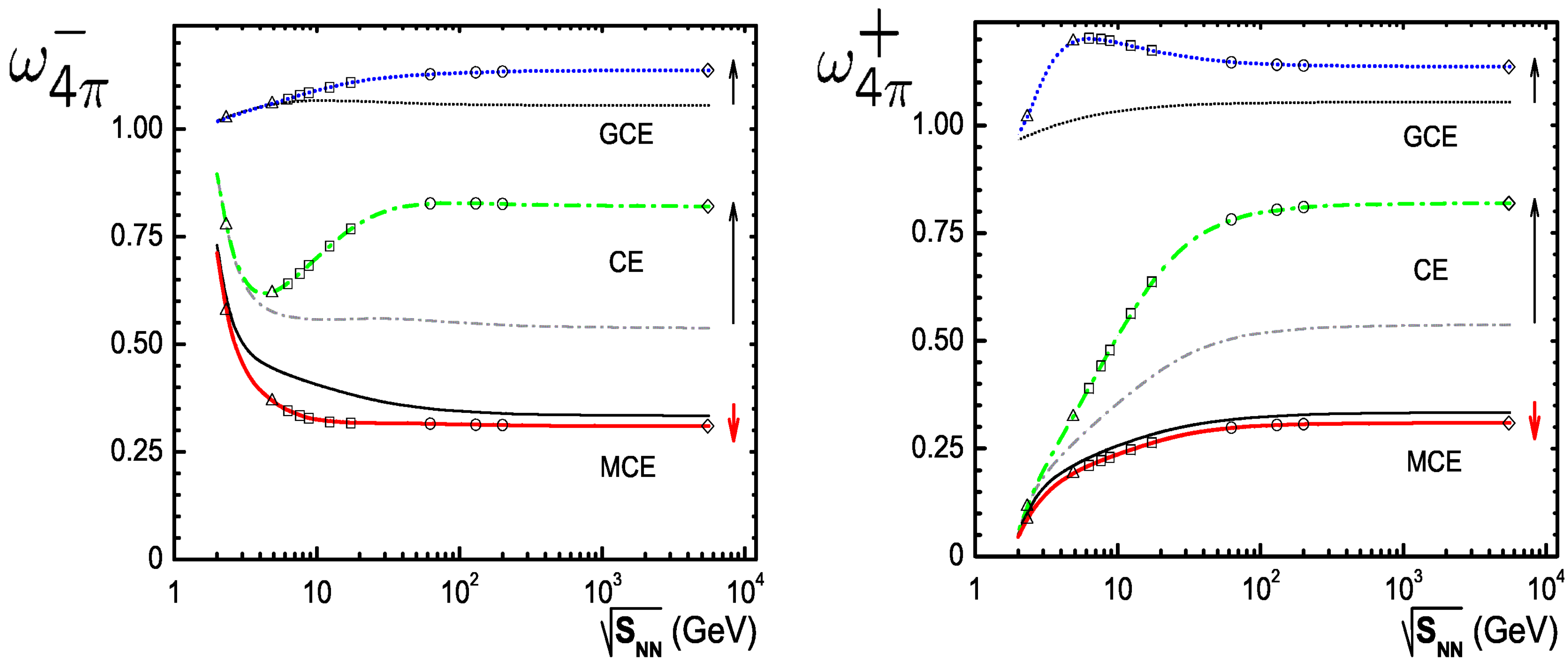


$$\mu_B\left(\sqrt{s_{NN}}\right) = \frac{1.308\ \mathrm{GeV}}{1 + 0.273\ \sqrt{s_{NN}}},$$

E/N = 1 GeV

Cleymans and Redlich, PRL 81 (1998)

Hadron-Resonance Gas



$$\omega^{\pm}_{4\pi} \equiv \frac{\langle(\Delta N_{\pm})^2\rangle}{\langle N_{\pm}\rangle} \qquad \omega^{\pm}_{acc} = 1 - q + q\,\omega^{\pm}_{4\pi}$$

Begun, Gazdzicki, M.I.G.,
Hauer, Konchakovsi, Lungwitz,
Phys. Rev. (2007)

Micro Canonical Ensemble with scaling Volume Fluctuations (MCE/sVF)

$$\boxed{\mathbf{P}_\alpha(\mathbf{X};\mathbf{E}) \;=\; \int_0^\infty \mathbf{dV}\;\mathbf{P}_\alpha(\mathbf{V})\;\mathbf{P}_{\mathbf{mce}}(\mathbf{X};\mathbf{E},\mathbf{V})}$$

Begun, Gazdzicki, M.I.G., Phys. Rev. C (2008) and (2009)

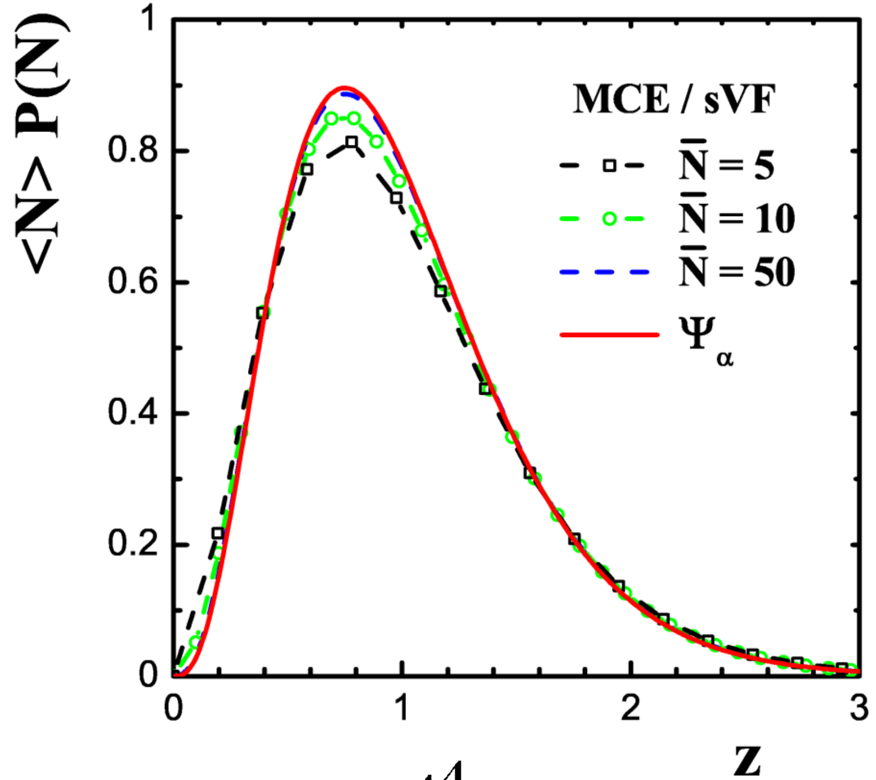
$$\mathbf{P}_\alpha(\mathbf{V}) \;=\; \frac{1}{\overline{\mathbf{V}}}\,\Phi_\alpha(\mathbf{V}/\overline{\mathbf{V}})$$

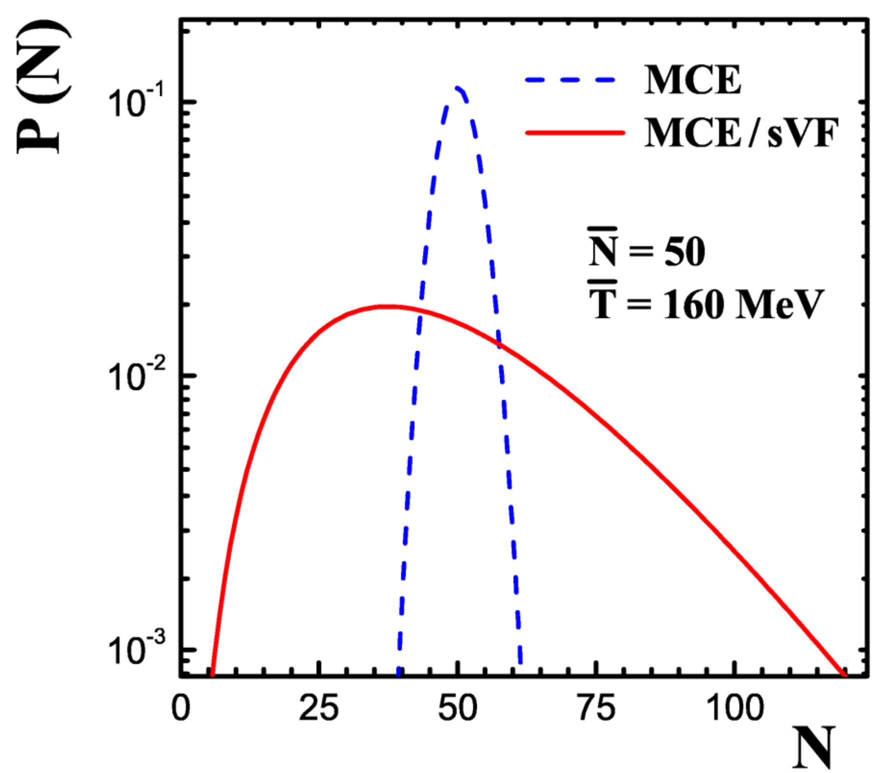
Scaling volume fluctuations selected to fit experimental multiplicity distribution

1) Describes the KNO-scaling of multiplicity distribution

2) Gives the Power Law of (transverse) momentum spectra

Multiplicity distribution in the MCE/sVF



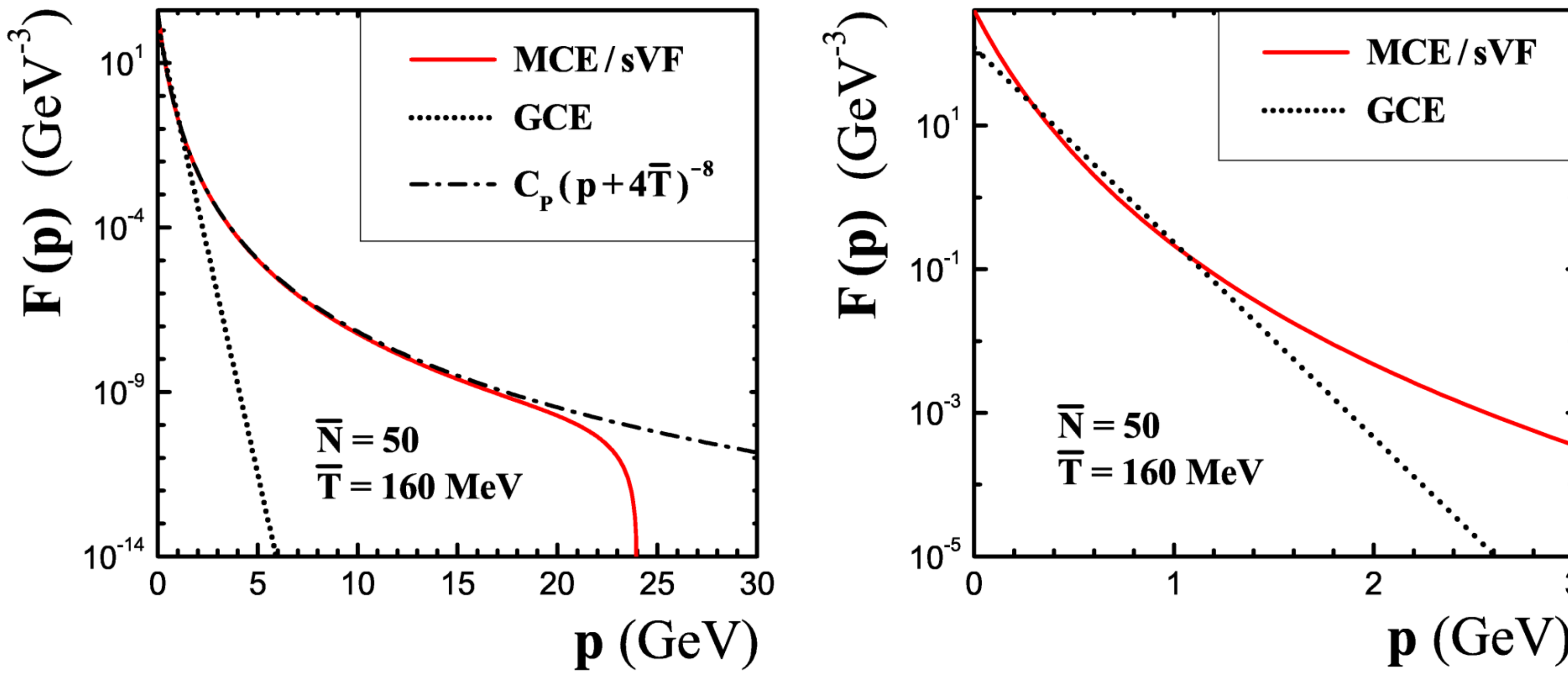


$$\Psi_\alpha(y) = \frac{4^4}{3!}\, y^3\, \exp(-4\,y)$$

$$z = N/\langle N\rangle_\alpha\,, \quad y = (V/\overline{V})^{1/4}$$

$$\boxed{\omega_\alpha = \frac{\langle N^2\rangle_\alpha - \langle N\rangle_\alpha^2}{\langle N\rangle_\alpha} \cong \frac{1}{4}\,\langle N\rangle_\alpha}$$

Power law in momentum spectrum



$$F_\alpha(p) = \frac{1}{\langle N \rangle_\alpha} \left\langle \frac{dN}{p^2 dp} \right\rangle_\alpha$$

Relation to Other Measures

$$\Phi = \frac{\sqrt{\langle A\rangle\langle B\rangle}}{\langle \mathbf{A}\rangle + \langle \mathbf{B}\rangle}\left[\left(\Sigma^{\mathrm{AB}}\right)^{1/2} - 1\right]$$

Gazdzicki, Mrowczynski (1992)

$$\nu_{\mathrm{dyn}}^{\mathrm{AB}} = \frac{\langle \mathrm{A}(\mathrm{A}-1)\rangle}{\langle \mathrm{A}\rangle^2} + \frac{\langle \mathrm{B}(\mathrm{B}-1)\rangle}{\langle \mathrm{B}\rangle^2} - 2\,\frac{\langle \mathrm{AB}\rangle}{\langle \mathrm{A}\rangle\langle \mathrm{B}\rangle}$$

Pruneau, Gavin, Voloshin (2002)

$$\nu_{dyn}[A,B] = \frac{< A+B >}{< A >< B >}[\Sigma[A,B]-1]$$

$\sqrt{s_{NN}}$	T	μ_B	$\rho[\pi^+,\pi^-]$	$\omega[\pi^+]$	$\omega[\pi^-]$	$\Sigma[\pi^+,\pi^-]$	$\langle R_{\pi\pi}\rangle/(\langle\pi^-\rangle+\langle\pi^+\rangle)$		
[GeV]	[MeV]	[MeV]	GCE	GCE	GCE	GCE	Eq.(47)	Eq.(41)	Eq. (42)
6.27	130.8	482.4	0.132	1.063	1.074	0.804	0.11	0.14	0.10
7.62	139.2	424.6	0.155	1.072	1.083	0.768	0.13	0.16	0.12
8.77	144.2	385.4	0.170	1.079	1.089	0.744	0.14	0.17	0.13
12.3	153.0	300.2	0.199	1.092	1.101	0.699	0.15	0.20	0.15
17.3	158.6	228.6	0.219	1.102	1.109	0.668	0.16	0.22	0.17
200	165.9	23.5	0.246	1.116	1.117	0.624	0.17	0.25	0.19
5500	166.0	0.87	0.246	1.117	1.117	0.624	0.17	0.25	0.19

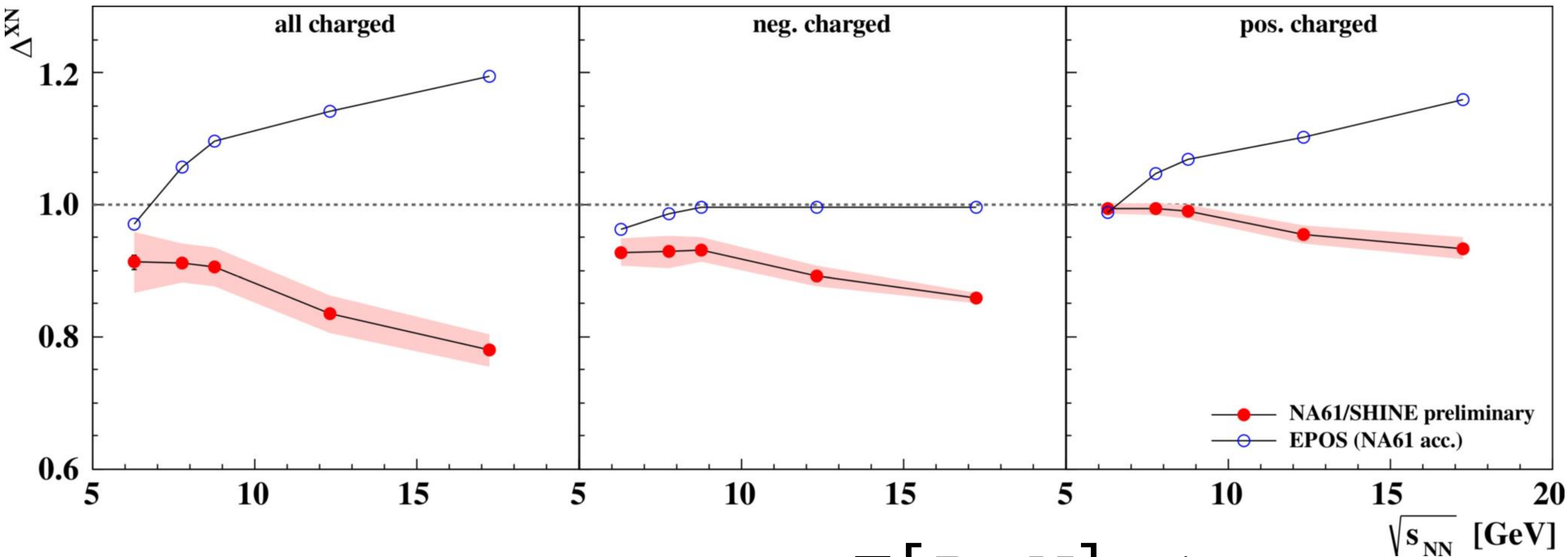
$$\Sigma\left[P_T, N\right] \cong 1.01,$$

p+p at 158 GeV/c

$$\Delta\left[P_T, N\right] \cong 0.82$$

M.I.G., Grebieszkow,
Phys. Rev.C (2014)

NA61/SHINE data in p+p reactions



Data: $\Sigma\left[P_T, N\right] \cong 1$