On the Importance of Electroweak Corrections for B Anomalies

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Plan of the talk

1. **Strategies to look for New Physics at low-energy**

2. **Anomalies in the semileptonic decays $\bar{B} \to D^{(*)}\ell\bar{\nu}$ and $B \to K\ell^+\ell^-$**

3. **Explaining the anomalies in NP scenarios**

4. **The importance of quantum effects**:
   - Running and matching effective Lagrangians
   - Z and W leptonic coupling modifications
   - generation of a purely leptonic effective Lagrangian
   - corrections to the semileptonic effective Lagrangian

5. **Observables**:
   - LFUV in $\bar{B} \to D^{(*)}\ell\bar{\nu}$ and $B \to K\ell^+\ell^-$
   - LFV B-decays: $B \to K^*\tau\mu$ and $B_s \to \tau\mu$
   - **LFUV in Z and τ decays: $Z \to \ell^+\ell^-$ and $\tau \to \ell\nu\bar{\nu}$**
   - **LFV τ decays: $\tau \to \mu\ell\ell$, $\tau \to \mu\rho$, ...**

6. **Conclusions and future prospects**

NP search strategies

- **High-energy frontier: A unique effort to determine the NP scale**
- **High-intensity frontier (flavor physics): A collective effort to determine the flavor structure of NP**

Where to look for New Physics at low-energy?

- **Processes very suppressed or even forbidden in the SM**
  - FCNC processes ($\mu \to e\gamma$, $\mu \to e$ in N, $\tau \to \mu\gamma$, $\tau \to 3\mu$, $B \to K\tau\mu$, $\cdots$)
  - CPV effects in the electron/neutron EDMs
  - FCNC & CPV in $B_{s,d}$ & D decay/mixing amplitudes
- **Processes predicted with high precision in the SM**
  - EWPO as $(g-2)_\mu$: $a_\mu^{exp} - a_\mu^{SM} \approx (3 \pm 1) \times 10^{-9}$, a discrepancy at 3σ!
  - LFUV in $M \to \ell\nu$ (with $M = \pi, K, B$), $B \to D^{(*)}\ell\nu$, $B \to K\ell\ell'$, τ and Z decays

Experimental status

- **Experimental data in B physics hints at non-standard LFU violations both in charged-current as well as neutral-current transitions:**
  - An overall 3.9σ violation from τ/ℓ universality ($\ell = \mu, e$) in the charged-current $b \to c$ decays [BaBar '13, Belle '15, LHCb '15, Fajfer, Kamenik and Nisandzic '12]

$$R^{\tau/\ell}_{D^{(*)}} = \frac{\mathcal{B}(\bar{B} \to D^{(*)}\tau\bar{\nu})_{\rm exp}/\mathcal{B}(\bar{B} \to D^{(*)}\tau\bar{\nu})_{\rm SM}}{\mathcal{B}(\bar{B} \to D^{(*)}\ell\bar{\nu})_{\rm exp}/\mathcal{B}(\bar{B} \to D^{(*)}\ell\bar{\nu})_{\rm SM}}$$

$$R^{\tau/\ell}_{D} = 1.37 \pm 0.17, \qquad R^{\tau/\ell}_{D^*} = 1.28 \pm 0.08$$

  - A 2.6σ deviation from μ/e universality in the neutral-current $b \to s$ transition

$$R^{\mu/e}_{K} = \frac{\mathcal{B}(B \to K\mu^+\mu^-)_{\rm exp}}{\mathcal{B}(B \to Ke^+e^-)_{\rm exp}} = 0.745^{+0.090}_{-0.074} \pm 0.036$$

while $(R^{\mu/e}_{K})_{SM} = 1$ up to few % corrections [Hiller et al,'07, Bordone, Isidori and Pattori, '16].

Fit results

Coeff.	best fit	1σ	2σ	$\chi^2_{\text{SM}} - \chi^2_{\text{b.f.}}$	pull
C_7^{NP}	-0.04	$[-0.07, -0.01]$	$[-0.10, 0.02]$	2.0	1.4
C_7'	0.01	$[-0.04, 0.07]$	$[-0.10, 0.12]$	0.1	0.2
C_9^{NP}	-1.07	$[-1.32, -0.81]$	$[-1.54, -0.53]$	13.7	3.7
C_9'	0.21	$[-0.04, 0.46]$	$[-0.29, 0.70]$	0.7	0.8
C_{10}^{NP}	0.50	$[0.24, 0.78]$	$[-0.01, 1.08]$	3.9	2.0
C_{10}'	-0.16	$[-0.34, 0.02]$	$[-0.52, 0.21]$	0.8	0.9
$C_9^{\text{NP}} = -C_{10}^{\text{NP}}$	-0.53	$[-0.71, -0.35]$	$[-0.91, -0.18]$	9.8	3.1

Table: Constraints on real WCs. The pull is defined as $\sqrt{\chi^2_{\text{SM}} - \chi^2_{\text{b.f.}}}$. [Altmannshofer & Straub, '15]

$$\mathcal{L}_{\text{eff}}^{\text{NC}} = \frac{4\,G_F}{\sqrt{2}} V_{tb} V_{ts}^* \frac{e^2}{16\pi^2} \sum_i (C_i O_i + C_i' O_i') + \text{h.c.}$$

$$O_7^{(\prime)} = \frac{m_b}{e} (\bar{s}\sigma_{\mu\nu} P_{R(L)} b) F^{\mu\nu}, \quad O_9^{(\prime)} = (\bar{s}\gamma_\mu P_{L(R)} b)(\bar{\ell}\gamma^\mu \ell), \quad O_{10}^{(\prime)} = (\bar{s}\gamma_\mu P_{L(R)} b)(\bar{\ell}\gamma^\mu \gamma_5 \ell)$$

[see also Hiller et al., '14, Hurth et al., '14, Descotes-Genon et al., '15]

Fit results

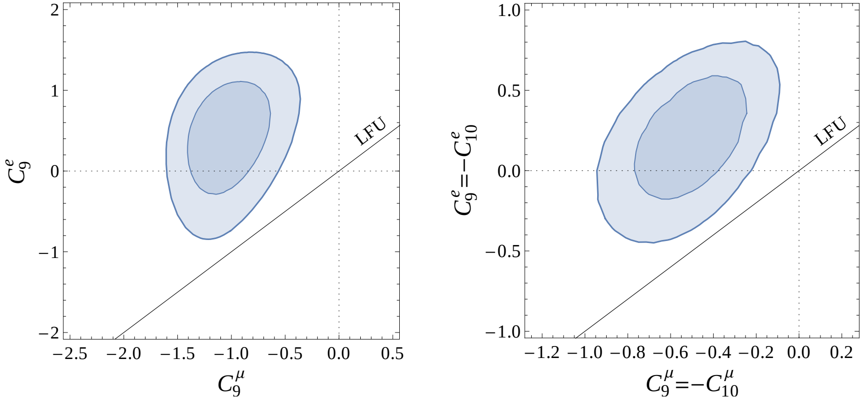


Figure: Best fit regions at 1 and 2σ in the plane C_9^μ vs. C_9^e (left) and $C_9^\mu = -C_{10}^\mu$ vs. $C_9^e = -C_{10}^e$ (right). The diagonal line corresponds to lepton flavour universality.

$$\mathcal{L}_{\rm eff}^{\rm NC} = \frac{4\,G_F}{\sqrt{2}} V_{tb} V_{ts}^* \frac{e^2}{16\pi^2} \sum_i (C_i O_i + C_i' O_i') + \text{h.c.}$$

$$O_9 = (\bar{s}\gamma_\mu P_L b)(\bar{\ell}\gamma^\mu \ell)\,, \qquad O_{10} = (\bar{s}\gamma_\mu P_L b)(\bar{\ell}\gamma^\mu \gamma_5 \ell)$$

[Altmannshofer & Straub, '15, see also Hiller et al., '14, Hurth et al., '14, Descotes-Genon et al., '15]

High-energy effective Lagrangian

- The explanation of the $R_K^{\mu/e}$ anomaly favours an effective 4-fermion operator involving left-handed currents, $(\bar{s}_L\gamma_\mu b_L)(\bar{\mu}_L\gamma_\mu\mu_L)$ [Hiller et al., '14, Hurth et al.,'14, Altmannshofer and Straub '14, Descotes-Genon et al., '15, ...]

- This naturally suggests to account also for the charged-current anomaly through a left-handed operator $(\bar{c}_L\gamma_\mu b_L)(\bar{\tau}_L\gamma_\mu\nu_L)$ which is related to $(\bar{s}_L\gamma_\mu b_L)(\bar{\mu}_L\gamma_\mu\mu_L)$ by the $SU(2)_L$ gauge symmetry [Bhattacharya et al., '14].

- This picture can work only if NP couples much more strongly to the third generation than to the first two. Two interesting scenarios are:

  - **Lepton Flavour Violating case:** NP couples in the interaction basis only to third generations. Couplings to lighter generations are generated by the misalignment between the mass and the interaction bases through small flavour mixing angles. LFU violation necessarily implies LFV [Glashow, Guadagnoli and Lane, '14] .

  - **Lepton Flavour Conserving case:** NP couples to different fermion generations proportionally to their mass squared [Alonso, '15]. The non-abelian leptonic flavour group is broken but $U(1)_e \times U(1)_\mu \times U(1)_\tau$ is preserved.

LFV case: high-energy effective Lagrangian

- In the energy window between the EW scale v and the NP scale Λ, NP effects are described by $\mathcal{L} = \mathcal{L}_{\rm SM} + \mathcal{L}_{\rm NP}$ with $\mathcal{L}$ invariant under $SU(2)_L \otimes U(1)_Y$.

$$\mathcal{L}_{\rm NP} = \frac{C_1}{\Lambda^2} \left(\bar{q}_{3L}\gamma^\mu q_{3L}\right)\left(\bar{\ell}_{3L}\gamma_\mu \ell_{3L}\right) + \frac{C_3}{\Lambda^2} \left(\bar{q}_{3L}\gamma^\mu \tau^a q_{3L}\right)\left(\bar{\ell}_{3L}\gamma_\mu \tau^a \ell_{3L}\right).$$

- After EWSB we move from the interaction to the mass basis through the unitary transformations ($V_u^\dagger V_d = V_{\rm CKM} \equiv V$) [Calibbi, Crivellin, Ota, '15]

$$u_L \to V_u u_L \qquad d_L \to V_d d_L \qquad \nu_L \to U_e \nu_L \qquad e_L \to U_e e_L\,,$$

$$\begin{aligned}
\mathcal{L}_{\rm NP} = \frac{1}{\Lambda^2}\Big[& (C_1 + C_3)\,\lambda^d_{ij}\lambda^e_{kl}\,(\bar{d}_{Li}\gamma^\mu d_{Lj})(\bar{e}_{Lk}\gamma_\mu e_{Ll}) \; + && B \to K\ell\ell' \\
& 2C_3\left(\lambda^{ud}_{ij}\lambda^e_{kl}\,(\bar{u}_{Li}\gamma^\mu d_{Lj})(\bar{e}_{Lk}\gamma_\mu \nu_{Ll}) + h.c.\right) && B \to D^{(*)}\ell\nu \\
& (C_1 - C_3)\,\lambda^d_{ij}\lambda^e_{kl}\,(\bar{d}_{Li}\gamma^\mu d_{Lj})(\bar{\nu}_{Lk}\gamma_\mu \nu_{Ll}) + \cdots \Big] && B \to K\nu\nu
\end{aligned}$$

$$\lambda^d_{ij} = V^*_{d3i} V_{d3j} \qquad \lambda^e_{ij} = U^*_{e3i} U_{e3j} \qquad \lambda^{ud}_{ij} = V^*_{u3i} V_{d3j}$$

Lesson: at tree-level τ LFU & LFV processes are not generated!!

Semileptonic effective Lagrangian

- Effective Lagrangian for $b \to s\ell\ell$ and $b \to s\nu\nu$ [Buchalla et al., '95]

$$\mathcal{L}_{\rm eff}^{\rm NC} = \frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \left(C_\nu^{ij} \mathcal{O}_\nu^{ij} + C_9^{ij} \mathcal{O}_9^{ij} + C_{10}^{ij} \mathcal{O}_{10}^{ij} \right) + h.c. \,,$$

$$\mathcal{O}_\nu^{ij} = \frac{e^2}{(4\pi)^2} (\bar{s}_L \gamma_\mu b_L)(\bar{\nu}_i \gamma^\mu (1-\gamma_5) \nu_j) \,, \qquad \mathcal{O}_{9(10)}^{ij} = \frac{e^2}{(4\pi)^2} (\bar{s}_L \gamma_\mu b_L)(\bar{e}_i \gamma^\mu (\gamma_5) e_j)$$

- By matching $\mathcal{L}_{\rm eff}^{\rm NC}$ with $\mathcal{L}_{\rm NP}$ [Alonso, Grinstein, Camalich, '14, '15 & Calibbi, Crivellin, Ota, '15]

$$(C_9)_{ij} = -\, C_{10}^{ij} = \frac{4\pi^2}{e^2 V_{tb} V_{ts}^*} \frac{v^2}{\Lambda^2} (C_1 + C_3) \, \lambda_{23}^d \lambda_{ij}^e + \cdots \,,$$

$$(C_\nu)_{ij} = \frac{4\pi^2}{e^2 V_{tb} V_{ts}^*} \frac{v^2}{\Lambda^2} (C_1 - C_3) \, \lambda_{23}^d \lambda_{ij}^e + \cdots$$

- Effective Lagrangian for $b \to c\ell\nu$ [Buchalla et al., '95]

$$\mathcal{L}_{\rm eff}^{\rm CC} = -\frac{4G_F}{\sqrt{2}} V_{cb} \, (C_L^{cb})_{ij} \, (\bar{c}_L \gamma_\mu b_L) \, (\bar{e}_{Li} \gamma^\mu \nu_{Lj}) + h.c.$$

- By matching $\mathcal{L}_{\rm eff}^{\rm CC}$ with $\mathcal{L}_{\rm NP}$ [Alonso, Grinstein, Camalich, '14, '15 & Calibbi, Crivellin, Ota, '15]

$$(C_L^{cb})_{ij} = \delta_{ij} - \frac{v^2}{\Lambda^2} \frac{\lambda_{23}^{ud}}{V_{cb}} C_3 \, \lambda_{ij}^e$$

Semileptonic observables

- $B \to K\ell\bar{\ell}$

$$R_K^{\mu/e} \approx \frac{|C_9^{\mu\mu} + C_9^{\rm SM}|^2}{|C_9^{ee} + C_9^{\rm SM}|^2} \approx 1 - 0.28\, \frac{(C_1 + C_3)}{\Lambda^2({\rm TeV})} \frac{\lambda_{23}^d\, |\lambda_{23}^e|^2}{10^{-3}}$$

$$R_K^{\mu/e} = 0.745^{+0.090}_{-0.074} \pm 0.036$$

- $R_{D^{(*)}}^{\tau/\ell}$

$$R_{D^{(*)}}^{\tau/\ell} = \frac{\sum_j |(C_L^{cb})_{3j}|^2}{\sum_j |(C_L^{cb})_{\ell j}|^2} \approx 1 - \frac{0.12\, C_3}{\Lambda^2({\rm TeV})} \left(1 + \frac{\lambda_{23}^d}{V_{cb}}\right) \lambda_{33}^e$$

$$R_D^{\tau/\ell} = 1.37 \pm 0.17, \qquad R_{D^*}^{\tau/\ell} = 1.28 \pm 0.08$$

- $B \to K\nu\bar{\nu}$

$$R_K^{\nu\nu} = \frac{\mathcal{B}(B \to K\nu\bar{\nu})}{\mathcal{B}(B \to K\nu\bar{\nu})_{\rm SM}} = \frac{\sum_{ij} |C_\nu^{\rm SM}\, \delta^{ij} + C_\nu^{ij}|^2}{3|C_\nu^{\rm SM}|^2} \le 4.3$$

$$\approx 1 + \frac{0.6\,(C_1 - C_3)}{\Lambda^2({\rm TeV})} \left(\frac{\lambda_{23}^d}{0.01}\right) + \frac{0.3\,(C_1 - C_3)^2}{\Lambda^4({\rm TeV})} \left(\frac{\lambda_{23}^d}{0.01}\right)^2$$

  - The correct pattern of deviation from the SM is reproduced for $C_3 < 0$, $\lambda_{23}^d < 0$ and $|\lambda_{23}^d / V_{cb}| < 1$. For $|C_3| \sim \mathcal{O}(1)$, we need $\Lambda \sim 1$ TeV and $|\lambda_{23}^e| \gtrsim 0.1$.

Construction of the low-energy effective Lagrangian: running and matching

- We use the renormalization group equations (RGEs) to evolve the effective lagrangian $\mathcal{L}_{\rm NP}$ from $\mu \sim \Lambda$ down to $\mu \sim 1$ GeV. This is done is three steps:
  - In the first step, the RGEs in the unbroken phase of the $SU(2) \otimes U(1)$ theory are used to compute the coefficients in the effective lagrangian down to a scale $\mu \sim m_Z$.
  - In the second step, the coefficients are matched to those of an effective lagrangian for the theory in the broken symmetry phase of $SU(2) \otimes U(1)$, that is $U(1)_{el}$.
  - In the third step, the coefficients of this effective lagrangian are computed at $\mu \sim 1$ GeV using the RGEs for the theory with only $U(1)_{el}$ gauge group.
- Then we take matrix elements of the relevant operators, using perturbative QCD for heavy quarks and chiral perturbation theory for light quark loops.The scale dependence of the RGE contributions cancels with that of the matrix elements.

Operator Product Expansion and the Renormalization Group

- **OPE**: if NP originates at $\Lambda \gg v = 246$ GeV, its effects are described above v by an effective Lagrangian invariant under the SM gauge group:

$$-\mathcal{L}_{NP}^{(0)} = \frac{1}{\Lambda^2} \sum_i C_i O_i^{(0)} + \ldots = \mathcal{H}_{NP}^{(0)}$$

- **Renormalization of composite operators** $O_i^{(0)} = Z_{ij} O_j$**:**

$$\mu \frac{d}{d\mu} O_i^{(0)} = 0 = \left(\mu \frac{d}{d\mu} Z_{ij} \right) O_j + Z_{ij} \left(\mu \frac{d}{d\mu} O_j \right)$$

- **Anomalous dimension** γ_{ij}**:**

$$\mu \frac{d}{d\mu} O_j = -\gamma_{ji} O_i(\mu) \qquad \gamma_{ji} = Z_{jk}^{-1} \left(\mu \frac{d}{d\mu} Z_{ki} \right)$$

- **Running of** C_i (from the condition $\mu \frac{d}{d\mu} \mathcal{H}_{NP} = 0$):

$$\mu \frac{d}{d\mu} C_i = \gamma_{ji} C_j \qquad\qquad C_i(\mu) \approx C_i(\Lambda) - \gamma_{ji}\, C_j(\Lambda) \log \frac{\Lambda}{\mu}$$

- **The amplitude of a physical process is** μ **independent:**

$$A = \langle \mathcal{H}_{NP} \rangle = \frac{1}{\Lambda^2} \sum_i C_i(\mu, \Lambda)\, \langle O_i(\mu) \rangle + \ldots$$

therefore the μ dependence of $C_i(\mu, \Lambda)$ has to cancel that of $\langle O_i(\mu) \rangle$.

Running effective Lagrangian

- In the energy window between the EW scale v and the NP scale Λ, NP effects are described by $\mathcal{L} = \mathcal{L}_{\rm SM} + \mathcal{L}_{\rm NP}$ with $\mathcal{L}$ invariant under $SU(2)_L \otimes U(1)_Y$.

$$\mathcal{L}_{\rm NP} = \frac{C_1}{\Lambda^2} (\bar{q}_{3L}\gamma^\mu q_{3L}) (\bar{\ell}_{3L}\gamma_\mu \ell_{3L}) + \frac{C_3}{\Lambda^2} (\bar{q}_{3L}\gamma^\mu \tau^a q_{3L}) (\bar{\ell}_{3L}\gamma_\mu \tau^a \ell_{3L}) \,.$$

Semileptonic operators	**Leptonic operators**
$[O^{(1)}_{\ell q}]_{prst} = (\bar{\ell}_{pL}\gamma_\mu \ell_{rL}) (\bar{q}_{sL}\gamma^\mu q_{tL})$	$[O_{\ell\ell}]_{prst} = (\bar{\ell}_{pL}\gamma_\mu \ell_{rL}) (\bar{\ell}_{sL}\gamma^\mu \ell_{tL})$
$[O^{(3)}_{\ell q}]_{prst} = (\bar{\ell}_{pL}\gamma_\mu \tau^a \ell_{rL}) (\bar{q}_{sL}\gamma^\mu \tau^a q_{tL})$	$[O_{\ell e}]_{prst} = (\bar{\ell}_{pL}\gamma_\mu \ell_{rL}) (\bar{e}_{sR}\gamma^\mu e_{tR})$
$[O_{\ell u}]_{prst} = (\bar{\ell}_{pL}\gamma_\mu \ell_{rL}) (\bar{u}_{sR}\gamma^\mu u_{tR})$	
$[O_{\ell d}]_{prst} = (\bar{\ell}_{pL}\gamma_\mu \ell_{rL}) (\bar{d}_{sR}\gamma^\mu d_{tR})$	
$[O_{qe}]_{prst} = (\bar{q}_{pL}\gamma_\mu q_{rL}) (\bar{e}_{sR}\gamma^\mu e_{tR})$	
Vector operators	**Hadronic operators**
$[O^{(1)}_{H\ell}]_{pr} = (\varphi^\dagger i \overleftrightarrow{D}_\mu \varphi) (\bar{\ell}_{pL}\gamma_\mu \ell_{rL})$	$[O^{(1)}_{qq}]_{prst} = (\bar{q}_{pL}\gamma_\mu q_{rL}) (\bar{q}_{sL}\gamma^\mu q_{tL})$
$[O^{(3)}_{H\ell}]_{pr} = (\varphi^\dagger i \overleftrightarrow{D}^a_\mu \varphi) (\bar{\ell}_{pL}\gamma_\mu \tau^a \ell_{rL})$	$[O^{(3)}_{qq}]_{prst} = (\bar{q}_{pL}\gamma_\mu \tau^a q_{rL}) (\bar{q}_{sL}\gamma^\mu \tau^a q_{tL})$
$[O^{(1)}_{Hq}]_{pr} = (\varphi^\dagger i \overleftrightarrow{D}_\mu \varphi) (\bar{q}_{pL}\gamma_\mu q_{rL})$	$[O^{(1)}_{qu}]_{prst} = (\bar{q}_{pL}\gamma_\mu q_{rL}) (\bar{u}_{sR}\gamma^\mu u_{tR})$
$[O^{(3)}_{Hq}]_{pr} = (\varphi^\dagger i \overleftrightarrow{D}^a_\mu \varphi) (\bar{q}_{pL}\gamma_\mu \tau^a q_{rL})$	$[O^{(1)}_{qd}]_{prst} = (\bar{q}_{pL}\gamma_\mu q_{rL}) (\bar{d}_{sR}\gamma^\mu d_{tR})$

Table: Minimal set of gauge-invariant operators involved in the RGE flow.

Leptonic Z-coupling modifications

- $\mathcal{L}_{\rm NP}$ induces modification of the W and Z couplings

$$\mathcal{L}_{\rm NP} = \frac{1}{\Lambda^2}\Big[(C_1 + C_3)\,\lambda^u_{ij}\lambda^e_{kl}\,(\bar{u}_{Li}\gamma^\mu u_{Lj})(\bar{\nu}_{Lk}\gamma_\mu \nu_{Ll}) + (C_1 - C_3)\,\lambda^u_{ij}\lambda^e_{kl}\,\bar{(}u_{Li}\gamma^\mu u_{Lj})(\bar{e}_{Lk}\gamma_\mu e_{Ll}) \; + \ldots\Big]$$

$$\mathcal{L}_Z = \frac{g_2}{c_W}\bar{e}_i\left(\not{Z} g^{ij}_{\ell L} P_L + \not{Z} g^{ij}_{\ell R} P_R\right) e_j + \frac{g_2}{c_W}\bar{\nu}_{Li}\,\not{Z}\, g^{ij}_{\nu L}\,\nu_{Lj}$$

$$\Delta g^{ij}_{\ell L} \simeq \frac{v^2}{\Lambda^2}\left(3y_t^2(C_1 - C_3)\lambda^u_{33} + g_2^2 C_3\right)\log\left(\frac{\Lambda}{m_Z}\right)\frac{\lambda^e_{ij}}{16\pi^2}$$

$$\Delta g^{ij}_{\nu L} \simeq \frac{v^2}{\Lambda^2}\left(3y_t^2(C_1 + C_3)\lambda^u_{33} - g_2^2 C_3\right)\log\left(\frac{\Lambda}{m_Z}\right)\frac{\lambda^e_{ij}}{16\pi^2}$$

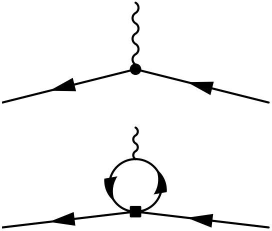


Figure: Z couplings with fermions. Upper: RGE induced coupling. Lower: one-loop diagram with a tree-level 4-fermion interaction.

- These expressions provide a good approximation of the **exact results** obtained adding to the **RGE** contributions from gauge and top yukawa interactions the **one-loop matrix element** with the Z four-momentum set on the mass-shell.

- The scale dependence of the RGE contribution cancels with that of the matrix element dominated by a quark loop.

- **LEP bounds on non-universal leptonic Z couplings** [PDG]

$$\frac{v_\tau}{v_e} = 0.959 \pm 0.029\,, \qquad \frac{a_\tau}{a_e} = 1.0019 \pm 0.0015$$

$v_\ell = g^{\ell\ell}_{\ell L} + g^{\ell\ell}_{\ell R}$ and $a_\ell = g^{\ell\ell}_{\ell L} - g^{\ell\ell}_{\ell R}$ are the vector and axial-vector couplings

$$\frac{v_\tau}{v_e} \simeq 1 - \frac{2\,\Delta g^{33}_{\ell L}}{(1-4s_W^2)} \approx 1 - 0.05\,\frac{(c_- + 0.2\,C_3)}{\Lambda^2(\mathrm{TeV})}$$

$$\frac{a_\tau}{a_e} \simeq 1 - 2\,\Delta g^{33}_{\ell L} \approx 1 - 0.004\,\frac{(c_- + 0.2\,C_3)}{\Lambda^2(\mathrm{TeV})}\,,$$

- **Number of neutrinos N_ν from the invisible Z decay width**

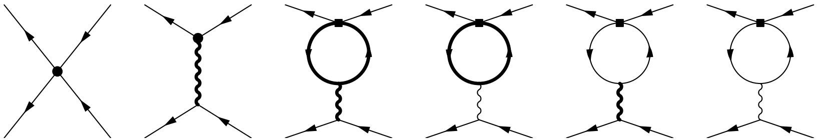
$$N_\nu = 2 + \left(\frac{g^{33}_{\nu L}}{g^{\rm SM}_{\nu L}}\right)^2 \simeq 3 + 4\,\Delta g^{33}_{\nu L} \approx 3 + 0.008\,\frac{(c_+ - 0.2\,C_3)}{\Lambda^2(\mathrm{TeV})}$$

to be compared with the experimental result [PDG]

$$N_\nu = 2.9840 \pm 0.0082$$

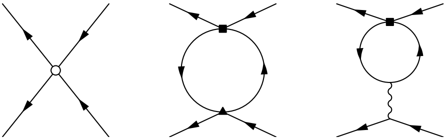
- $\mathcal{B}(Z \to \mu^\pm \tau^\mp)$ is always well below the current experimental bound.

Purely leptonic effective Lagrangian



(1.a) (1.b) (1.c) (1.d) (1.e) (1.f)

Figure: Diagrams contributing to the Wilson coefficient of $(\bar{e}_{iL}\gamma_\mu e_{jL})(\bar{e}_{kL}\gamma^\mu e_{nL})$ above m_{EW}. Thick (thin) lines denote heavy (light) fields. The tree level diagrams (1.a) and (1.b) arise from RGE effects (full circle). The loop-induced diagrams (1.c)–(1.f) come from tree-level four-fermion interactions (square) and cancel the μ dependence of (1.a), (1.b).



(2.a) (2.b) (2.c)

Figure: Diagrams contributing to the Wilson coefficient of $(\bar{e}_{iL}\gamma_\mu e_{jL})(\bar{e}_{kL}\gamma^\mu e_{nL})$ below m_{EW}. Only light fields are present. The four-fermion interaction (empty circle) of (2.a) is the unknown of the matching procedure. The triangle in (2.b) denotes the SM Fermi interaction. In the matching condition, (2.b) is canceled by (1.e) and (2.c) by (1.f).

Purely leptonic effective Lagrangian

- **Quantum effects generate a purely leptonic effective Lagrangian**:

$$\mathcal{L}_{\rm eff}^{\rm NC} = -\frac{4G_F}{\sqrt{2}}\lambda_{ij}^e\left[(\overline{e}_{Li}\gamma_\mu e_{Lj})\sum_\psi \overline{\psi}\gamma^\mu\psi\left(2g_\psi^Z \mathbf{c_t^e} - Q_\psi \mathbf{c_\gamma^e}\right) + h.c.\right]$$

$$\mathcal{L}_{\rm eff}^{\rm CC} = -\frac{4G_F}{\sqrt{2}}\lambda_{ij}^e\left[\mathbf{c_t^{cc}}(\overline{e}_{Li}\gamma_\mu \nu_{Lj})(\overline{\nu}_{Lk}\gamma^\mu e_{Lk} + \overline{u}_{Lk}\gamma^\mu V_{kl} d_{Ll}) + h.c.\right]$$

where $\psi = \{\nu_{Lk}, e_{Lk,Rk}, u_{L,R}, d_{L,R}, s_{L,R}\}$ and $g_\psi^Z = T_3(\psi) - Q_\psi \sin^2\theta_W$.

$$\mathbf{c_t^e} = \mathbf{y_t^2}\frac{3}{32\pi^2}\frac{v^2}{\Lambda^2}(C_1 - C_3)\lambda_{33}^u \log\frac{\Lambda^2}{m_t^2}$$

$$\mathbf{c_t^{cc}} = \mathbf{y_t^2}\frac{3}{16\pi^2}\frac{v^2}{\Lambda^2}C_3\,\lambda_{33}^u \log\frac{\Lambda^2}{m_t^2}$$

$$\mathbf{c_\gamma^e} = \frac{\mathbf{e^2}}{48\pi^2}\frac{v^2}{\Lambda^2}\left[(3C_3 - C_1)\log\frac{\Lambda^2}{\mu^2} - (C_1 + C_3)\lambda_{33}^d \log\frac{m_b^2}{\mu^2} + 2(C_1 - C_3)\left(\lambda_{33}^u \log\frac{m_t^2}{\mu^2} + \lambda_{22}^u \log\frac{m_c^2}{\mu^2}\right)\right]$$

- Top-quark yukawa interactions affect both neutral and charged currents.
- Gauge interactions are proportional to e^2 and to the e.m. current.
- The μ dependence is removed by the matrix elements in the low energy theory.

LFU violation in $\tau \to \ell\bar{\nu}\nu$

- **LFU breaking effects in $\tau \to \ell\bar{\nu}\nu$**

$$R_\tau^{\tau/e} = \frac{\mathcal{B}(\tau \to \mu\nu\bar{\nu})_{\rm exp}/\mathcal{B}(\tau \to \mu\nu\bar{\nu})_{\rm SM}}{\mathcal{B}(\mu \to e\nu\bar{\nu})_{\rm exp}/\mathcal{B}(\mu \to e\nu\bar{\nu})_{\rm SM}}$$

$$R_\tau^{\tau/\mu} = \frac{\mathcal{B}(\tau \to e\nu\bar{\nu})_{\rm exp}/\mathcal{B}(\tau \to e\nu\bar{\nu})_{\rm SM}}{\mathcal{B}(\mu \to e\nu\bar{\nu})_{\rm exp}/\mathcal{B}(\mu \to e\nu\bar{\nu})_{\rm SM}}$$

- $R_\tau^{\tau/\ell}$: experiments vs. theory

$$R_\tau^{\tau/\mu} = 1.0022 \pm 0.0030\,, \quad R_\tau^{\tau/e} = 1.0060 \pm 0.0030$$ [HFAG, '14]

$$R_\tau^{\tau/\ell} \simeq 1 + 2\, c_t^{cc} \lambda_{33}^e \approx 1 + \frac{0.008\, C_3}{\Lambda^2(\mathrm{TeV})}\, \lambda_{33}^e$$

- $R_{D^{(*)}}^{\tau/\ell}$: experiments vs. theory

$$R_D^{\tau/\ell} = 1.37 \pm 0.17, \qquad R_{D^*}^{\tau/\ell} = 1.28 \pm 0.08$$

$$R_{D^{(*)}}^{\tau/\ell} \approx 1 - \frac{0.12\, C_3}{\Lambda^2(\mathrm{TeV})} \lambda_{33}^e$$

Strong tension between $R_\tau^{\tau/\ell}$ and $R_D^{\tau/\ell}$!!

- **LFV τ decays**

$$\mathcal{B}(\tau \to 3\mu) \approx 5 \times 10^{-8} \frac{c_-^2}{\Lambda^4(\mathrm{TeV})} \left(\frac{\lambda_{23}^e}{0.3}\right)^2$$

$$\mathcal{B}(\tau \to \mu\rho) \approx 5 \times 10^{-8} \frac{(c_- - 0.28C_3)^2}{\Lambda^4(\mathrm{TeV})} \left(\frac{\lambda_{23}^e}{0.3}\right)^2$$

$$\mathcal{B}(\tau \to \mu\pi) \approx 8 \times 10^{-8} \frac{c_-^2}{\Lambda^4(\mathrm{TeV})} \left(\frac{\lambda_{23}^e}{0.3}\right)^2$$

- **LFV B decays**

$$\mathcal{B}(B \to K\tau\mu) \approx 4 \times 10^{-8} \left|C_9^{\mu\tau}\right|^2 \approx 10^{-7} \left|\frac{C_9^{\mu\mu}}{0.5}\right|^2 \left|\frac{0.3}{\lambda_{23}^e}\right|^2,$$

since $C_9^{\mu\mu}/C_9^{\mu\tau} \approx \lambda_{23}^e$ and $|C_9^{\mu\mu}| \approx 0.5$ from $R_K^{e/\mu} \approx 0.75$.

- **Experimental bounds** [HFAG]:

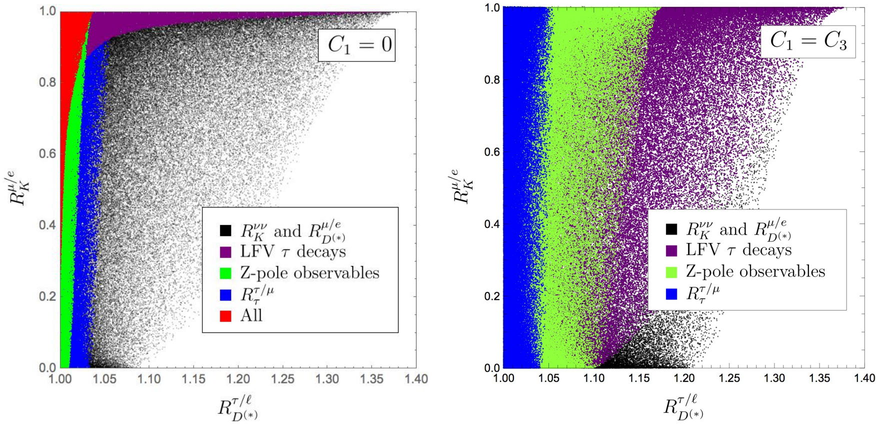
$$\mathcal{B}(\tau \to 3\mu)_{\mathrm{exp}} \leq 2.1 \times 10^{-8}$$

$$\mathcal{B}(\tau \to \mu\rho)_{\mathrm{exp}} \leq 1.2 \times 10^{-8}$$

$$\mathcal{B}(\tau \to \mu\pi)_{\mathrm{exp}} \leq 2.7 \times 10^{-8}$$

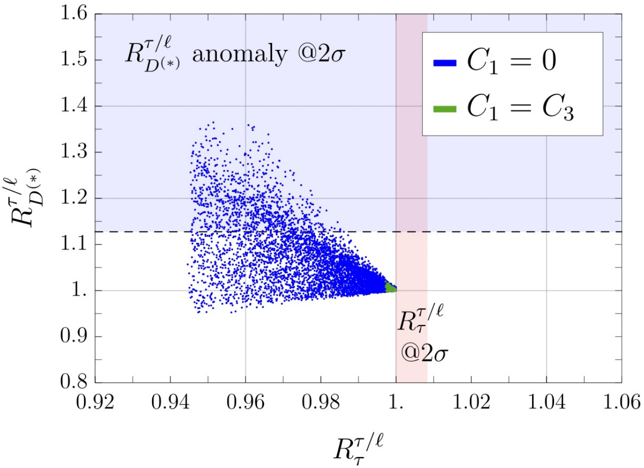
$$\mathcal{B}(B \to K\tau\mu)_{\mathrm{exp}} \leq 4.8 \times 10^{-5}$$

$R_K^{\mu/e}$ vs. $R_{D^{(*)}}^{\tau/\ell}$



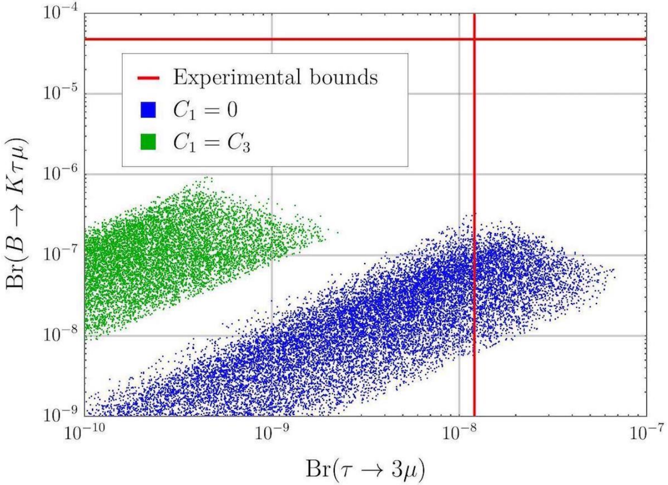
$R_K^{\mu/e}$ vs. $R_{D^{(*)}}^{\tau/\ell}$. The allowed regions are coloured according to the legend.

$R_\tau^{\tau/\ell}$ vs. $R_{D^{(*)}}^{\tau/\ell}$



$$R_\tau^{\tau/\mu} = 1.0022 \pm 0.0030\,, \quad R_\tau^{\tau/e} = 1.0060 \pm 0.0030$$ [HFAG, '14]

$\mathcal{B}(B \to K\tau\mu)$ vs. $\mathcal{B}(\tau \to 3\mu)$



$\mathcal{B}(B \to K\tau\mu)$ vs. $\mathcal{B}(\tau \to 3\mu)$ imposing all the experimental bounds except $R^{\tau/\ell}_{D^{(*)}}$.

Conclusions and future prospects

- **Important questions in view of ongoing/future experiments are**:
  - What are the expected deviations from the SM predictions induced by TeV NP?
  - Which observables are not limited by theoretical uncertainties?
  - In which case we can expect a substantial improvement on the experimental side?
  - What will the measurements teach us if deviations from the SM are [not] seen?

- **(Personal) answers**:
  - The expected deviations from the SM predictions induced by NP at the TeV scale with generic flavor structure are already ruled out by many orders of magnitudes. Therefore, we can expect any size of deviation below the current bounds.
  - LFV processes, leptonic EDMs and LFU observables do not suffer from theoretical limitations and there are still excellent prospects for experimental improvements.
  - The observed LFU breaking effects in $B \to D^{(*)}\ell\nu$, $B \to K\ell\ell'$ might be true NP signals. It's worth to look for LFU breaking effects in $B \to \ell\nu$ and $B \to K\tau\tau$.
  - Large LFU breaking effects in $B \to D^{(*)}\ell\nu$ and $B \to K\ell\ell'$ Z τ are typically associated with large LFU breaking effects in $\tau \to \ell\nu\nu$ and in Z pole observables.
  - If LFU breaking effects arise from LFV sources, the most sensitive LFV channels are typically not B-decays but τ decays such as $\tau \to \mu\ell\ell$ and $\tau \to \mu\rho, \cdots$.