Equation of state and kinetic properties of nucleon gas from UrQMD transport model in the box

Anton Motornenko

Taras Shevchenko National University of Kiev, Kiev, Ukraine
Frankfurt Institute for Advanced Studies, Frankfurt am Main, Germany
Department of Physics, University of Oslo, Oslo, Norway

In collaboration with:

M. I. Gorenstein, A. G. Magner, L. Bravina,
E. Zabrodin, and **H. Stöcker**

Subject of study

Simple system – single-component gas of nucleons.

- system reached equilibrium
- $T = 10 \div 50$ MeV
- $N_p = N_n$ – symmetric nuclear matter, no isospin effects
- only elastic collisions
- interaction cross-section σ_{int} is fixed
- density $n = (0.5 \div 3) \cdot n_0$

Motivation – to figure out how simple systems are treated within complicated transport models (UrQMD), so the first step to understand behavior of multicomponent hadronic systems within transport models will be done.

UrQMD model

Is a microscopic transport model used to simulate relativistic heavy ion collisions in a wide range of collision energies.

Includes:

- All known hadrons and some unknown resonances;
- Strings.

References:

S.A. Bass *et al.*, Prog. Part. Nucl. Phys. **41**, 255 (1998); M. Bleicher *et al.*, J. Phys. G **25**, 1859 (1999);

H. Petersen, M. Bleicher, S.A. Bass, and H. Stöcker, arXiv:0805.0567 [hep-ph].

UrQMD model "Black disk" interaction mechanism

- particles do move on a **classical** straight-line trajectories
- collisions only occur only if both:
  - particles have reached the minimal distance d between them (allowed by their classical trajectories)
  - that minimal distance d between particles is less than $d < d_{\rm int} = \sqrt{\sigma_{\rm int}/\pi}$, where $\sigma_{\rm int}$ is particle interaction cross-section

So the particles can be considered as disks moving on a straight trajectories and interaction occurs only when they do intersect.

UrQMD model "Black disk" interaction mechanism

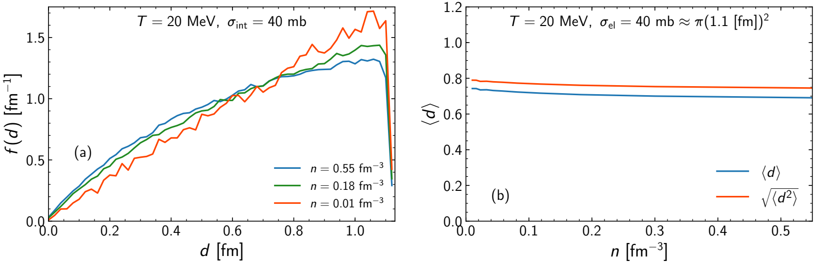


Figure: Distribution of distances d between particles at the time of their interaction for a different densities n. Interaction cross-section is foxed to $\sigma_{\rm el} = 40$ mb, temperature $T = 20$ MeV.

UrQMD model Simulations of equilibrated nucleon gas

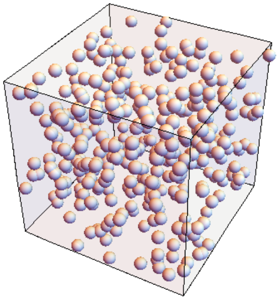


Figure: Motion of 400 nucleons inside $20 \times 20 \times 20$ fm^3 UrQMD box, T=20 MeV, $\Delta t = 100$ fm/c, scale: 10 fm/c = 1 sec.

UrQMD provides possibility to study system in a closed box, so energy density is conserved in contradiction to the Heavy-Ions collisions where system expansion is fast. System closeness allows thermalization.
After time $t_{\mathrm{eq}} \leq 150$ fm/c system reaches thermal equilibrium via elastic collisions, so:

$$\langle E \rangle = \frac{3}{2} T \tag{1}$$

UrQMD model Simulations of equilibrated nucleon gas

System properties were set at the range of values that are typical for nuclear matter:

N	L (fm)	n (fm^{-3})	T (MeV)	σ_{int} (mb)	t_{eq} (fm/c)
400	$9 \div 40$	$0.06 \div 0.5$	$10 \div 50$	$10 \div 80$	$\lesssim 150$

Table: Properties of nucleon system studied within UrQMD-box.

The thermal (Boltzmann) particle spectra had served as a signature of system equilibration.

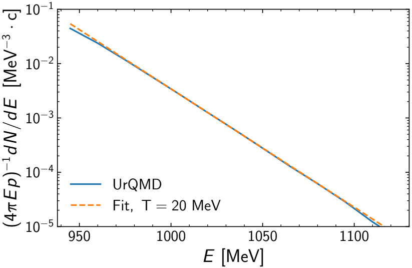


Figure: Energy spectra of nucleons inside UrQMD box with nucleon density $n = 0.05$ fm^{-3} and temperature T=20 MeV. The fit with with thermal distribution with T=20 MeV is also presented (dashed).

Shear viscosity η of nucleon gas Green-Kubo formalism

For system in equilibrium one could measure response coefficients using Green-Kubo formalism. For shear viscosity η:

$$\eta \ = \ \frac{V}{T}\int_0^\infty dt \ \langle \pi^{xy}(t)\pi^{xy}(0)\rangle_t \tag{2}$$

where $\langle \pi^{xy}(t)\pi^{xy}(0)\rangle$ is a self-correlator of a non-diagonal spatial components of stress-energy tensor $T^{\mu\nu}$:

$$\pi^{ij} = T^{ij} - \delta^{ij} T^{ij}$$

$$T^{ij} = \frac{1}{V}\langle \int\limits_V d^3x \ \frac{p^i(x)p^j(x)}{p^0(x)}\rangle_t = \frac{1}{V}\langle \sum_{k=0}^{N} \frac{p^i_k p^j_k}{p^0_k}\rangle_t \tag{3}$$

Shear viscosity η of nucleon gas Autocorrelation function

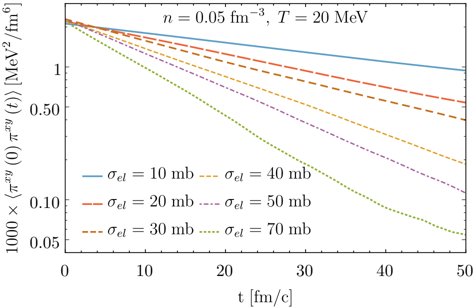


Figure: Autocorrelation function measured within UrQMD box simulations for different values of particle interaction cross-section σ.

The autocorrelation function $C(t) = \langle \pi^{xy}(t)\pi^{xy}(0)\rangle_t$ has exponential form:

$$C(t) = C(0) \cdot e^{-t/\tau}\ , \qquad (4)$$

where τ can be considered as fluctuation damping time. So the following equation for η can be obtained:

$$\eta = \frac{V}{T} C(0) \cdot \tau, \qquad (5)$$

where $C(0)$ and τ are functions of (σ, n, T).

Shear viscosity η of nucleon gas **Chapman-Enskog approximation**

In the Chapman-Enskog (CE) approximation the following expression for η can be obtained:

$$\eta_{\mathrm{CE}} = \frac{5}{16}\frac{\sqrt{\pi m T}}{\sigma_{\mathrm{int}}}\ , \quad \sigma_{\mathrm{int}} = \pi\, d^2 \tag{6}$$

Assumptions:

- isotropic cross-section
- $\sigma = \pi d^2$ as for gas of hard spheres
- frequent collisions regime (free path λ is smaller then system size L – $\lambda < L$)

In CE approximation η is not dependent on density n.

Shear viscosity η of nucleon gas Results

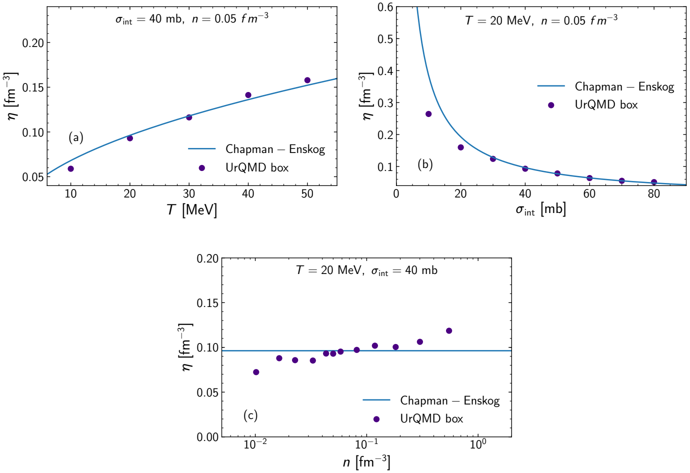
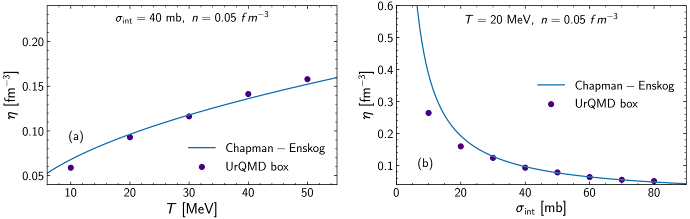
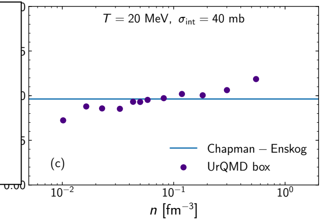


Figure: Shear viscosity η of infinite nucleon matter as function of temperature T (a), interaction cross-section $\sigma_{\rm int}$ (b), and density n (c) calculated within UrQMD box using Green-Kubo formalism. Comparison with predictions of Chapman-Enskog approximation is presented.

Shear viscosity η of nucleon gas Results



Good agreement with CE predictions – UrQMD does include excluded volume effects?



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Equation of state study

Summary of previous slides:

- There is non-zero mean distance $\langle d \rangle \approx 0.7$ fm between particles at the time of their interaction
- CE approximation for gas of hard spheres is in agreement with results from Green-Kubo formalism obtained form microscopical UrQMD simulations and so $d = \sqrt{\sigma_{\rm int}/\pi}$

One expects excluded volume (EV) equation of state (EoS):

$$P = \frac{nT}{1 - bn}, \quad b = \frac{16}{3}\pi \left(\frac{d}{2}\right)^3 \tag{7}$$

where d is a particle hard-core diameter.

Equation of state study **Pressure**

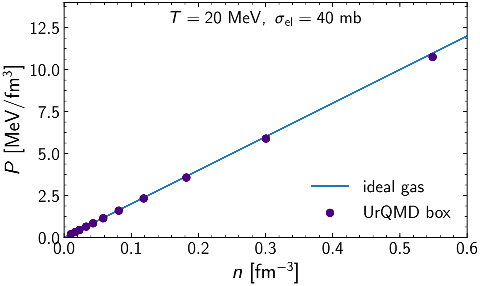

Figure: Pressure P of nucleon matter as function of density n calculated from UrQMD infinite box simulations and comparison with predictions of ideal gas equation of state.

Pressure is calculated as:

$$
\begin{aligned}
P &= 1/3 \sum_{i=1}^{3} T^{ii} \\
&= 1/3 \sum_{i=1}^{3} \langle p^i v^i \rangle_t
\end{aligned}
\tag{8}
$$

Ideal gas EoS ($P = nT$) even at extreme densities $n = 0.5$ fm^{-3}.

Particle number fluctuations

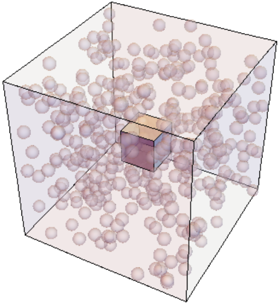

Figure: Cell with size $L_{\text{cell}} = 4$ fm inside UrQMD box of size $L_{\text{box}} = 20$ fm.

Another property of EV EoS – particle number fluctuations in grand canonical ensemble:

$$\omega = \frac{\langle N^2 \rangle - \langle N \rangle^2}{\langle N \rangle} = (1 - bn)^2 \qquad (9)$$

GCE inside closed box can be realized by considering cell with size L_{cell} inside box, so cell exchanges particles with external reservoir (whole box).

Equation of state study **Particle number fluctuations**

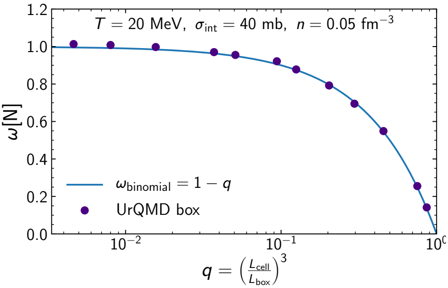

Figure: Scaled variance ω of particle number fluctuations inside a cell of an infinite box, q – ratio between cell and box volumes.

Expectation: $\omega_{\mathrm{EV}} = (1 - bn)^2$, but $b = 0$ is observed.

If cell size is big enough ($V_{\mathrm{cell}} \gg b$) then one expects binomial distribution for number of particles that got into cell:

$$P(\text{in cell}) = q, \quad q \equiv \frac{V_{\mathrm{cell}}}{V_{\mathrm{box}}} = \left(\frac{L_{\mathrm{cell}}}{L_{\mathrm{box}}}\right)^3 \tag{10}$$

Scaled variance for binomial statistics:

$$\omega = 1 - q \tag{11}$$

However this behavior is reproduced for wide range of cell sizes. Ideal gas EoS is observed again.

Ideal gas EoS?! Discussion

- Interaction cross-section $\sigma_{\rm int}$ in UrQMD model is only related to probability of particle interaction, but does not carry any information about particle structure or interaction potential
- Particles in UrQMD can be packed with any desired density and still no deviations from IG EoS, because interactions doesn't change with scale, the probability of collisions is just getting larger with increase of density
- Success of CE approximation is since it considers constant $\sigma_{\rm int}$ from collision term $I_{\rm coll}$ in Boltzmann equation:

$$I_{\rm coll} = \int (f_1' f_2' - f_1 f_2) \sigma_{\rm int} F d\Omega' \frac{d^3 p_2}{p_2^0} \ , \tag{12}$$

so it takes into account probability of interaction, but not particle structure or interaction potential

Summary

- Chapman-Enskog approximation is valid in a broad range of system parameters
- "Black disk" interaction mechanism produce kinetic properties of a non-ideal gas
- But still ideal gas EoS even at extreme densities
- No effects of simplest possible particle interaction (EV) for EoS
- UrQMD pretends to describe very dense and strongly interacting systems (Heavy-Ion collisions), but still that model considers these complicated systems as mixtures of point-like particles that are just out of equilibrium + effects of detailed balance violation