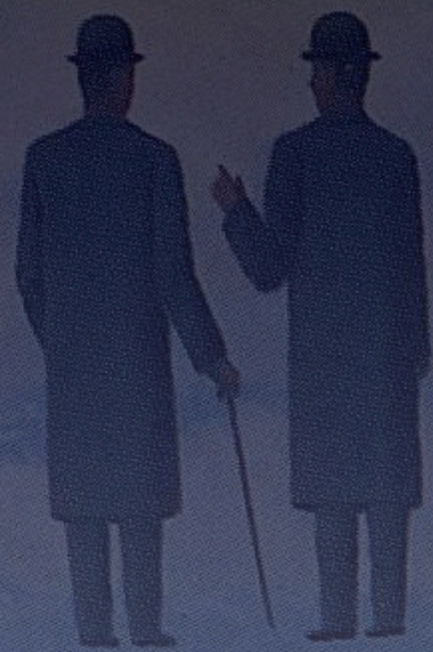
Axions and GR



Alfredo Urbano
CERN - TH Department

DaMESyFLa in the Higgs era
SISSA, Trieste 16 March 2017

The hydrogen atom

The hydrogen atom

$$i\hbar\frac{\partial}{\partial t}\Psi(t,r,\theta,\phi) = \left[-\frac{\hbar^2}{2\mu}\Delta + V(r)\right]\Psi(t,r,\theta,\phi)$$

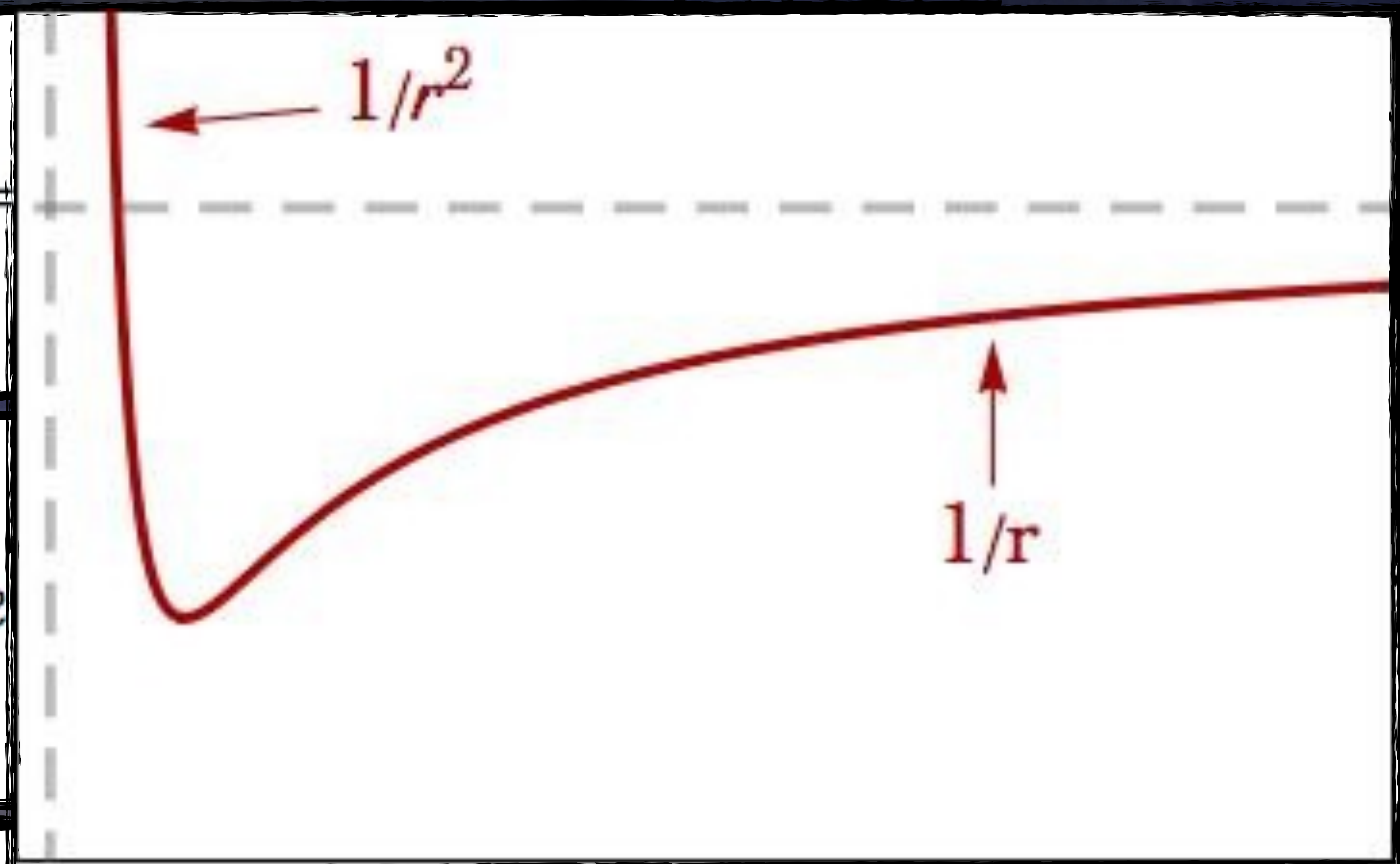
$$\Psi(t,r,\theta,\phi) = e^{-i\omega t}\frac{R(r)}{r}Y_l^m(\theta,\phi)$$

$$\left\{-\frac{\hbar^2}{2\mu}\frac{d^2}{dr^2} + \left[-\frac{e^2}{4\pi\epsilon_0 r} + \frac{\hbar^2 l(l+1)}{2\mu r^2}\right]\right\}R(r) = \omega R(r)$$

The hydrogen atom

$$i\hbar\frac{\partial}{\partial t}\Psi(t,r,\theta,\phi) =$$

$$\Psi(t,r,\theta,\phi) = e$$



$$\left\{-\frac{\hbar^2}{2\mu}\frac{d^2}{dr^2}+\left[-\frac{e^2}{4\pi\epsilon_0 r}+\frac{\hbar^2 l(l+1)}{2\mu r^2}\right]\right\}R(r) = \omega R(r)$$

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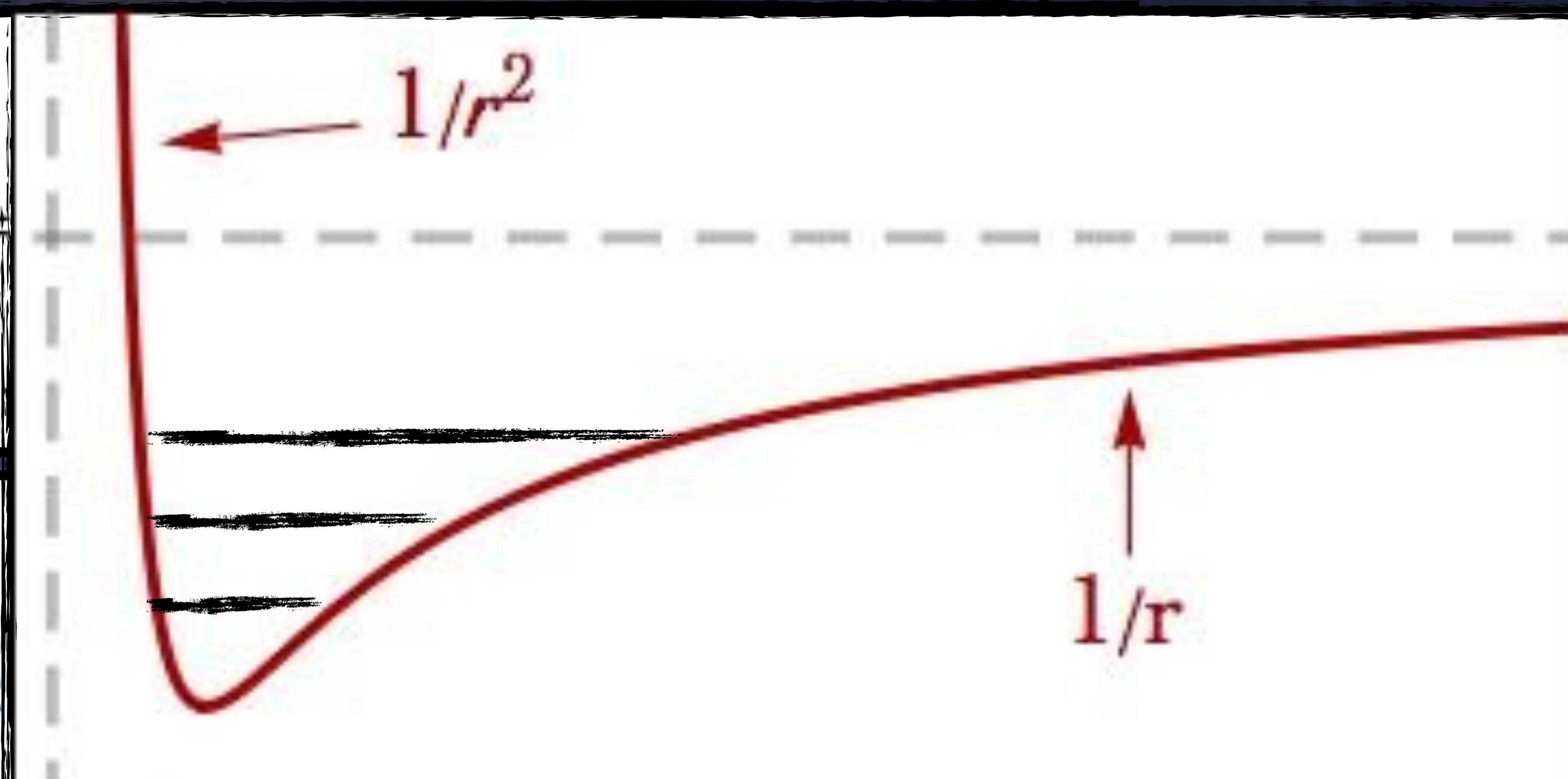


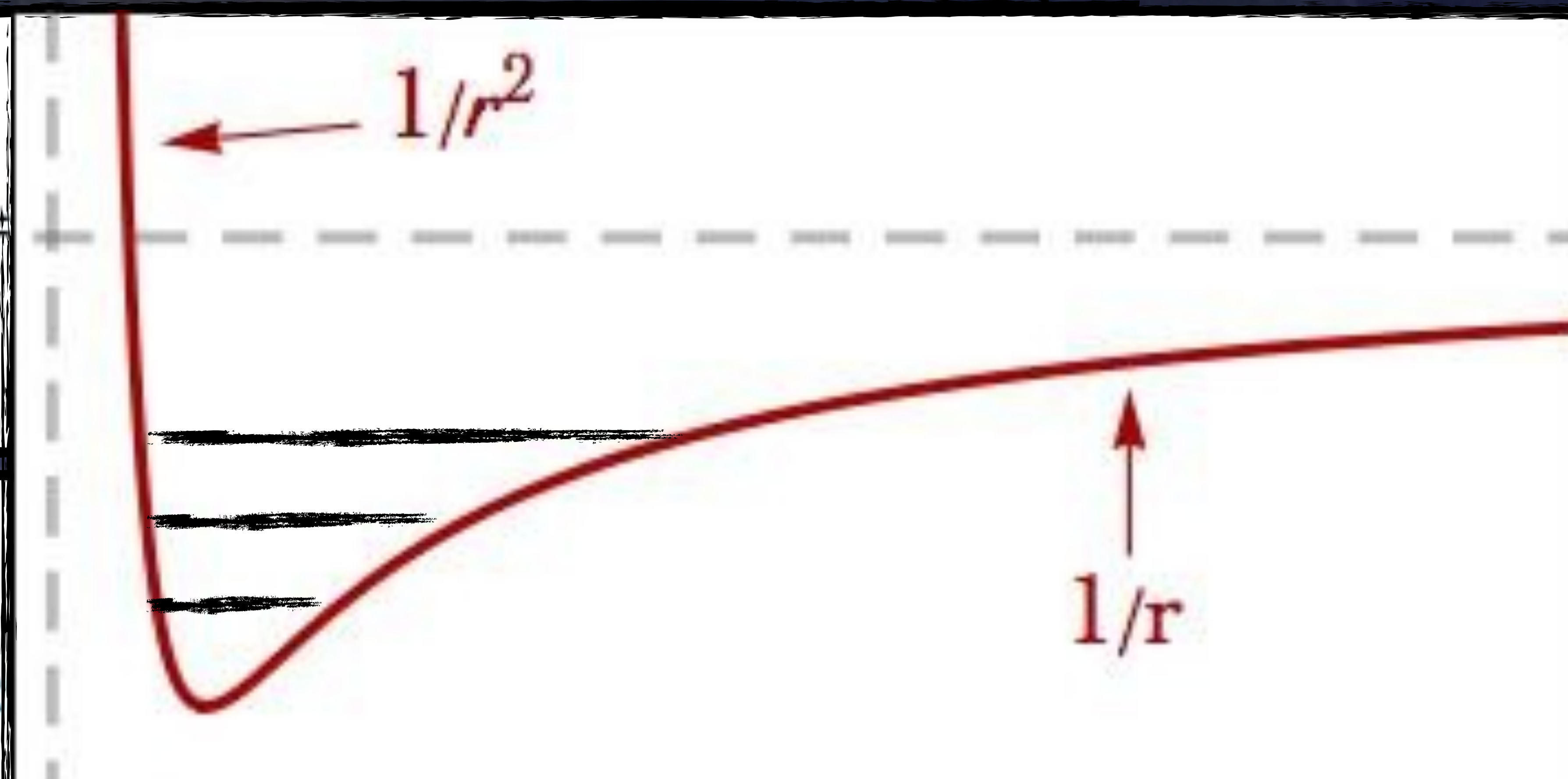
$$\Psi(t,r,\theta,\phi) = e$$

$$\{-$$

$$\omega_{(l,n)} = -\left(\frac{e^2}{4\pi\epsilon_0\hbar}\right)^2 \frac{\mu}{2\hbar(l+n+1)^2}$$

The hydrogen atom

$$i\hbar\frac{\partial}{\partial t}\Psi(t,r,\theta,\phi) =$$



$$\Psi(t,r,\theta,\phi) = e$$

$$\{-$$

$$\boxed{e^{-i\omega t}}$$

$$\omega_{(l,n)} = -\left(\frac{e^2}{4\pi\epsilon_0\hbar}\right)^2 \frac{\mu}{2\hbar(l+n+1)^2}$$

Real frequencies - bound states

Scalar field in Schwarzschild BG

Scalar field in Schwarzschild BG

$$ds^2 = -\left(1 - \frac{2M}{r}\right) dt^2 + \frac{dr^2}{1 - \frac{2M}{r}} + r^2 \left(d\theta^2 + \sin^2\theta d\phi^2\right)$$

$$\Box\Phi = \mu^2\Phi$$

$$\Phi(t, r, \theta, \phi) = \sum_{l,m} Y_l^m(\theta, \phi) e^{-i\omega t} \frac{R(r)}{r}$$

Scalar field in Schwarzschild BG

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$$dr_* = \frac{dr}{1-\frac{2M}{r}} \quad \Longrightarrow \quad r_* = r + 2M\ln\left(\frac{r}{2M} - 1\right)$$

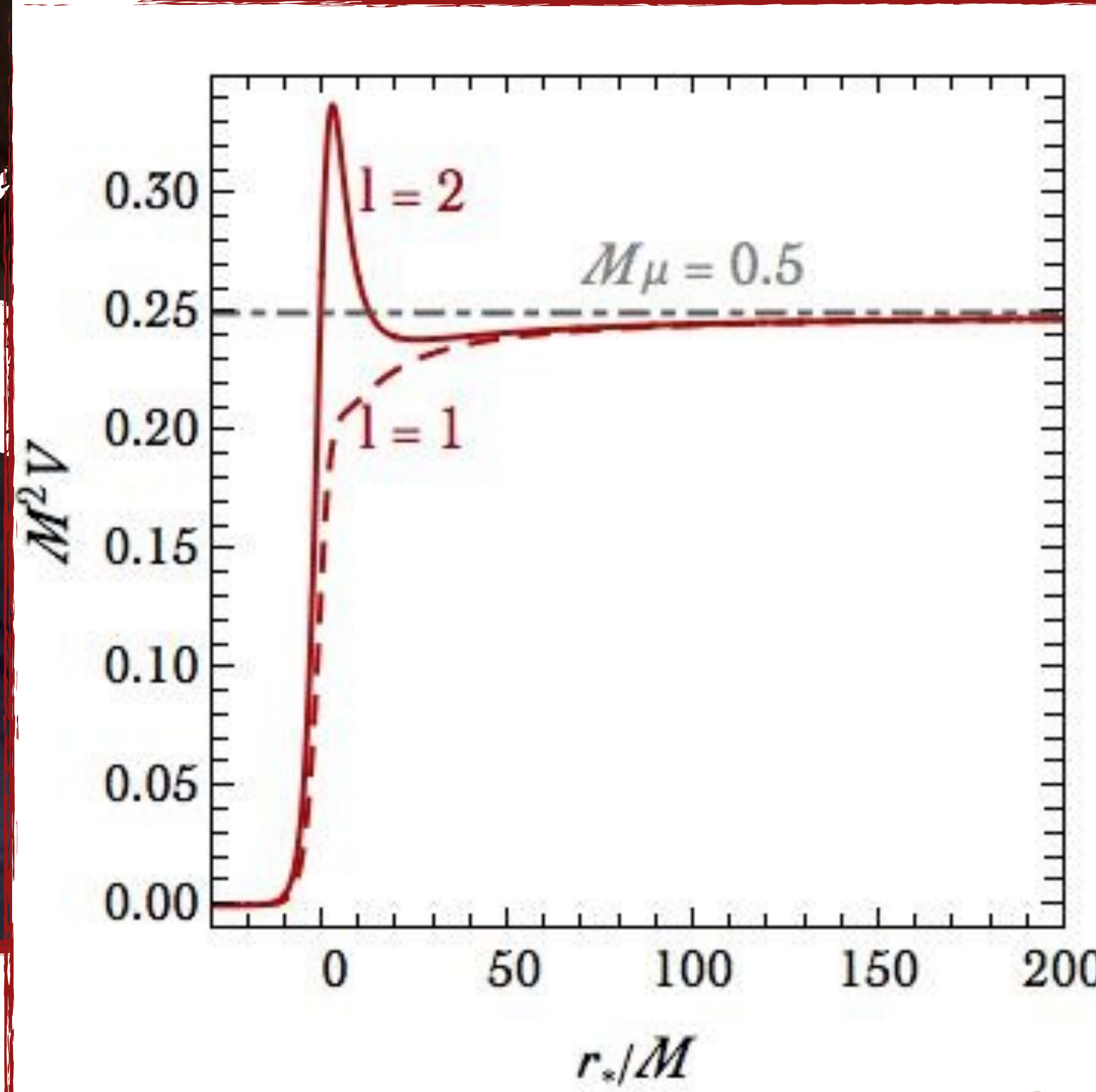
$$\Phi(t,r,\theta,\phi) = \sum_{l,m} Y_l^m(\theta,\phi)e^{-i\omega t}\frac{R(r)}{r}$$

Scalar field in Schwarzschild BG

$$\left[-\frac{d^2}{dr_*^2}+V_{\text{eff}}(r)\right]R(r)=\omega^2 R(r)$$

$$V_{\text{eff}}(r)=\left(1-\frac{2M}{r}\right)\left[\frac{2M}{r^3}+\frac{l(l+1)}{r^2}+\mu^2\right]$$

Scalar fi
Schwarzsc

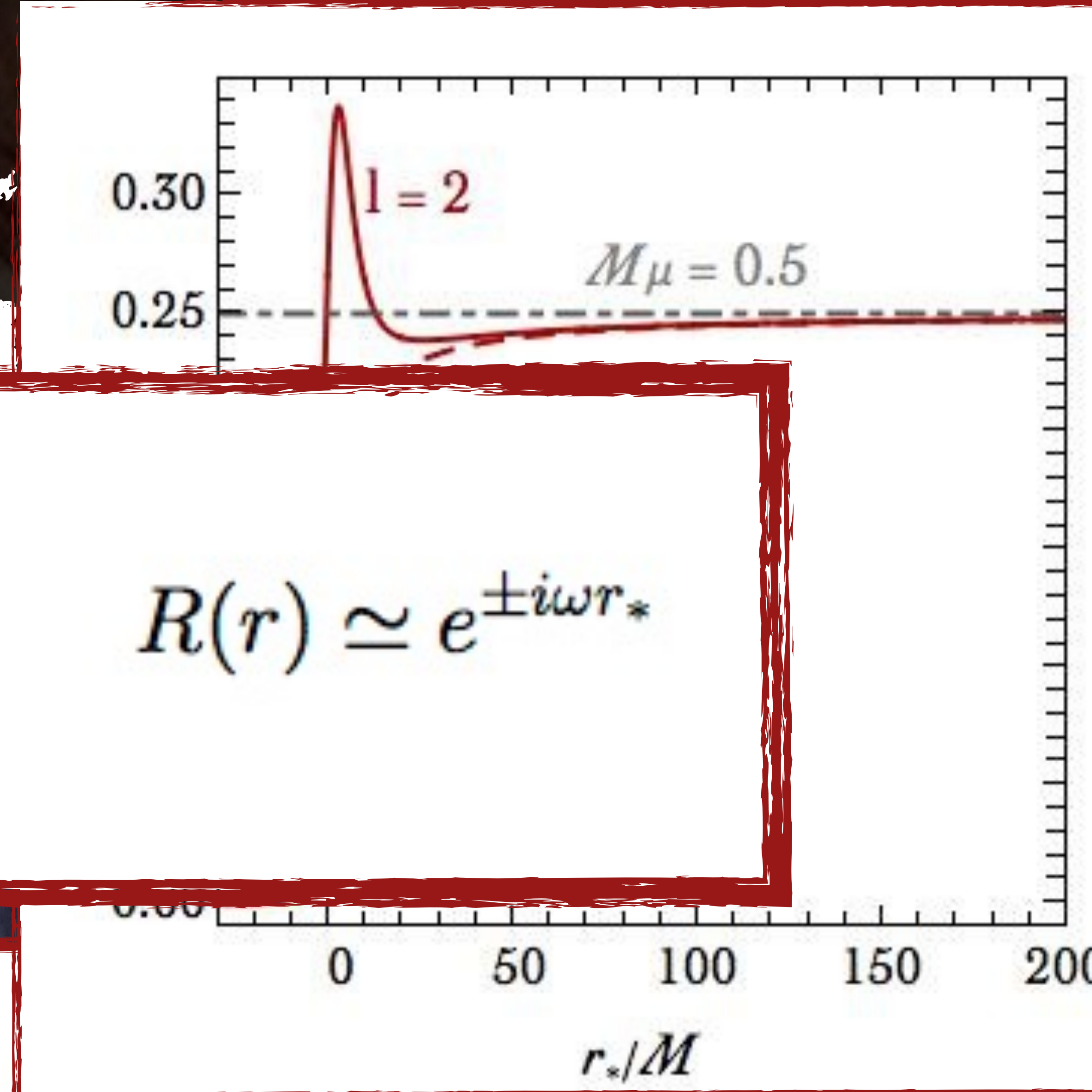


$$\left[-\frac{d^2}{dr_*^2} + V_{\rm eff}(r)\right] R(r) = \omega^2 R(r)$$

$$V_{\rm eff}(r) = \left(1 - \frac{2M}{r}\right)\left[\frac{2M}{r^3} + \frac{l(l+1)}{r^2} + \mu^2\right]$$

Scalar fi

$$-\frac{d^2R(r)}{dr_*^2} \simeq \omega^2 R(r) \qquad \Longrightarrow \qquad R(r) \simeq e^{\pm i\omega r_*}$$



$$\left[-\frac{d^2}{dr_*^2} + V_{\text{eff}}(r)\right] R(r) = \omega^2 R(r)$$

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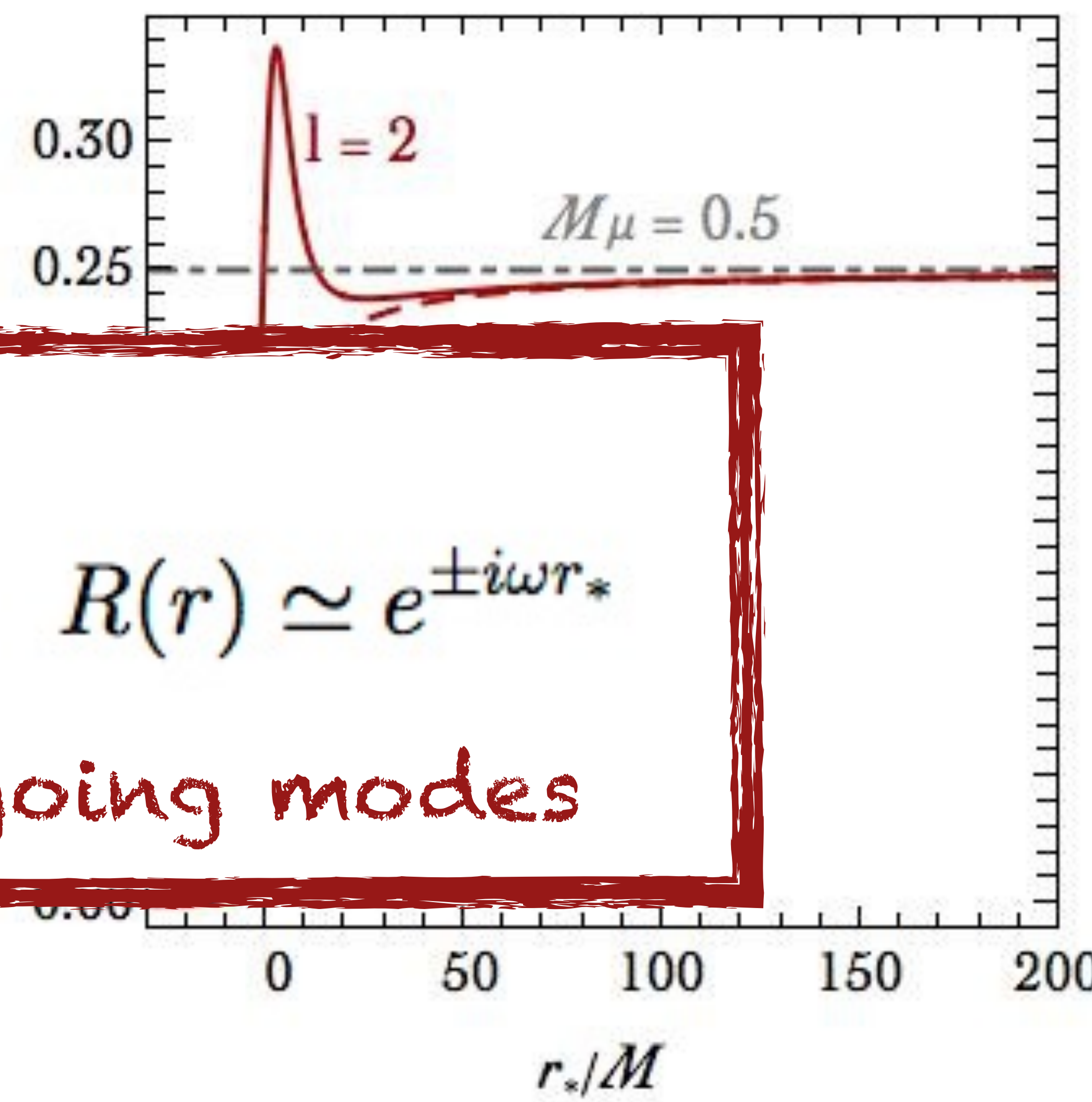
Scalar fi



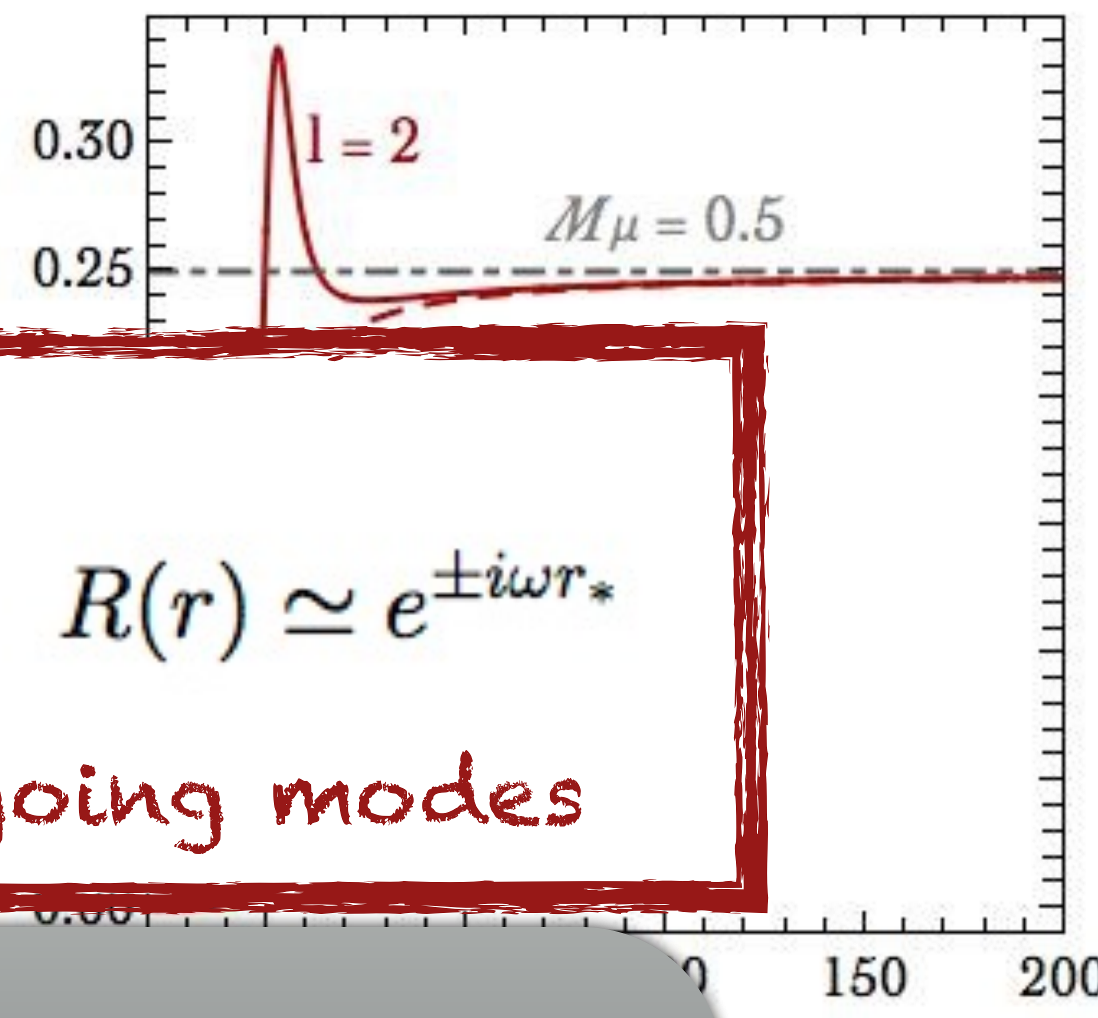
$$-\frac{d^2R(r)}{dr_*^2} \simeq \omega^2 R(r) \qquad \Longrightarrow \qquad R(r) \simeq e^{\pm i\omega r_*}$$

At the horizon we impose ingoing modes

$$\left[-\frac{d^2}{dr_*^2} + V_{\text{eff}}(r)\right] R(r) = \omega^2 R(r)$$

$$V_{\text{eff}}(r) = \left(1 - \frac{2M}{r}\right)\left[\frac{2M}{r^3} + \frac{l(l+1)}{r^2} + \mu^2\right]$$

Scalar fi



$$-\frac{d^2R(r)}{dr_*^2} \simeq \omega^2 R(r) \quad \Longrightarrow \quad R(r) \simeq e^{\pm i\omega r_*}$$

At the horizon we impose ingoing modes

Physically,
we expect no real bound state to exist,
since there is an energy flux
into the black hole.
It is more reasonable to expect
modes that decay with time

Scalar field in Schwarzschild BG

$\ell = 1$

μ	ω
0.1	$0.09987 - 1.5182 \times 10^{-11} i$
0.2	$0.19895 - 4.0586 \times 10^{-8} i$
0.3	$0.29619 - 9.4556 \times 10^{-6} i$
0.4	$0.38955 - 5.6274 \times 10^{-4} i$
0.5	$0.47759 - 5.5441 \times 10^{-3} i$

$\ell = 2$

μ	ω
0.1	$0.09994 - 8.6220 \times 10^{-17} i$
0.2	$0.19954 - 5.9249 \times 10^{-14} i$
0.3	$0.29844 - 4.9002 \times 10^{-11} i$
0.4	$0.39619 - 1.1703 \times 10^{-8} i$
0.5	$0.49219 - 1.2271 \times 10^{-6} i$
0.6	$0.58541 - 6.9974 \times 10^{-5} i$
0.7	$0.67385 - 1.4987 \times 10^{-3} i$
0.8	$0.75788 - 8.1511 \times 10^{-3} i$

in

$$\Phi(t,r,\theta,\phi) = \sum_{l,m} Y_l^m(\theta,\phi) e^{-i\omega t} \frac{R(r)}{r}$$

$$e^{-i\omega t} = e^{-i(\omega_{\mathrm{R}} + i\omega_{\mathrm{I}})t} = e^{-i\omega_{\mathrm{R}} t} e^{\omega_{\mathrm{I}} t}$$

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The Horizon boundary condition only permits the existence of QUASI-BOUND STATES around the Schwarzschild solution
(notice, however, that these can be very long lived, especially for small masses)

$\ell = 1$

μ	ω
0.1	$0.09987 - 1.5182 \times 10^{-11} i$
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0.8	$0.75788 - 8.1511 \times 10^{-3} i$

Scalar field in Kerr BG

$$ds^2_{\text{Kerr}} = -\left(1 - \frac{2Mr}{\rho^2}\right) dt^2 - \frac{4Mra\sin^2\theta}{\rho^2} d\phi dt + \frac{\rho^2}{\Delta} dr^2 + \rho^2 d\theta^2$$
$$+ \left(r^2 + a^2 + \frac{2Mra^2\sin^2\theta}{\rho^2}\right)\sin^2\theta d\phi^2$$

$$\Box\Phi = \mu^2\Phi$$

$$\Phi(t,r,\theta,\phi) = \sum_{l,m} e^{im\phi} S_{lm}(\theta) e^{-i\omega t} R_{ln}(r)$$

$$r_\pm = M(1 \pm \sqrt{1 - \tilde{a}^2})$$

r+ is the position of the event horizon

$$\rho^2 = r^2 + a^2\cos^2\theta$$

$$\Delta = r^2 - 2Mr + a^2$$

$$a = J/M \qquad \tilde{a} = a/M$$

If $a = M$ ($\tilde{a} = 1$) extreme Kerr BH

Equation defining the spheroidal harmonics functions

M. Abramowitz, and I. A. Stegun, Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables. Dover, New York, 1965; E. Berti, V. Cardoso, and M. Casals, Phys. Rev. D 73 (2006), 024013

with regularity boundary conditions that discretized the allowed value of the "quantum numbers" l and m

$$\Phi(t,r,\theta,\phi) = \sum_{l,m} e^{im\phi} S_{lm}(\theta) e^{-i\omega t} R_{ln}(r)$$

$$\frac{1}{\sin\theta}\frac{d}{d\theta}\left(\sin\theta\frac{dS_{lm}}{d\theta}\right) + \left[a^2\left(\omega^2-\mu^2\right)\cos^2\theta - \frac{m^2}{\sin^2\theta} + \Lambda_{lm}\right] S_{lm} = 0$$

Solutions to Laplace's equation when phrased in ellipsoidal coordinates.

$$u_{nl}(r^*) = (r^2 + a^2)^{1/2} R_{nl}(r)$$

$$\frac{d^2u}{dr^{*2}} + \left[\omega^2 - V(\omega)\right] u = 0$$

$$\Phi(t, r, \theta, \phi) = \sum_{l,m} e^{im\phi} S_{lm}(\theta) e^{-i\omega t} R_{ln}(r)$$

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$$\Phi(t, r, \theta, \phi) = \sum_{l,m} e^{im\phi} S_{lm}(\theta) e^{-i\omega t} R_{ln}(r)$$

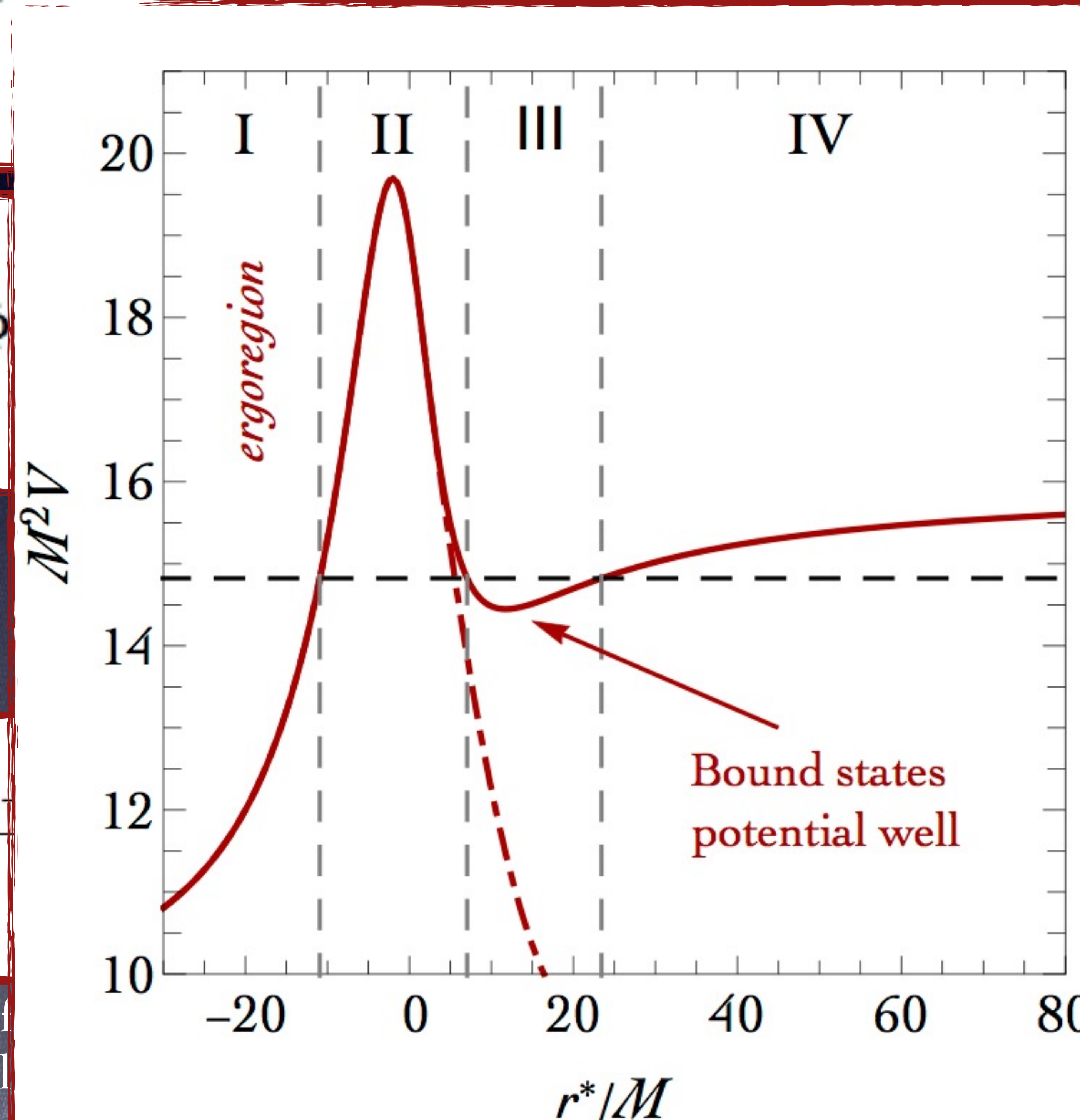
$$V = \frac{\Delta\mu^2}{r^2 + a^2} + \frac{4Mram\omega - a^2m^2 + \Delta\left[\lambda_{lm} + (\omega^2 - \mu^2)a^2\right]}{(r^2 + a^2)^2} + \frac{\Delta(3r^2 - 4Mr + a^2)}{(a^2 + r^2)^3} - \frac{3r^2\Delta^2}{(r^2 + a^2)^4}$$

An analogous equation first arose in the study of the electronic spectrum of the hydrogen molecule W. G. Baber and H. R. Hassé, Proc. Camb. Phil. Soc. 25 (1935), 564; G. Jaffé, Z. Phys. A87 (1934) 535

$$u_{nl}(r^*) = (r^2 + a^2)^{1/2} R_{nl}(r)$$

$$\frac{d^2u}{dr^{*2}} + \left[\omega^2 - V(\omega)\right] u = 0$$

Φ



$$V = \frac{\Delta\mu^2}{r^2 + a^2} + \frac{4Mram\omega - a^2m^2 + \Delta\left[\lambda_{lm}\right.}{(r^2 + a^2)^2}$$

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W. G. Baber and H. R. Hassé, Proc. Camb. Phil
535

$\mu = 0.3; \ell = 1$

$\tilde{a}$	$m=-1$	$m=0$	$m=1$
0.1	$0.29618 - 1.19213 \times 10^{-5} i$	$0.29619 - 9.39767 \times 10^{-6} i$	$0.29620 - 7.30823 \times 10^{-6} i$
0.5	$0.29613 - 2.51902 \times 10^{-5} i$	$0.29612 - 8.00351 \times 10^{-6} i$	$0.29625 - 1.66155 \times 10^{-6} i$
0.9	$0.29607 - 4.44672 \times 10^{-5} i$	$0.29620 - 4.68608 \times 10^{-6} i$	$0.29630 + 1.46971 \times 10^{-8} i$
0.95	$0.29600 - 4.70610 \times 10^{-5} i$	$0.29620 - 4.08878 \times 10^{-6} i$	$0.29630 + 2.72170 \times 10^{-8} i$

$\mu = 0.4; \ell = 1$

$\tilde{a}$	$m=-1$	$m=0$	$m=1$
0.1	$0.38948 - 6.62132 \times 10^{-4} i$	$0.38955 - 5.61203 \times 10^{-4} i$	$0.38963 - 4.67614 \times 10^{-4} i$
0.5	$0.38926 - 1.08538 \times 10^{-3} i$	$0.38955 - 5.23330 \times 10^{-4} i$	$0.39001 - 1.53007 \times 10^{-4} i$
0.9	$0.38914 - 1.52116 \times 10^{-3} i$	$0.38954 - 4.26952 \times 10^{-4} i$	$0.39045 - 4.34117 \times 10^{-6} i$
0.95	$0.38913 - 1.57507 \times 10^{-3} i$	$0.38954 - 4.09975 \times 10^{-4} i$	$0.39050 - 5.71763 \times 10^{-7} i$

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Exp decay in time
(quasi-bound states)

$$e^{-i\omega t} = e^{-i(\omega_{\mathrm{R}} + i\omega_{\mathrm{I}})t} = e^{-i\omega_{\mathrm{R}} t} e^{\omega_{\mathrm{I}} t}$$

$\mu = 0.3; \ell = 1$

$\tilde{a}$	$m=-1$	$m=0$	$m=1$
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"Superradiant instability"

$$e^{-i\omega t} = e^{-i(\omega_{\rm R} + i\omega_{\rm I})t} = e^{-i\omega_{\rm R} t} e^{\omega_{\rm I} t}$$

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$$a_{\text{crit}} \simeq \frac{2\mu r_+ M}{m}$$

If a > a_{crit} the imaginary part of the frequency is positive and the mode grows instead of decaying

$\mu = 0.3; \ell = 1$

$\tilde{a}$	$m = -1$	$m = 0$	$m = 1$
0.1	$0.29618 - 1.19213 \times 10^{-5} i$	$0.29619 - 9.39767 \times 10^{-6} i$	$0.29620 - 7.30823 \times 10^{-6} i$
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$$a_{\text{crit}} \simeq \frac{2\mu r_+ M}{m}$$

Modes that exist precisely at the critical frequency have zero imaginary part and hence are BOUND STATES analogue to the ones in the hydrogen atom!

The no-hair conjecture

Ruffini-Wheeler (1971)

Collapse leads to equilibrium black holes uniquely determined by M, J, Q

asymptotically measured quantities subject to a Gauss law

and no other independent characteristics ("hair")

The no-hair conjecture

Bekenstein (1972)

J. D. Bekenstein, Phys. Rev. Lett. 28 (1972) 452

Assumption 1:

Canonical scalar field, minimally coupled to Einstein gravity

Bekenstein (1972)

J. D. Bekenstein, Phys. Rev. Lett. 28 (1972) 452

Assumption 2:

The scalar field inherits the space-time symmetries

$$\partial_t \Phi = 0 = \partial_\phi \Phi$$

Bekenstein (1972)

J. D. Bekenstein, Phys. Rev. Lett. 28 (1972) 452

Assumption 3:

The potential V obeys the condition:

$$\Phi V' \geqslant 0$$

A generic mass term does respect assumption 3

$$V = \frac{\mu^2}{2}\Phi^2 \quad \Longrightarrow \quad \Phi^2\mu^2 \geqslant 0$$

Bekenstein (1972)

J. D. Bekenstein, Phys. Rev. Lett. 28 (1972) 452

Assumption 2:

The scalar field inherits the space-time symmetries

$$\partial_t \Phi = 0 = \partial_\phi \Phi$$

It seems natural to assume that the scalar field has the same symmetries as the geometry. However, this condition is not mandatory.

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Assumption 2:

The scalar field inherits ... s

The symmetry argument is strictly valid for the energy-momentum tensor

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G_N T_{\mu\nu}$$

$$\Phi(t,r,\theta,\phi) = \sum_{l,m} e^{im\phi} S_{lm}(\theta) e^{-i\omega t} R_{ln}(r)$$

The harmonic time and azimuthal angular dependence allows the EMT to respect the spacetime symmetries

The symmetry argument is strictly valid for the energy-momentum tensor

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G_N T_{\mu\nu}$$


Superradiance

Superradiance is by no means exclusive to black hole physics, but it can occur in the scattering of bosonic fields by rotating (and also charged) black holes.

For a review:

R. Brito, V. Cardoso and P. Pani, "Superradiance",

Lect. Notes Phys. 906 (2015) pp.1, [arXiv:1501.06570 [gr-qc]]

In black hole physics, superradiant amplification, leading to energy and angular momentum (or charge) extraction from the black hole, was first discussed:

- from a thermodynamic viewpoint J. Bekenstein, Phys. Rev. D7 (1973) 949-953;
- in the scattering of scalar J. M. Bardeen, W. H. Press and S. A. Teukolsky, Astrophys. J. 178 (1972) 347; A. Starobinski, Zh. Eksp. Teor. Fiz. 64 (1973) 48. (Sov. Phys. - JETP, 37, 28, 1973), electromagnetic and gravitational waves by a rotating black hole A. Starobinski and S. M. Churilov, Zh. Eksp. Teor. Fiz. 65 (1973) 3. (Sov. Phys. - JETP, 38, 1, 1973)

Superradiance

Initial state

Event horizon

Superradiance

Initial state

Event horizon

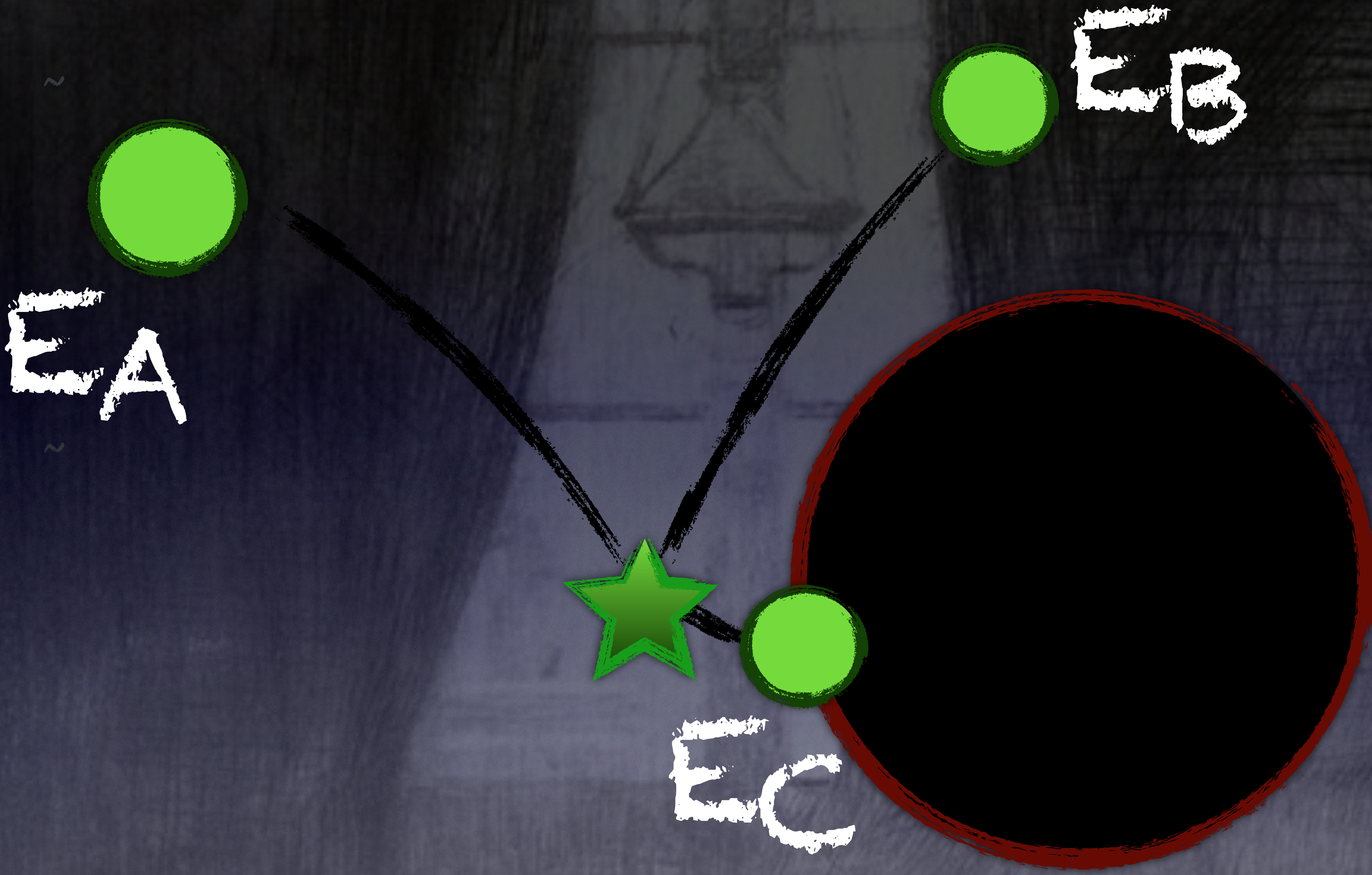
Final state

Event horizon

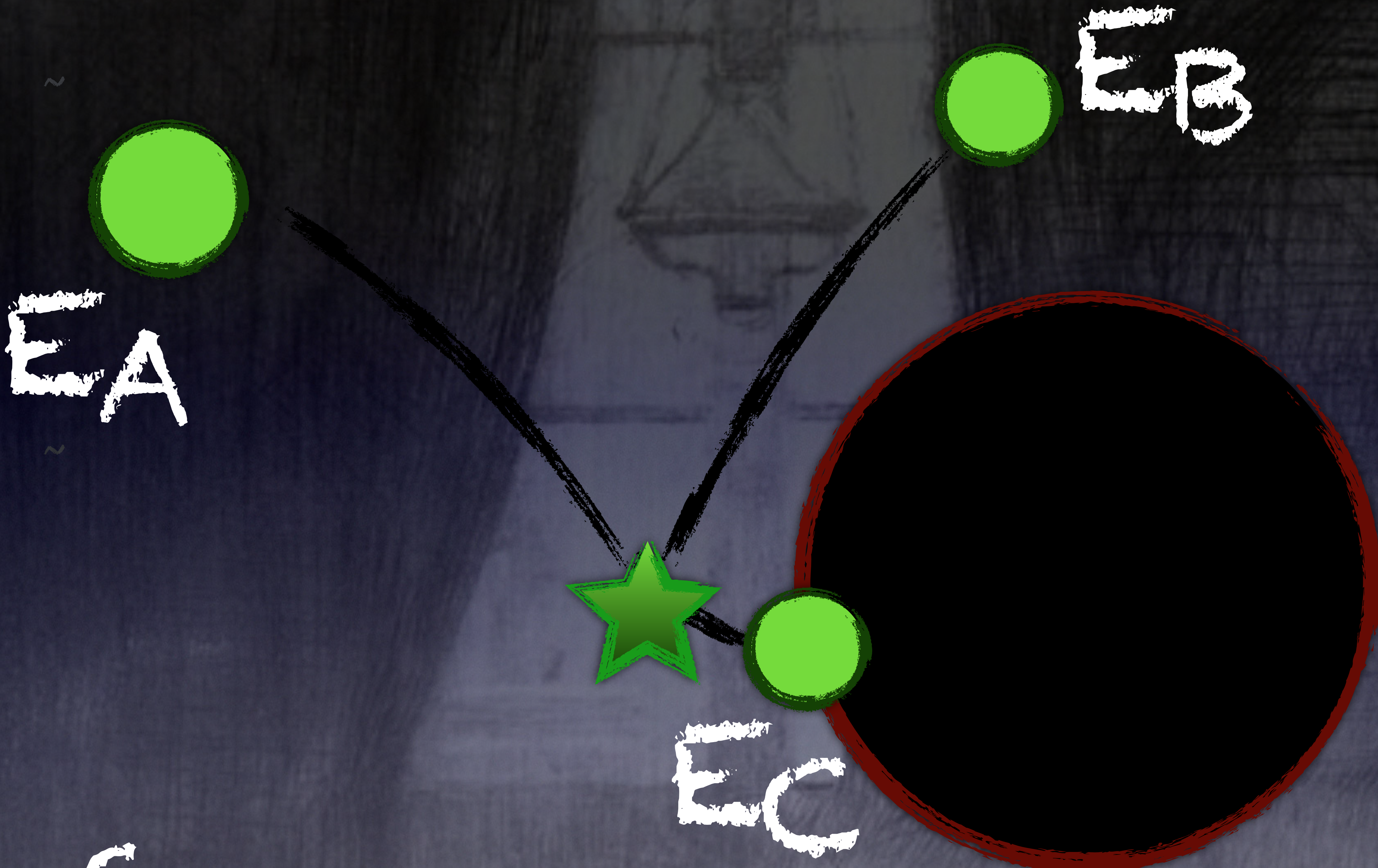
$|R|^2 > |I|^2$

Superradiance

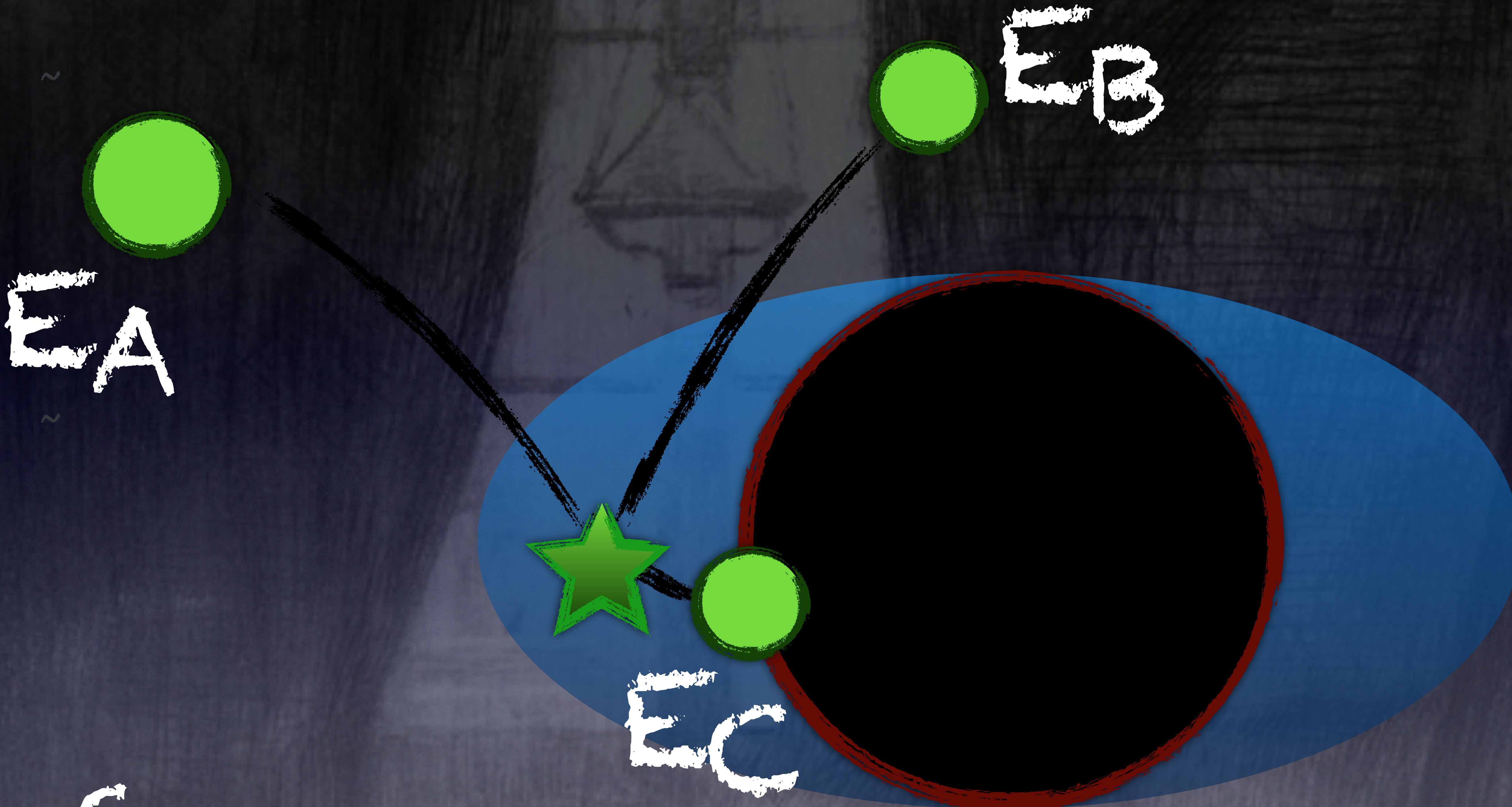

EA
Superradiance



Superradiance



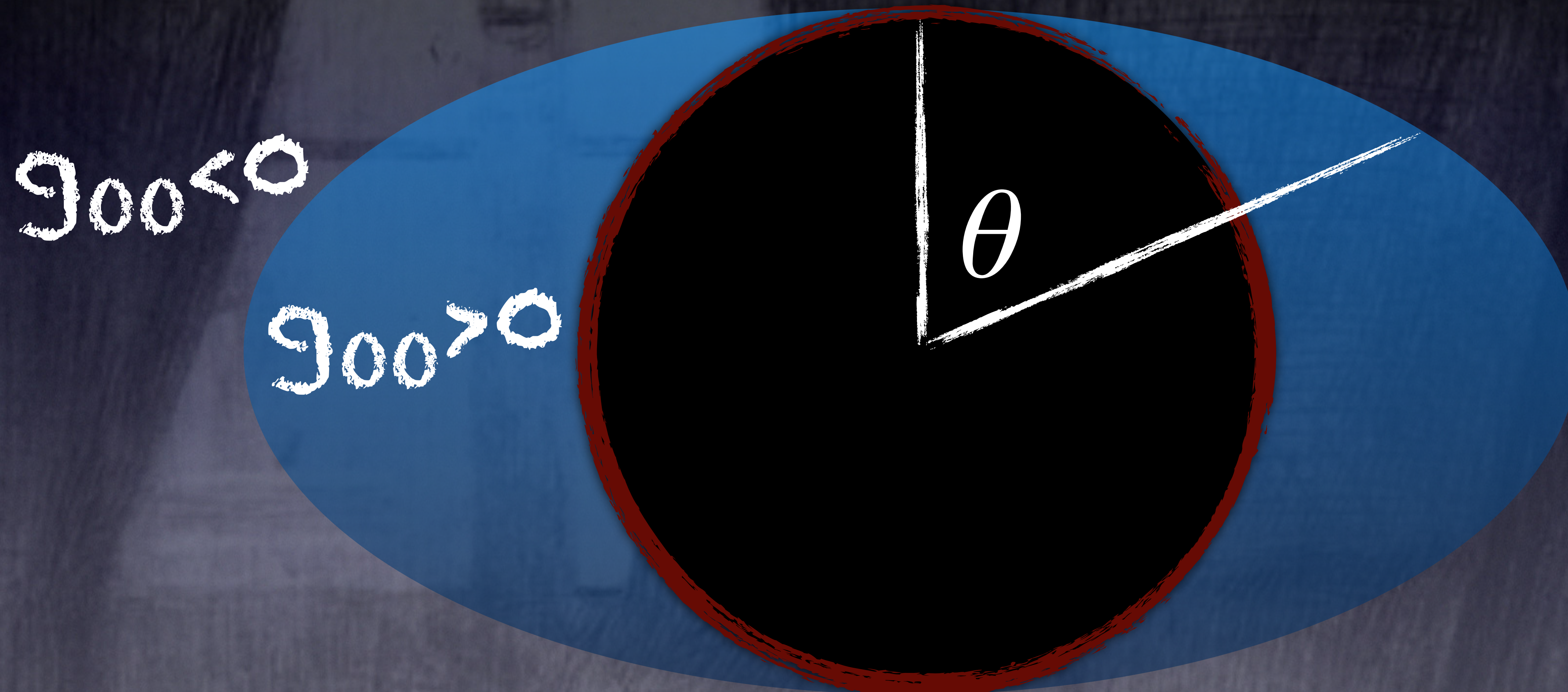
If $E_C < 0$ $\longrightarrow$ $E_B > E_A$



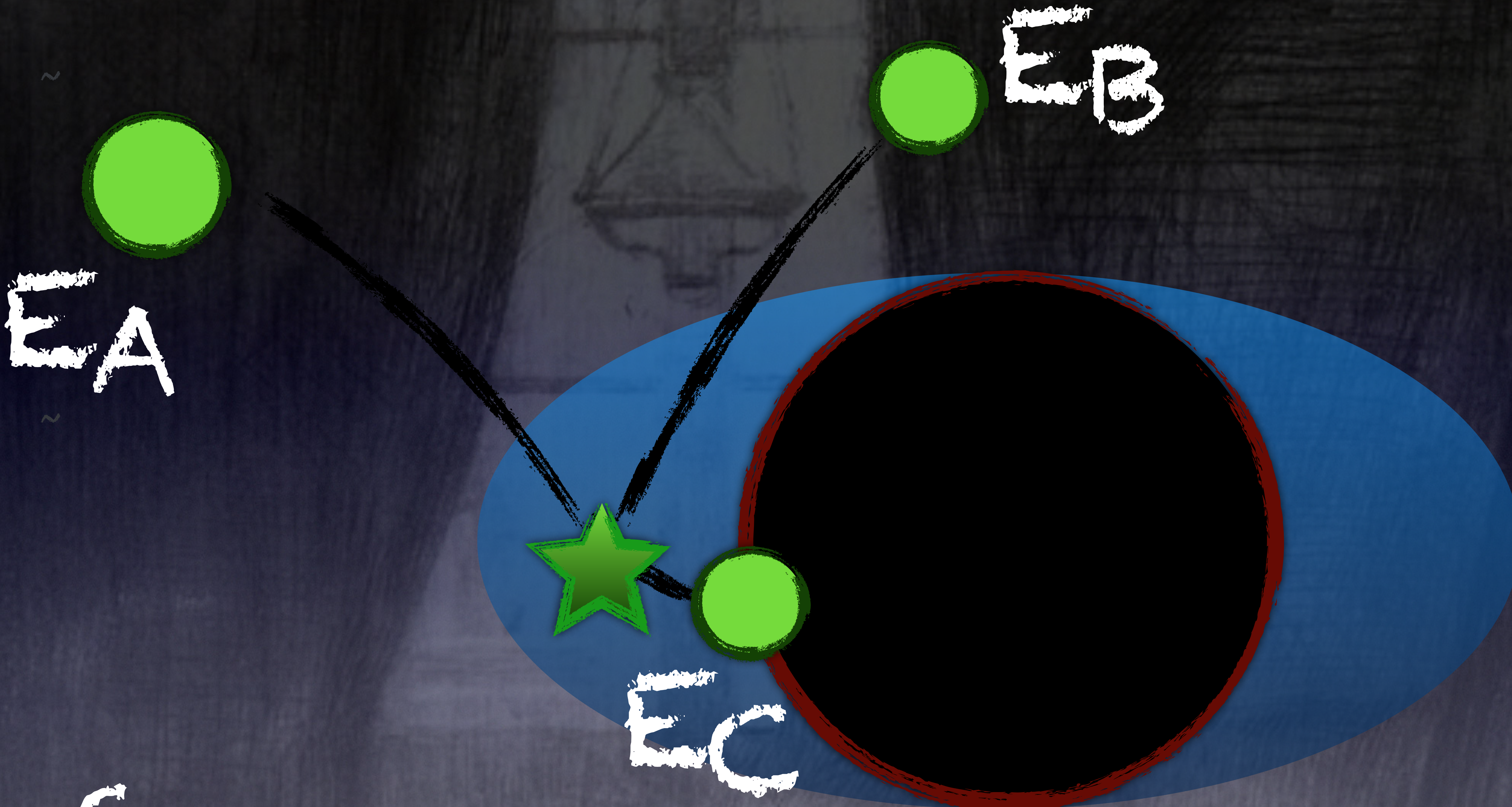
If $E_C < 0$

$\longrightarrow E_B > E_A$

$$g_{00} = -\left(1 - \frac{2Mr}{\rho^2}\right) = -\left(\frac{r^2 + a^2\cos^2\theta - 2Mr}{r^2 + a^2\cos^2\theta}\right) = 0$$



$$r_{\text{ergo}}(\theta) = M + \sqrt{M^2 - a^2\cos^2\theta}$$



If $E_C < 0 \longrightarrow E_B > E_A$

The "gravitational atom"

The "gravitational atom"

$$\Phi(t,r,\theta,\phi) = \sum_{l,m} e^{im\phi} S_{lm}(\theta) e^{-i\omega t} R_{ln}(r)$$

orbital	$\nu \equiv n+l+1$	n	l	m
1s	1	0	0	0
2s	2	1	0	0
2p	2	0	1	0,±1
3s	3	2	0	0
3p	3	1	1	0,±1
3d	3	0	2	0,±1,±2
4s	4	3	0	0
4p	4	2	1	0,±1
4d	4	1	2	0,±1,±2
4f	4	0	3	0,±1,±2,±3
...	...	...	...	...

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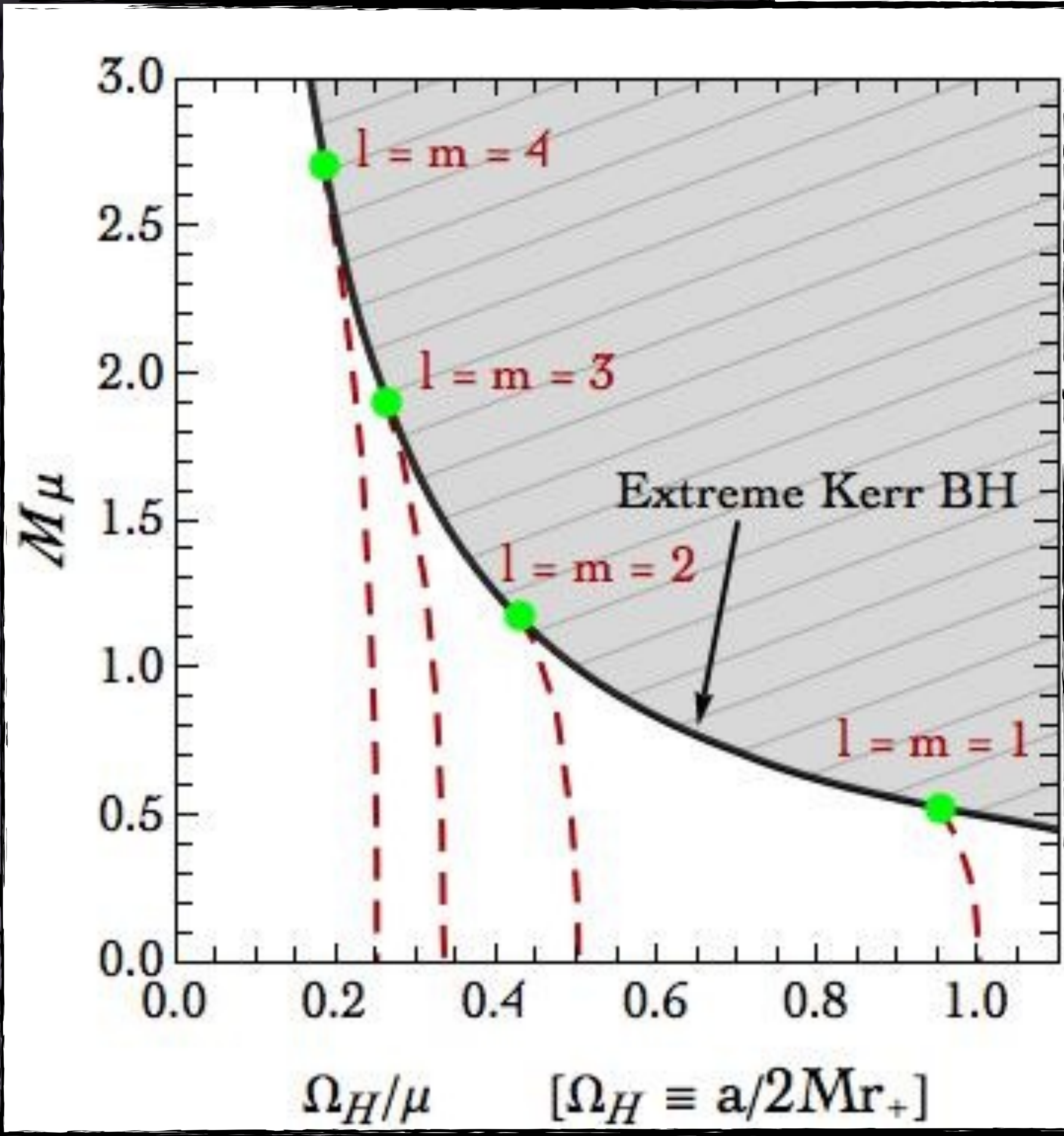

orbital	$\nu \equiv n+l+1$	n	l	m
1s	1	0	0	0
2s	2	1	0	0
2p	2	0	1	0,±1
3s	3	2	0	0
3p	3	1	1	0,±1
3d	3	0	2	0,±1,±2
4s	4	3	0	0
4p	4	2	1	0,±1
4d	4	1	2	0,±1,±2
4f	4	0	3	0,±1,±2,±3
...	...	...	...	...

S. Hod,
Phys. Rev. D86 (2012) 104026,
arXiv:1211.3202 [gr-qc]

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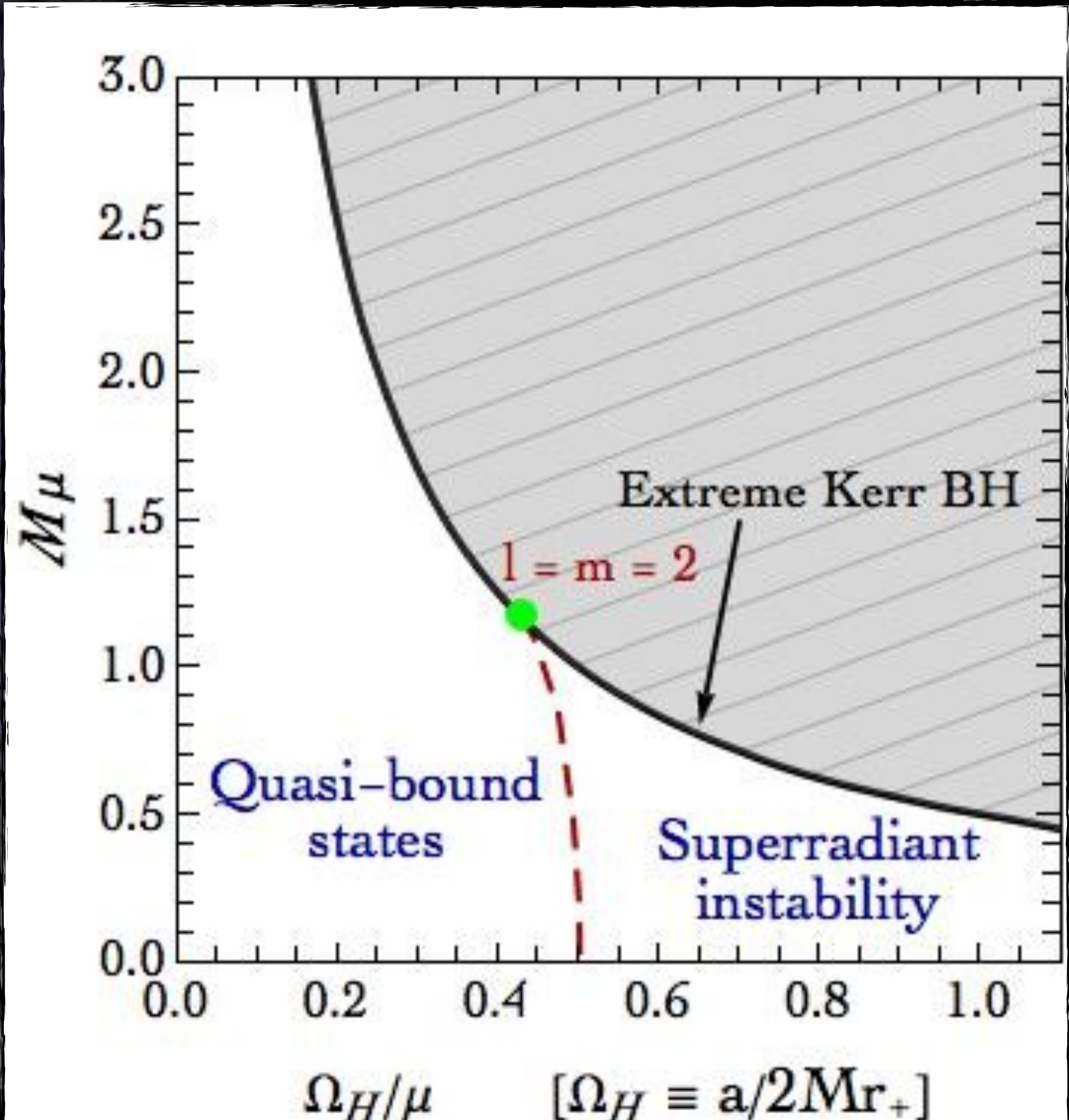

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3p	3	1	1	0,±1
3d	3	0	2	0,±1,±2
4s	4	3	0	0
4p	4	2	1	0,±1
4d	4	1	2	0,±1,±2
4f	4	0	3	0,±1,±2,±3
…	…	…	…	…

C. A. R. Herdeiro and E. Radu, Phys. Rev. Lett. 112 (2014) 221101, [arXiv:1403.2757 [gr-qc]].

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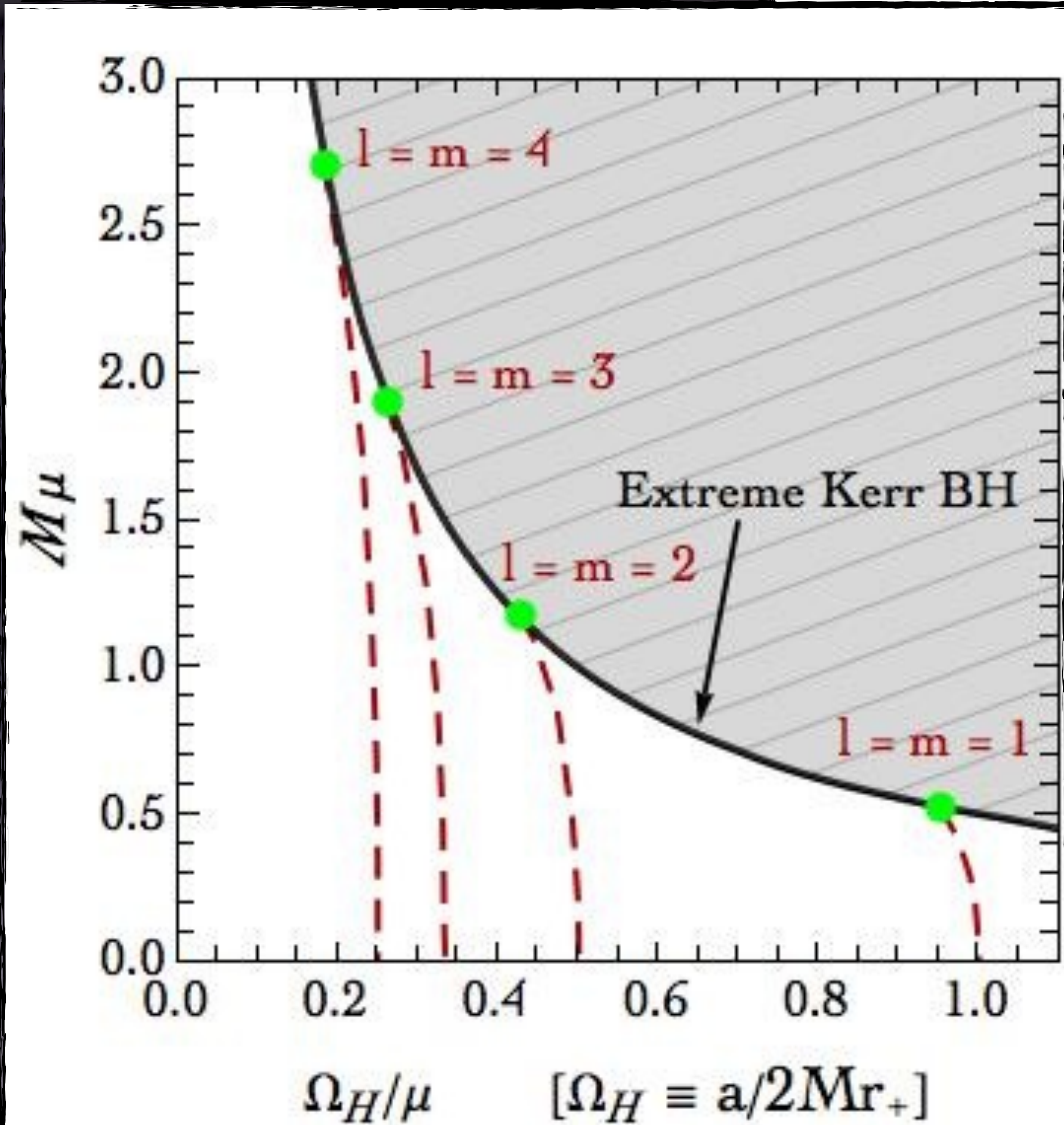

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...	...	...	...	...

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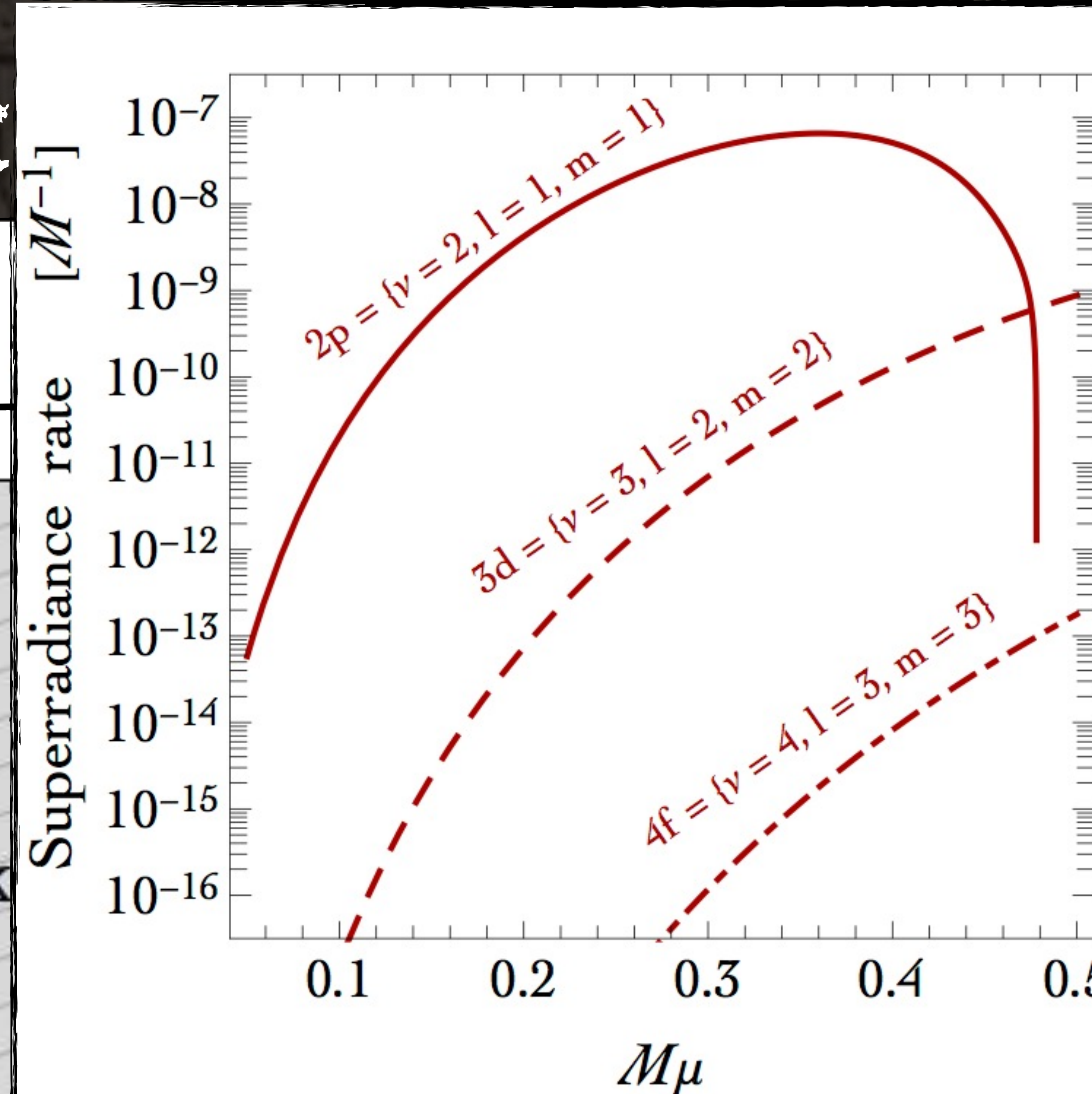
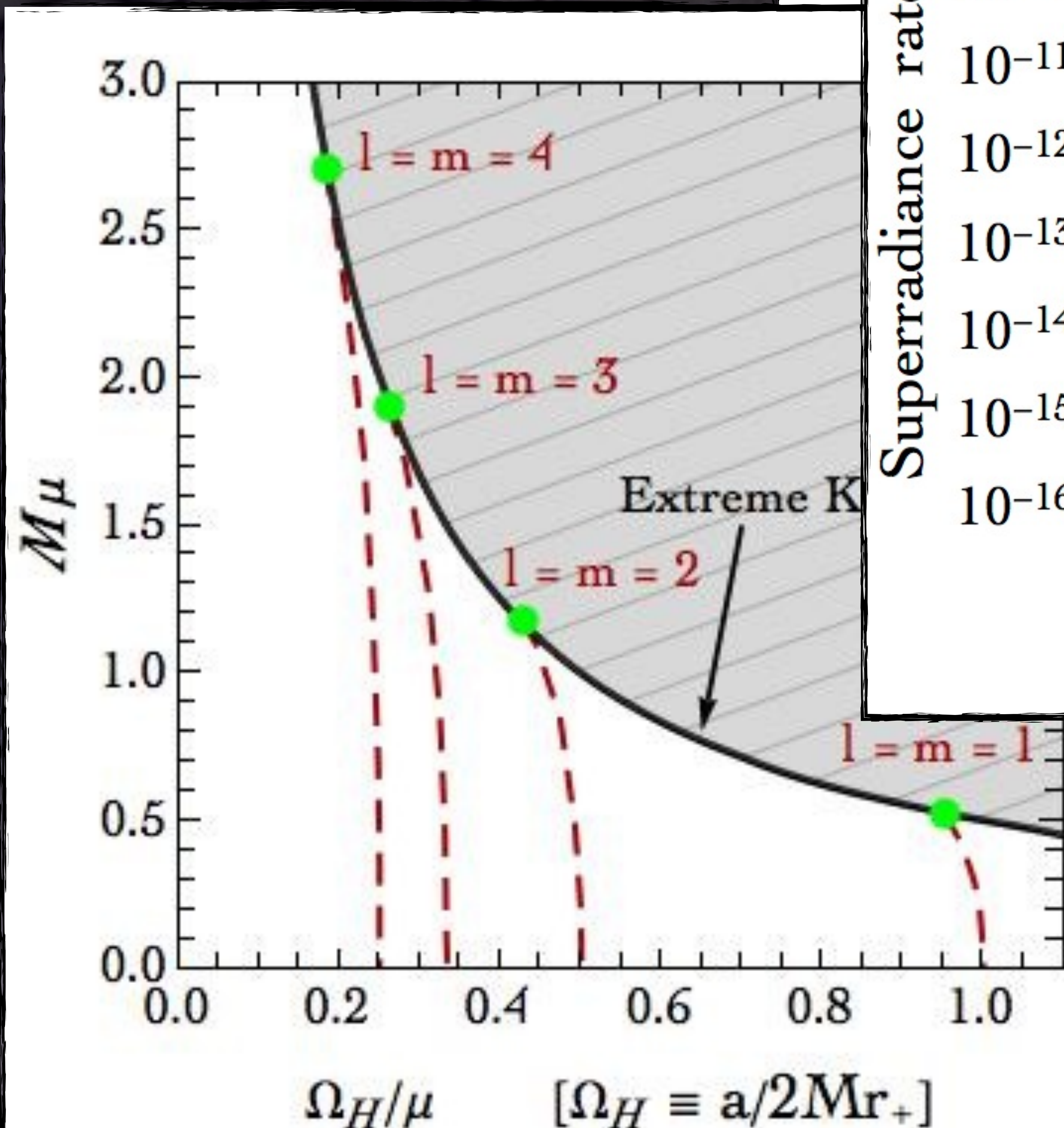
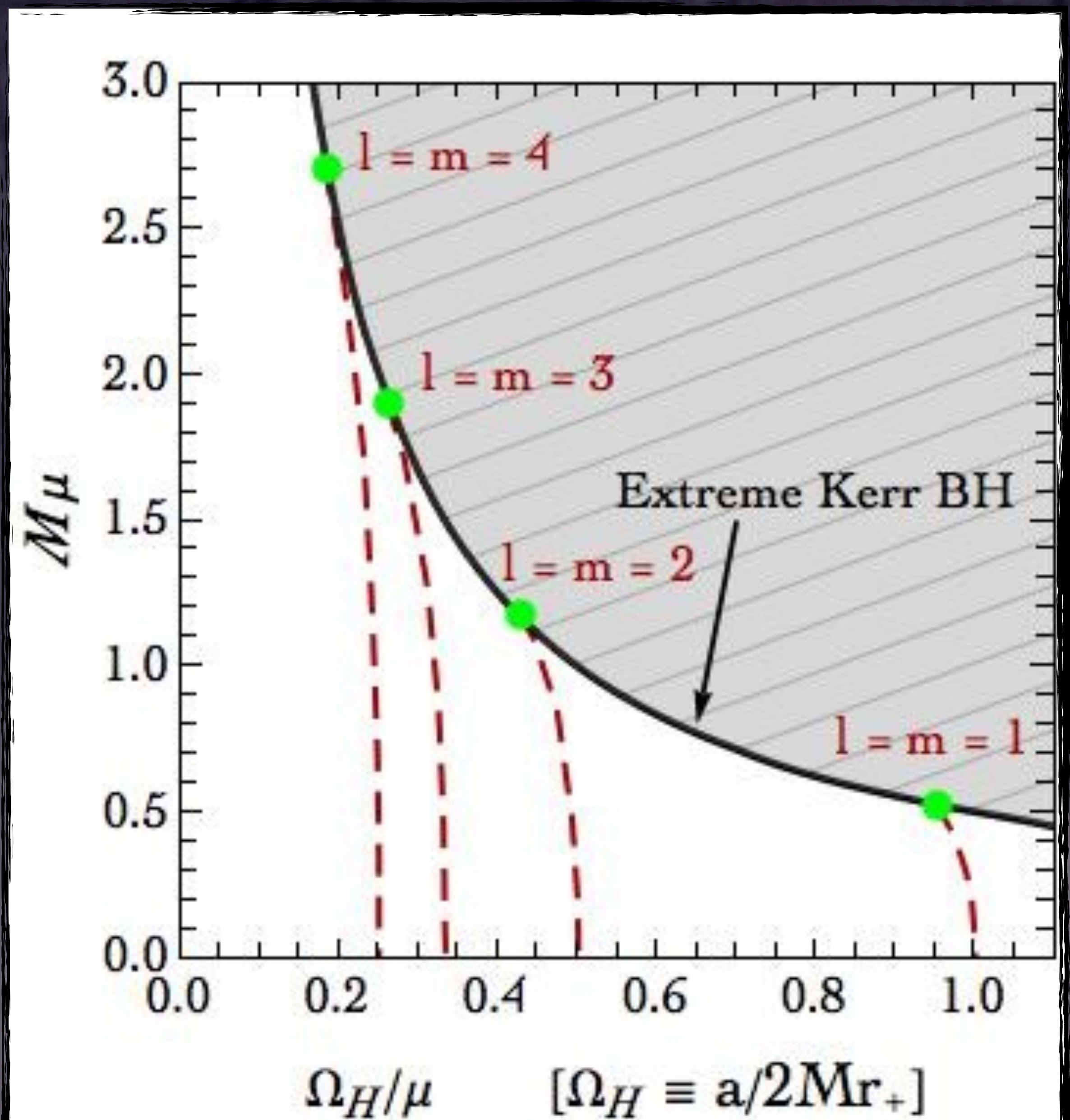


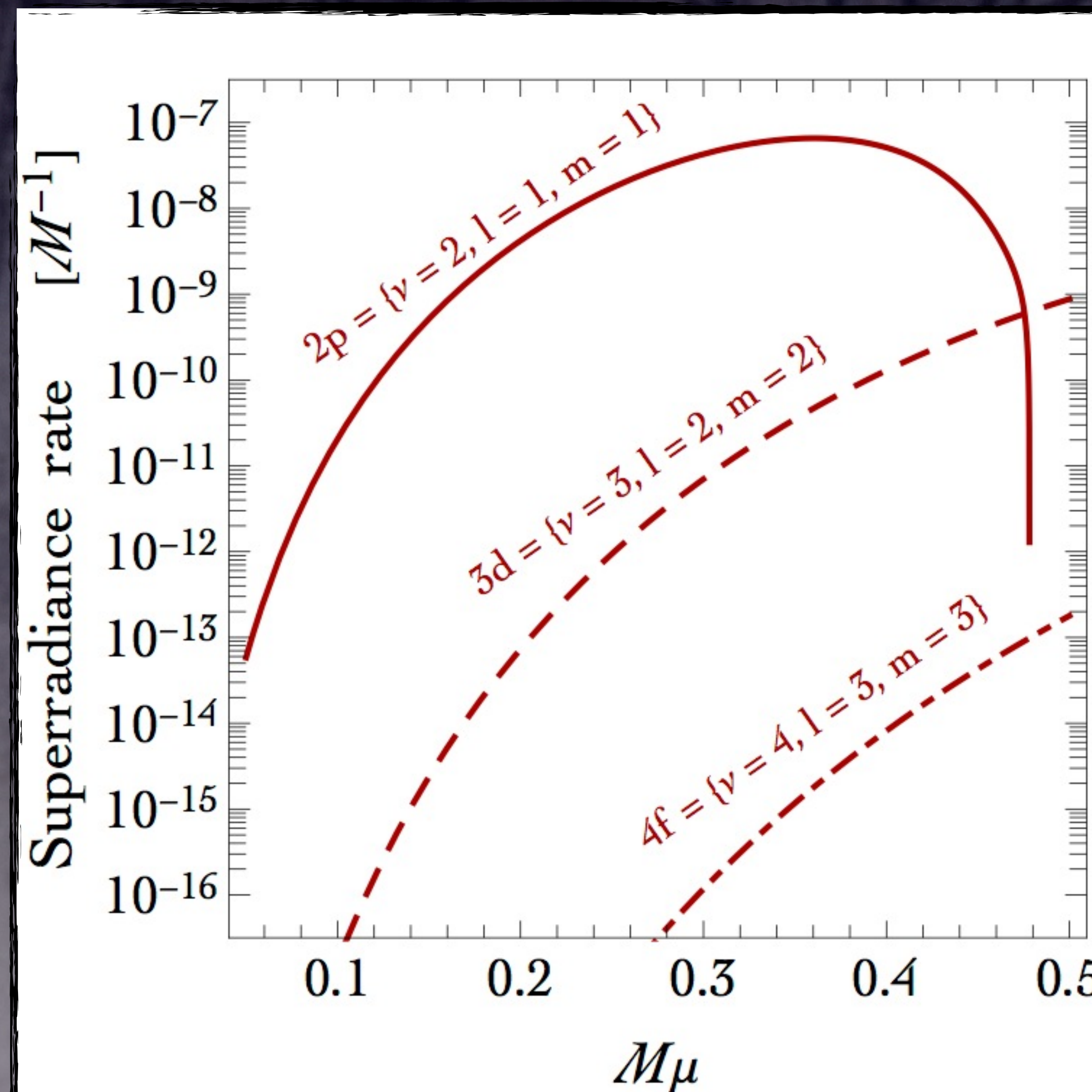


4f	4	0	3	0,±1,±2,±3
…	…	…	…	…

Arvanitaki and S. Dubovsky
Phys. Rev. D 83, 044026
arXiv:1004.3558

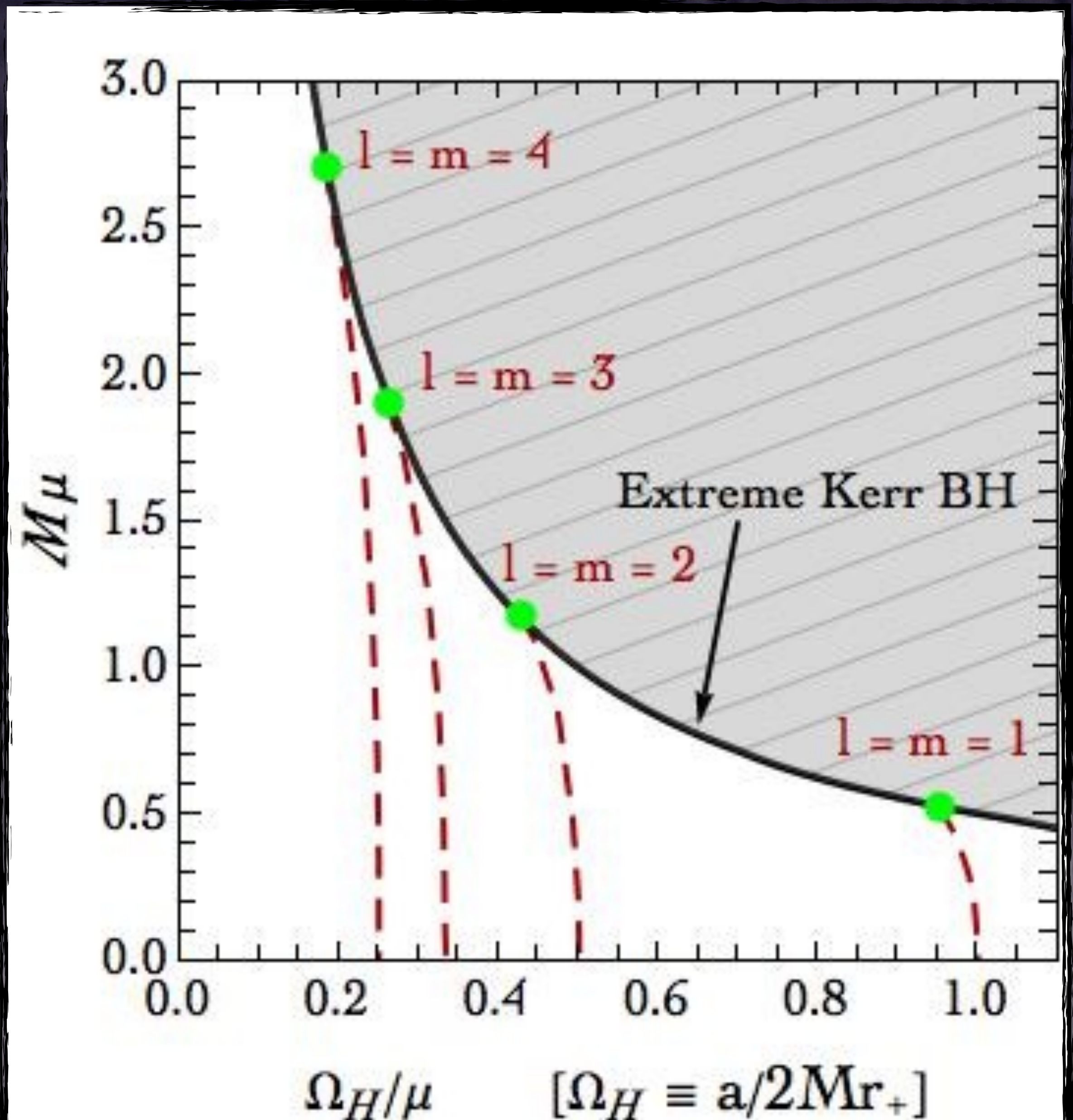
1) We start with a maximally rotating BH. The BH loses its spin favoring the formation of the 2p scalar field configuration.

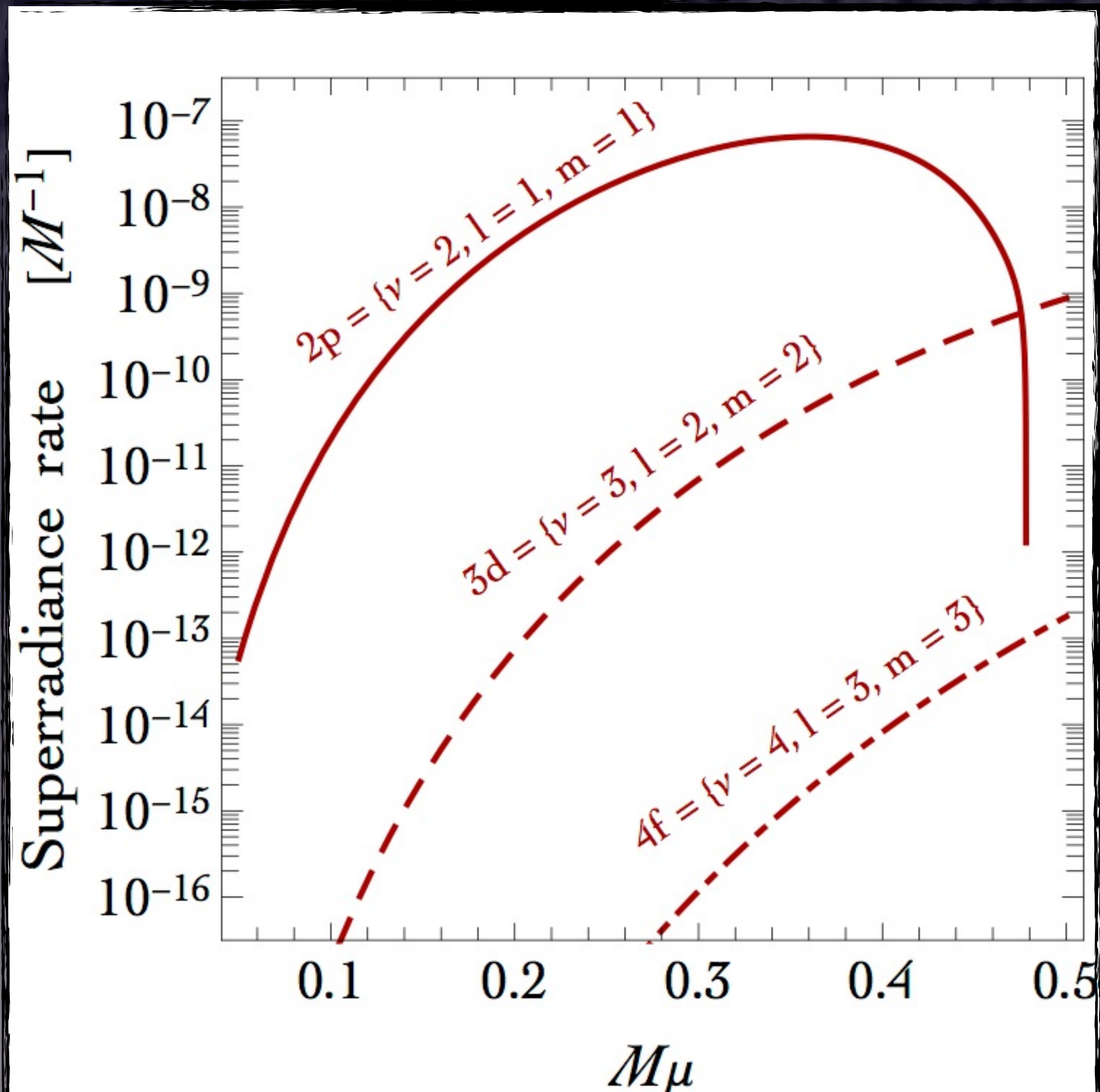




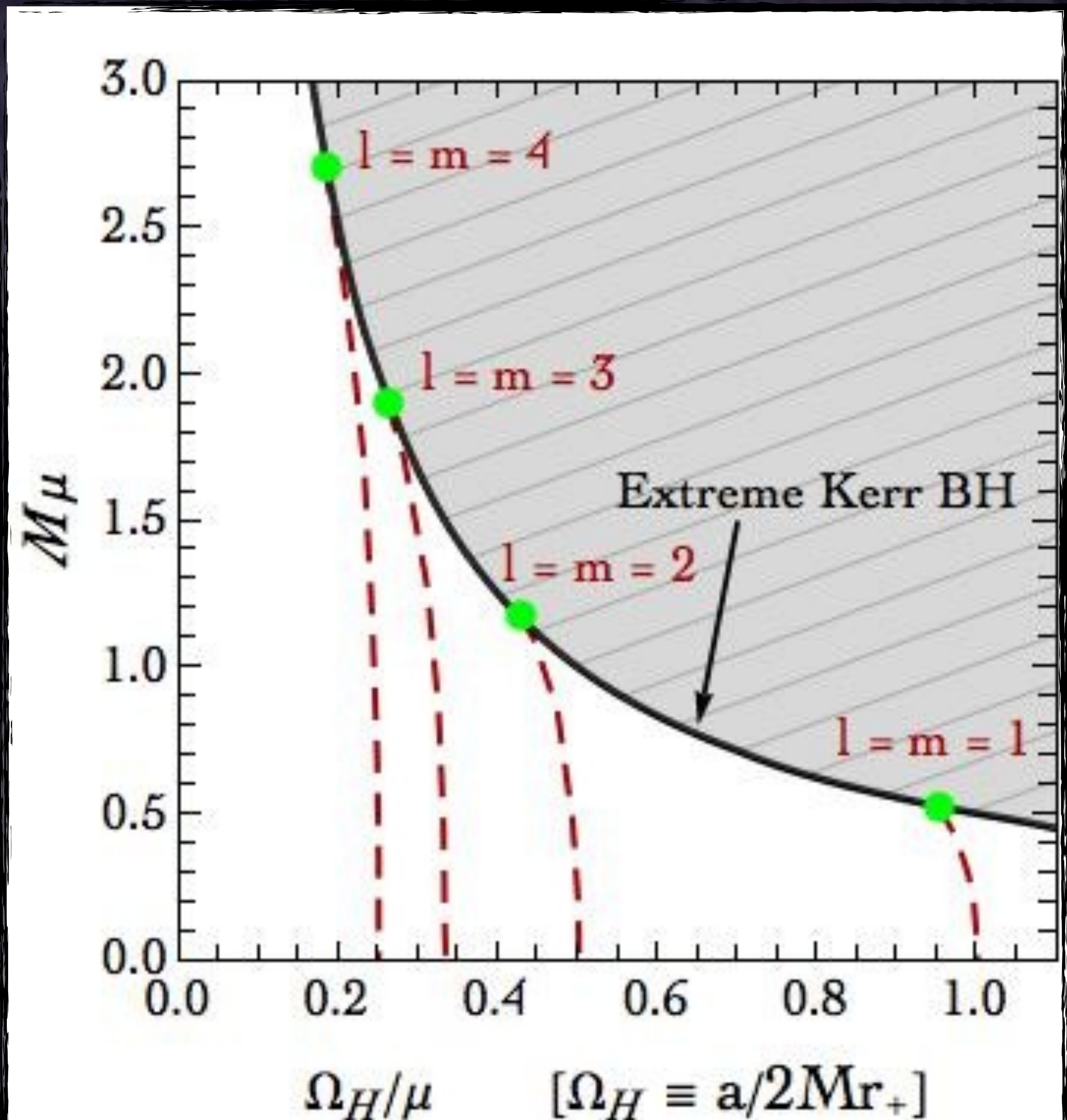
2) The spin-down of the BH continues until it reaches the critical threshold. We said that, at this point, the 2p state is stable.

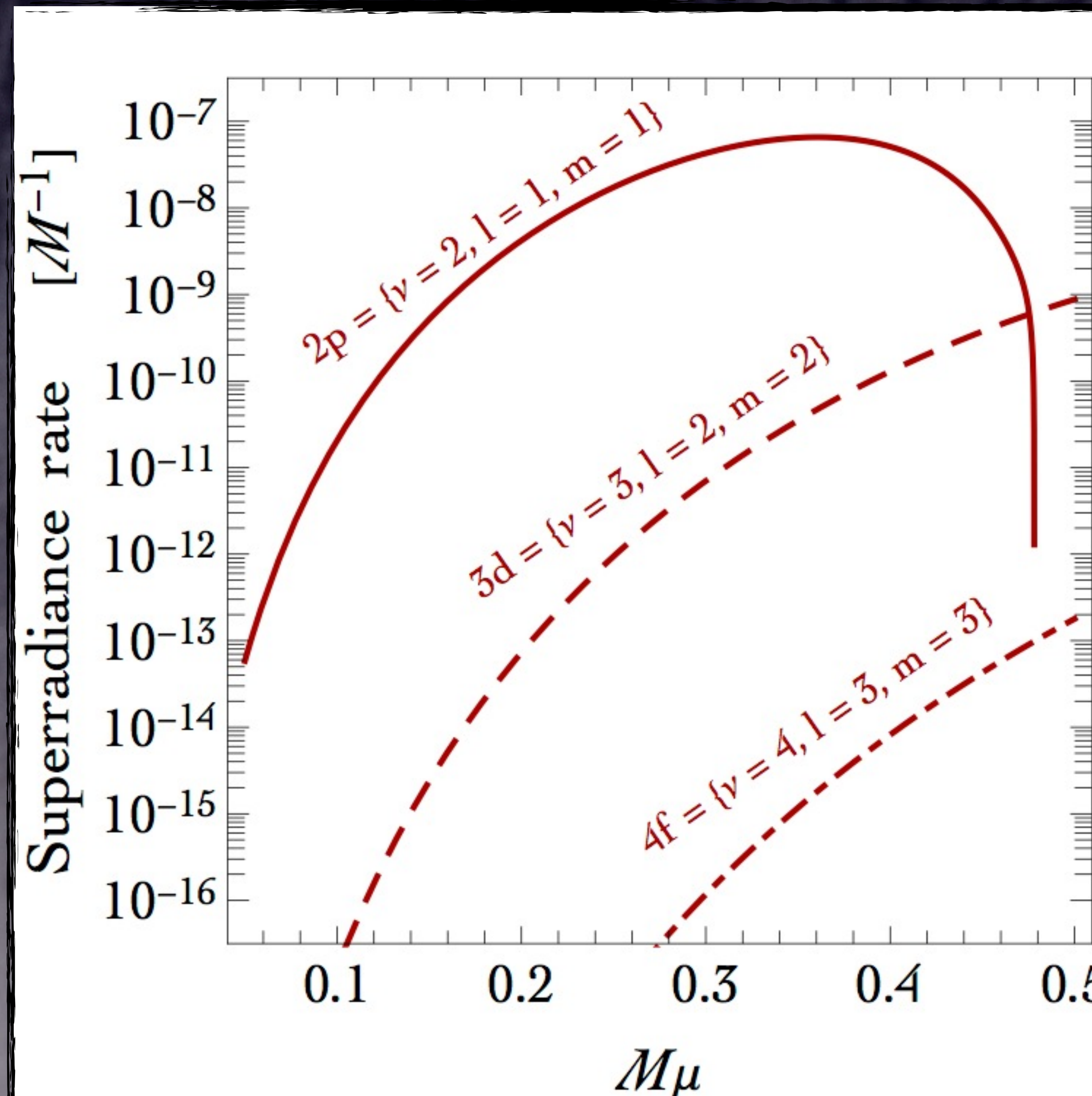
However, this is not entirely true...





3) ...the 2p scalar field configuration loses its energy (annihilation into gravitons and self-interactions are the main effects).





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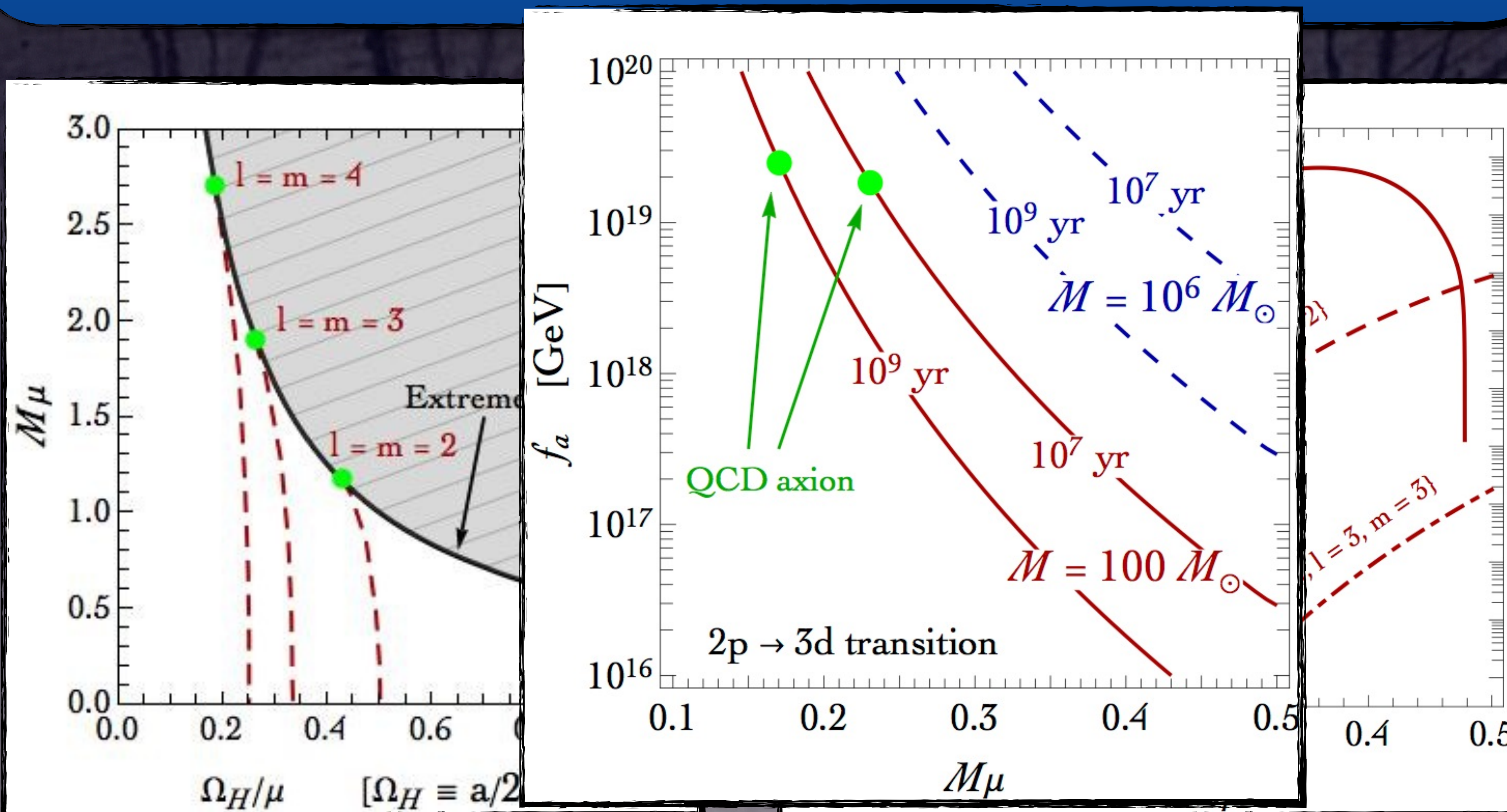
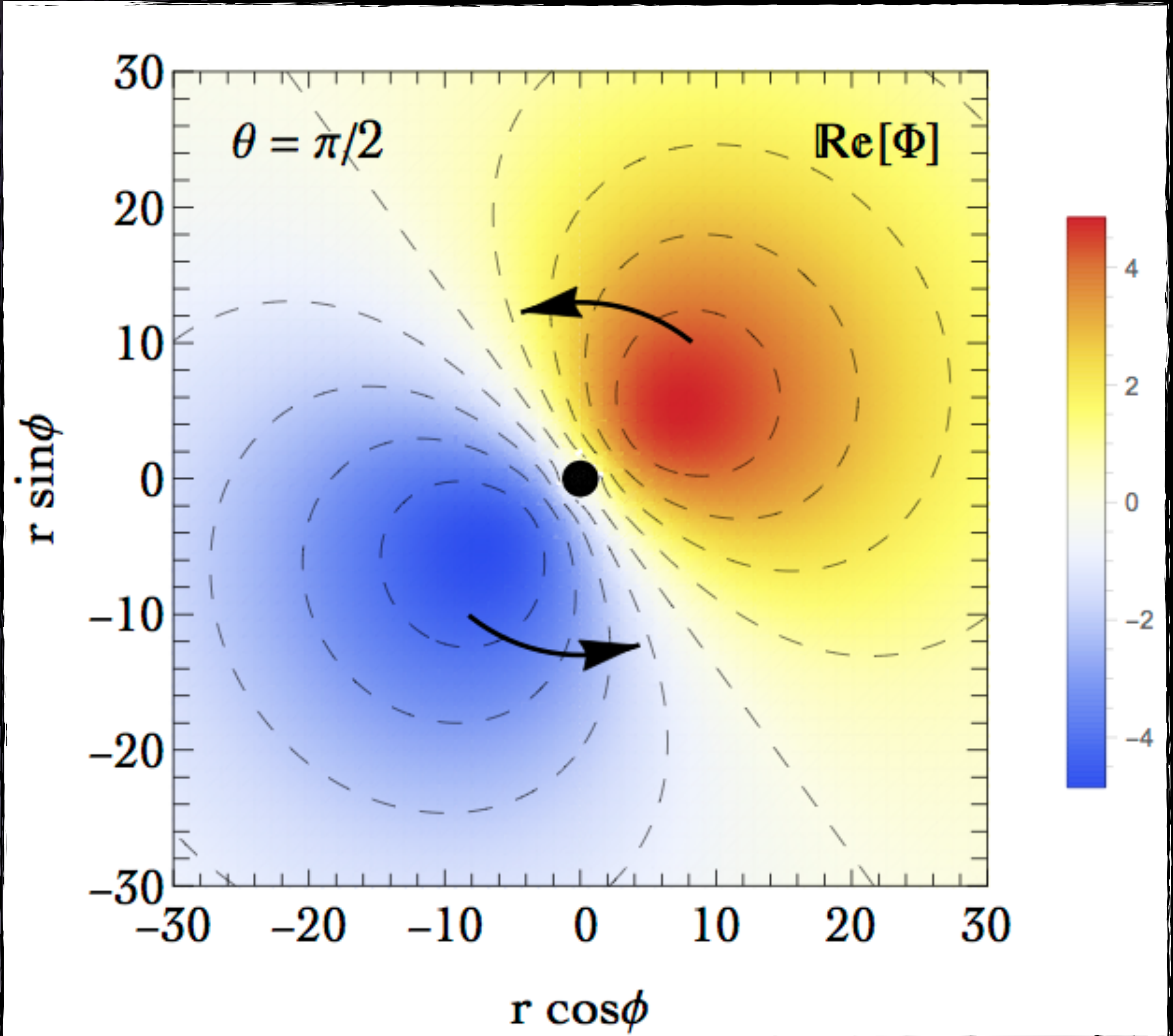


$$M\mu = 7.5 \times 10^{-2} \times \left(\frac{M}{10\, M_\odot}\right) \times \left(\frac{\mu}{10^{-12}\,\mathrm{eV}}\right)$$

Stationary scalar field configuration ("scalar cloud") around a Kerr BH in the 2p level

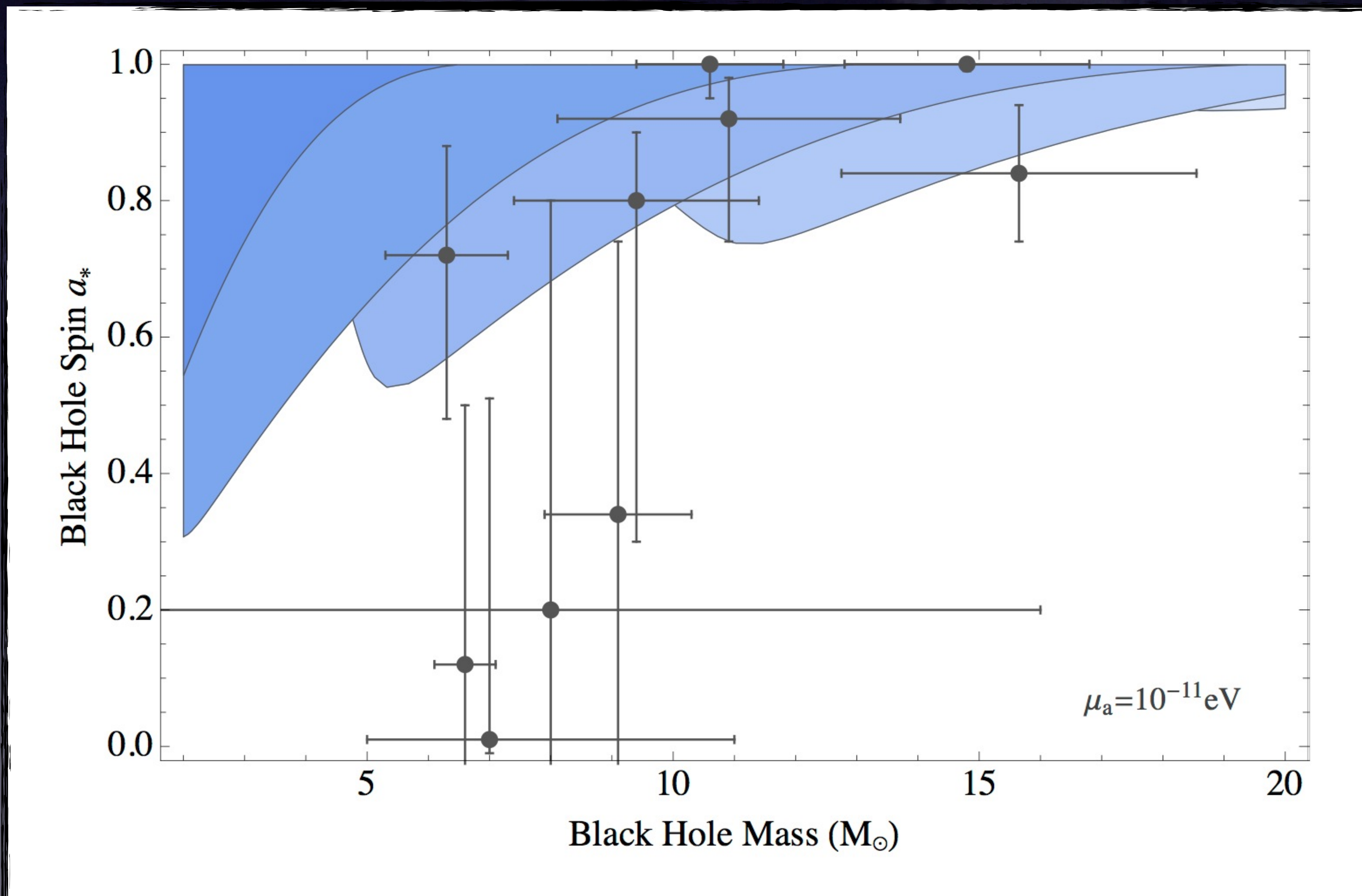


Axions and GR

Axions and GR

Black hole spin and mass measurements from X-ray binaries

Arvanitaki, Baryakhtar, Huang, Phys.Rev. D91 (2015) no.8, 084011, [arxiv/1411.2263]

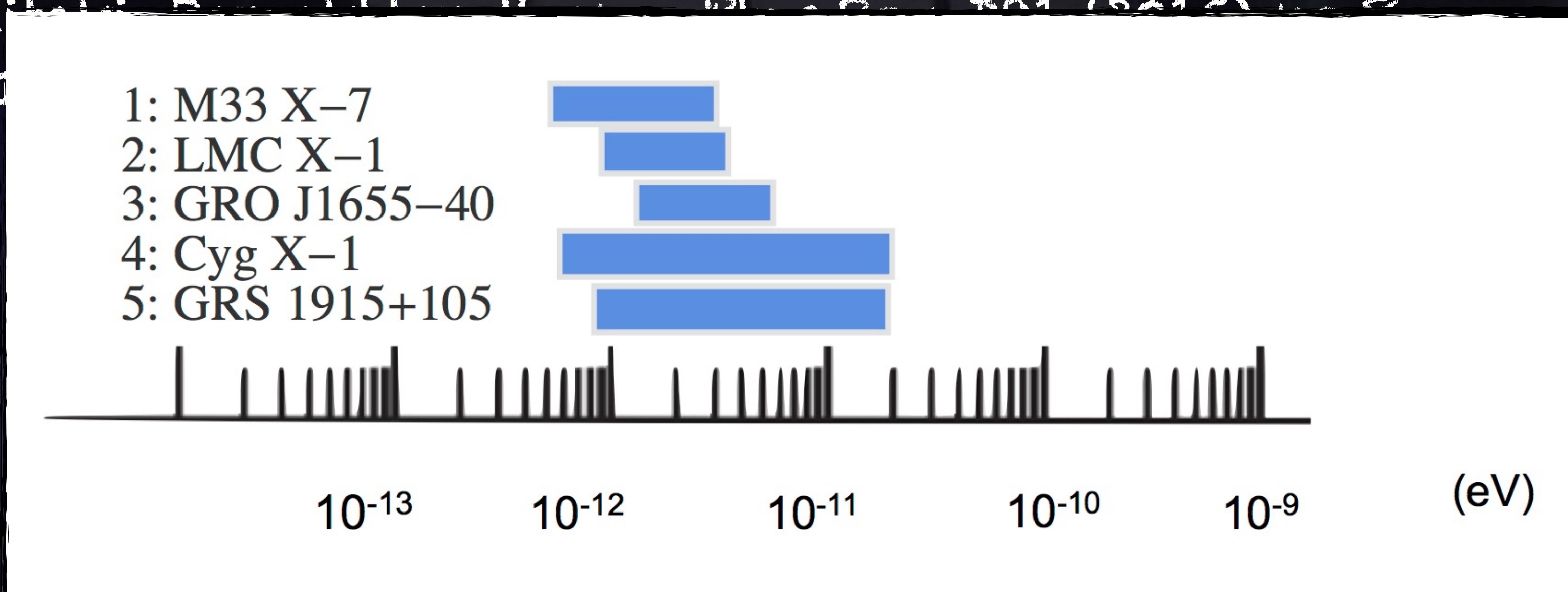


Axions and GR

Black hole spin and mass measurements from X-ray binaries

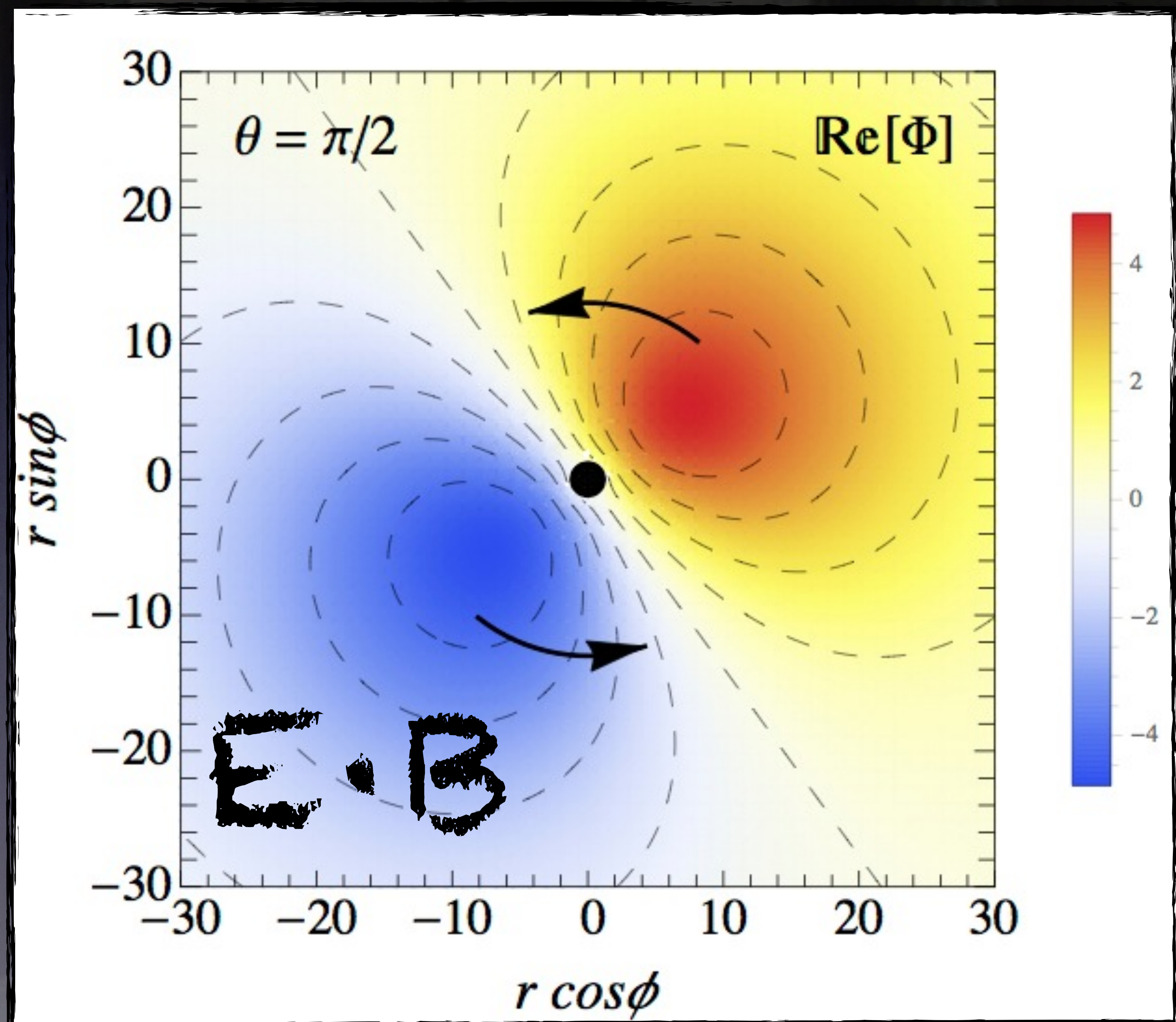
Arvani

0840

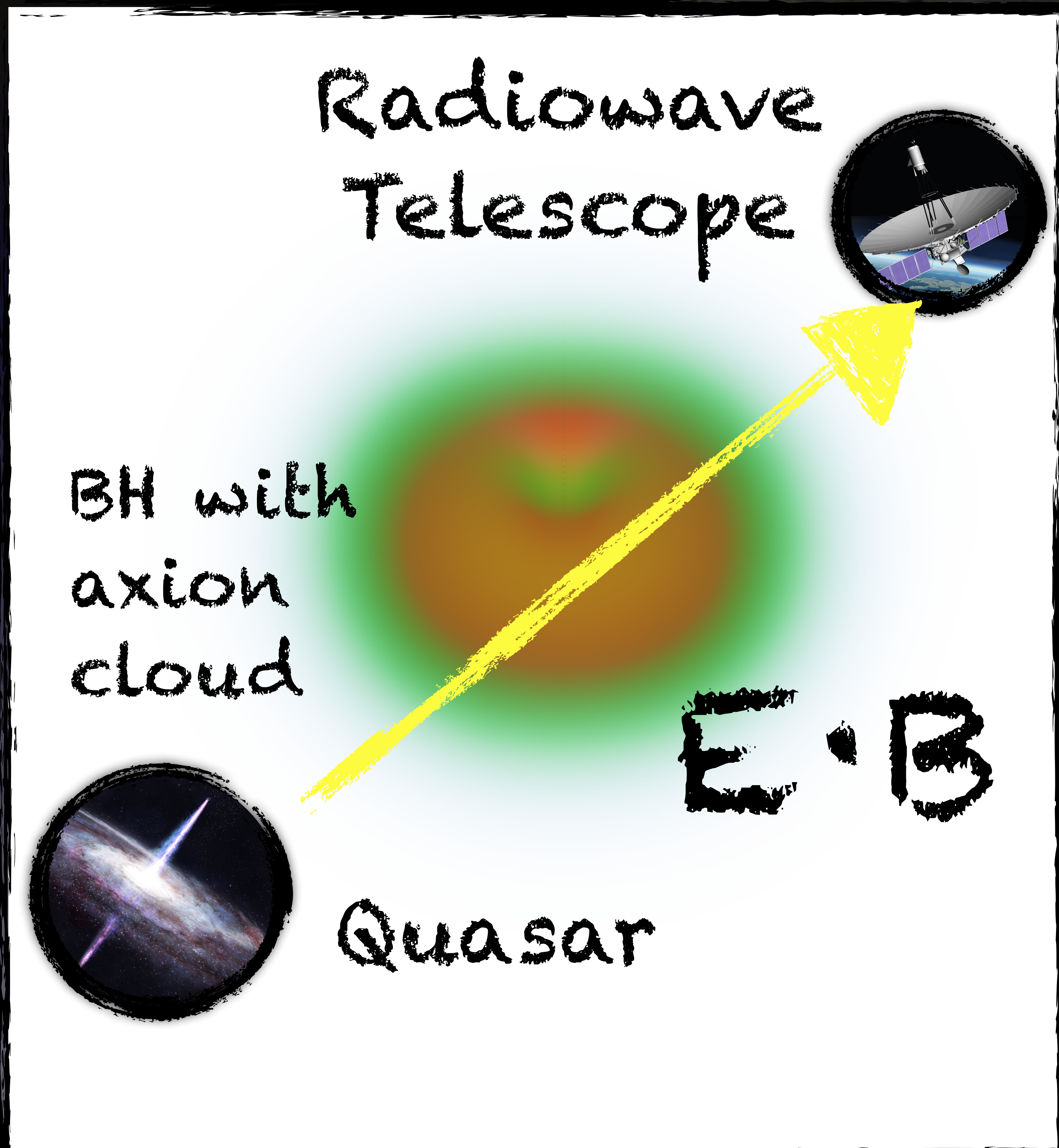


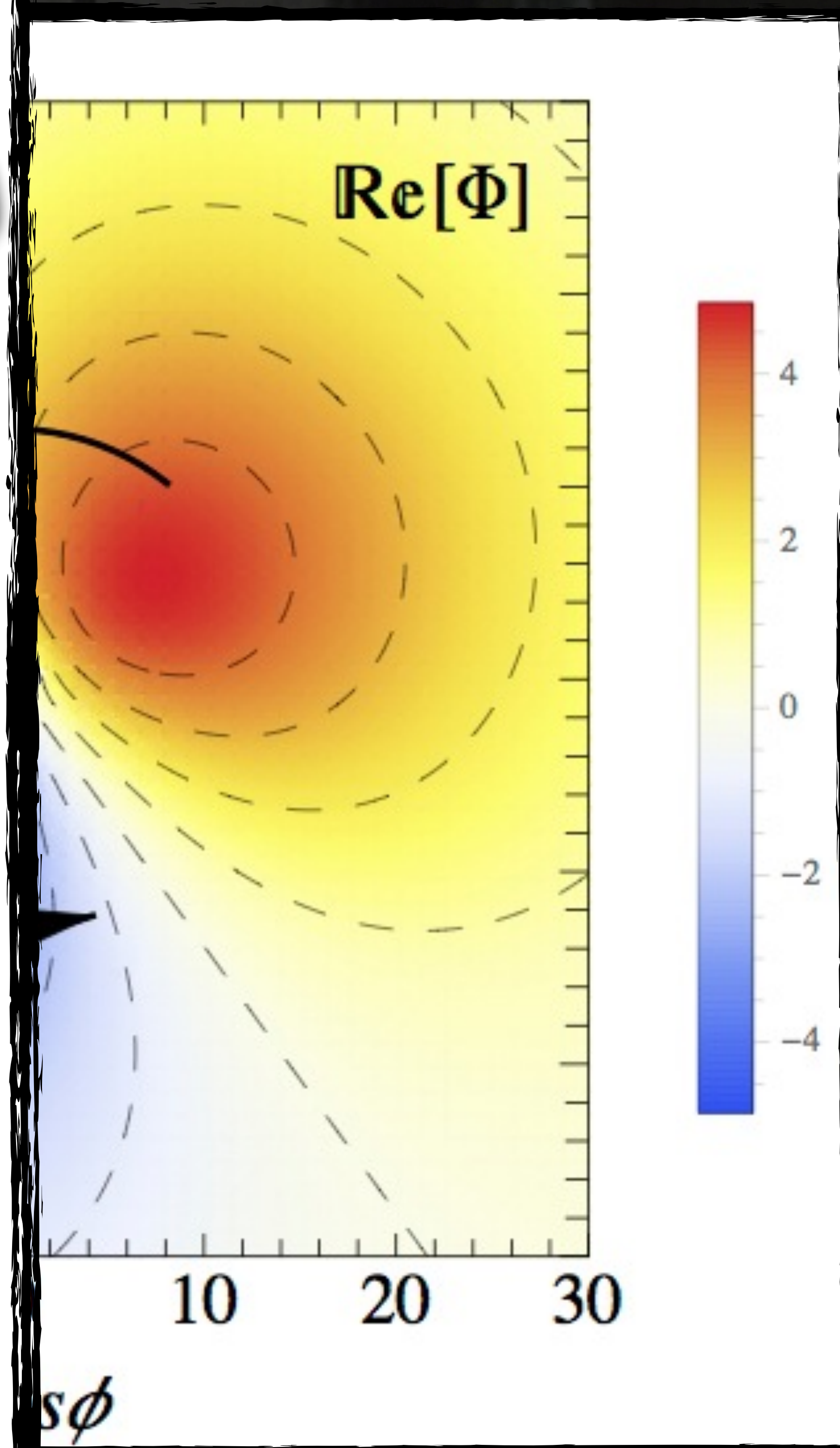
$$2 \times 10^{-11} > \mu_a > 6 \times 10^{-13} \text{ eV}$$
$$3 \times 10^{17} < f_a < 1 \times 10^{19} \text{ GeV}$$

Axions and GR

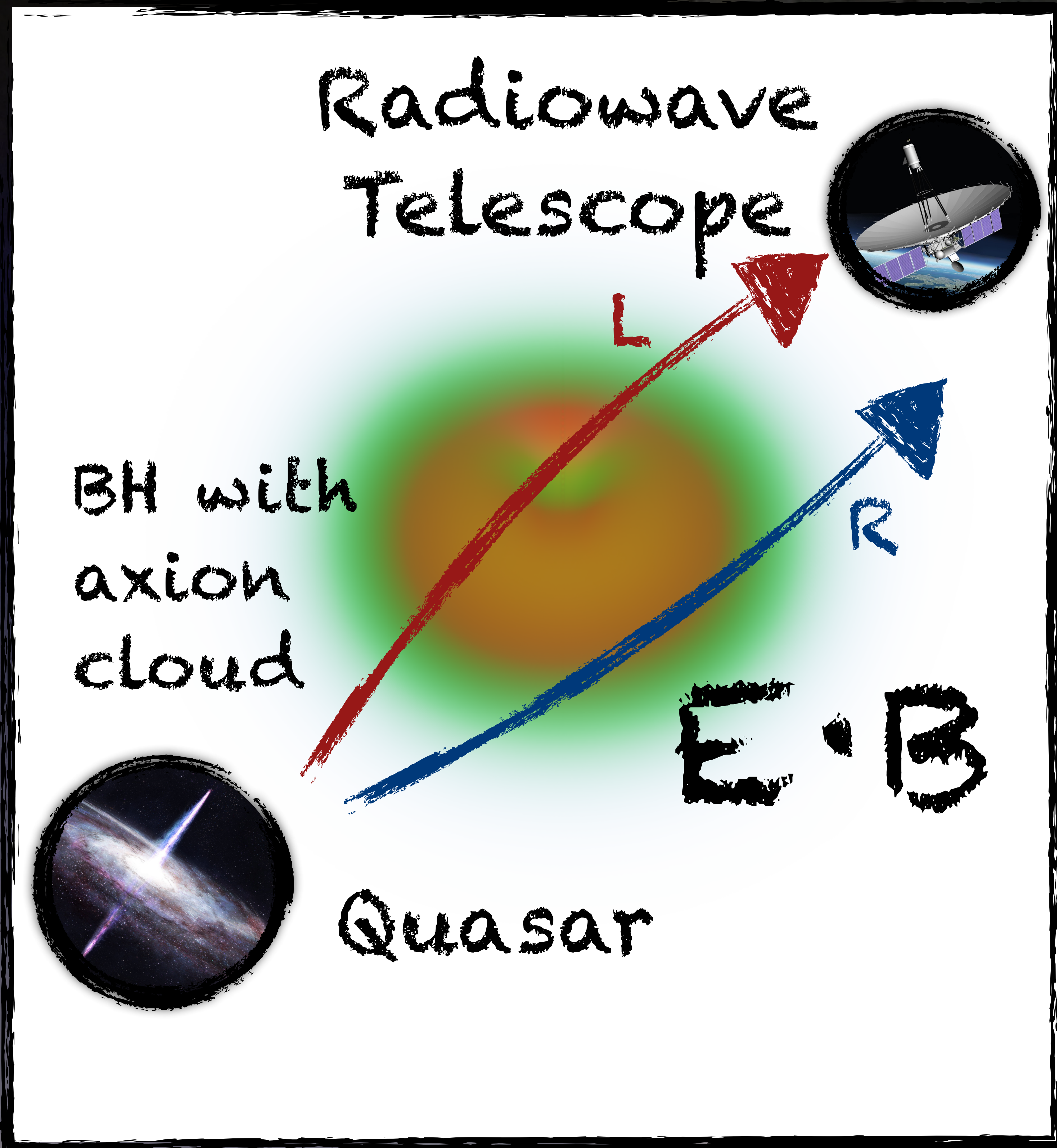


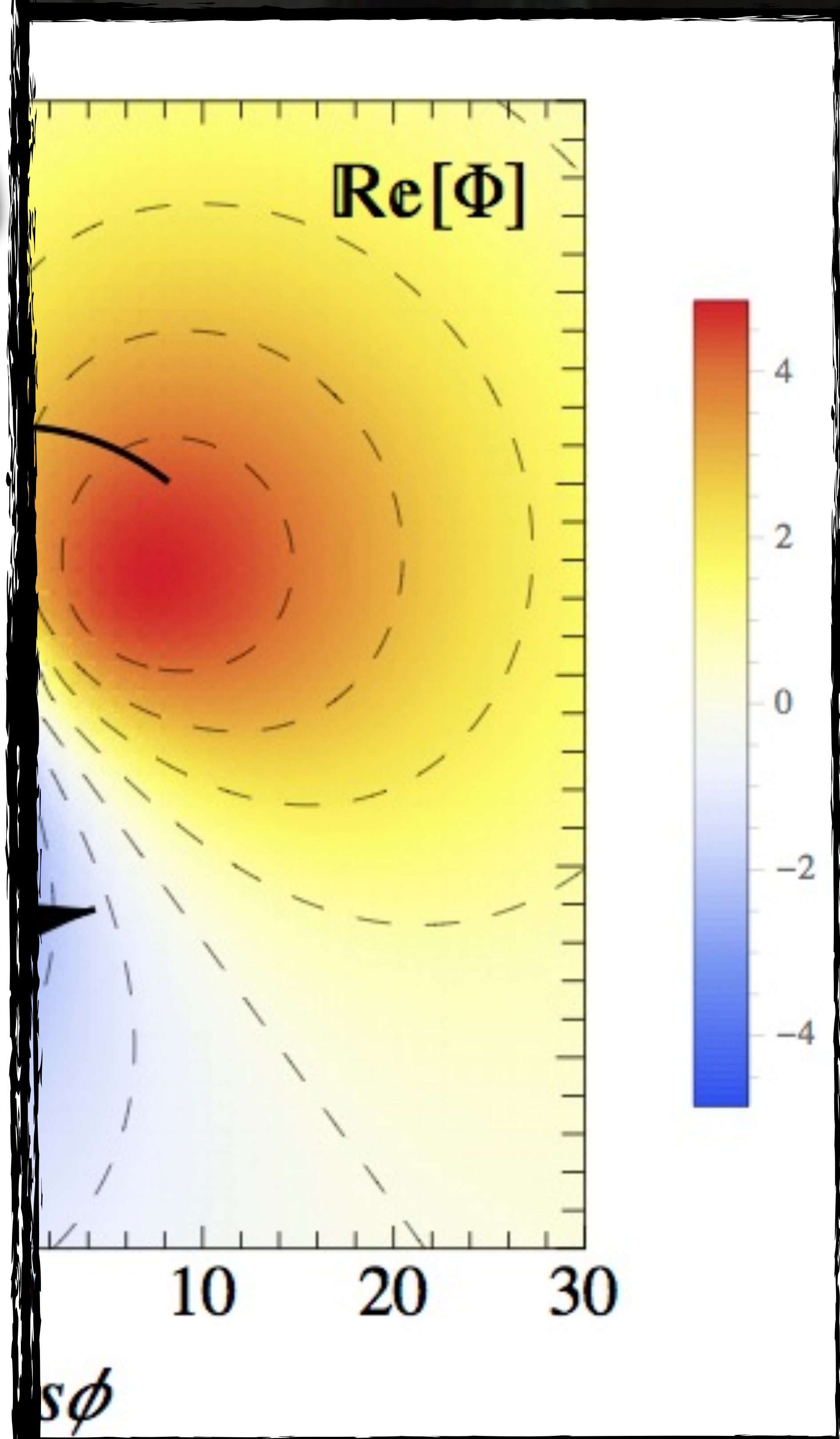
Axions and GR





Axions and GR

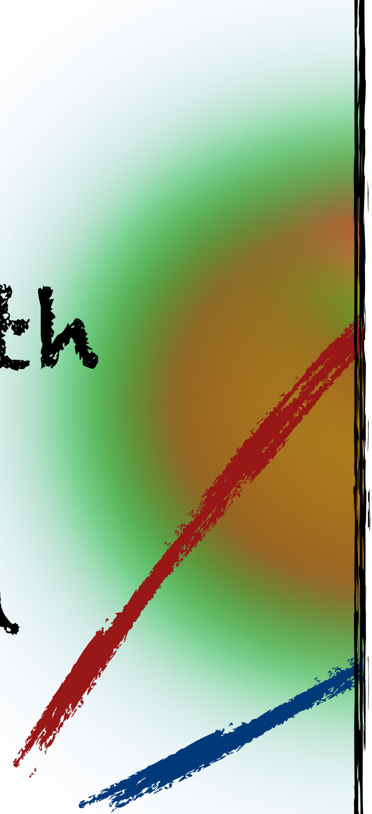




Axio

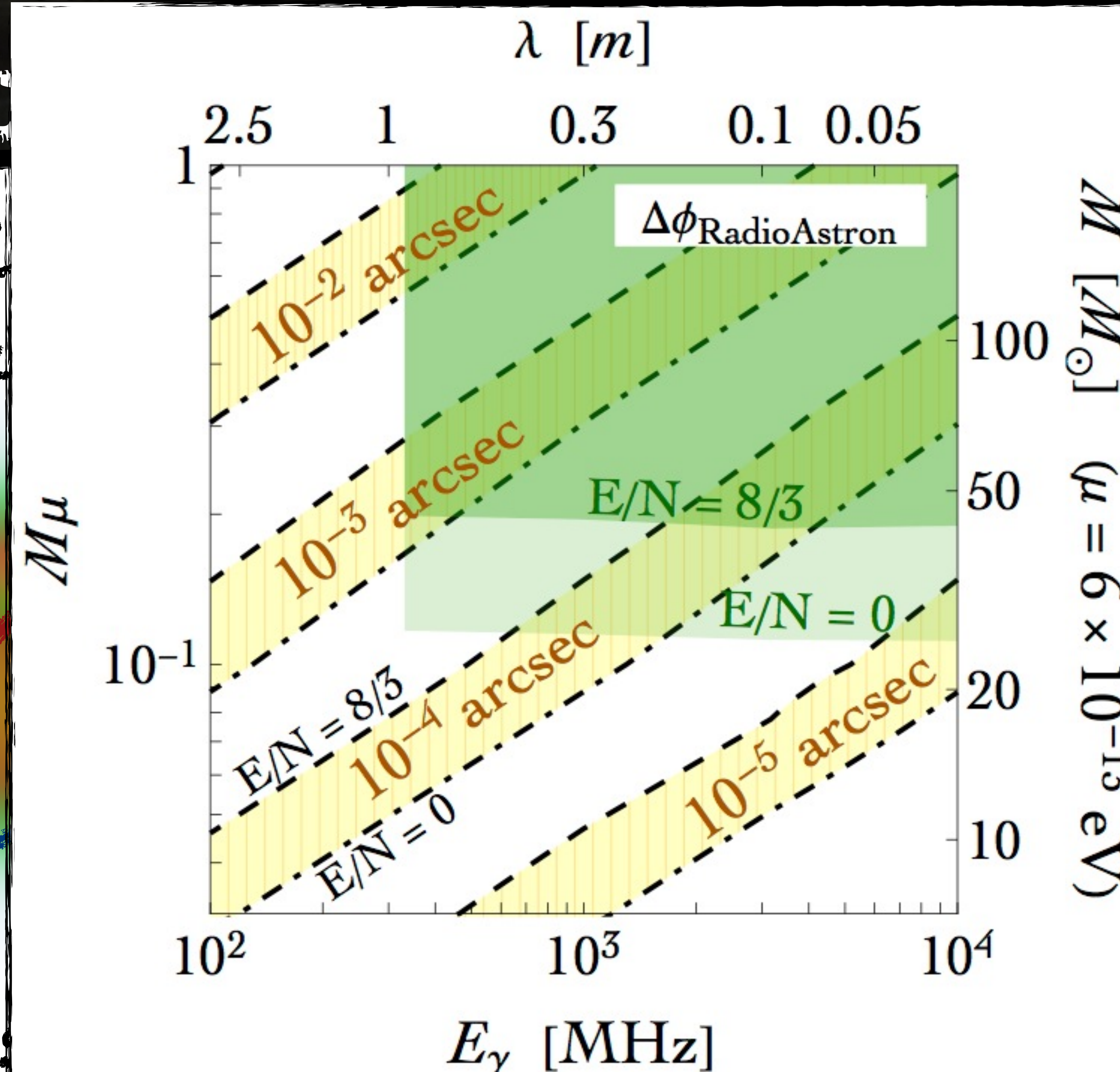
Radi

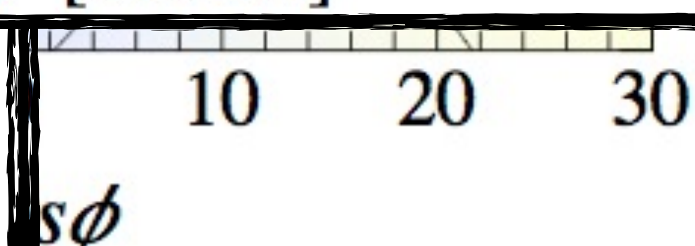
Tele

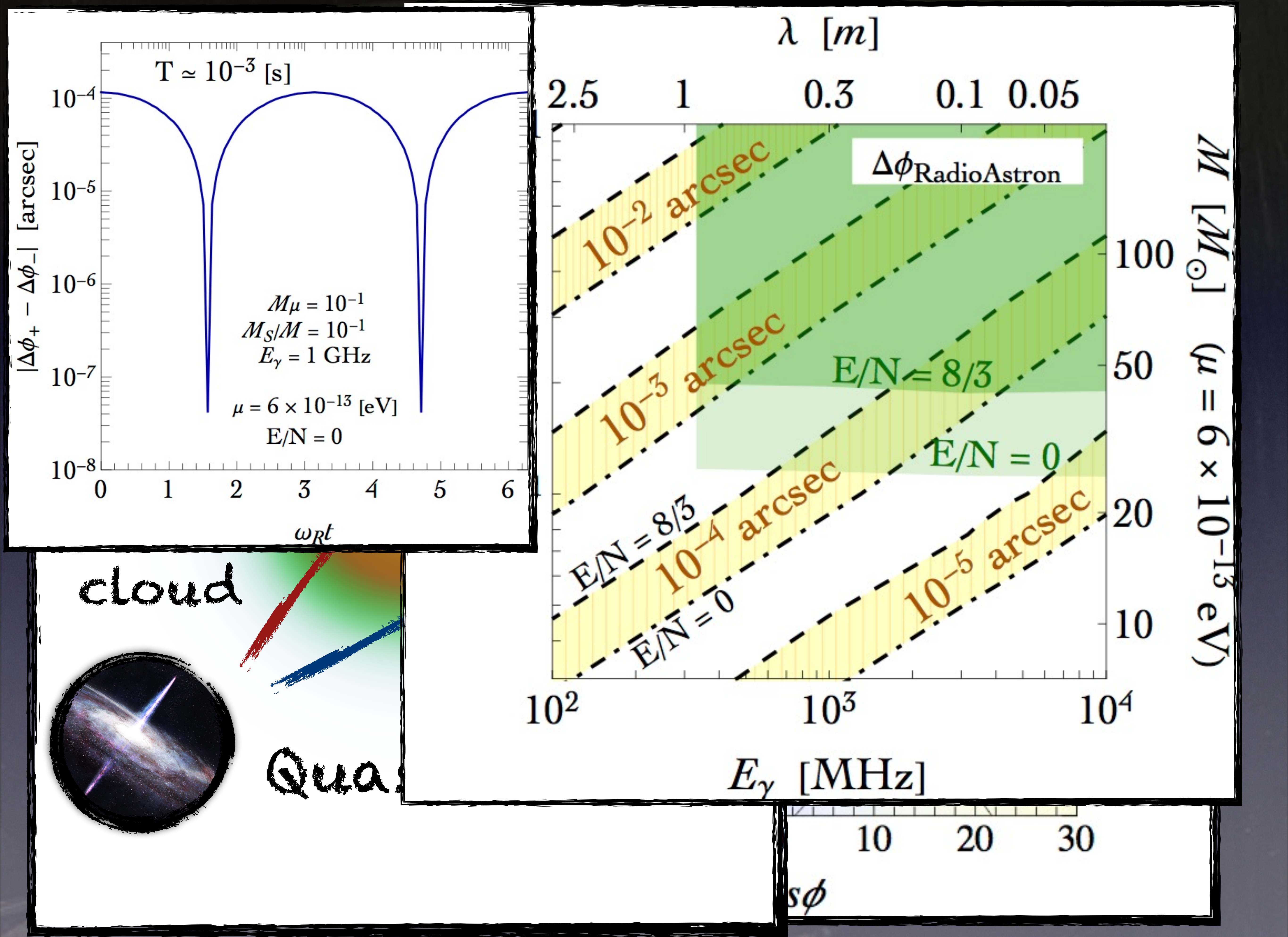


BH with axion cloud

Qua

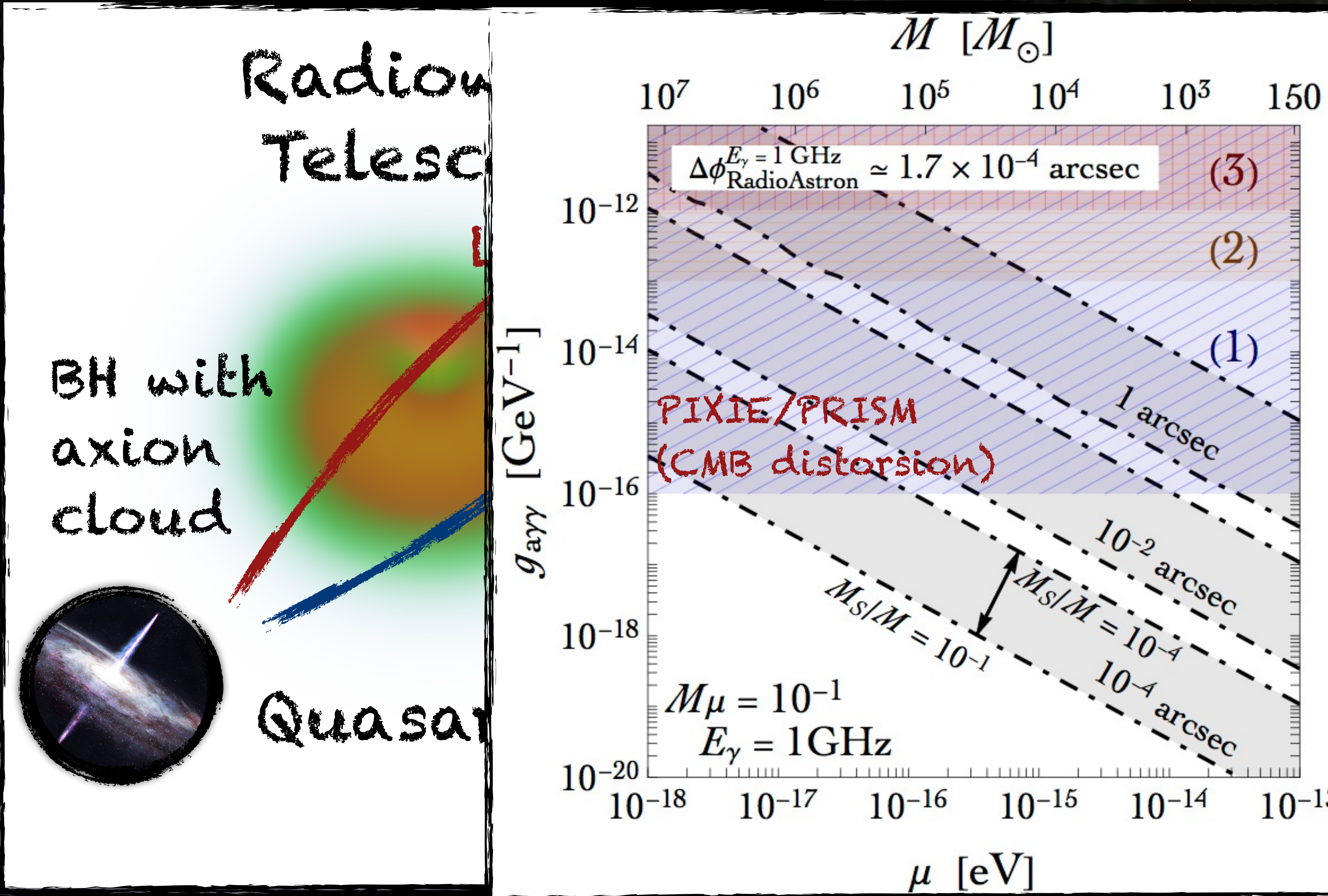




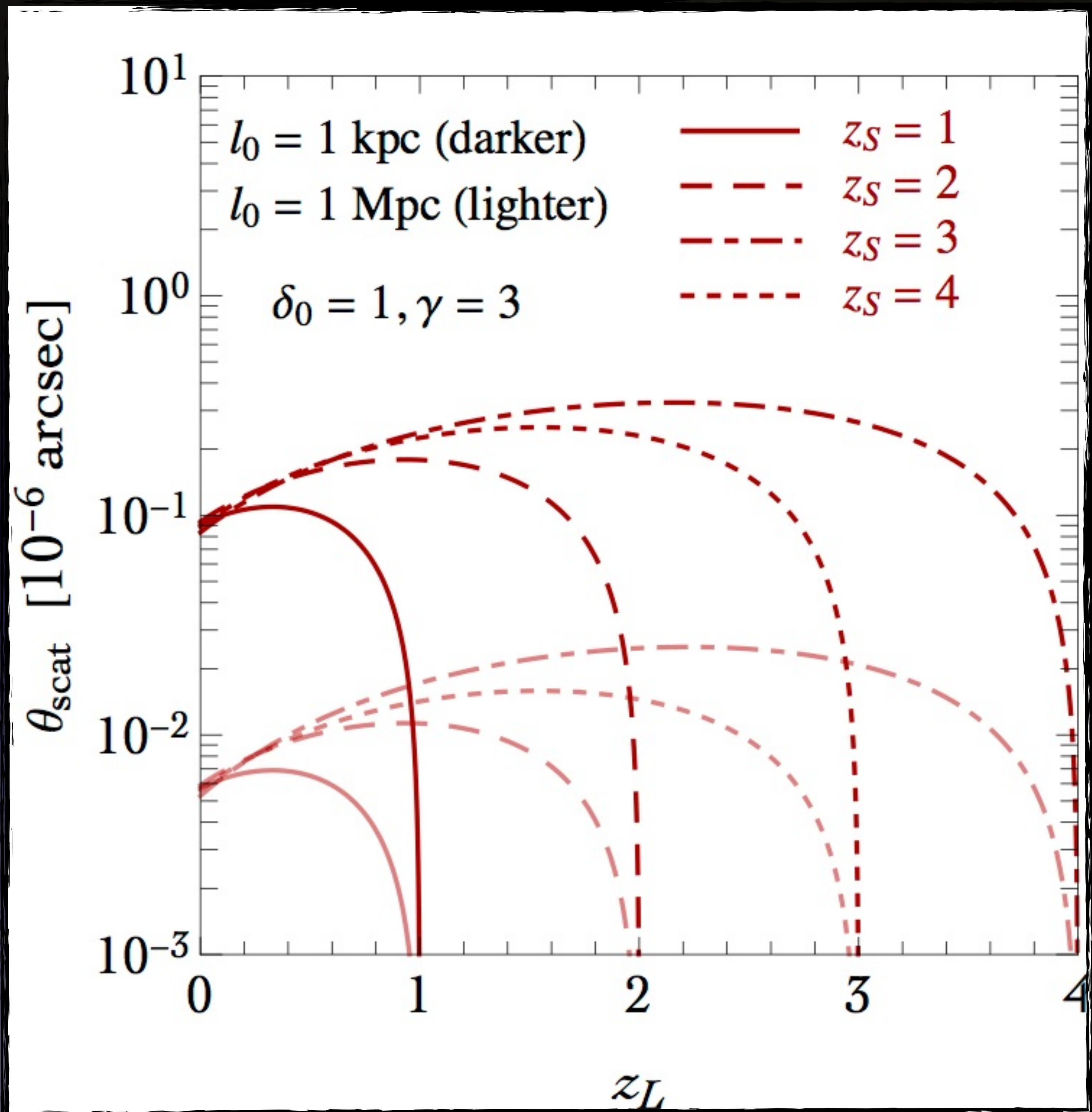


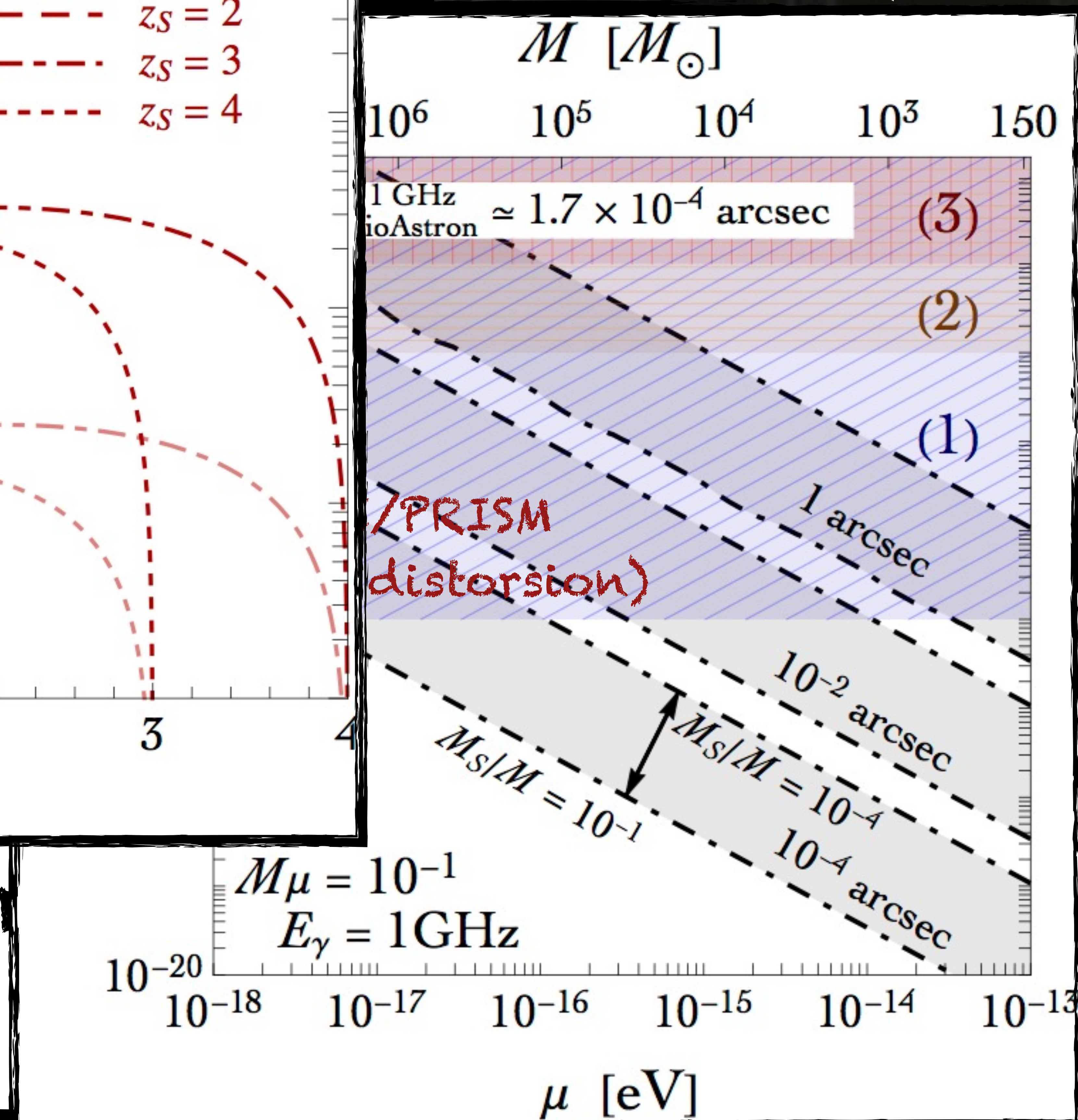
T ≃ 10^-3 [s]
|Δφ+ − Δφ−| [arcsec]
Mμ = 10^-1
M_S/M = 10^-1
E_γ = 1 GHz
μ = 6 × 10^-13 [eV]
E/N = 0
ω_R t
cloud
Qua
λ [m]
2.5 1 0.3 0.1 0.05
Δφ_RadioAstron
10^-2 arcsec
10^-3 arcsec
10^-4 arcsec
10^-5 arcsec
E/N = 8/3
E/N = 0
E/N = 8/3
E/N = 0
M [M_⊙] (μ = 6 × 10^-13 eV)
E_γ [MHz]
10 20 30
sφ

Axions and GR

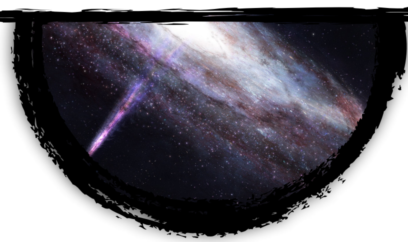


...nd GR





/PRISM
distorsion)

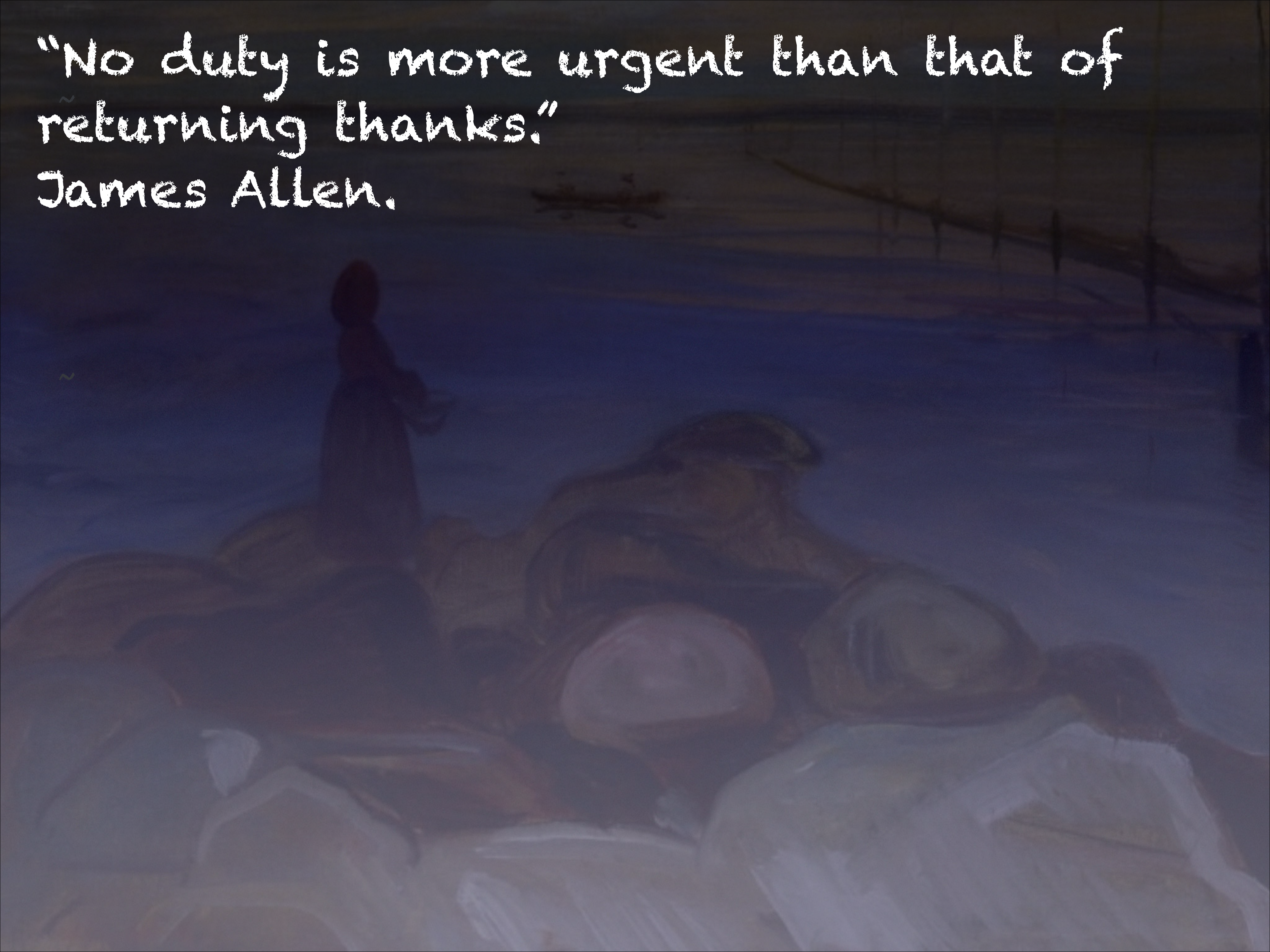


Quasar

Outlook

BSM and GR

Outlook


"No duty is more urgent than that of returning thanks."
James Allen.