Application of light-by-light sum rules to the charmonium system

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Introduction

- Sum rules are a tool to constrain the hadron masses and decay constants.
- As these rules are consequences of general principles as analitycity and unitarity, they should be model independent.
- In this work, we will examine the application of the sum rules to well-known potential models of the charmonium system.

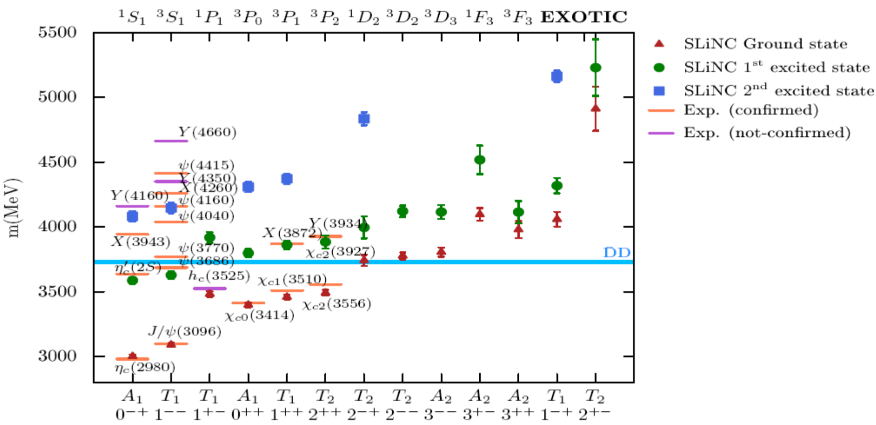
Derivation of the sum rule

We check the first sum rule that involves helicity difference cross section for the process of $\gamma\gamma$ fusion: $\gamma\gamma \to q\bar{q}$

$$\int_{s_0}^{\infty} \frac{\mathrm{d}s}{s}(\sigma_2 - \sigma_0) = 0$$

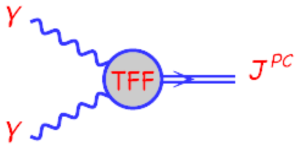
$$\sigma = \frac{(2\pi)^4}{2} \int \frac{\mathrm{d}\tilde{\mathbf{q}}}{(2\pi)^3} \frac{\delta(\tilde{\mathbf{k_1}} + \tilde{\mathbf{k_2}} - \tilde{\mathbf{q}})\delta(k_1^0 + k_2^0 - q^0)}{2(k_1^0)(2k_2^0)(2q^0)} |\mathcal{M}|^2 = \pi\delta(s - M^2)\frac{|\mathcal{M}|^2}{M^2}$$

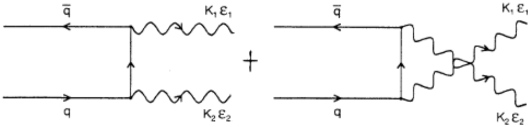
Charmonium spectrum (Preliminary)



Single meson production in $\gamma\gamma$ collisions

• produced meson has C $= +1$
• by Landau-Yang theorem final state J=1 is forbidden as both photons are real
• the main contribution comes from $J = 0$: 0^{-+}(pseudoscalar) and 0^{++} (scalar) and $J = 2$: 2^{++}(tensor) mesons.





$$\mathcal{M} = 4\pi\alpha\sqrt{3}e_Q^2 \frac{\sqrt{2M}}{\sqrt{2m}\sqrt{2m}} \int \frac{\mathrm{d}\tilde{\mathbf{p}}}{(2\pi)^3} \sum_{s_1 s_2} \bar{v}_{s_1}(-\tilde{\mathbf{p}})[\not{\epsilon}_2 \frac{\not{p} - \not{k}_1 - m}{(p-k_1)^2 + m^2} \not{\epsilon}_1 + \{1 \leftrightarrow 2\}]u_{s_2}(\tilde{\mathbf{p}})\psi_M^{s_1 s_2}$$

The meson wave function for the state with the orbital momentum L, spin S and total momentum J has the form

$$\psi_M^{s_1 s_2}(\tilde{\mathbf{p}}) = \sum_{m_l m_s} \langle L S m_l m_s | J m_j \rangle \langle \frac{1}{2} \frac{1}{2} s_1 s_2 | S m_s \rangle Y_{L m_l}(\theta, \varphi) R_M(|\tilde{\mathbf{p}}|)$$

$$\sum_{s_1 s_2} \bar{v}_{s_1}(-\tilde{\mathbf{p}}) \hat{\mathcal{O}} u_{s_2}(\tilde{\mathbf{p}}) \langle \frac{1}{2} \frac{1}{2} s_1 s_2 | S m_s \rangle =$$

$$= \frac{1}{\sqrt{E_{-\tilde{\mathbf{p}}} + m}} \frac{1}{\sqrt{E_{\tilde{\mathbf{p}}} + m}} Tr \left[(\not{p} - m) \hat{\mathcal{O}} (\not{p} + m) \frac{1 + \gamma_0}{2\sqrt{2}} \Pi_{S m_s} \right]$$

with $\Pi_{00} = \gamma_5, \ \Pi_{1 m_s} = \not{e}_{m_s}$

$$e^{\mu}_{m_s} = \begin{cases} (0, 0, 0, 1) & \text{if } m_s = 0 \\ (0, \mp 1, -i, 0)/\sqrt{2} & \text{if } m_s = \pm 1 \end{cases}$$

$$d\Gamma = \frac{1}{2}\frac{1}{32\pi^2}\frac{1}{(2J+1)}\sum_{\lambda_1\lambda_2}|\mathcal{M}_{\lambda_1\lambda_2}|^2\frac{M/2}{M^2}d\Omega$$

After simplifying in the non-relativistic limit we obtain

$$\Gamma({}^1S_0 \to \gamma\gamma) = \frac{3\alpha^2 {e_Q}^4 |R_{\mathbf{n}S}(0)|^2}{{m_Q}^2}$$

$$\Gamma({}^3P_0 \to \gamma\gamma) = \frac{27\alpha^2 {e_Q}^4 \left|R'_{\mathbf{n}P}(0)\right|^2}{{m_Q}^4}$$

$$\Gamma({}^3P_2 \to \gamma\gamma) = \frac{36\alpha^2 {e_Q}^4 \left|R'_{\mathbf{n}P}(0)\right|^2}{5{m_Q}^4}$$

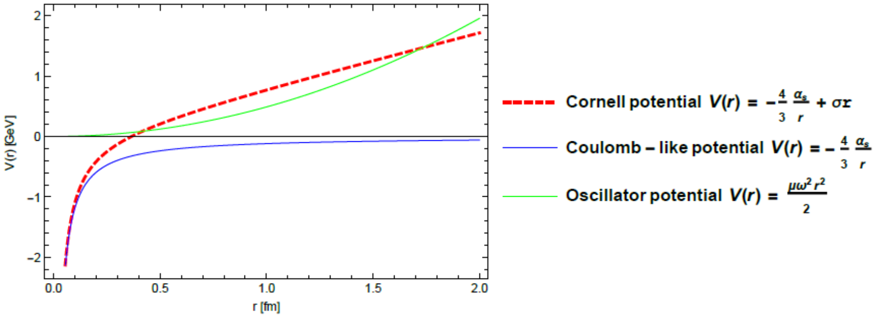
For pseudoscalar and scalar mesons $\sigma_0 = 16\pi^2\delta(s - M^2)\frac{\Gamma_{\gamma\gamma}}{M}$ and $\sigma_2 = 0$

For tensor mesons $\sigma_0 = 16\pi^2\delta(s - M^2)\frac{5\Gamma_{\gamma\gamma}(\Lambda=0)}{M}$ $\sigma_2 = 16\pi^2\delta(s - M^2)\frac{5\Gamma_{\gamma\gamma}(\Lambda=2)}{M}$

Predominantly ($\sim 95\%$) tensor mesons are produced in a state of the helicity $\Lambda = 2 \Rightarrow$

$$-\sum_{n_{ps}} 16\pi^2 \frac{\Gamma_{^1S_0\to\gamma\gamma}}{M_{ps}^3} - \sum_{n_s} 16\pi^2 \frac{\Gamma_{^3P_0\to\gamma\gamma}}{M_s^3} + \sum_{n_t} 16\pi^2 \frac{\Gamma_{^3P_2\to\gamma\gamma}}{M_t^3} = 0$$

Potential for the charmonium system



Thus,at first we will study toy models with explicit and well-known expressions for radial functions.

Oscillator model

$$R_{n_r,l}(r) = C_{n_r,l} r^l e^{-\frac{\mu\omega r^2}{2}} {}_1F_1[-n_r, l+\tfrac{3}{2}; \mu\omega r^2]$$

Here n_r is a radial quantum number and $C_{n_r,l}$ is the normalization constant.
Oscillator frequency ω and $\mu = \frac{m_c}{2}$ are fit parameters.

The energy levels (and thus masses of the states) can be found from $E = \omega(2n_r + l + \frac{3}{2})$

$$C_{n,l}^2 = 2\lambda^{l+\frac{3}{2}} \frac{\Gamma(-l-\frac{1}{2})}{\Gamma(-n-l-\frac{1}{2})} \frac{(-1)^n}{n!} \frac{1}{\Gamma(l+\frac{3}{2}) {}_2F_1[n+l+\frac{3}{2},1;1;0]}$$

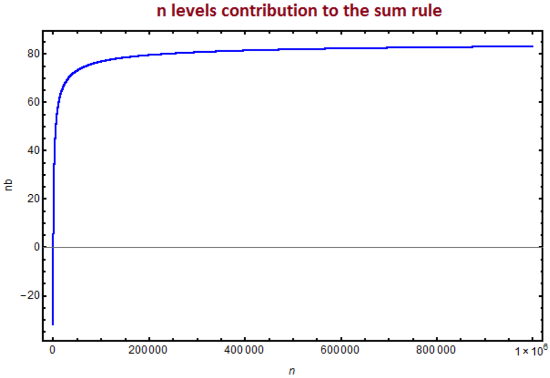
It is convenient to go to the dimensionless parameter $x = \frac{\omega}{m_c}$

To stay in the non-relativistic limit $nx \ll 1$

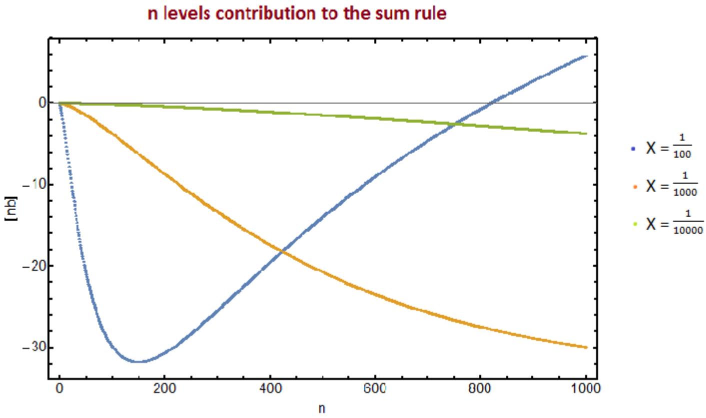
The sum rule takes the form:

$$-\frac{16\pi\alpha^2 e_Q^4}{m_c^2}\left(\frac{x}{2}\right)^{\frac{3}{2}}\sum_{n=0}^{\infty}\frac{1}{(1+xn)^3}\left[3\sqrt{n}+\frac{3}{2\sqrt{n}}-2xn^{\frac{3}{2}}-4x\sqrt{n}-\frac{3x}{2\sqrt{n}}\right]=0$$

- The sum rule goes to the limit of 85.6436 nb.

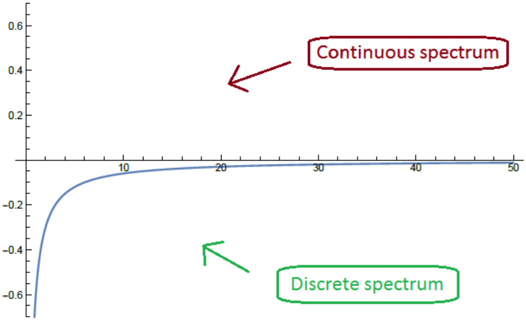


Still remember, that considering the non-relativistic limit $nx \ll 1$



Coulomb-like potential model

$$V(r) = -\frac{4}{3}\frac{\alpha_s}{r}, \alpha_s = 0.45$$



Discrete spectrum

$$R(r) = -\sqrt{\frac{4\tilde{\alpha}^3(n-l-1)!}{n^4[(n+l)!]^3}} e^{-\frac{\tilde{\alpha}r}{n}} (\frac{2\tilde{\alpha}r}{n})^l L_{n+l}^{2l+1}(\frac{2\tilde{\alpha}r}{n})$$

$\tilde{\alpha} = \mu\alpha_s$, $n = n_r + l + 1$ the main quantum number

$$E_n = -\frac{\tilde{\alpha}^2}{2\mu n^2} = -\frac{\mu{\alpha_s}^2}{2n^2}$$

$M(\mathbf{1}^1S_0) = 2.4257$ Gev

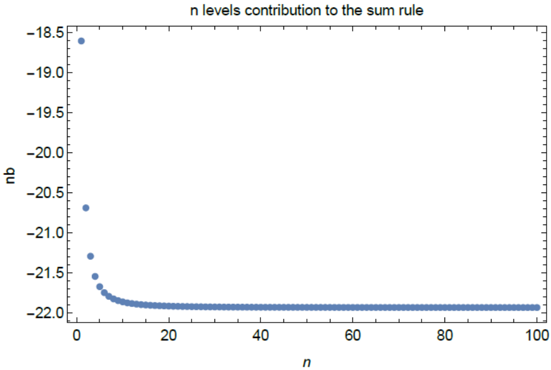
$M(\mathbf{1}^3P_0) = 2.51143$ Gev

$M(\mathbf{1}^3P_2) = 2.51143$ Gev

$\Gamma(1^1S_0) = 4.33055$ keV

$\Gamma(1^3P_0) = 3.6539 \times 10^{-2}$ keV

$\Gamma(1^3P_2) = 9.74373 \times 10^{-3}$ keV

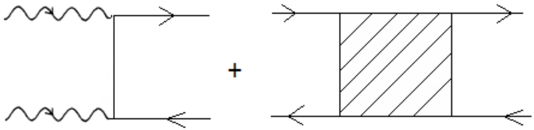


Goes to the limit of -21.9338 nb

Continuous spectrum

$$n \to -\frac{i}{k}$$

$$R = \frac{2ke^{\frac{\pi}{2k}}\left|\Gamma(l+1-\frac{i}{k})\right|}{(2l+1)!}(2kr)^l e^{-ikr}{}_1F_1(\frac{i}{k}+l+1, 2l+2, 2ikr)$$



Mesons	m_M (MeV)	$\Gamma_{\gamma\gamma}$ (keV)	$\int_{s_0}^{\infty} \frac{\mathrm{d}s}{s}(\sigma_2 - \sigma_0)$ (nb)
$\eta_c(1S)$	2980.3 $\pm$ 1.2	6.7 $\pm$ 0.9	-15.6 $\pm$ 2.1
$\chi_{c0}(1P)$	3414.75 $\pm$ 0.31	2.32 $\pm$ 0.13	-3.6 $\pm$ 0.2
$\chi_{c2}(1P)$	3556.2 $\pm$ 0.09	0.50 $\pm$ 0.06	3.4 $\pm$ 0.4
Sum resonances			-15.8 $\pm$ 2.1
Continuum			15.1
Resonances+Continuum			-0.7 $\pm$ 2.1

Table 1: $\gamma\gamma$ sum rule contributions of the $c\bar{c}$ mesons based on the PDG values of the meson masses and 2γ decay widths $\Gamma_{\gamma\gamma}$.

Mesons	m_M (MeV)	$\Gamma_{\gamma\gamma}$ (keV)	$\int_{s_0}^{\infty} \frac{ds}{s}(\sigma_2 - \sigma_0)$ (nb)
$\eta_c(1S)$	2425.7	4.33	-18.65
$\chi_{c0}(1P)$	2511.43	0.036	-0.14
$\chi_{c2}(1P)$	2511.43	0.0097	0.19
Sum resonances			-18.6
Continuum			?
Resonances+Continuum			?

Table 2: $\gamma\gamma$ sum rule contributions of the $c\bar{c}$ mesons calculated for the Coulomb toy potential.

Conclusions and Outlook

- In the non-relativistic limit sum rule works for discrete spectrum!
- Behaviour of the contribution to the sum rule of ground states in Coulomb potential is similar to the experimental.

As a next step:

- plan to include the continuous spectrum
- apply sum rule to the Cornell potential