Magnetic structure of light mesons in lattice QCD

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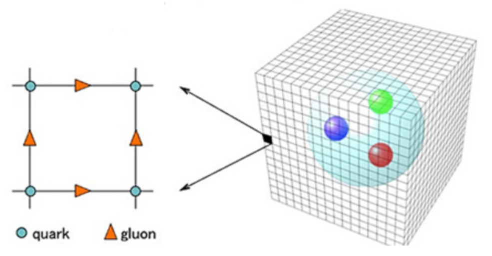
1 Introduction

Quantum Chromodynamics in abelian magnetic field of hadron scale is a rich area for exploration.Today it is possible to create a strong magnetic field of $15 - 100m_\pi^2 \sim 0.27 - 2.5\,Gev^2$ in terrestrial laboratories (LHC, RHIC, NICA, FAIR) during non-central heavy ion collisions. Our studies aims at revealing the effects that can appear in such experiments. Currently there exist significant discrepancies between the theoretical predictions, different phenomenological models and experimentally obtained data. For most mesons the magnetic parameters like polarizabilities, hyperpolarizabilities and magnetic moments(g-factors) has not been found experimentally yet.

2 Lattice QCD



$x \Rightarrow an, \psi(x) \Rightarrow \psi(n)$

a is the lattice spacing, $n \in Z$, $N_s^3 \times N_t$ is the lattice volume.

1. Make a transition to Euclidean space $x_0 \to it$.

2. Exchange the Lagrangian of the theory by it's discretized version.

$$S_{QCD} \to iS_{QCD}^E \Rightarrow e^{iS_{QCD}} \to e^{-S_{QCD}^E}.$$

3. We generate numerically gauge field ensembles of gluonic configurations with weight $e^{-S_{QCD}^E}$, (corresponding to Boltzmann distribution) using Monte-Carlo methods.

3 Numerical calculations

For each meson we construct interpolation operators with given quantum numbers. Then we calculate correlation functions of these operators in Euclidean space in the external $U(1)$ magnetic field and in the presence of vacuum $SU(3)$ nonabelian gluon fields. $\langle\bar{\psi}(x)O_\mu\psi(x)\bar{\psi}(y)O_\nu\psi(y)\rangle_A$ (where $O_\mu = \gamma_\mu, \gamma_\nu$ for the vector particle and γ_5 for pion, $\mu, \nu = 1,..,4$ are Lorenz indices)

4 Calculations of energies

We generate $200 - 300$ $SU(3)$ statistically independent lattice gauge configurations for lattice volumes 16^4, 18^4, 20^4 and lattice spacings $a = 0.105\ fm$, $0.115\ fm$ and $0.125\ fm$. Solve the Dirac equation numerically, where field is a sum of non-abelian $SU(3)$ gluonic field and $U(1)$ abelian constant magnetic field.

Calculate the propagators:

$$D^{-1}(x,y) = \sum_{k<M} \frac{\psi_k(x)\psi_k^\dagger(y)}{i\lambda_k + m}. \qquad (1)$$

where $M = 50$ is the number of the lowest eigenmodes.

$$\langle\bar{\psi}O_1\psi\bar{\psi}O_2\psi\rangle_A = -Tr[O_1 D^{-1}(x,y)O_2 D^{-1}(y,x)]$$

We make 3-dimentional Fourier transformation of correlators and consider zero momentum $\mathbf{p} = 0$ because we are interested in the ground energy state

$$\tilde{C}(n_t) = \langle\psi^\dagger(\mathbf{0},n_t)O_1\psi(\mathbf{0},n_t)\psi^\dagger(\mathbf{0},0)O_2\psi(\mathbf{0},0)\rangle_A =$$

$$\sum_k \langle 0|O_1|k\rangle\langle k|O_2^\dagger|0\rangle e^{-n_t a E_k}$$

We obtain the ground state energy E_0 from the fiting correlation functions:

$$\tilde{C}_{fit}(n_t) = 2A_0 e^{-N_T a E_0/2}\cosh((\frac{N_T}{2} - n_t)aE_0)$$

π meson: $C^{PSPS} = \langle\bar{\psi}(\mathbf{0},n_t)\gamma_5\psi(\mathbf{0},n_t)\bar{\psi}(\mathbf{0},0)\gamma_5\psi(\mathbf{0},0)\rangle$,
ρ meson: $C_i^{VV} = \langle\bar{\psi}(\mathbf{0},n_t)\gamma_i\psi(\mathbf{0},n_t)\bar{\psi}(\mathbf{0},0)\gamma_i\psi(\mathbf{0},0)\rangle$,
where $x, y, z \to i = 1, 2, 3$

ρ^0 meson with $s_z = 0 \longleftarrow C_{zz}^{VV}$

$C^{VV}(s_z = \pm 1) = C_{xx}^{VV} + C_{yy}^{VV} \pm i(C_{xy}^{VV} - C_{yx}^{VV})$. The Energy of a point-like particle in constant magnetic field directed along z are described by The Landau level(for ground state energy $n = 0$, $p_z = 0$):

$$E^2 = E^2(B=0) + pz^2 + (2n+1)qB - gs_z qB$$

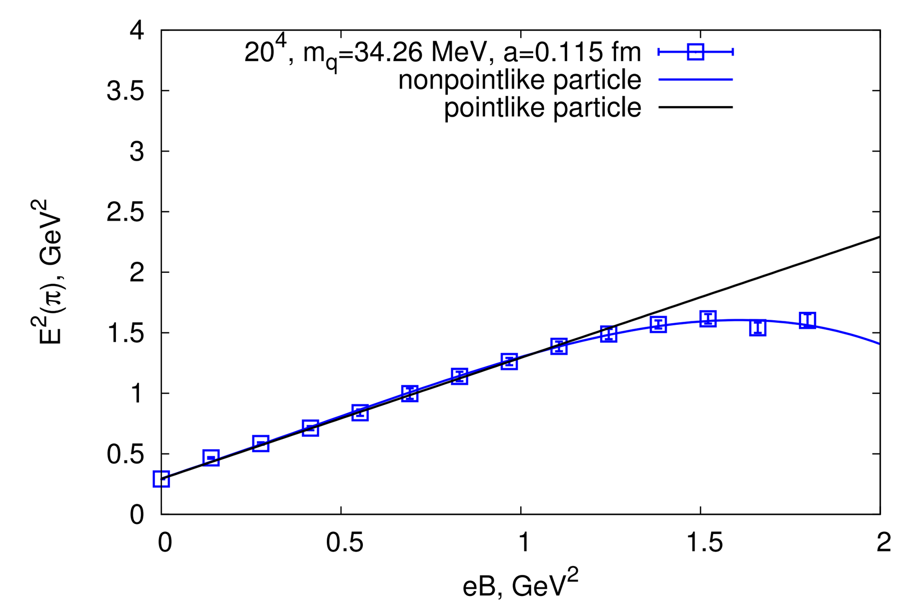
While in our calculations we took into account the quark structure of a particle and introduced the term with magnetic polarizabilities:

$$E^2 = E^2(B=0) + |qB| - gs_z qB - 4\pi m\beta(qB)^2 - 4\pi m\beta^{1h}(qB)^4,$$

where $q = -e$, β is the magnetic polarizability, β^{1h} - the magnetic hyperpolarizability,g - g-factor . We have obtained the magnetic polarizability and hyperpolarizability for charged pion

Magnetic dypole polarizability:
$\beta_\pi = -(2.06 \pm 0.76) \times 10^{-4}\ fm^3, V = 18^4, a = 0.086\ fm;$
COMPASS (CERN): $\beta_\pi = -(2.0 \pm 0.6_{stat} \pm 0.7_{syst}) \times 10^{-4} fm^3$,
ChPT (two loops): $\beta_\pi = -2.77 \times 10^{-4}\ fm^3$.
Magnetic hyperpolarizability:
$\beta_m^{1h} = (1.3 \pm 0.2) \times 10^{-7}\ fm^7,\ V = 20^4,\ a = 0.115\ fm.$

Figure 1: The ground state energy of charged π^- meson. Black line corresponds to the Landau level picture point-like particle, Blue line corresponds to formula for case non point-like particle



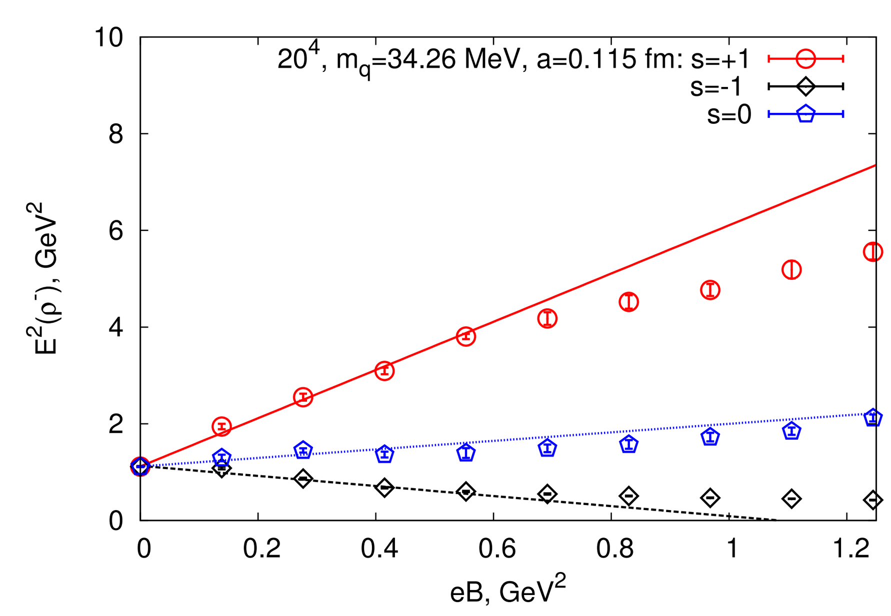
We have explored the energy dependence of neutral pion off the external constant magnetic field and have found its dipole magnetic polarizability and hyperpolarizability. For neutral pion the energy level are described by the following formula:

$$E^2 = E^2(B=0) - 4\pi m\beta(qB)^2 - 4\pi m\beta^{1h}(qB)^4,$$

Figure 2: The values of magnetic polarizabilities of the π^0 meson for the various quark masses, the lattice volumes 18^4 and 16^4 and the lattice spacing $a = 0.115 fm$.

V_{latt}	m_d (MeV)	a (fm)	β_m (GeV^{-3})	χ^2/d.o.f.
16^4	34.26	0.115	0.064 ± 0.006	0.828
18^4	11.99	0.115	0.046 ± 0.009	0.947
18^4	17.13	0.115	0.061 ± 0.003	0.137
18^4	25.70	0.115	0.067 ± 0.002	0.200
18^4	34.26	0.115	0.071 ± 0.009	3.670
18^4	$m_d = 0$ *extr.*	0.115	0.043 ± 0.005	0.576

Figure 3: The ground state energy of charged ρ meson. Lines correspond to point-like meson energies

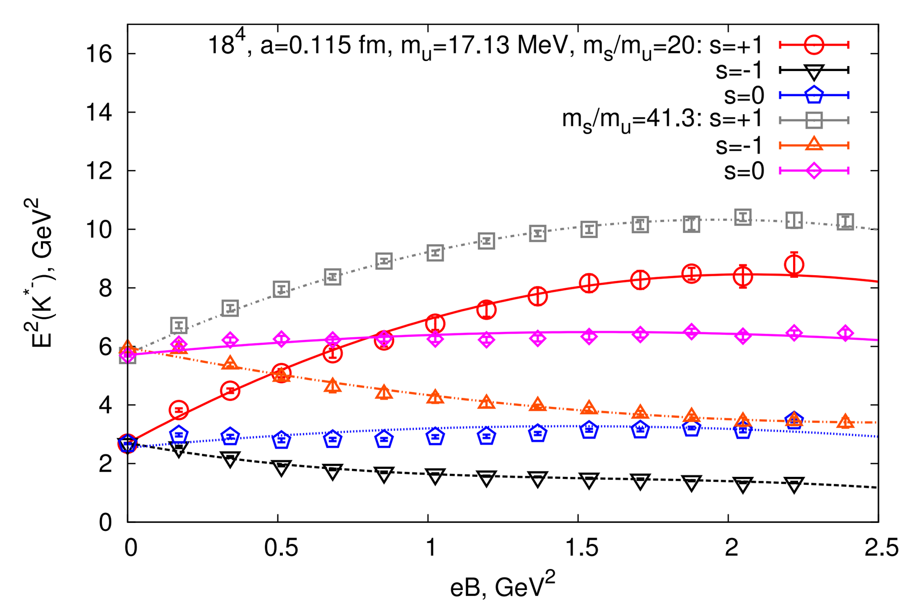


At the magnetic fields $eB \in [0, 0.5]\ Gev^2$ our data agree with the Landau levels(lines on the fig) within the errors. At larger magnetic fields the deviation from the Landau levels has been observed, because the energy of ρ meson has nonzero contibution of the terms with magnetic polarizabilities.

Tensor of magnetic dipole polarizability at rest for the $\rho^\pm$ meson.
$m_q = 34.26\ MeV,\ V = 18^4,\ a = 0.095\ fm:$
$\beta_m(s = \pm 1) = -(7.7 \pm 1.1) \times 10^{-4}\ fm^3$,
$\beta_m(s = 0) = -(2.1 \pm 0.6) \times 10^{-4}\ fm^3$.
$m_q = 17.13\ MeV,\ V = 18^4,\ a = 0.115\ fm:$
$\beta_m(s = \pm 1) = -(6.3 \pm 1.1) \times 10^{-4}\ fm^3$,
$\beta_m(s = 0) = -(2.1 \pm 0.7) \times 10^{-4}\ fm^3$.

We have also explored the energy of the vector $K^{*\pm}(1^{--})$ meson in a strong magnetic field for various spin projections on the field direction. Although the ratio of the strange quark mass to the light quark mass is sufficiently high we observe the same qualitative behaviour of the energy levels for the $K^{*\pm}$ meson as for the $\rho^\pm$ meson.

Figure 4: The ground state energy of charged K^* meson.Lines correspond to not point-like meson energies



The g-factor of $\rho^\pm$ is estimated in the chiral limit: for lattice 18^4,$a = 0.115 fm$ after $m_q \to 0$: $g_{lat} = 2.11 \pm 0.10$
experiment ($e^+e^- \to \pi^+\pi^- 2\pi^0$): $g_{exp} = 2.1 \pm 0.5$
D. G. Gudino and G. T.Sanchez, (2013),?Int.J. of Mod.Phys.A,30:1550114 arXiv: 1305.6345 [hep-ph]
Relativistic quark model : $g \approx 2.37$ A. M. Badalian, Yu. A. Simonov(ITEP), Phys. Rev. D 87, 074012 (2013)
QCD sum rules: $g = 2.4 \pm 0.4$ T. M. Aliev at al., Phys.Lett.B678

5 Conclusions

We explore the energy dependence of light mesons versus the background Abelian magnetic field on the base of quenched SU(3) lattice gauge theory and calculate the magnetic dipole polarizabilities and hyperpolarizabilities of charged and neutral π and ρ mesons for various lattice volumes and lattice spacings. At low magnetic fields our data agree with the picture of the Landau levels within the error range. At magnetic fields $eB > [0.3 \div 0.5]\,Gev^2$ the non-linear terms in the magnetic field give contributions to the energy providing clear indications of the nonzero magnetic dipole polarizability and the hyperpolarizabilities of the mesons. The functional dependences of the energy on the field for various spin projections $s_z = -1, 0$ and $+1$ coincide with the theoretical expectations. There are no evidences in favour of charged vector meson condensation or tachyonic mode existence at large magnetic fields. The g-factor of $\rho^\pm$ is estimated in the chiral limit. For more details see : Phys.Lett. B761 (2016) 393-398, arXiv:1511.09316 and arXiv:1608.03472 and references therein.

6 Acknowledgements

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