

# PRECISION TESTS AND FINE TUNING IN TWIN HIGGS MODELS

Roberto Contino  
Scuola Normale Superiore, Pisa



Based on: RC, D. Greco, R. Mahbubani, R. Rattazzi and R. Torre, [arXiv:1702.00797](https://arxiv.org/abs/1702.00797)

‘DaMESyFla in the Higgs Era’ - 15-17 March 2017, SISSA

# Naturalness and the scale of New Physics

- When viewing the SM as an effective field theory, the Higgs mass is the observable most sensitive to the New Physics scale

$$\delta m_h^2 = \frac{3y_t^2}{4\pi^2}\Lambda_t^2 - \frac{9g^2}{32\pi^2}\Lambda_g^2 - \frac{3g'^2}{32\pi^2}\Lambda_{g'}^2 - \frac{3\lambda_h}{8\pi^2}\Lambda_h^2 + \dots$$

New physics expected at the scale

$$\Lambda^2 \sim \frac{4\pi^2}{3y_t^2} \times \frac{m_h^2}{\epsilon} \implies \Lambda \sim 0.45 \sqrt{\frac{1}{\epsilon}} \text{ TeV} \quad \epsilon = \text{Fine Tuning}$$

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For example, consider the MSSM with high-scale SUSY breaking:

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**Super-Soft Models**

[ Higgs mass fully generated at around the weak scale ]

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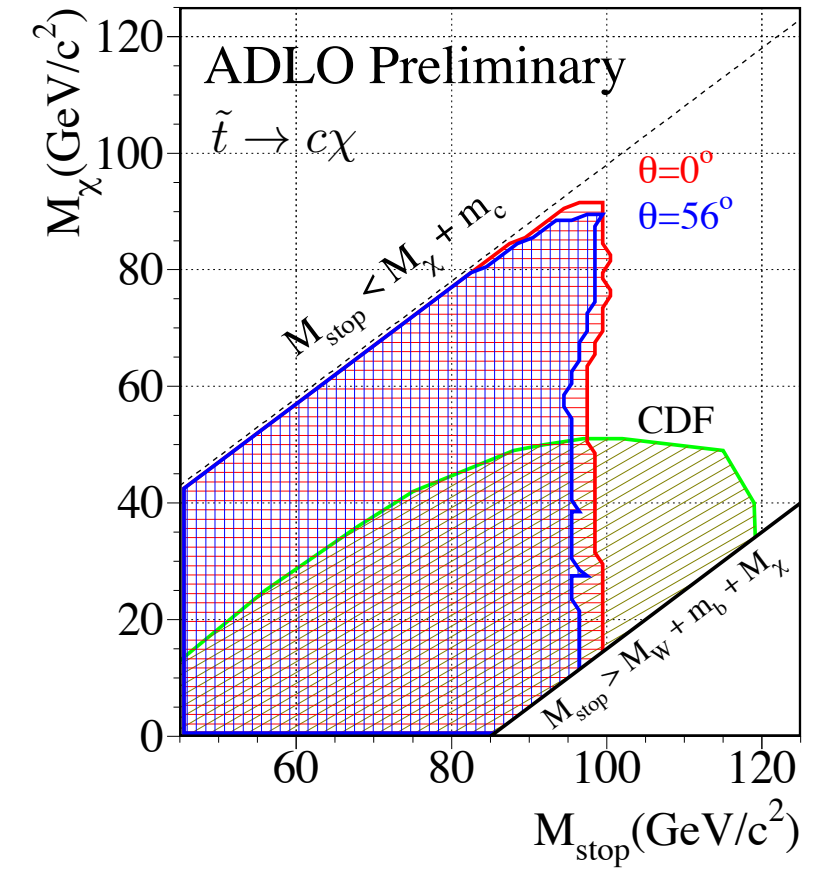
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**Soft Models**

[ R. Rattazzi, talk at “Neutral Naturalness”, CERN 2015 ]

- Super-Soft natural models (e.g. SUSY) already constrained at LEP2

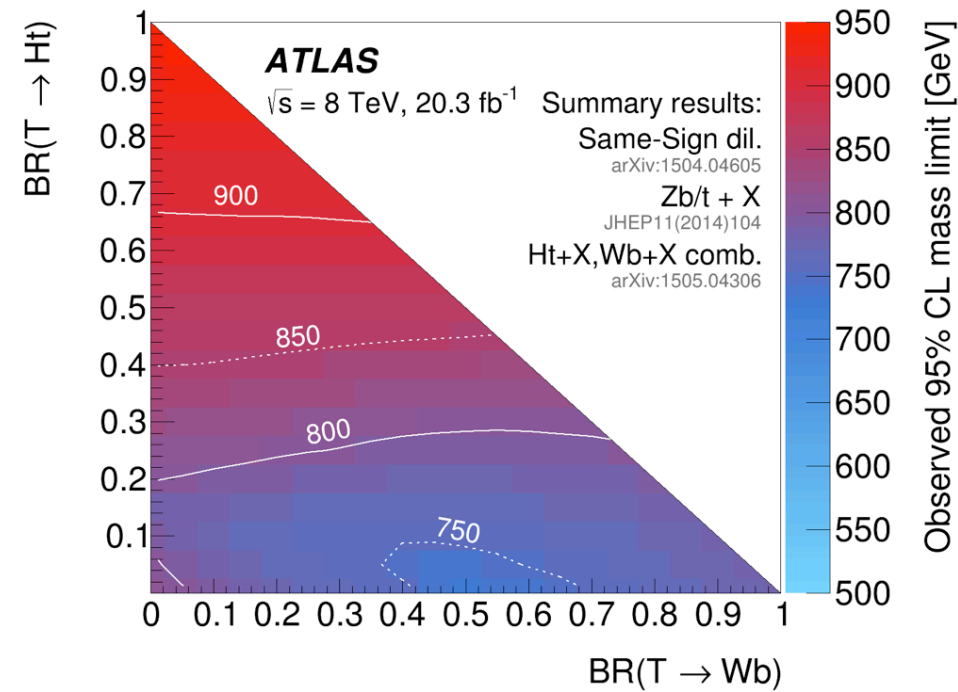
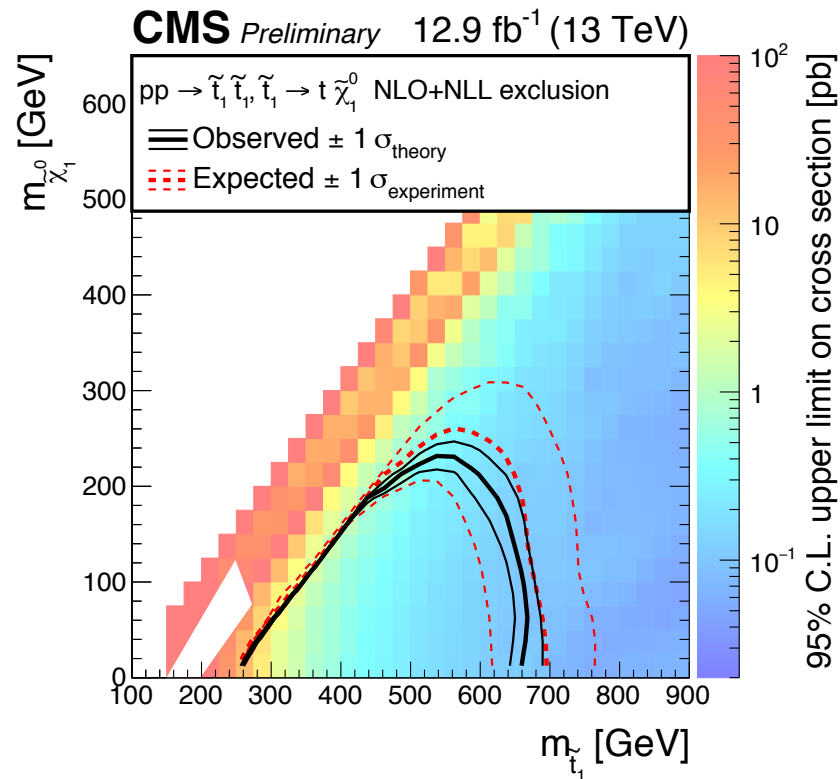
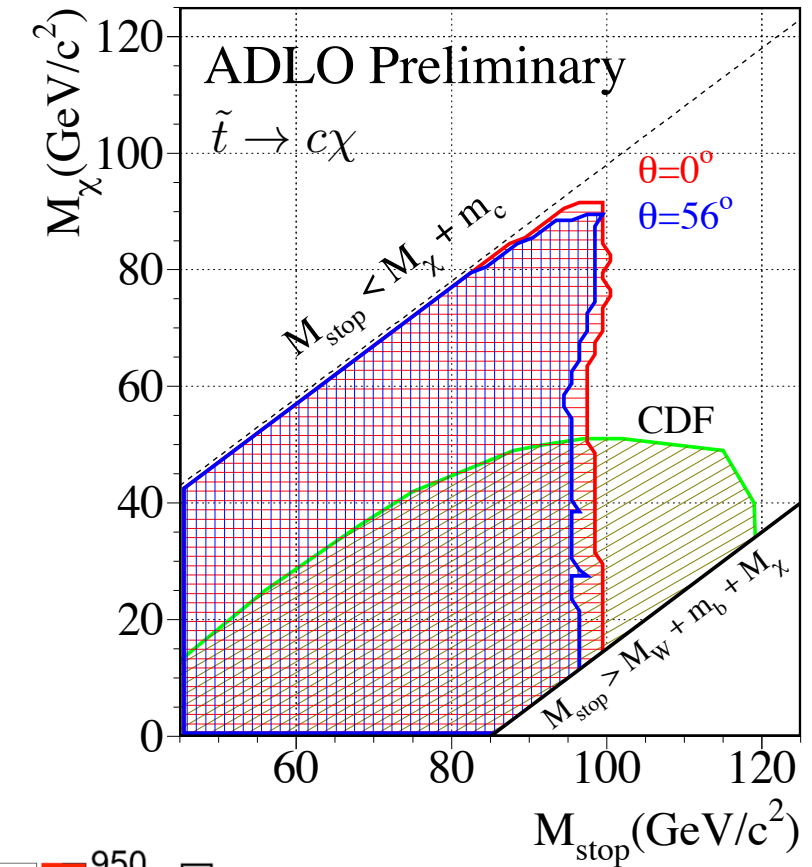
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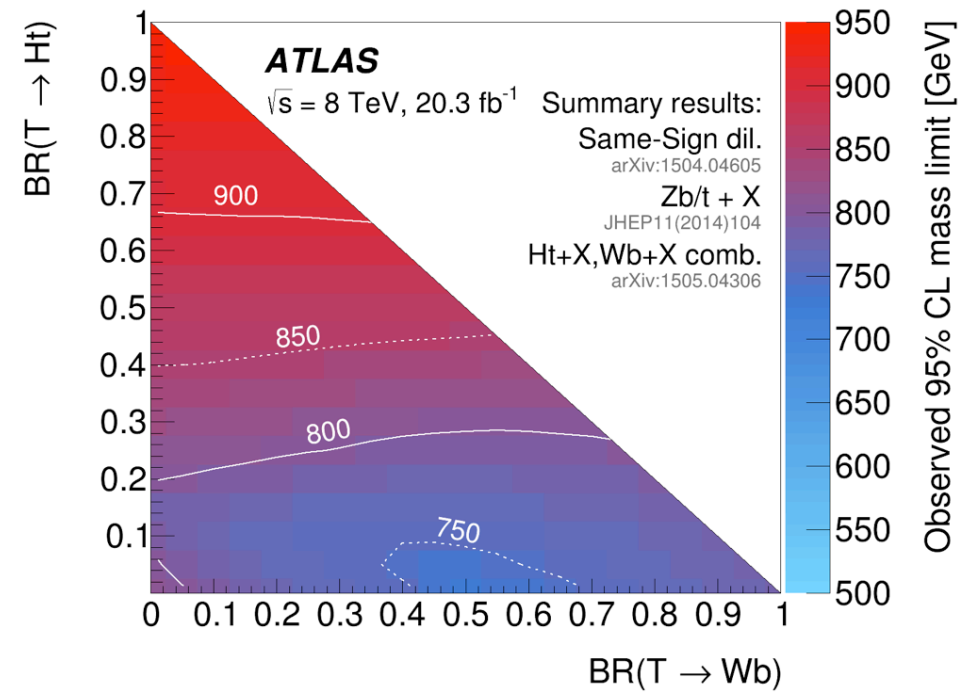
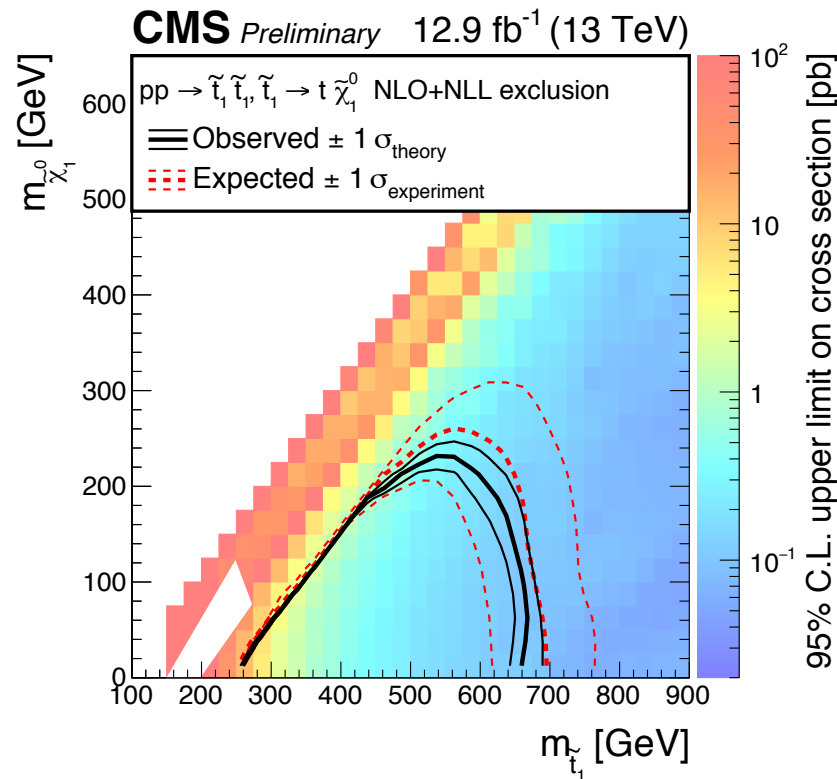
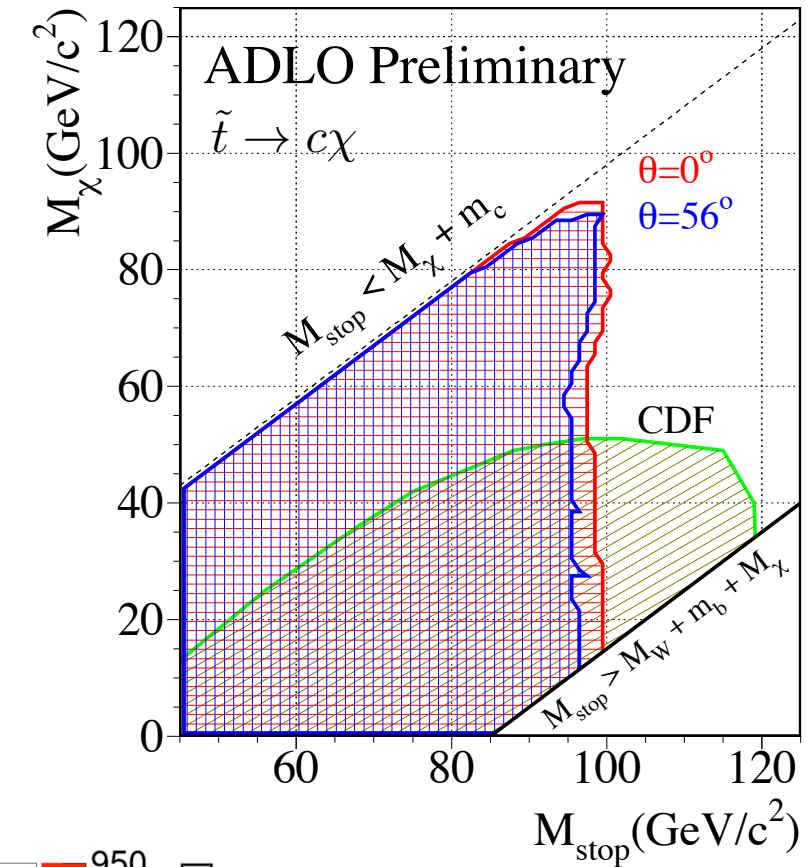
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👉 Both kind kind of theories are now confined into fine-tuned territory

The Twin Higgs paradigm: *Higgs mass saturated by new states  
**neutral** under the SM gauge group*

Naive difficulty: i) *How to relate the coupling of the new states to  $y_t$ ?*  
ii) *Make sure that 2-loop QCD corrections do not spoil the cancellation*



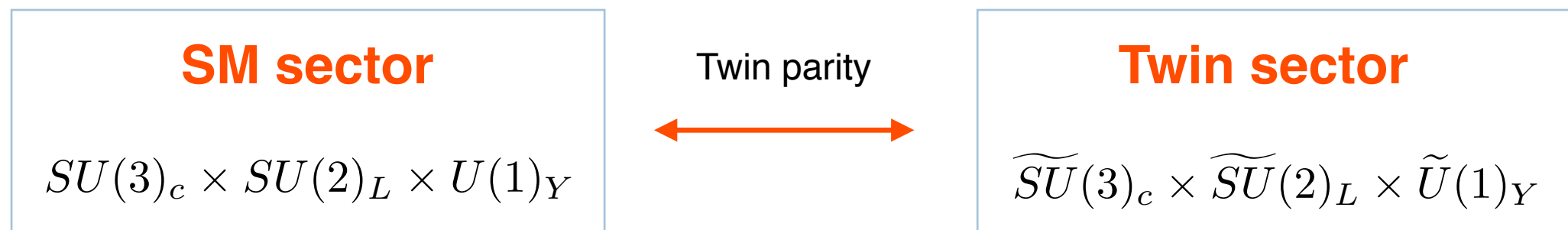
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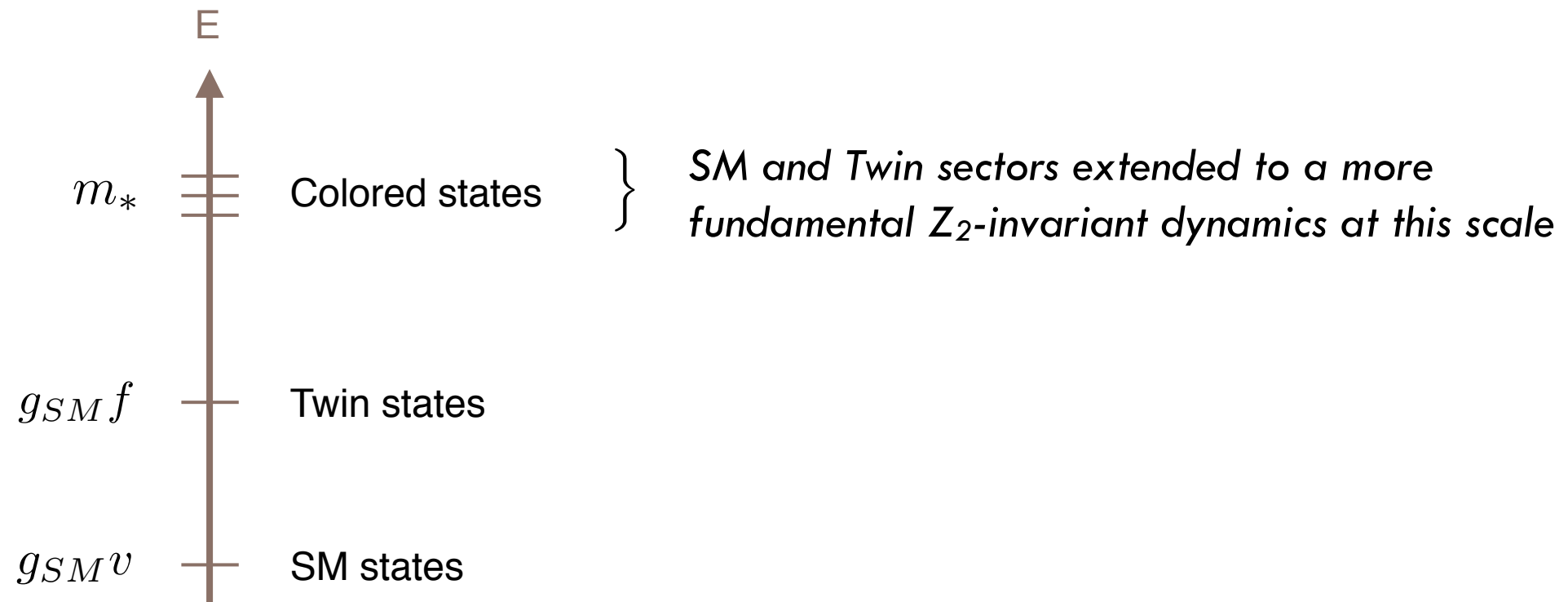


Twin Higgs idea: *the SM sector related to a copy through a  $Z_2$  (Twin) parity*

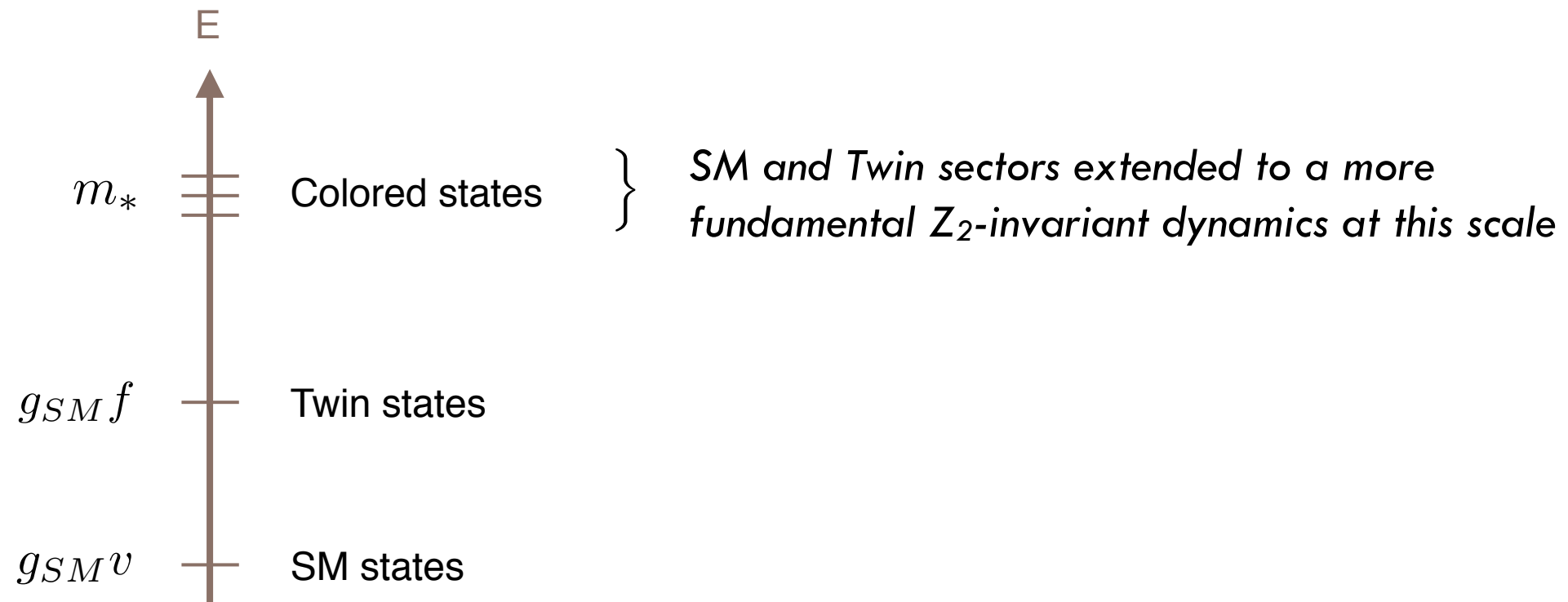
[ Chacko, Goh, Harnik, PRL 96 (2006) 231802 ]



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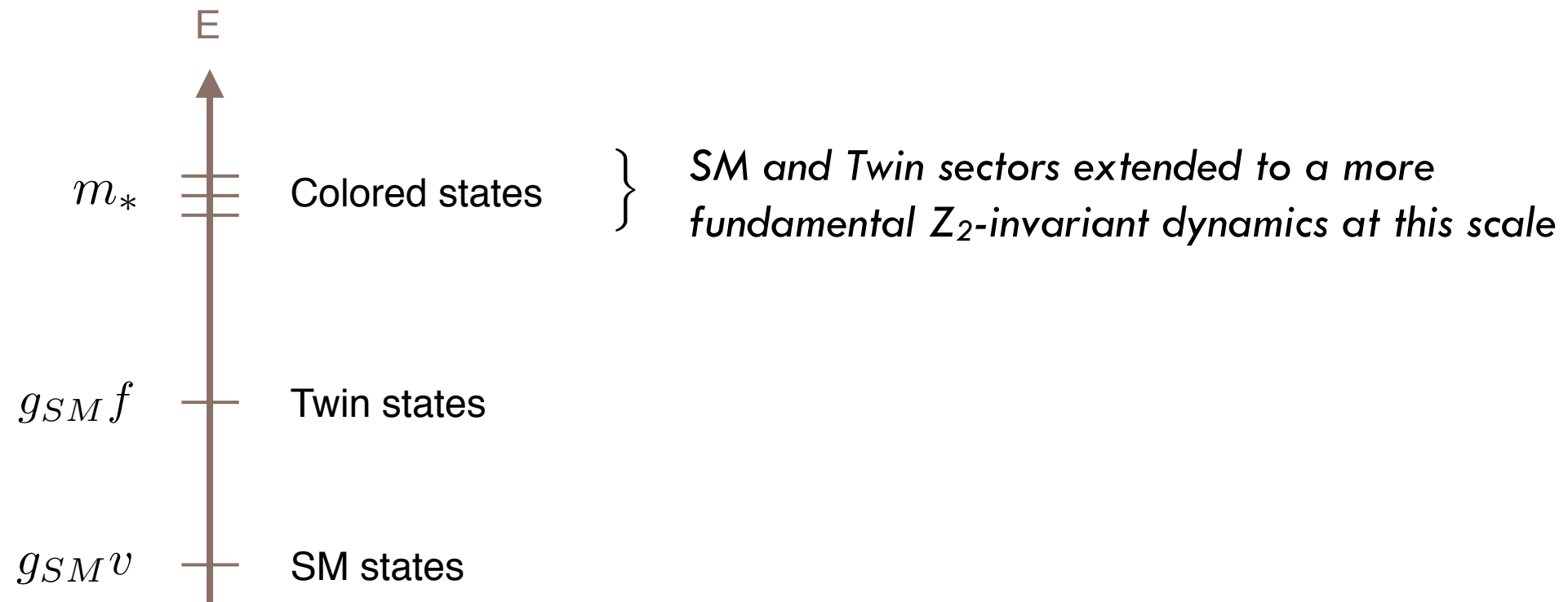
Most general  $Z_2$ -invariant potential:

- i) has  $SO(4) \times \widetilde{SO}(4)$  accidental invariance
- ii) mass term has larger  $SO(8)$  invariance

$$V(H, \tilde{H}) = -m_\phi^2(|H|^2 + |\tilde{H}|^2) + \frac{\lambda_\phi}{2}(|H|^2 + |\tilde{H}|^2)^2 + \frac{\hat{\lambda}_h}{4}(|H|^4 + |\tilde{H}|^4) + \dots$$



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👉 Consider scenarios where  $SO(8)$ -breaking terms are small,  
and let's analyze first the  $SO(8)$ -invariant limit

$$V(\phi) = -m_\phi^2 |\phi|^2 + \frac{\lambda_\phi}{2} |\phi|^4$$

$$\phi = \begin{pmatrix} H \\ \tilde{H} \end{pmatrix} \quad \langle \phi \rangle = f = \frac{m_\phi}{\sqrt{\lambda_\phi}}$$

$$\phi = e^{i\pi/f} \begin{pmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{pmatrix}$$

In the unitary gauge:

$$SO(8) \rightarrow SO(7) \supset SU(2)_L \times U(1)_Y$$

**7 Nambu-Goldstone bosons**

- 3 NGB eaten to give mass to  $\widetilde{W}$
- Twin photon remains massless
- one massless  $SU(2)$  doublet  $\mathcal{H}$

$$H^\dagger H = f^2 \sin^2(h/f)$$

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$$(h^2 \equiv \mathcal{H}^\dagger \mathcal{H})$$

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Cancellation in the mass term (due to accidental  $SO(8)$  from  $Z_2$  invariance)

[ Chacko, Goh, Harnik ]

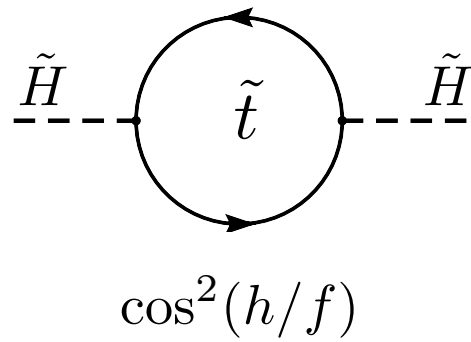
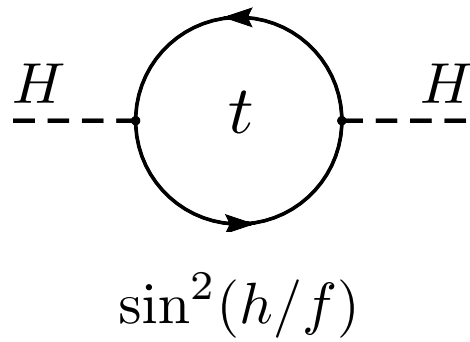
The figure shows two Feynman diagrams for the decay of a Higgs boson into two photons. The left diagram represents the decay via a  $W$  loop, with the amplitude labeled  $\sin^2(h/f)$ . The right diagram represents the decay via a  $\widetilde{W}$  loop, with the amplitude labeled  $\cos^2(h/f)$ . Both diagrams show a Higgs boson (solid line) decaying into two photons (wavy lines) through a loop of  $W$  or  $\widetilde{W}$  bosons. The external lines are labeled  $H$  and  $\widetilde{H}$ .

$$\delta V \sim \frac{m_*^2}{16\pi^2} \left( g^2 |H|^2 + \tilde{g}^2 |\tilde{H}|^2 \right) = \frac{m_*^2 f^2 g^2}{16\pi^2} \left( \sin^2 \frac{h}{f} + \cos^2 \frac{h}{f} \right) = const.$$

A similar cancellation occurs in the correction to the mass term from fermions:

$$\mathcal{L} \supset y_t \bar{q}_L H t_R + \tilde{y}_t \bar{\tilde{q}}_L \tilde{H} \tilde{t}_R + h.c.$$

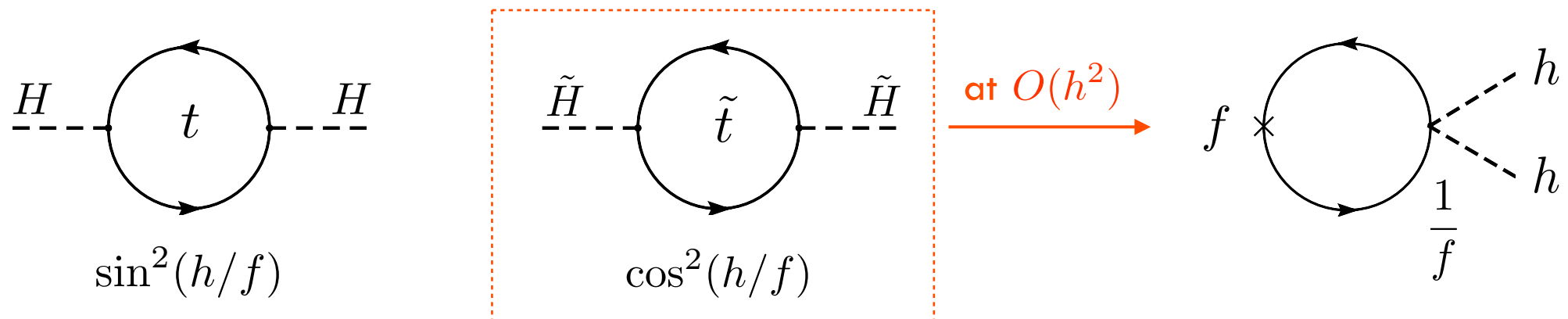
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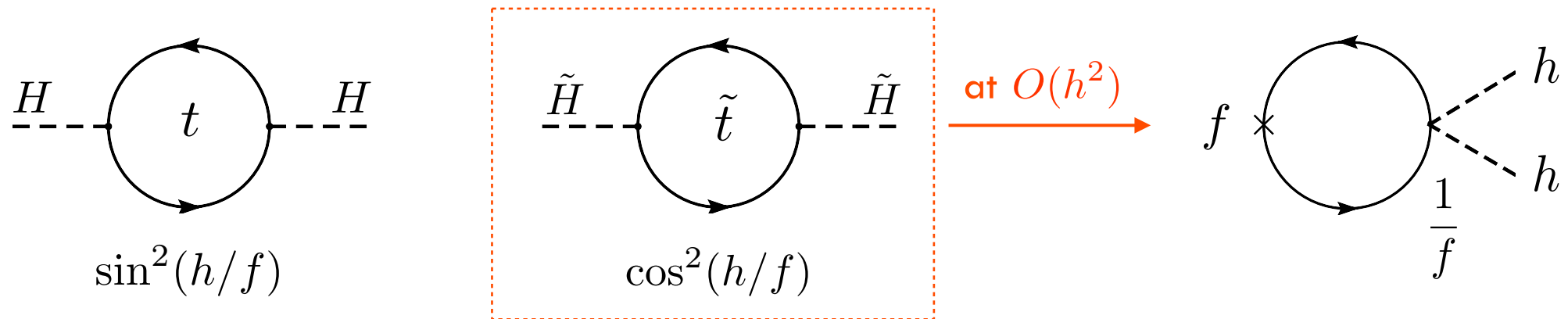
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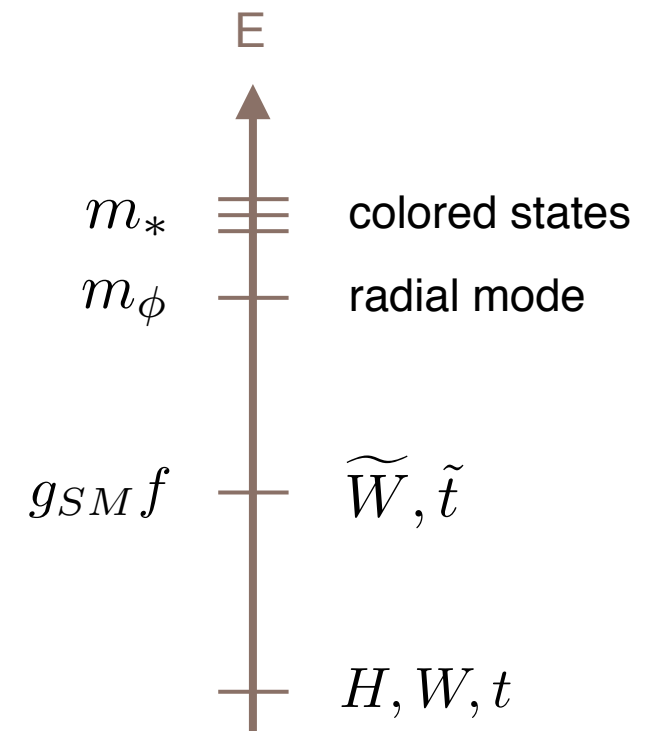


Mass of twin states:  $m_{\tilde{W}}^2 = \frac{1}{4} g^2 f^2$

$$m_{\tilde{t}} = \frac{y_t}{\sqrt{2}} f$$

Mass of radial mode:  $m_\phi^2 = f^2 \lambda_\phi \lesssim m_*^2$

Massless twin photon can be removed by  
not gauging  $\tilde{U}(1)$  (small  $Z_2$  breaking)



- Effect of the SO(8)-breaking terms

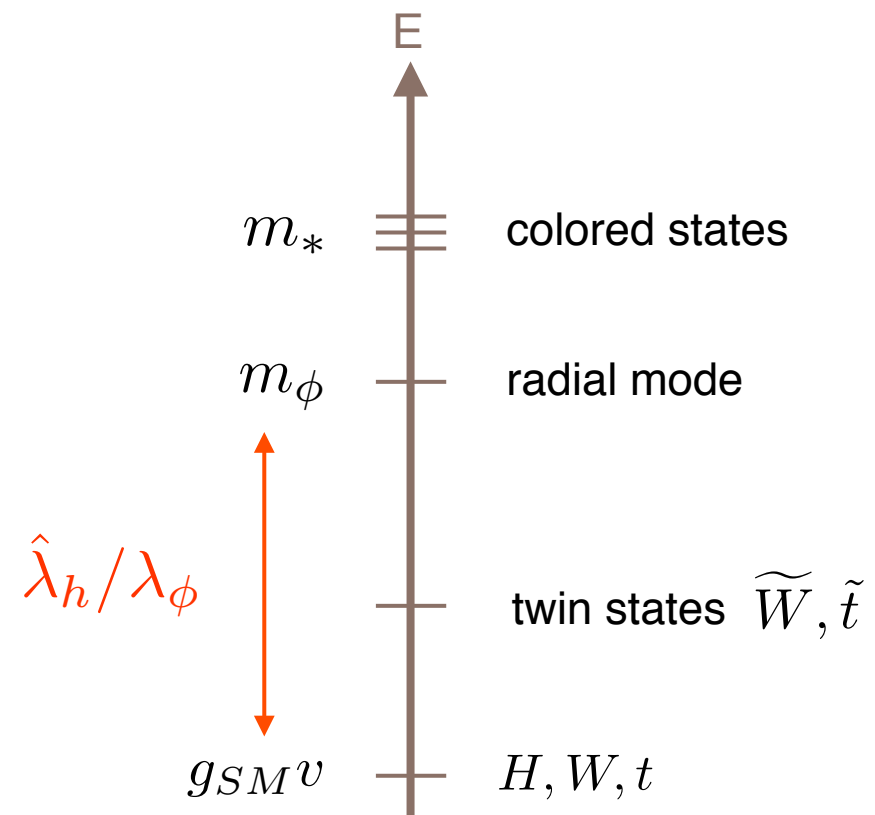
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$\tilde{H}^2 = f^2 - H^2$  at LO in  $\hat{\lambda}_h/\lambda_\phi$

A non-vanishing SO(8)-breaking quartic gives the NGB a potential:

$$\lambda_h \simeq \hat{\lambda}_h$$

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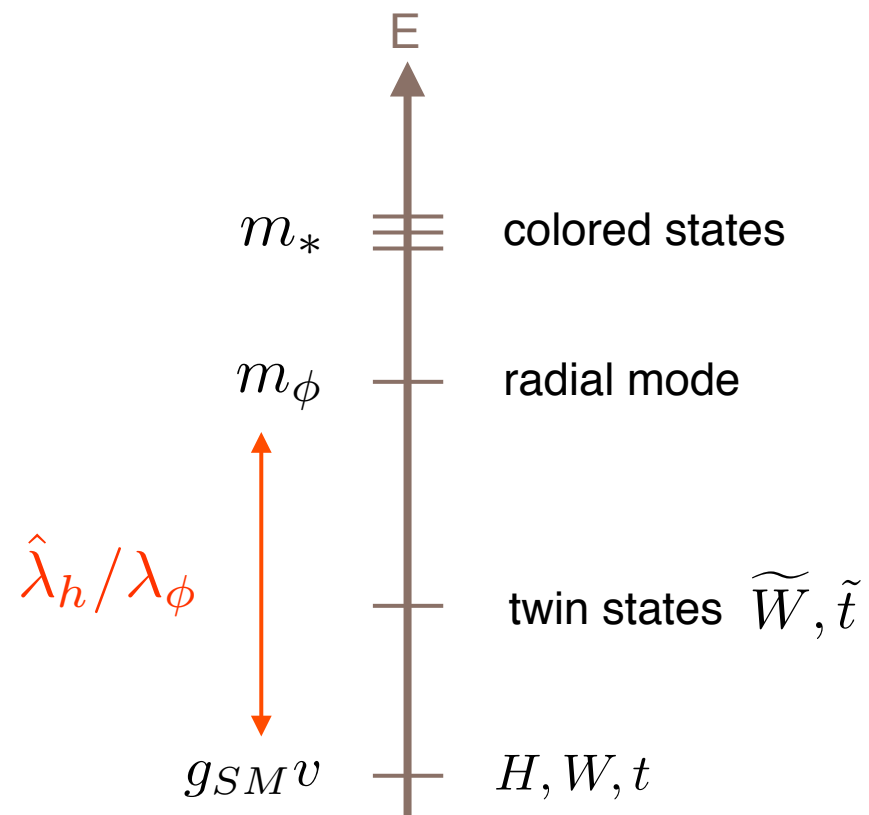
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Need to relate  $m_\phi$  to  $m_*$





## Theories with $m_\phi \sim m_*$ (Sub-Hypersoft)

Consider the case in which  $m_\phi \sim m_*$

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Then:

$$m_*^2 \sim \left( \frac{2g_*^2}{\lambda_h} \right) \frac{m_h^2}{\epsilon} \longrightarrow \lambda_h \sim \frac{3y_t^4}{4\pi^2} \log \frac{m_*^2}{m_t m_{\tilde{t}}} \Rightarrow m_*^2 \sim \frac{4\pi^2}{3y_t^2} \times \frac{m_h^2}{\epsilon} \times \frac{g_*^2}{y_t^2} \frac{1}{\log \frac{m_*^2}{m_t m_{\tilde{t}}}}$$

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👉 To gain in FT,  $SO(8)$ -breaking terms must **not** be generated at  $O(g_{SM}^2)$

This can be ensured through symmetries and selection rules of the UV dynamics [ Barbieri, Greco, Rattazzi, Wulzer, JHEP 1508 (2015) 161 ]

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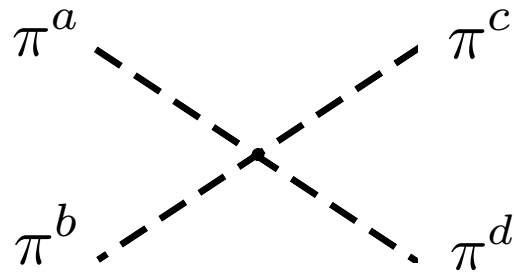
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👉 Q: *How large  $g_*$  can be ?*

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$$\mathcal{A}(\pi^a \pi^b \rightarrow \pi^c \pi^d) = \frac{s}{f^2} \delta^{ab} \delta^{cd} + \frac{t}{f^2} \delta^{ac} \delta^{bd} + \frac{u}{f^2} \delta^{ad} \delta^{bc}$$

$$7 \times 7 = 1 + 21 + 27$$

Decomposing into partial wave amplitudes:

$$a_{j=0}^1 = \frac{N-2}{32\pi} \frac{s}{f^2} \quad \text{for } SO(N)/SO(N-1)$$

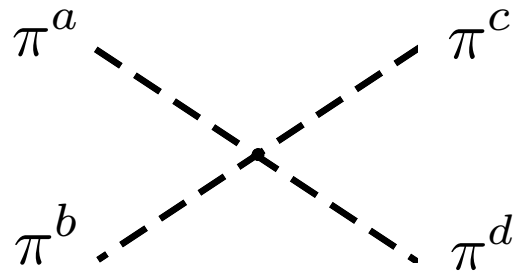
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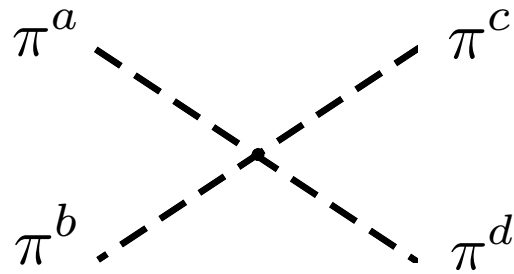
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*large size of multiplets lowers strong scale compared to naive expectation*

$$m_* \sim 0.45 \text{ TeV} \times \frac{g_*}{y_t} \times \frac{1}{\sqrt{\log \frac{m_*^2}{m_t m_{\tilde{t}}}}} \times \sqrt{\frac{1}{\epsilon}} \quad \xrightarrow{g_* \lesssim 5}$$

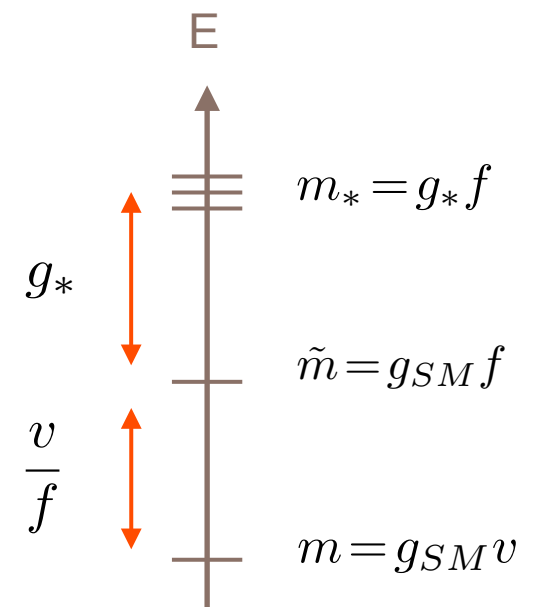
$$m_* \lesssim (3-4) \text{ TeV} \times \sqrt{\frac{0.1}{\epsilon}}$$

*... just beyond the LHC reach*

# EW and Higgs precision physics

Ratio of colored/twins obtained through a large  $g_*$  at fixed  $f$

-----> effects scaling with  $f$  do *not* decouple



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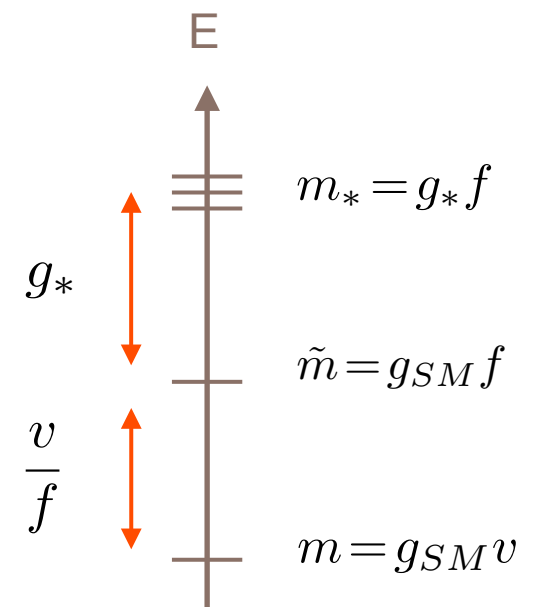
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$$\frac{\delta c}{c} \sim \frac{v^2}{f^2}$$



$$\xi \equiv \frac{v^2}{f^2} \lesssim 0.1 - 0.2$$

same constraint as for CH models



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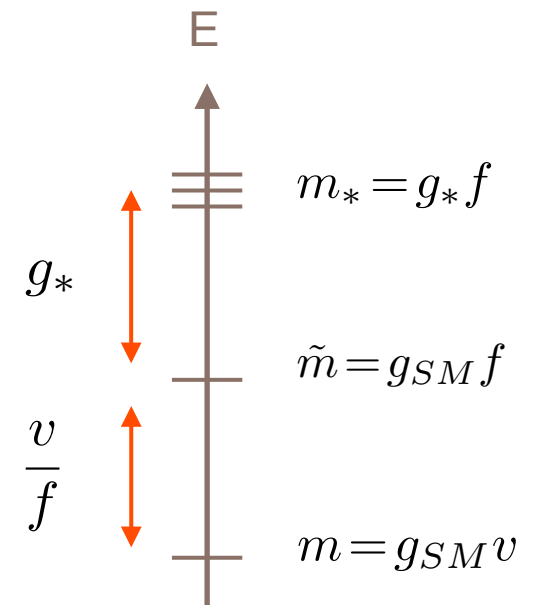
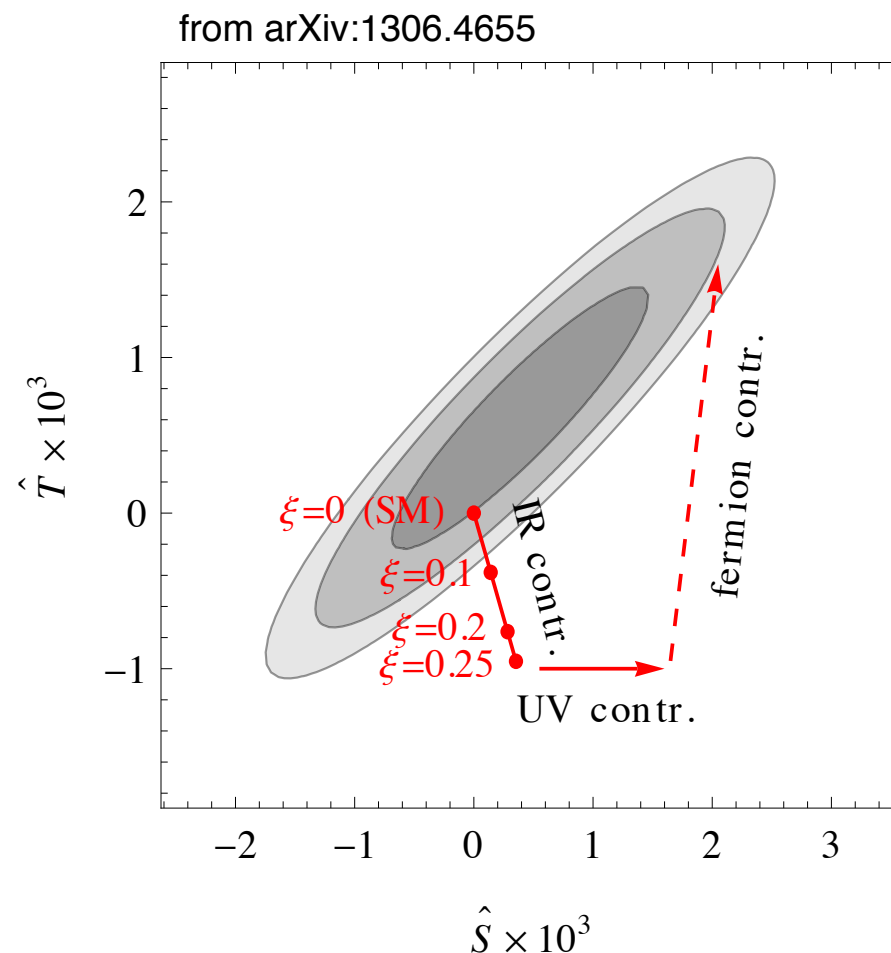
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IR contribution from Higgs compositeness is a non-decoupling one



Fermion contribution

$$\Delta \hat{T}_\Psi \sim \frac{3y_t^2}{16\pi^2} \frac{y_t^2 v^2}{m_*^2}$$

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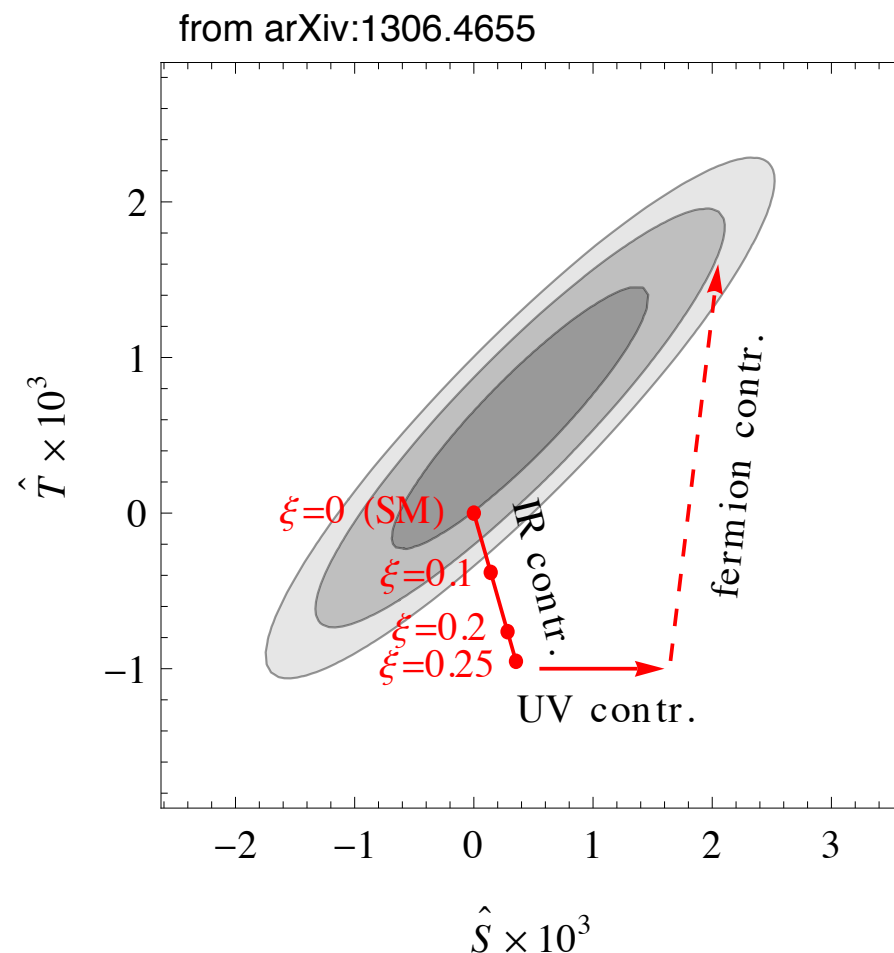
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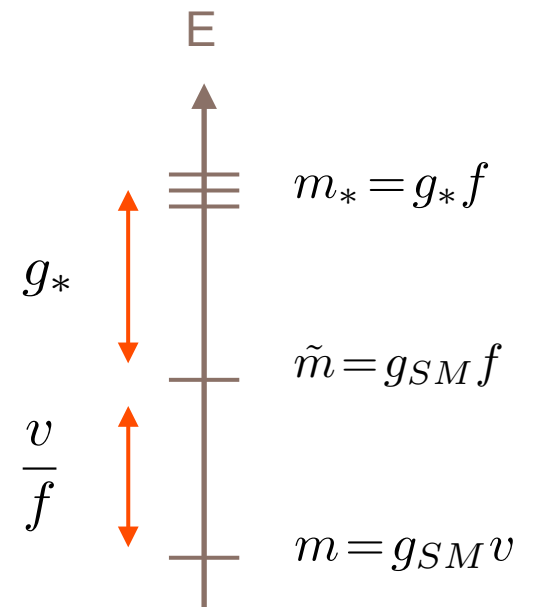


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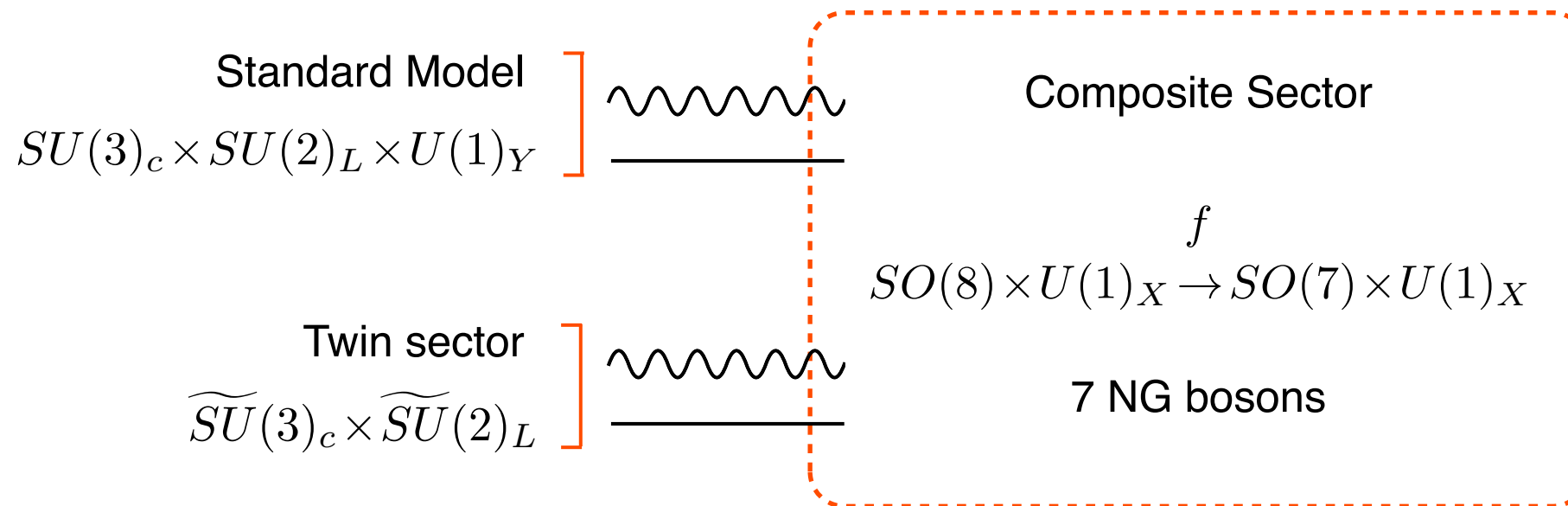
is a decoupling one

Can EWPT be satisfied in Composite TH theories ?



# Phenomenology of an SO(8) Twin Higgs model

[ R.C., D. Greco, R. Mahbubani, R. Rattazzi and R. Torre arXiv:1702.00797]



$$\mathcal{L}_{mix} = g W_\mu J^\mu + g' B_\mu J_B^\mu + \tilde{g} \widetilde{W}_\mu J^\mu + y_L \bar{q}_L \mathcal{O}_q + y_R \bar{t}_R \mathcal{O}_t + \tilde{y}_L \bar{\tilde{q}}_L \tilde{\mathcal{O}}_q + \tilde{y}_R \bar{\tilde{t}}_R \tilde{\mathcal{O}}_t + h.c.$$

$$J_B^\mu = J_{3R}^\mu + J_X^\mu$$

Partial compositeness:

$$J^\mu, \longrightarrow \rho^\mu = \mathbf{28} \text{ of } SO(8)$$

$$J_X^\mu \longrightarrow \rho_X^\mu = \mathbf{1} \text{ of } SO(8)$$

$$\mathcal{O}_{q,t}, \tilde{\mathcal{O}}_{q,t} \longrightarrow \Psi, \tilde{\Psi} = \mathbf{8} \text{ of } SO(8)$$

# Higgs potential at NLO

- Higgs potential generated at the scale  $m_*$  by 1-loop threshold corrections

$$\delta V_B = \frac{3g_\rho^2 g'^2}{512\pi^2} f^4 \sin^2(h/f) \quad (\text{from } Z_2 \text{ breaking})$$

$$\delta V_\Psi = \frac{N_c f^4}{128\pi^2} \left( y_L^4 F_1 + \tilde{y}_t^4 \tilde{F}_1 \right) (\sin^4(h/f) + \cos^4(h/f)) \quad (F_1, \tilde{F}_1 \text{ are } \mathcal{O}(1) \text{ functions})$$



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By making the field redefinition  $H \rightarrow H' = f \frac{H}{\sqrt{H^\dagger H}} \sin(\sqrt{H^\dagger H}/f)$

one gets the effective Lagrangian ( $\tilde{y}_0 = \tilde{y}_2 = \tilde{y}_4 = y_1, \tilde{c}_2 = \tilde{c}_4 = 0$ ):

$$\mathcal{L}_H = |D_\mu H|^2 + \frac{1}{2f^2} [\partial_\mu (H^\dagger H)]^2 + \mu^2 H^\dagger H - \lambda_h (H^\dagger H)^2$$

$$\mathcal{L}_t = -y_1 \bar{q}_L H^c t_R + h.c.$$

$$\mathcal{L}_{\tilde{t}} = -\frac{f}{\sqrt{2}} \left( \tilde{y}_0 - \frac{\tilde{y}_2}{2} \frac{H^\dagger H}{f^2} - \frac{\tilde{y}_4}{8} \frac{(H^\dagger H)^2}{f^4} + \dots \right) \bar{\tilde{t}} \tilde{t} + \bar{\tilde{t}} i \not{\partial} \tilde{t} \left( \tilde{c}_2 \frac{H^\dagger H}{f^2} + \frac{\tilde{c}_4}{6} \frac{(H^\dagger H)^2}{f^4} \right)$$

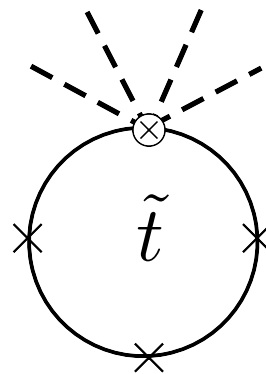
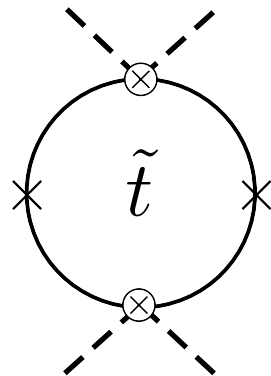
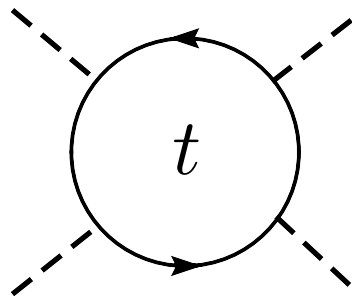
RG evolution from  $m_*$  down to  $\mu \sim m_h, m_t$  encodes the bulk of radiative corrections:

$$\frac{m_t}{G_F^{-1/2}} = \frac{1}{\sqrt{2}} y_1(\mu) \qquad \frac{m_h^2}{G_F^{-1}} = 8\lambda_h(\mu)(1 - \xi) \qquad \xi = \frac{v^2}{f^2}$$

$$\beta_{y_1} = \frac{1}{16\pi^2} \left( \frac{9}{4} y_1^3 - 4g_S^2 y_1 \right)$$

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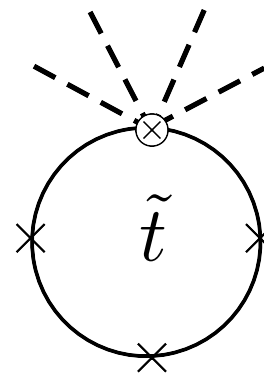
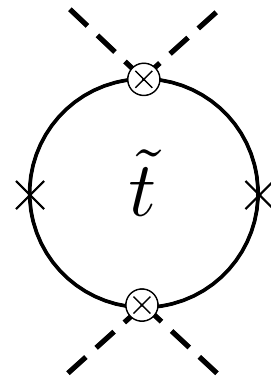
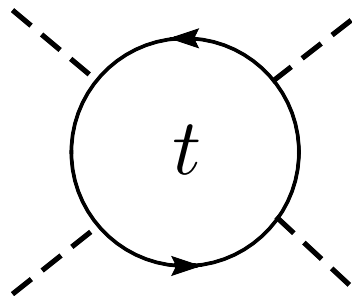
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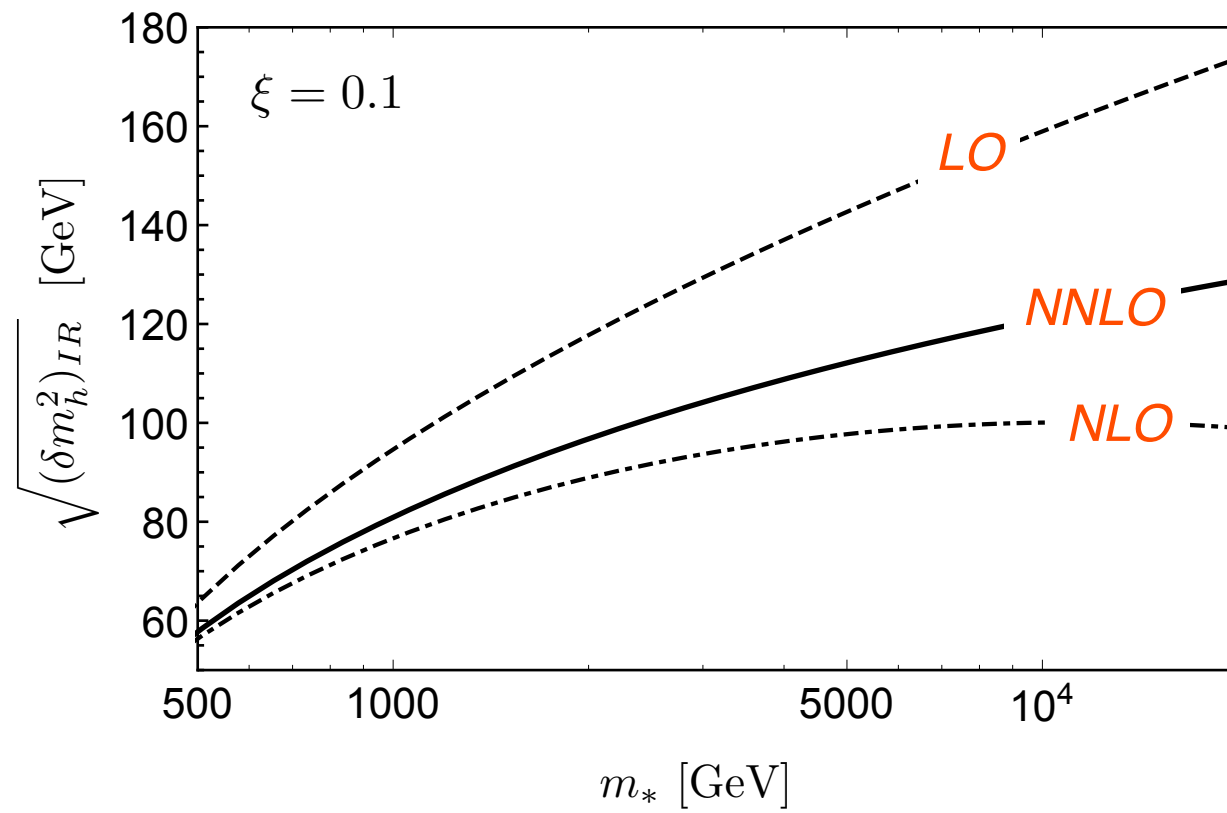
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Twin top operators  
up to D=7 contribute  
and must be included

RG equations are solved at Next-to-Leading order in a combined perturbative expansion in  $(\alpha \log)$  and  $\xi$



Ex: for  $m_* = 5 \text{ TeV}$  and  $\xi = 0.1$

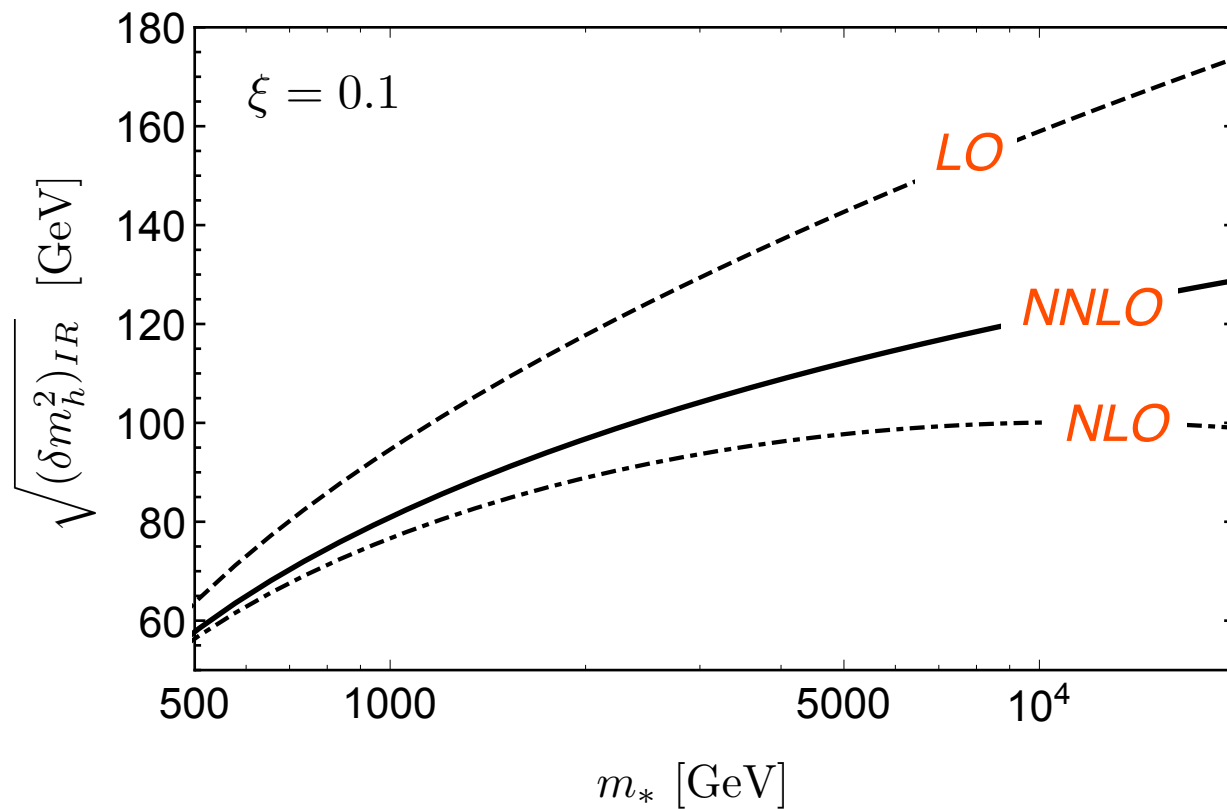
NLO:  $-32\%$

NNLO:  $+15\%$

NNLO curve taken from

Greco and Mimouni, arXiv:1609.05922

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IR contribution almost accounts  
for the whole Higgs mass,  
UV threshold are sub-dominant

Ex: for  $m_* = 5 \text{ TeV}$  and  $\xi = 0.1$

IR = 74% ( 47% SM + 27% twin tops)

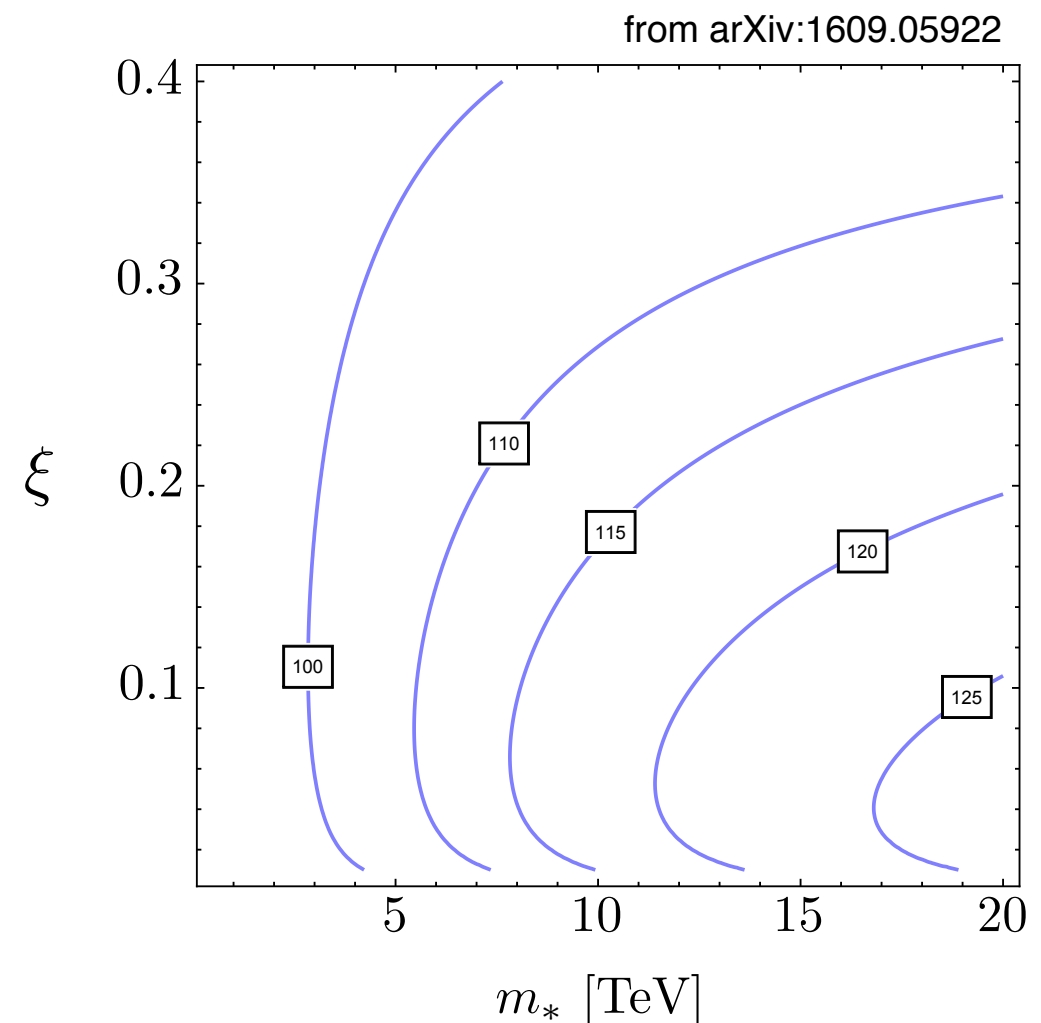
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# EW and Higgs precision physics

- 1-loop contributions to EWPO from Twin states are subleading
- Corrections parametrically the same as in CH models (with singlet  $t_R$ )

$$\Delta\hat{S} = \frac{g^2}{2g_\rho^2}\xi + \frac{g^2}{192\pi^2}\xi \log \frac{m_*^2}{m_h^2}$$

$$\Delta\hat{T} = a_{UV} \frac{y_L^2}{16\pi^2} N_c \frac{y_L^2 v^2}{M_\Psi^2} + a_{IR} \frac{y_t^2}{16\pi^2} N_c \frac{y_L^2 v^2}{M_\Psi^2} \log \frac{M_\Psi^2}{m_t^2} - \frac{3g_1^2}{64\pi^2} \xi \log \frac{m_*^2}{m_h^2}$$

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$a_{UV}, a_{IR}, b_{UV}, b_{IR}$   
coefficients of  $O(1)$

For recent analyses of EWPT in CH models see:

C. Grojean, O. Matsedonskyi and G. Panico, JHEP 10 (2013) 160

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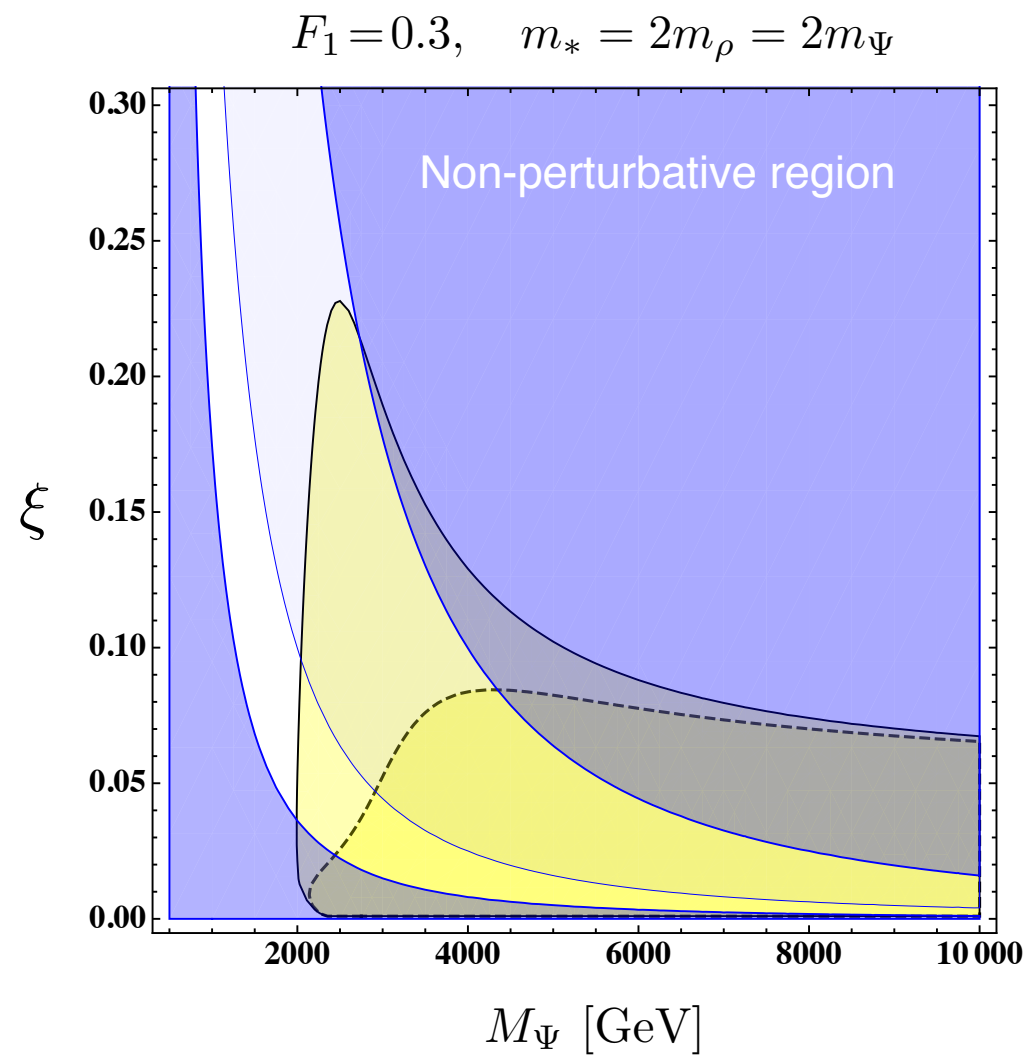
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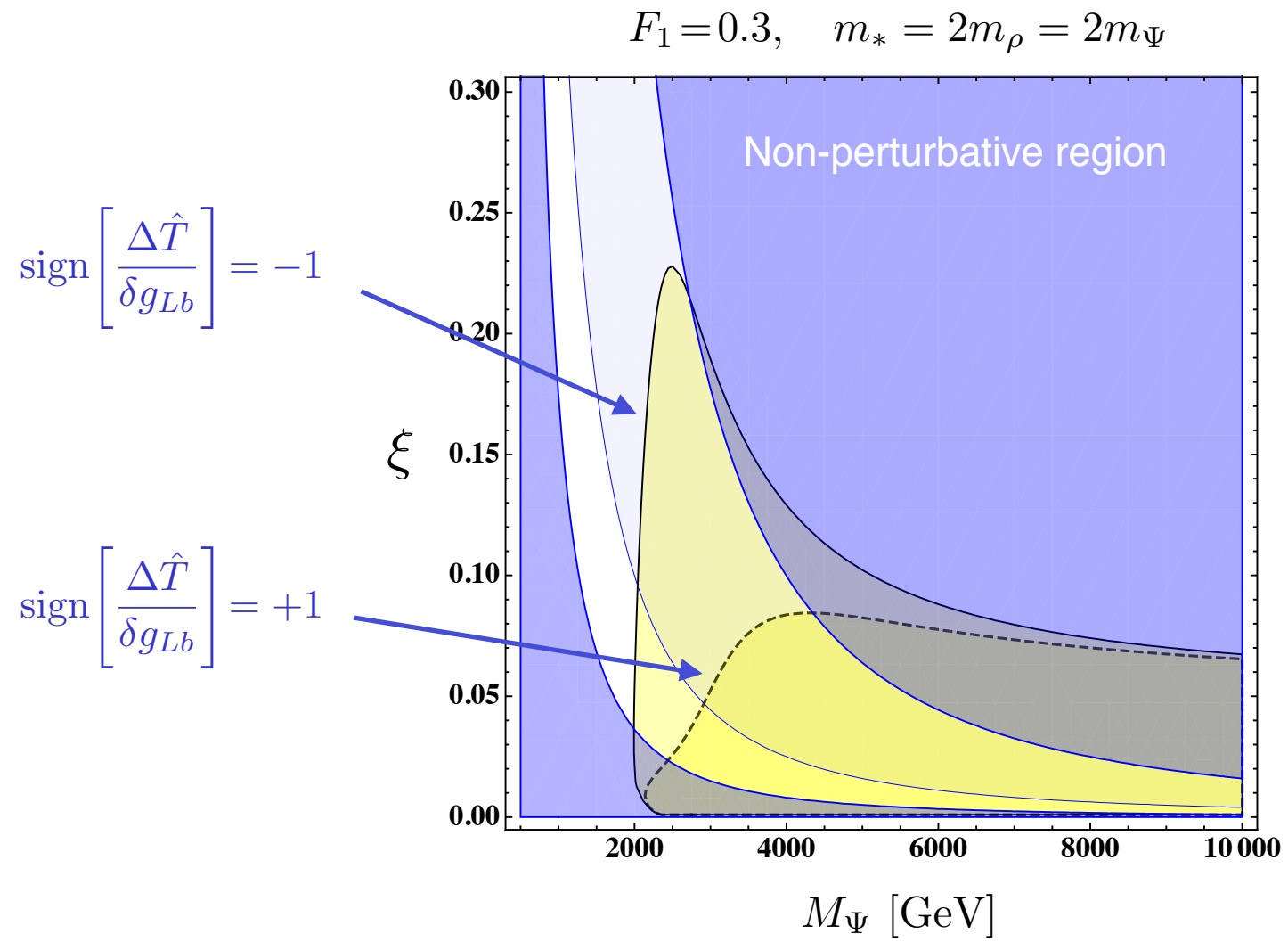
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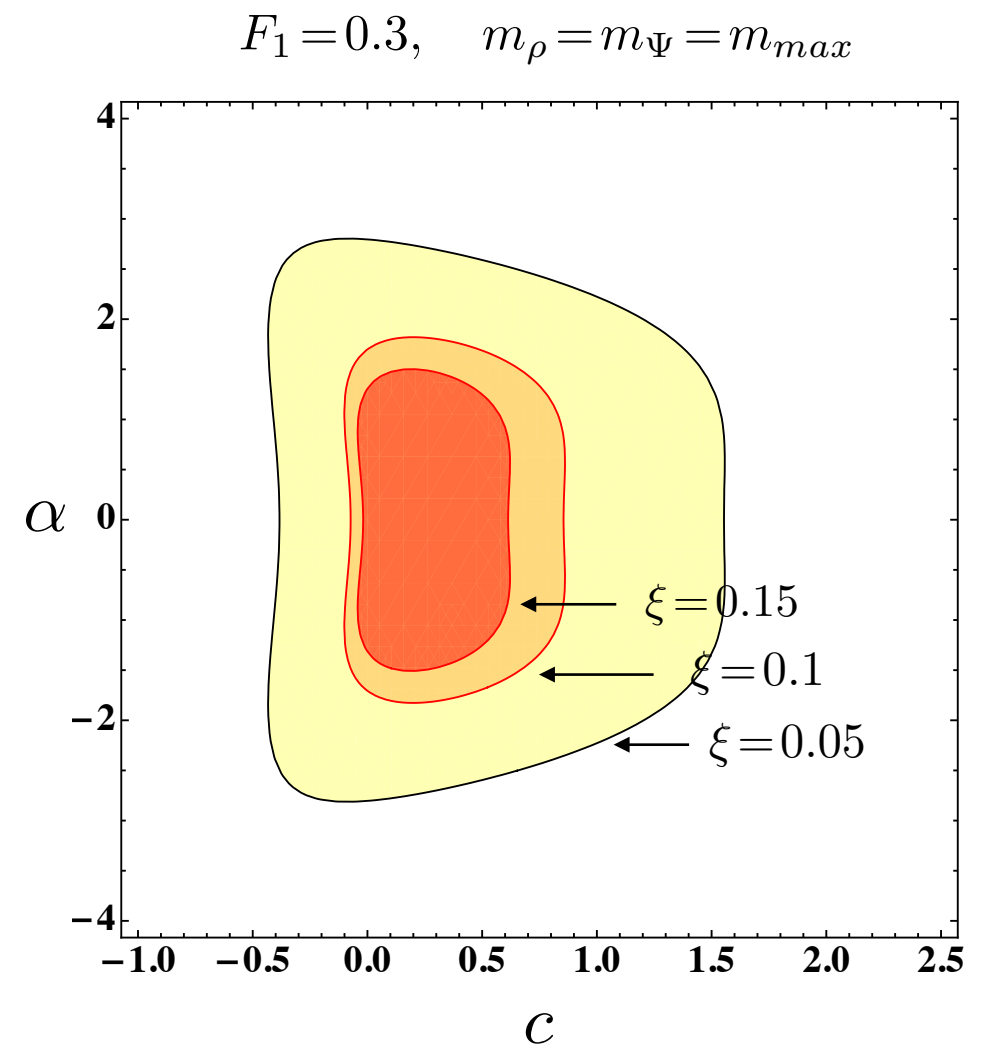
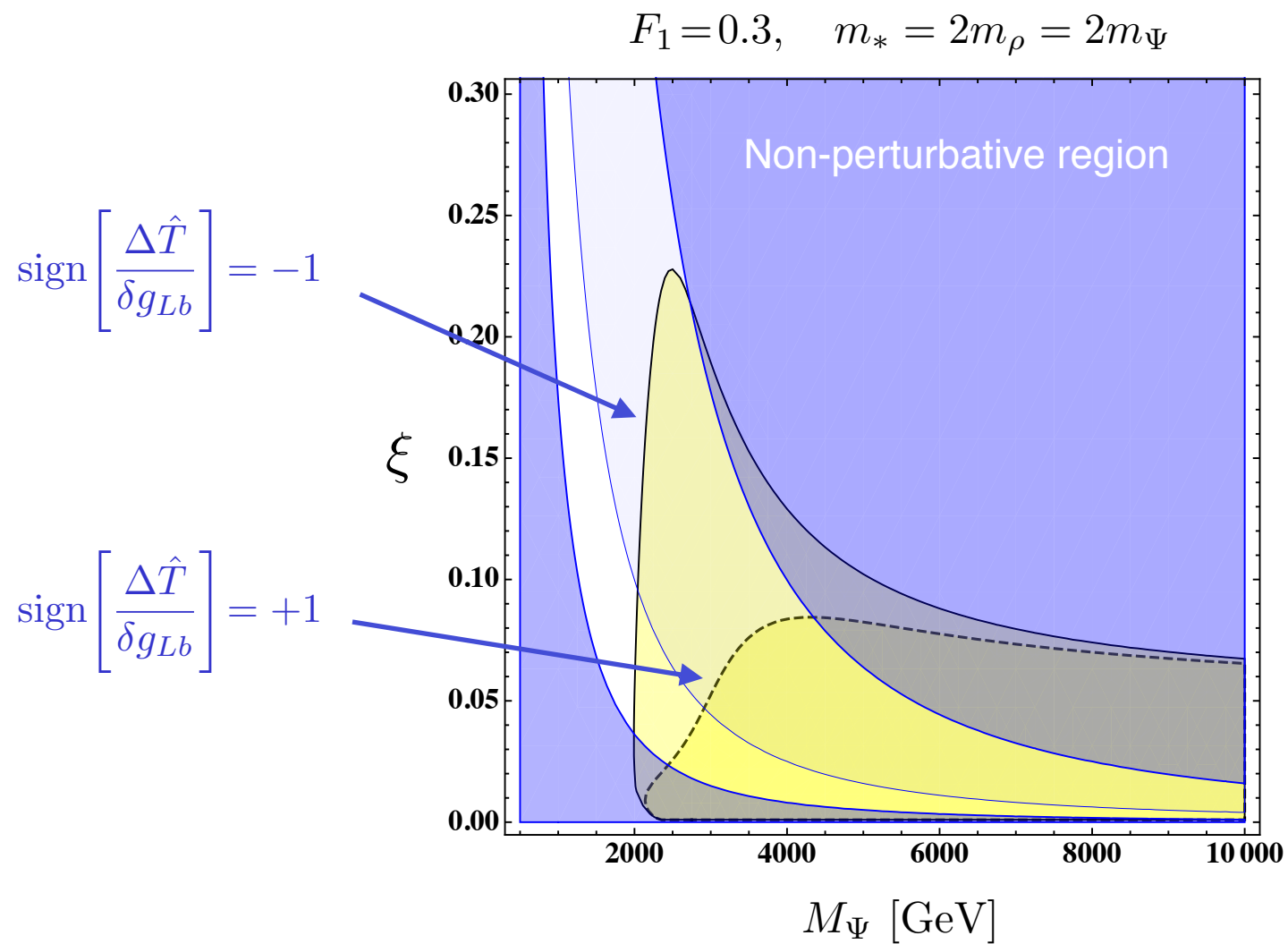
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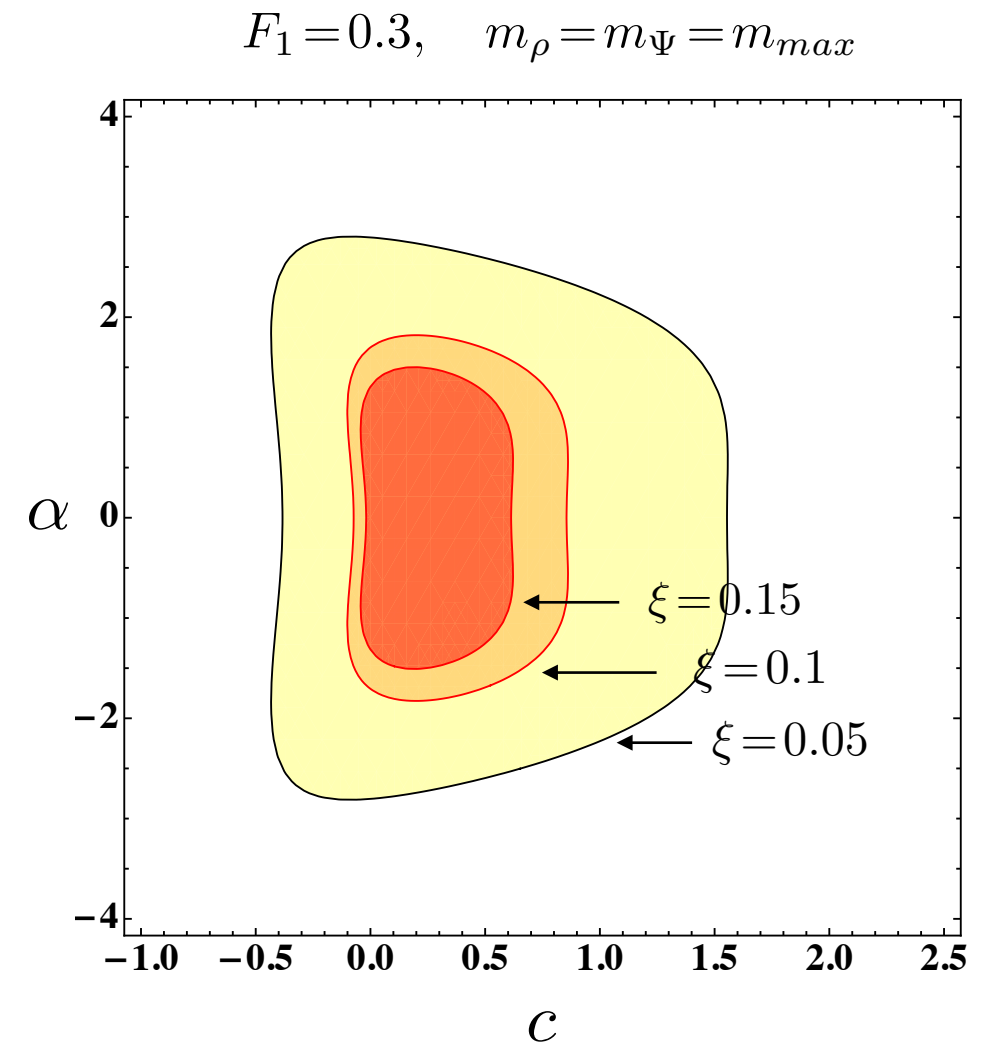
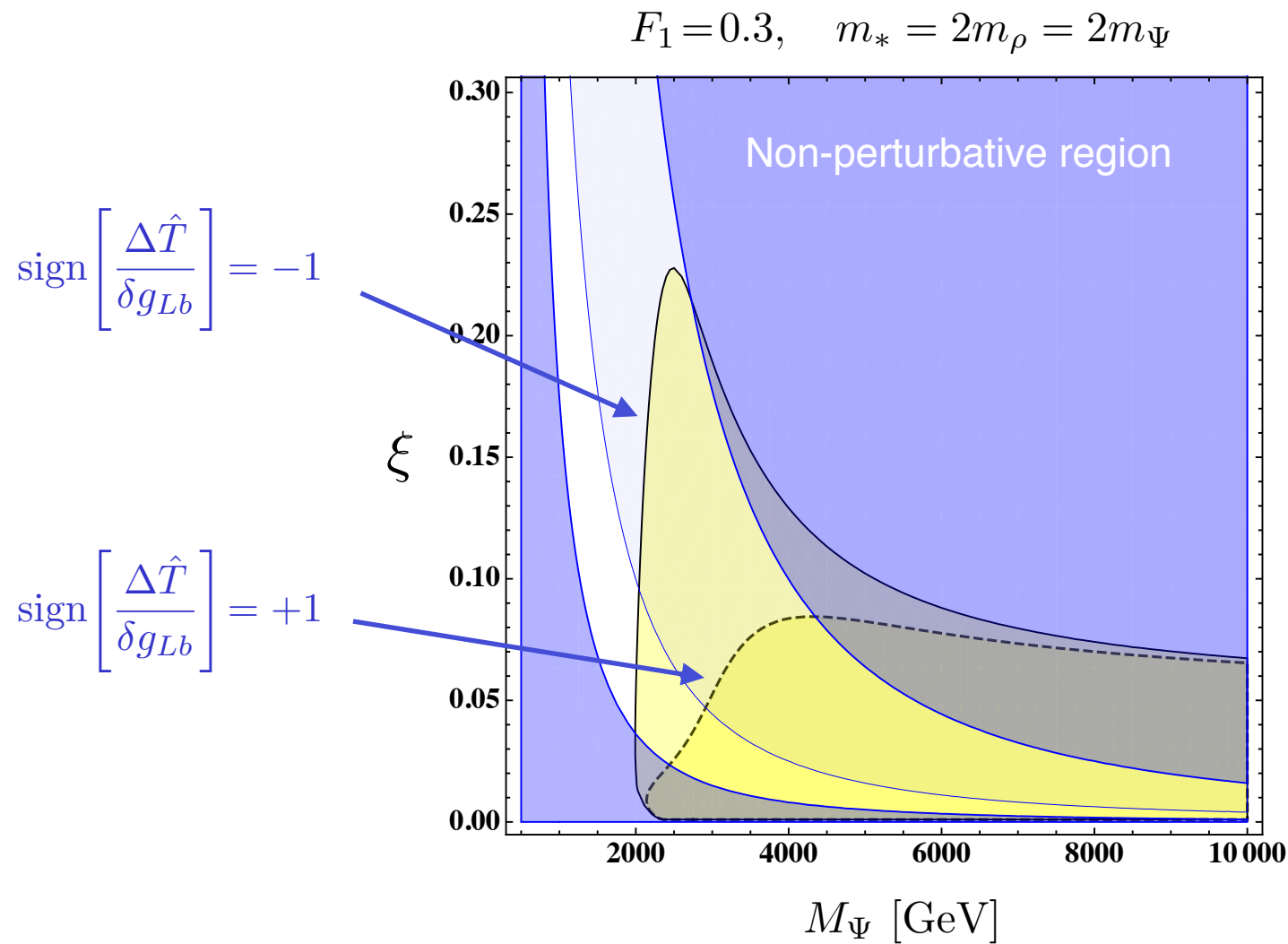
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$$\mathcal{L} \supset \alpha \bar{\Psi}(\rho_\mu - E_\mu)\gamma^\mu \Psi + c \bar{\Psi} d_\mu \gamma^\mu \Psi$$

$$\alpha, c = O(1)$$

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**Moral:** once the perturbative bound is satisfied, EWPT can be passed in a sizable portion of the parameter space

$$\mathcal{L} \supset \alpha \bar{\Psi}(\rho_\mu - E_\mu)\gamma^\mu \Psi + c \bar{\Psi} d_\mu \gamma^\mu \Psi$$

$$\alpha, c = O(1)$$

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Maximal FT gain for **strongly coupled** UV dynamics
- $Z_2$  parity alone *not* sufficient to guarantee gain in FT: one needs accidental  $SO(8)$  at  $O(g_{SM})^2$

Condition on symmetries/selection rules of UV dynamics is required

Ex:  $SO(8)/SO(7)$  works,  $SU(4)/SU(3)$  does not



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Ex:  $SO(8)/SO(7)$  works,  $SU(4)/SU(3)$  does not

- Perturbativity bound on  $g_*$  made stringent by large multiplicity of states required for realistic models. Naive estimates give:  $m_*/f \lesssim 3 - 5$

This bound to be compared with  $m_*/f \lesssim 1.5$  in CH models from Higgs mass

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- Naively, larger  $M_\Psi$  in tension with EWPT (because of too small  $\Delta\hat{T}_\Psi$ )

In practice,  $\xi \sim 0.2$  still allowed (though borderline) for  $M_\Psi \lesssim 4 \text{ TeV}$

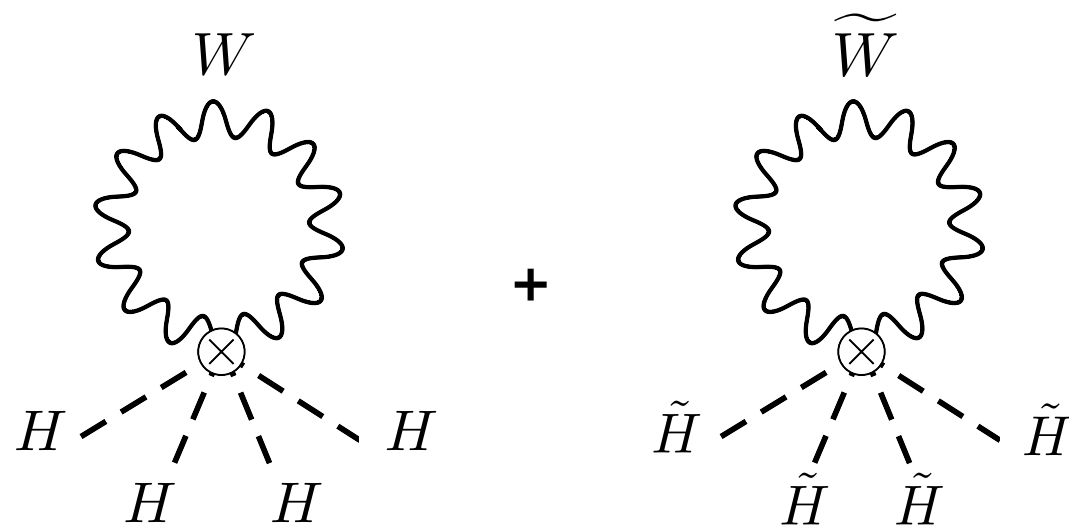


Extra slides

# On the size of SO(8)-breaking quartic term

In general, interactions of the type  $\left(H^\dagger \overleftrightarrow{D}_\mu H\right)^2 + \left(\tilde{H}^\dagger \overleftrightarrow{D}_\mu \tilde{H}\right)^2$   
 $\left(H^\dagger \overleftrightarrow{D}_\mu H\right) \left(\tilde{H}^\dagger \overleftrightarrow{D}_\mu \tilde{H}\right)$

are not SO(8) invariant and can generate a quartic at  $O(g^2)$ :



The diagram shows two Feynman diagrams separated by a plus sign. The left diagram features a loop of  $W$  bosons (represented by a wavy line) with four external Higgs lines ( $H$ , represented by dashed lines) meeting at a vertex marked with a cross in a circle. The right diagram is identical but with  $\tilde{W}$  bosons and  $\tilde{H}$  Higgs fields. To the right of these diagrams is an expression for the resulting quartic term:

$$\sim \frac{m_*^2}{16\pi^2} \times \frac{g^2 g_*^2}{m_*^2} \left( |H|^4 + |\tilde{H}|^4 \right)$$

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$$+ \quad \sim \frac{m_*^2}{16\pi^2} \times \frac{g^2 g_*^2}{m_*^2} \left( |H|^4 + |\tilde{H}|^4 \right)$$



Symmetries and selection rules of the UV dynamics  
 can forbid the SO(8)-breaking terms at  $O(g_{SM}^2)$

- Whether or not  $\text{SO}(8)$ -breaking terms are generated at  $O(g_{SM}^2)$  can be determined solely based on *symmetries* and *spurion* quantum numbers

[ Barbieri, Greco, Rattazzi, Wulzer, JHEP 1508 (2015) 161 ]



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[ Barbieri, Greco, Rattazzi, Wulzer, JHEP 1508 (2015) 161 ]

For example, consider the case:

- $SO(8)$ -invariant UV dynamics
- coset  $SO(8)/SO(7)$
- gauge contribution to the potential

spurion transforms as  $28 = 21 + 7$  of  $SO(7)$

$$\mathcal{G}^a = U^\dagger(\pi) g T^a U(\pi)$$

$$\begin{array}{l} 21 \times 21 \supset 1 \\ 7 \times 7 \supset 1 \end{array} \quad \left. \vphantom{\begin{array}{l} 21 \times 21 \supset 1 \\ 7 \times 7 \supset 1 \end{array}} \right\} \rightarrow 1 \text{ non-trivial invariant}$$

$$\left( \text{Tr}[T_{(7)}^{\hat{a}} \mathcal{G}^a] \right)^2 + \left( \text{Tr}[T_{(7)}^{\hat{a}} \tilde{\mathcal{G}}^a] \right)^2 = g^2 \sin^2(h/f) + \tilde{g}^2 \cos^2(h/f) \quad \checkmark$$

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[ Barbieri, Greco, Rattazzi, Wulzer, JHEP 1508 (2015) 161 ]

For example, consider the case:

- $SU(4)$ -invariant UV dynamics
- coset  $SU(4)/SU(3)$
- gauge contribution to the potential

spurion transforms as  $\mathbf{15} = \mathbf{8} + (\mathbf{3} + \bar{\mathbf{3}}) + \mathbf{1}$  of  $SU(3)$

$$\mathcal{G}^a = U^\dagger(\pi) g T^a U(\pi)$$

$$\begin{array}{l} \mathbf{8} \times \mathbf{8} \supset \mathbf{1} \\ (\mathbf{3} + \bar{\mathbf{3}}) \times (\mathbf{3} + \bar{\mathbf{3}}) \supset \mathbf{1} \\ \mathbf{1} \times \mathbf{1} = \mathbf{1} \end{array} \quad \left. \vphantom{\begin{array}{l} \mathbf{8} \times \mathbf{8} \supset \mathbf{1} \\ (\mathbf{3} + \bar{\mathbf{3}}) \times (\mathbf{3} + \bar{\mathbf{3}}) \supset \mathbf{1} \\ \mathbf{1} \times \mathbf{1} = \mathbf{1} \end{array}} \right\} \rightarrow 2 \text{ non-trivial invariants}$$

$$\left( \text{Tr}[T_{(\mathbf{3}+\bar{\mathbf{3}})}^{\hat{a}} \mathcal{G}^a] \right)^2 + \left( \text{Tr}[T_{(\mathbf{3}+\bar{\mathbf{3}})}^{\hat{a}} \tilde{\mathcal{G}}^a] \right)^2 = g^2 (3 \sin^2(h/f) - \sin^4(h/f)) + \tilde{g}^2 (3 \cos^2(h/f) - \cos^4(h/f))$$

$$\left( \text{Tr}[T_{(\mathbf{1})}^{\hat{a}} \mathcal{G}^a] \right)^2 + \left( \text{Tr}[T_{(\mathbf{1})}^{\hat{a}} \tilde{\mathcal{G}}^a] \right)^2 = g^2 \sin^4(h/f) + \tilde{g}^2 \cos^4(h/f)$$

✗

# Hypersoft Theories

Consider the case in which  $m_\phi^2 \sim \text{loop} \times m_*^2$

Examples: 1) Theories where  $\phi$  itself is a pNGB

2) SUSY with soft masses  $m_*$  generated at a scale  $\sim m_*$  where  $\phi$  is massless

Then:

$$m_h^2 \sim \frac{\lambda_h}{2\lambda_\phi} \frac{1}{\epsilon} \left( \frac{3y_t^2}{4\pi^2} + \frac{5\lambda_\phi}{16\pi^2} \right) m_*^2$$

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for  $\lambda_\phi \gg y_t^2$

as naturally expected if

$SO(8)$ -preserving  $\gg SO(8)$ -breaking

# Super-Hypersoft Theories

Variant of the Hypersoft case where leading correction to  $m_\phi$  comes from the top quark:

Example: Approximate SUSY in the scalar sector below  $m_*$

$$m_h^2 \sim \frac{\lambda_h}{2\lambda_\phi} \frac{1}{\epsilon} \left( \frac{3y_t^2}{4\pi^2} + \frac{5\lambda_\phi}{16\pi^2} \right) m_*^2 \quad \Rightarrow \quad m_*^2 \sim \frac{4\pi^2}{3y_t^2} \times \frac{m_h^2}{\epsilon} \times \boxed{\frac{g_*^2}{\lambda_h}} \quad \text{gain in FT}$$

$$m_* \sim 1.4 \text{ TeV} \frac{g_*}{\sqrt{2}y_t} \sqrt{\frac{1}{\epsilon}}$$