Hadron Spectra, Scattering Properties and Decays from Basis Light Front Quantization

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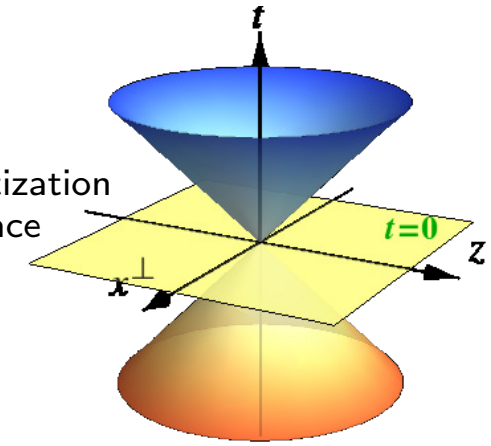
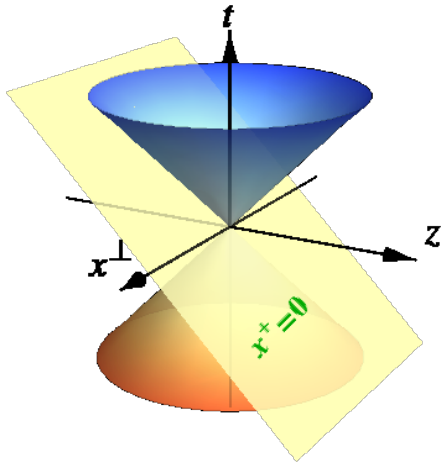
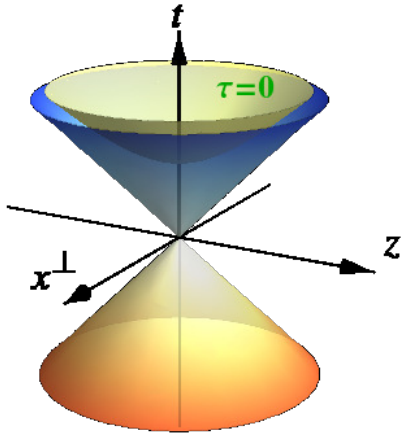
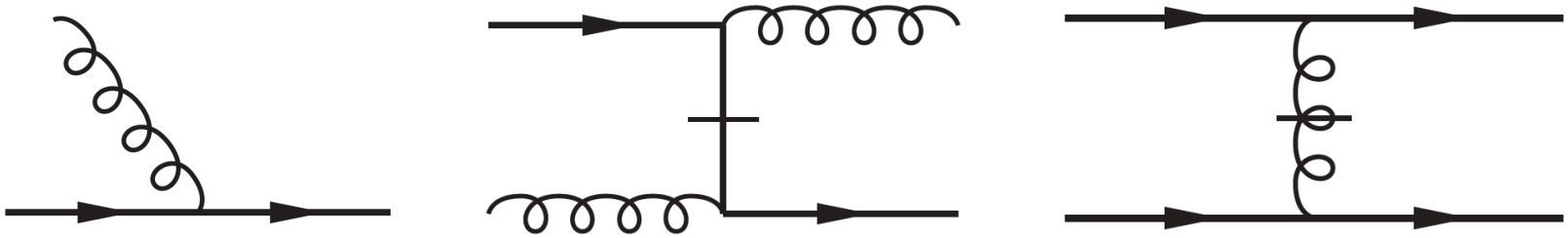
Dirac's Forms of Relativistic Dynamics [Dirac, Rev.Mod.Phys. '49]

Front form defines QCD on the light front (LF) $x^+ \triangleq t + z = 0$.

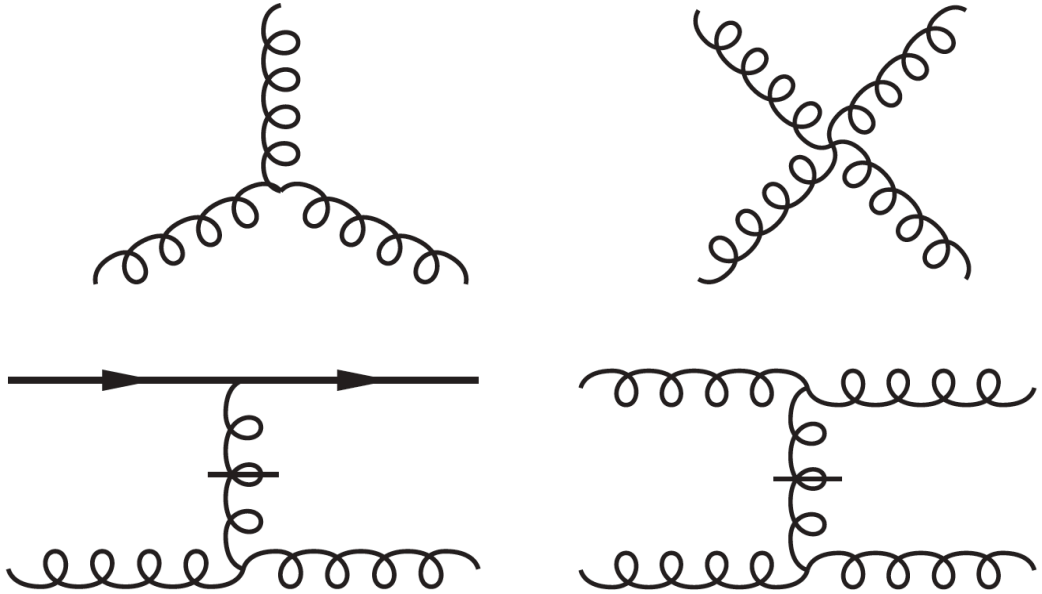
$P^\pm \triangleq P^0 \pm P^3$, $\vec{P}^\perp \triangleq (P^1, P^2)$, $x^\pm \triangleq x^0 \pm x^3$, $\vec{x}^\perp \triangleq (x^1, x^2)$, $E^i = M^{+i}$, $E^+ = M^{+-}$, $F^i = M^{-i}$, $K^i = M^{0i}$, $J^i = \frac{1}{2}\epsilon^{ijk} M^{jk}$.

	instant form	front form	point form
time variable	$t = x^0$	$x^+ \triangleq x^0 + x^3$	$\tau \triangleq \sqrt{t^2 - \vec{x}^2 - a^2}$
quantization surface			
Hamiltonian	$H = P^0$	$P^- \triangleq P^0 - P^3$	P^μ
kinematical	$\vec{P}, \vec{J}$	$\vec{P}^\perp, P^+, \vec{E}^\perp, E^+, J_z$	$\vec{J}, \vec{K}$
dynamical	$\vec{K}, P^0$	$\vec{F}^\perp, P^-$	$\vec{P}, P^0$
dispersion relation	$p^0 = \sqrt{\vec{p}^2 + m^2}$	$p^- = (\vec{p}_\perp^2 + m^2)/p^+$	$p^\mu = mv^\mu \ (v^2 = 1)$

Light Front (LF) Hamiltonian Defined by its Elementary Vertices in LF Gauge



QED & QCD



QCD

Light-Front Wavefunctions (LFWFs)

$$|\psi_h(P,j,\lambda)\rangle = \sum_n \int [\mathrm{d}\mu_n]\, \psi_{h/n}(\{\vec{k}_{i\perp}, x_i, \lambda_i\}_n)|\{\vec{p}_{i\perp}, p_i^+, \lambda_i\}_n\rangle$$

LFWFs are *frame-independent* (boost invariant) and depend only on the relative variables: $x_i \equiv p_i^+/P^+, \vec{k}_{i\perp} \equiv \vec{p}_{i\perp} - x_i\vec{P}_\perp$

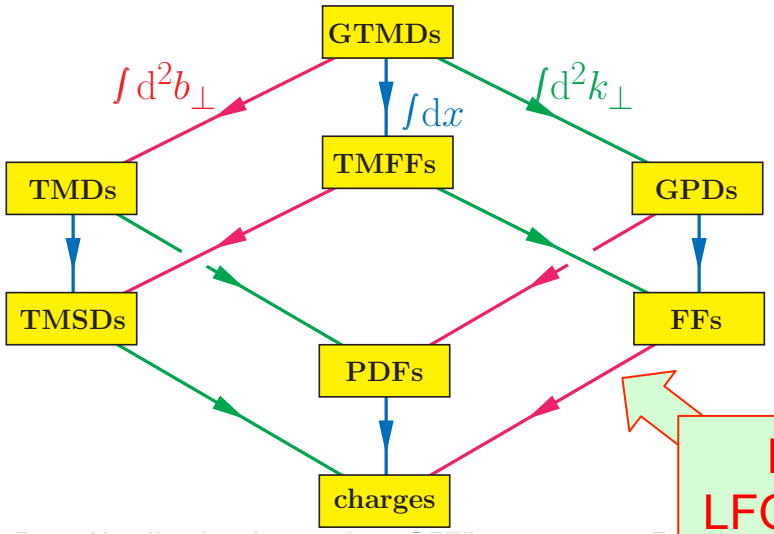
LFWFs provides intrinsic information of the structure of hadrons:

- structure functions (e.g., PDF) are the square of LFWFs
- form factors (e.g., E&M, gravitational ...) are the overlap of LFWFs

LFWFs are indispensable for exclusive processes in DIS [Lepage '80]

"Hadron Physics without LFWFs is like Biology without DNA!"

— Stanley J. Brodsky



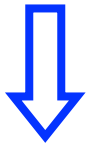
Route map for LFQCD applications

$\vec{k}_\perp \leftrightarrow \vec{r}_\perp,\ \vec{\Delta}_\perp \leftrightarrow \vec{b}_\perp$

[Lorcé & Pasquini '11]

Discretized Light Cone Quantization

Pauli & Brodsky c1985



Basis Light Front Quantization*

$$\phi(\vec{x}) = \sum_{\alpha} \left[f_\alpha(\vec{x}) a_\alpha^+ + f_\alpha^*(\vec{x}) a_\alpha \right]$$

where $\{a_\alpha\}$ satisfy usual (anti-) commutation rules.

Furthermore, $f_\alpha(\vec{x})$ are arbitrary except for conditions:

Orthonormal: $\int f_\alpha(\vec{x}) f_{\alpha'}^*(\vec{x}) d^3x = \delta_{\alpha\alpha'}$

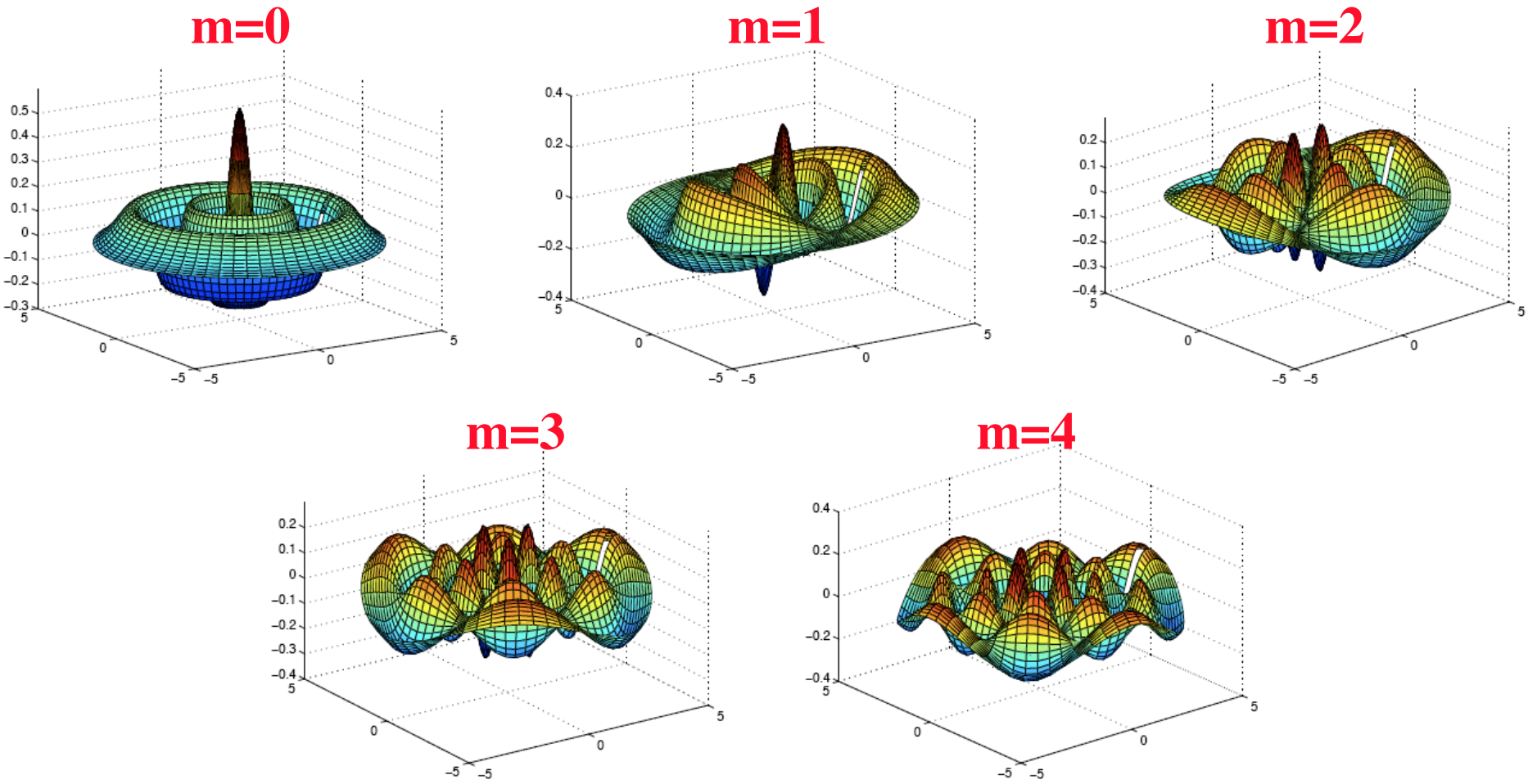
Complete: $\sum_{\alpha} f_\alpha(\vec{x}) f_\alpha^*(\vec{x}') = \delta^3(\vec{x} - \vec{x}')$

=> Wide range of choices for $f_a(\vec{x})$ and our initial choice is

$$f_\alpha(\vec{x}) = N e^{ik^+x^-} \Psi_{n,m}(\rho,\varphi) = N e^{ik^+x^-} f_{n,m}(\rho) \chi_m(\varphi)$$

*J.P. Vary, H. Honkanen, J. Li, P. Maris, S.J. Brodsky, A. Harindranath, G.F. de Teramond, P. Sternberg, E.G. Ng and C. Yang, PRC 81, 035205 (2010). ArXiv:0905:1411

Set of Transverse 2D HO Modes for n=4



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BLFQ
Symmetries & Constraints

Baryon number $\sum_i b_i = B$

Charge $\sum_i q_i = Q$

Angular momentum projection (M-scheme) $\sum_i (m_i + s_i) = J_z$

Longitudinal momentum (Bjorken sum rule) $\sum_i x_i = \sum_i \frac{k_i}{K} = 1$

Transverse mode regulator (2D HO) $\sum_i (2n_i + |m_i| + 1) \leq N_{\max}$

Global Color Singlets (QCD)

Light Front Gauge

Optional Fock-Space Truncation

$H \to H + \lambda H_{CM}$

All $J \geq J_z$ states in one calculation

Finite basis regulators

Preserve transverse boost invariance

Light-Front Regularization and Renormalization Schemes

1. Regulators in BLFQ (Ω, N_{max}, K)
2. Additional Fock space truncations (if any)
3. Counterterms identified/tested*
4. Sector-dependent renormalization**
5. SRG & OLS in NCSM*** - adapted to BLFQ (future)

*D. Chakrabarti, A. Harindranath and J.P. Vary,
Phys. Rev. D **69**, 034502 (2004)
*P. Wiecki, Y. Li, X. Zhao, P. Maris and J.P. Vary,
Phys. Rev. D **91**, 105009 (2015)

**V. A. Karmanov, J.-F. Mathiot, and A. V. Smirnov,
Phys. Rev. D **77**, 085028 (2008); Phys. Rev. D **86**, 085006 (2012)
**Y. Li, V.A. Karmanov, P. Maris and J.P. Vary,
Phys. Letts. B. 748, **278** (2015); arXiv: 1504.05233

***B.R. Barrett, P. Navratil and J.P. Vary,
Prog. Part. Nucl. Phys. **69**, 131 (2013)

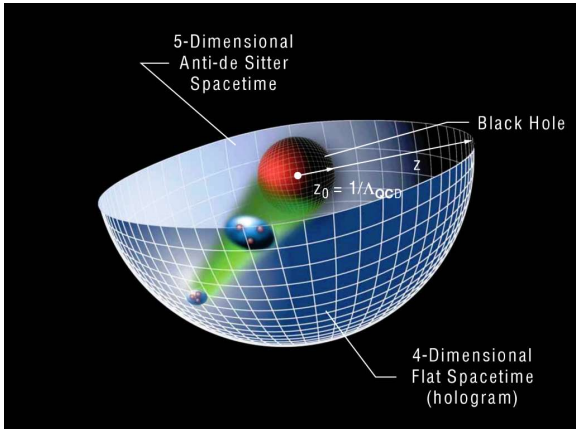
Light-Front Schrödinger equation for quark – antiquark systems

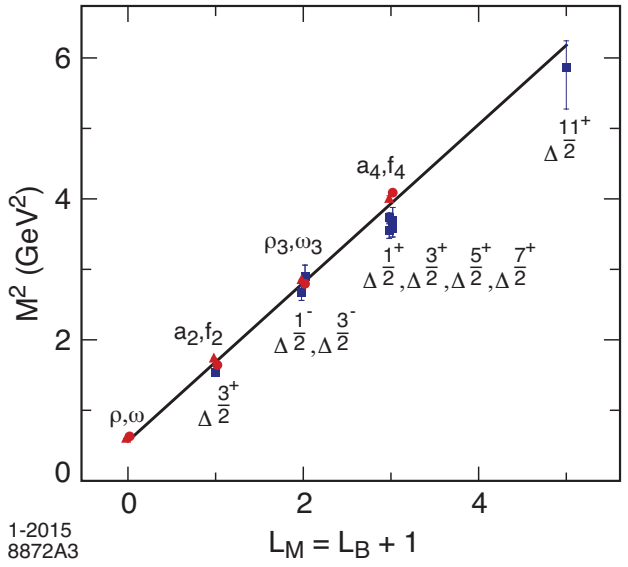
$$M_h^2\,\psi^{\lambda}_{s\bar{s}/h}(x,\vec{k}_\perp) = \Big[\underbrace{\frac{\vec{k}_\perp^{\,2}+m_q^2(x,k_\perp)}{x}+\frac{\vec{k}_\perp^{\,2}+m_{\bar{q}}^2(x,k_\perp)}{1-x}}_{\text{kinetic energy w. running masses}}\Big]\psi^{\lambda}_{s\bar{s}/h}(x,\vec{k}_\perp)$$

$$+\sum_{s',\bar{s}'}\int_0^1\frac{\mathrm{d}x'}{2x'(1-x')}\int\frac{\mathrm{d}^2k'_\perp}{(2\pi)^3}\underbrace{V_{s'\bar{s}',s\bar{s}}(x,\vec{k}_\perp,x',\vec{k}'_\perp)}_{\text{effective relativistic interaction}}\psi^{\lambda}_{s'\bar{s}'/h}(x',\vec{k}'_\perp)$$

What is the effective relativistic interaction V for QCD?

- Light-Front Holography [Brodsky, de Téramond et al]
- Renormalization Group Procedure for Effective Particles [Głazek et al]
- Okubo-Suzuki-Lee RG, pQCD, Lattice QCD, relativized NR potentials, ...

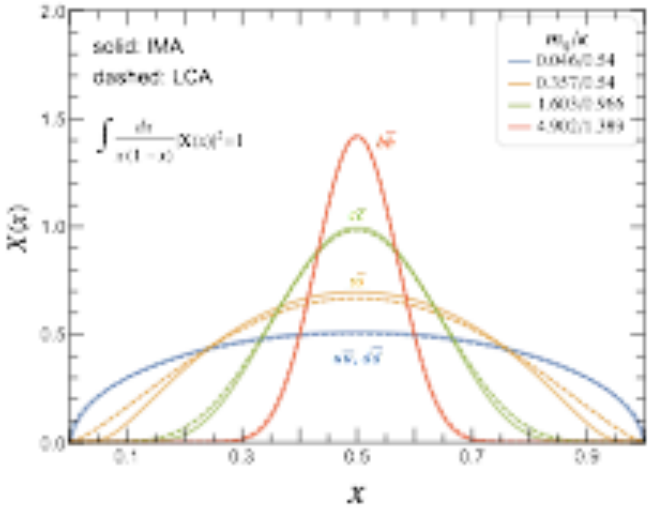


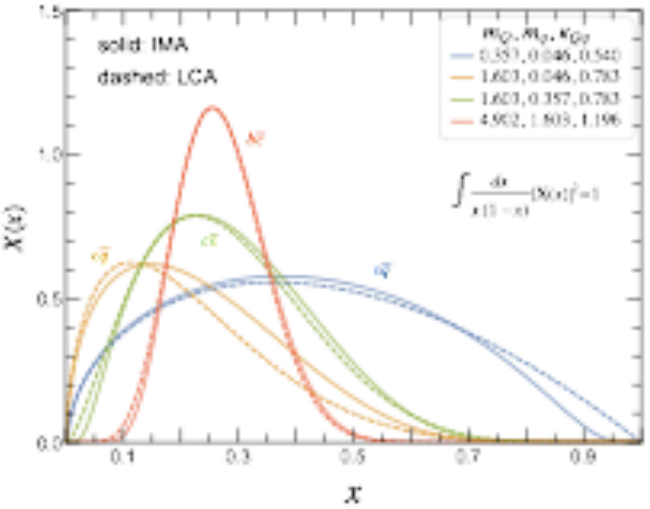


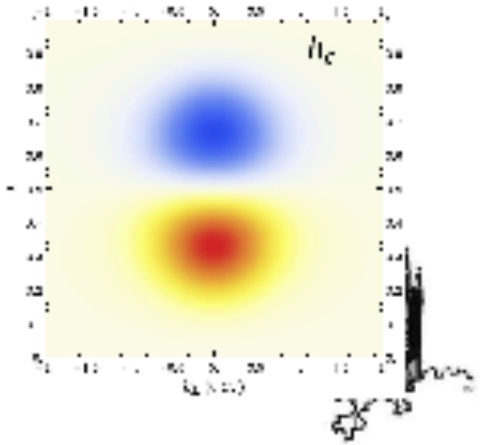
Basis Light-Front Holography

$$V = \overbrace{\underbrace{\kappa^2\zeta_\perp^2}_{\text{LFH}} - \underbrace{\frac{\kappa^4}{4m_q^2}\partial_x\left(x(1-x)\partial_x\right)}_{\text{longitudinal confinement}}}^{\text{long-distance}} - \overbrace{\underbrace{\frac{C_F 4\pi\alpha_s(Q^2)}{Q^2}\bar{u}_s(k)\gamma^\mu u'_s(k')\bar{v}_{\bar{s}'}(\bar{k}')\gamma^\nu v_{\bar{s}}(\bar{k})d_{\mu\nu}}_{\text{Krautgärtner-Pauli-Wölz one-gluon exchange w. running coupling}}}^{\text{short-distance}}$$

- Implement quark masses with longitudinal degree of freedom: identical to LFH in both chiral limit and NR limit; $X(x) \sim x^a(1-x)^b$
- Introduce short-distance physics with spinor structure [cf. Głazek '17]
- Self-energy and dynamical chiral symmetry breaking to be implemented
- Basis light-front quantization [Vary et al, PRC 2010]

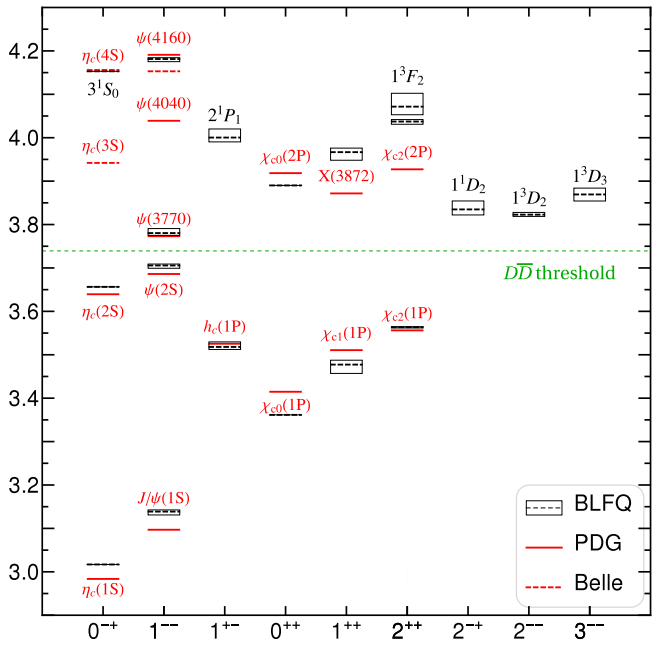


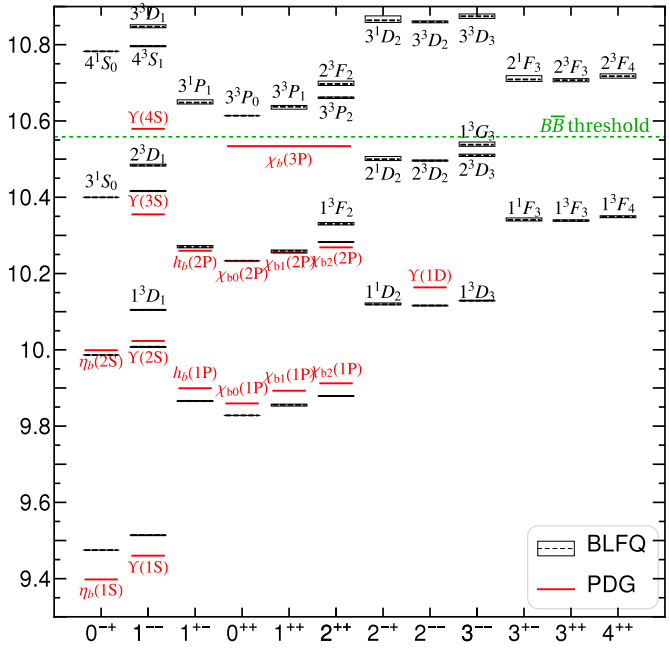


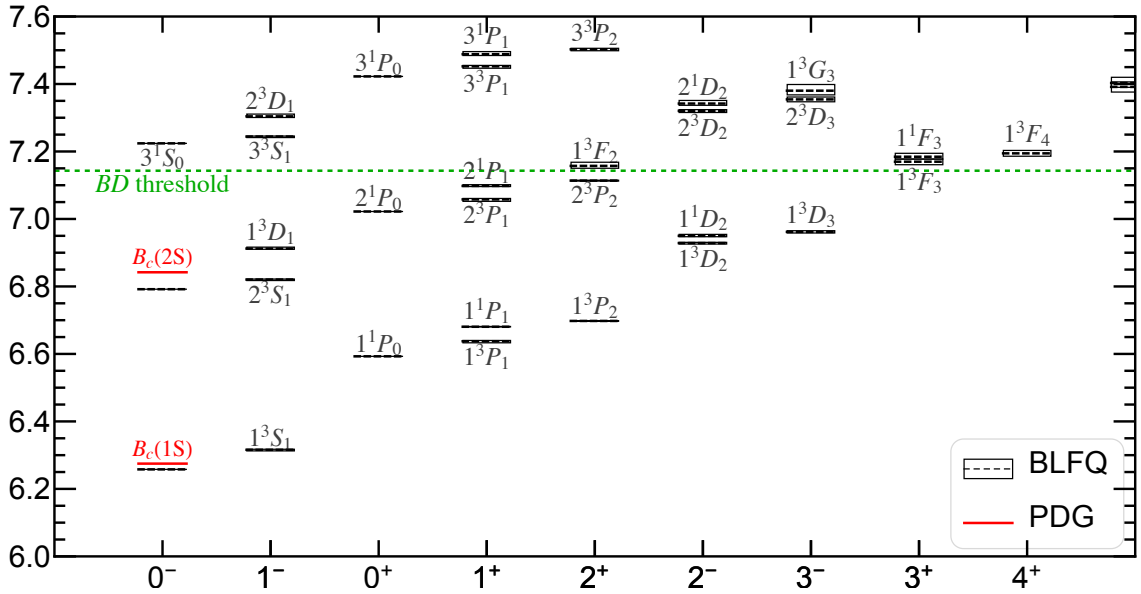


Spectroscopy

[Li, Maris & Vary, PRD '17; Yang, Li, Maris & Vary, in preparation]







fitting parameters:

$m_c, m_b, \kappa_{qQ} = c\sqrt{M_{qQ}}$

(HQET, [cf. Dosch '17])

rms deviation: 31–38 MeV

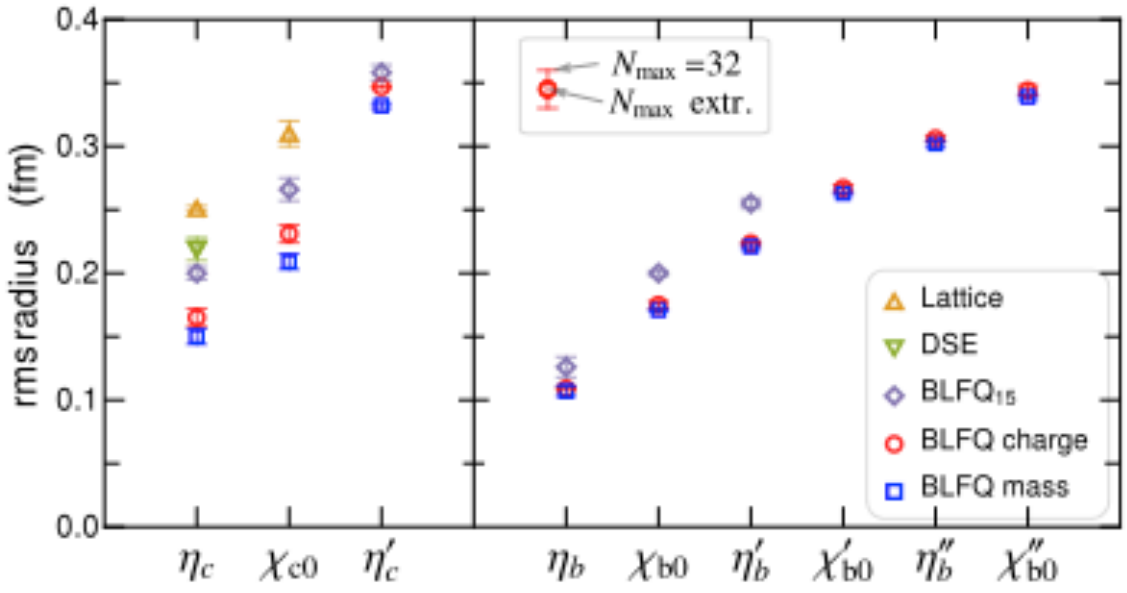
Radii

[YL, Maris & Vary, PRD'17]

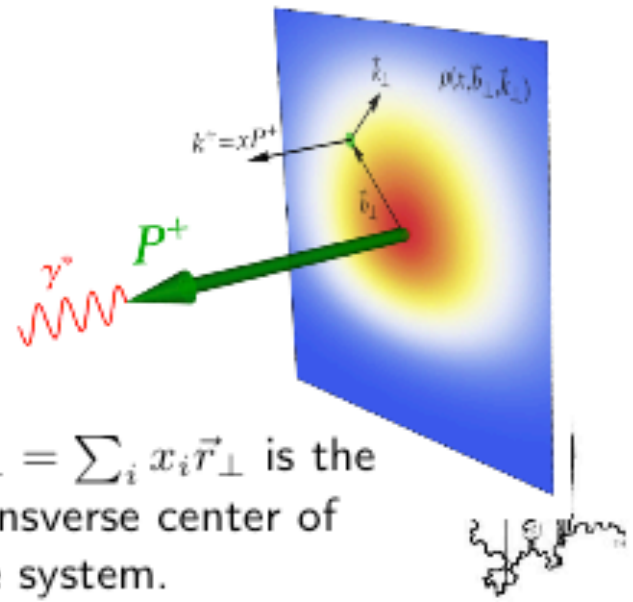
The r.m.s. charge and mass radii – long-distance physics: [YL, FBS '17]

$$\langle r_{\rm c}^2\rangle \triangleq -6F'_{\rm em}(0) = \frac{3}{2}\langle b_\perp^2\rangle, \quad \langle r_{\rm m}^2\rangle \triangleq -6F'_{\rm gr}(0) = \frac{3}{2}\langle \zeta_\perp^2\rangle$$

- $\vec{b}_\perp^2 = \sum_i e_i(\vec{r}_\perp - \vec{R}_\perp)^2 \Rightarrow \vec{b}_\perp = (1-x)\vec{r}_\perp$: Burkardt's impact parameter
- $\vec{\zeta}_\perp^2 = \sum_i x_i(\vec{r}_{i\perp} - \vec{R}_\perp)^2 \Rightarrow \vec{\zeta}_\perp = \sqrt{x(1-x)}\vec{r}_\perp$: Brodsky & de Téramond's holographic coordinate [Burkardt '01, Brodsky '06]
- Difference between charge and mass radii is a relativistic effect



[DSE: Maris '07; Lattice: Dudeck '06; BLFQ$_{15}$: YL et al, PLB '16]

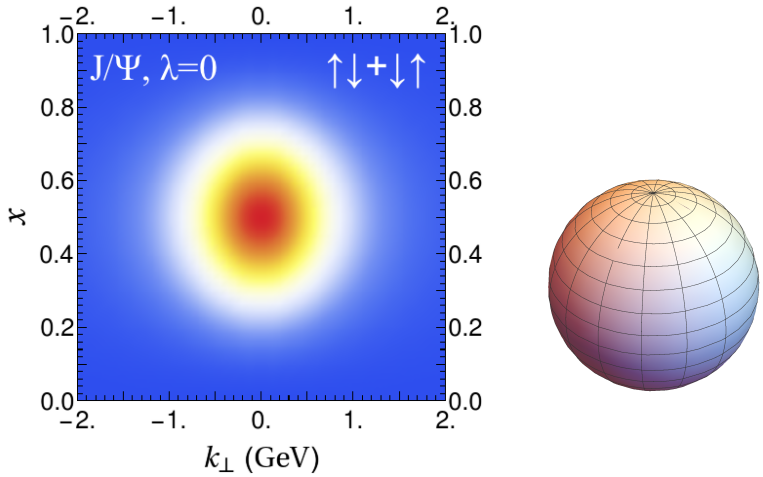


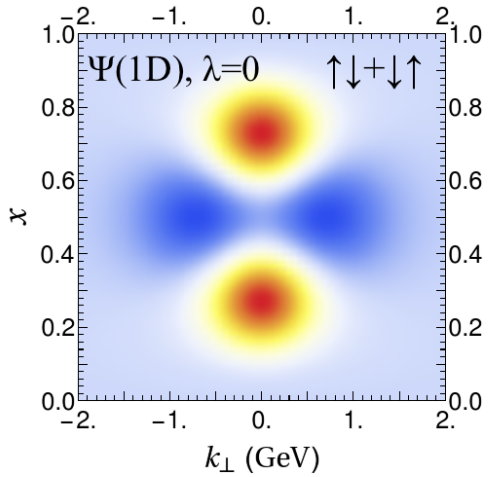
$\vec{R}_\perp = \sum_i x_i \vec{r}_\perp$ is the transverse center of the system.

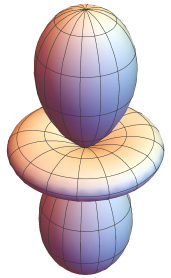
Wave functions

[Li, Maris & Vary, PRD '17]

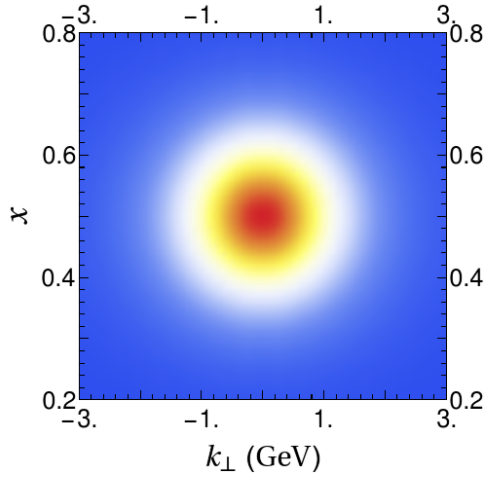
angular excitation of a vector meson: J/ψ vs $\psi(1D)$



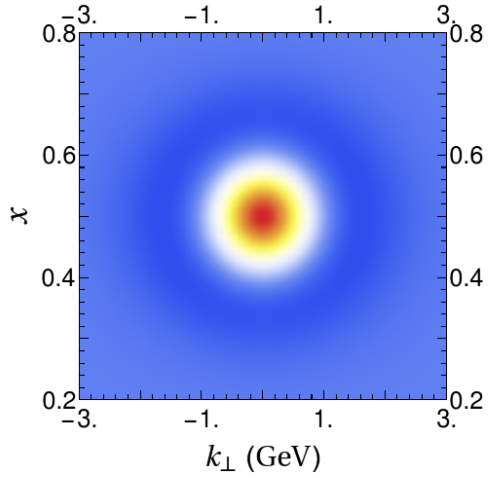




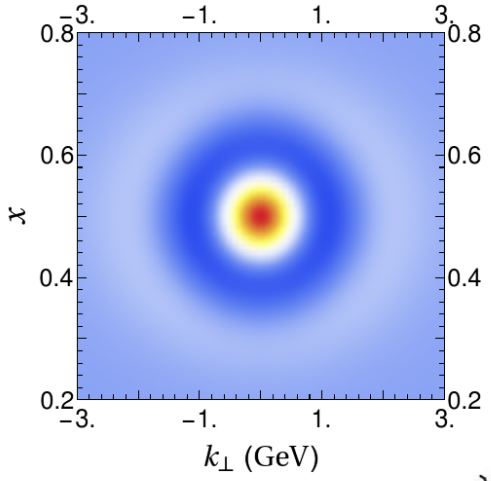
radial excitations of a pseudo scalar: $\psi_{\uparrow\downarrow-\downarrow\uparrow}$



$\eta_b(1S)$



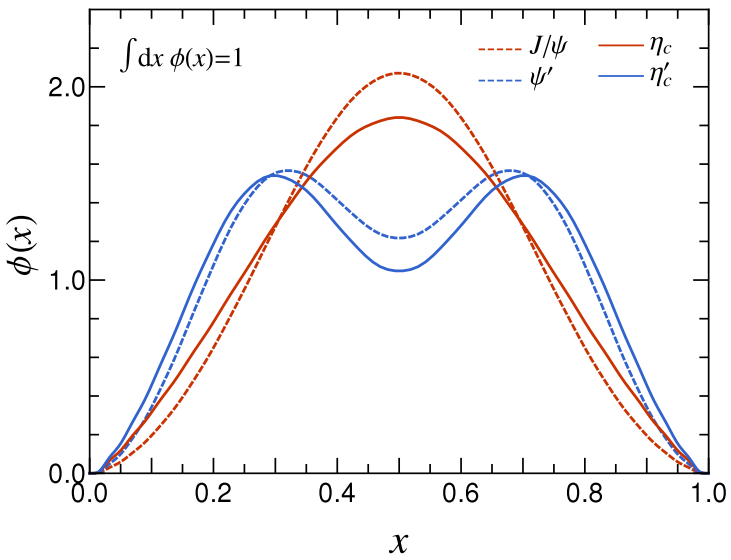
$\eta_b(2S)$

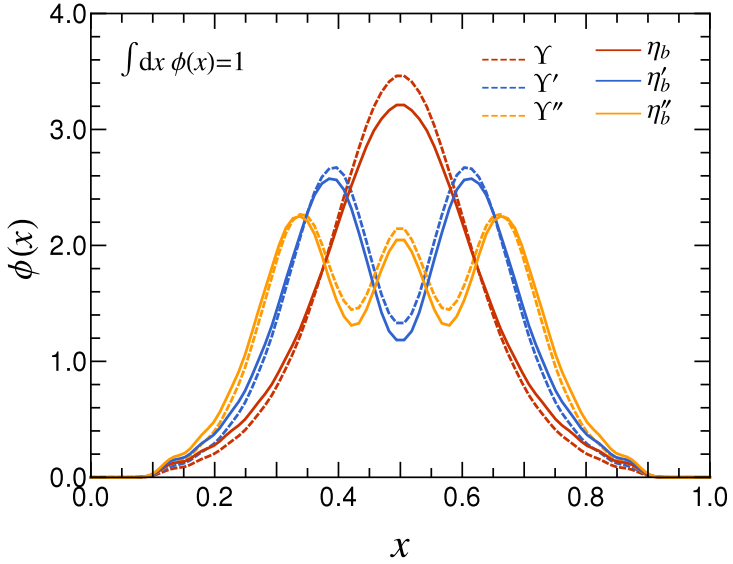


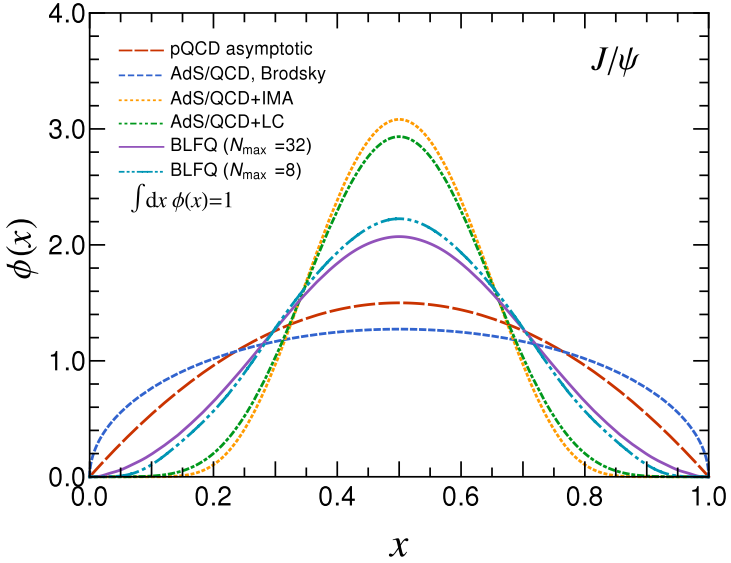
$\eta_b(3S)$

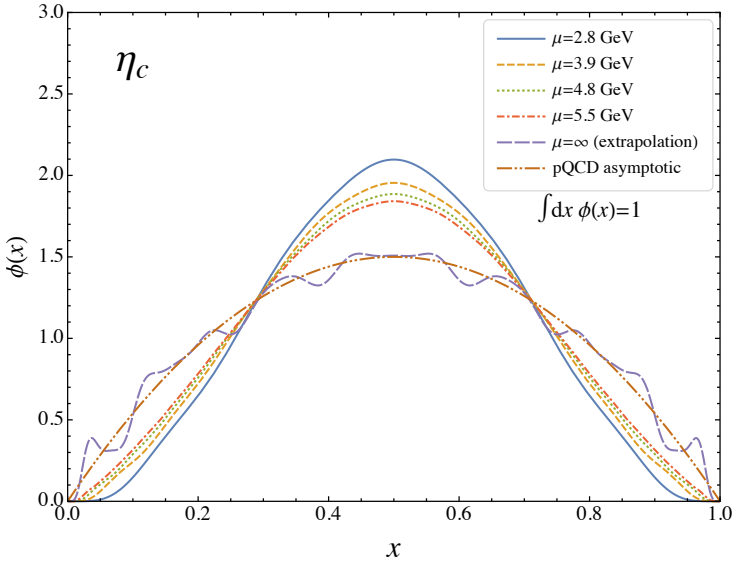
Distribution amplitudes

[Li, Maris & Vary, PRD '17]









Tantalizing result for scale evolution through naïve basis extrapolation ($N_{\max}$ extrapolation) – how to compare with ERBL evolution?

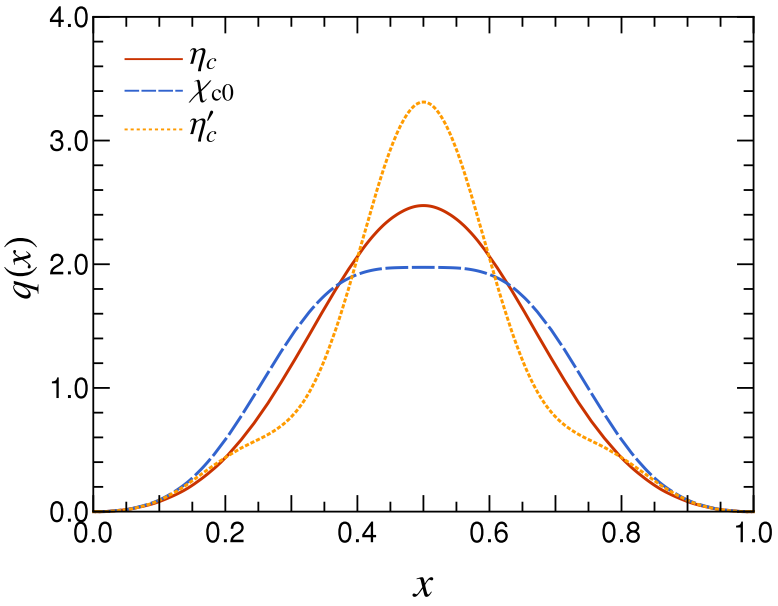
Parton distribution functions

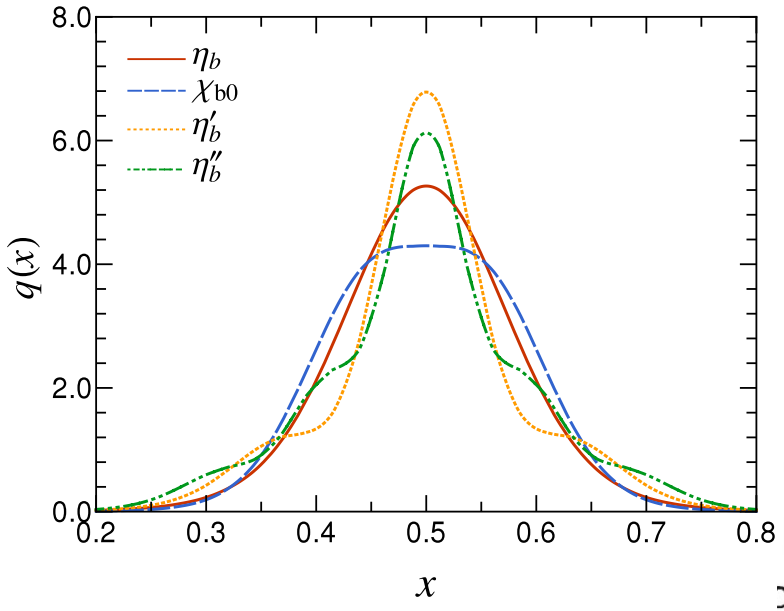
Parton distribution function $q(x;\mu)$ is the probability of finding a collinear quark carrying momentum fraction x up to scale μ.

LFWF representation

$$q(x;\mu) = \frac{1}{x(1-x)} \sum_{s,\bar{s}} \int^{\lesssim \mu^2} \frac{\mathrm{d}^2 k_\perp}{2(2\pi)^3} \left|\psi_{s\bar{s}}(x, \vec{k}_\perp)\right|^2.$$

PDF vs DA of excited states **(probability vs amplitude)**





Radiative transitions of heavy quarkonia

- M1 transition

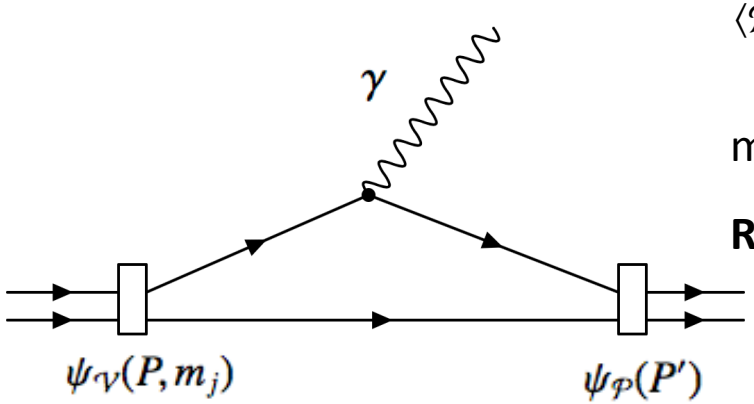
Meijian Li, et al., in preparation

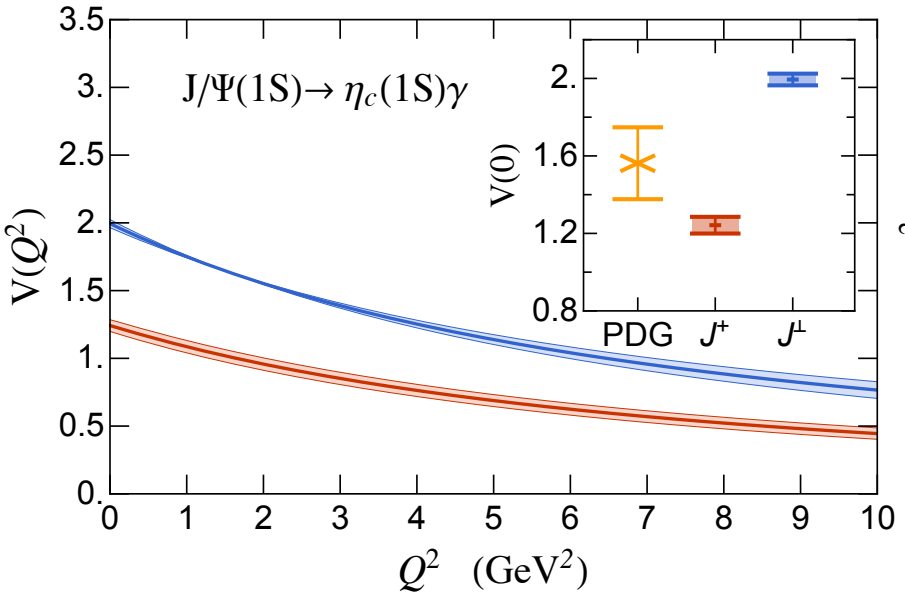
$$\langle \mathcal{P}(P')|\, J^{\mu}(0)\, |\mathcal{V}(P,m_j)\rangle = \frac{2V(Q^2)}{m_{\mathcal{P}}+m_{\mathcal{V}}}\epsilon^{\mu\alpha\beta\sigma}P'_{\alpha}P_{\beta}e_{\sigma}(P,m_j)$$

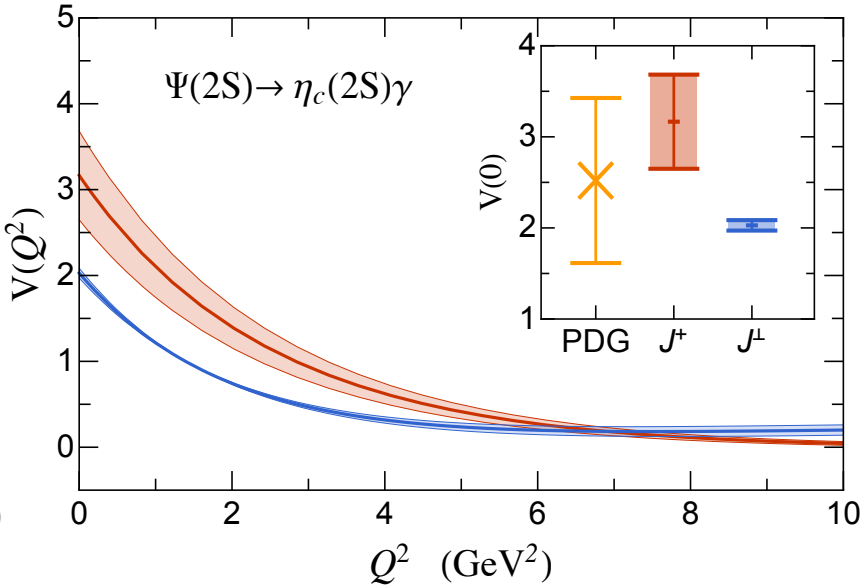
momentum transfer, $q^{\mu} = P'^{\mu} - P^{\mu}$, $Q^2 \equiv -q^2$

Radiative decay width:

$$\Gamma(\mathcal{V}\to\mathcal{P}\gamma) = \frac{\alpha_{EM}(m_{\mathcal{V}}^2-m_{\mathcal{P}}^2)^3}{(2m_{\mathcal{V}})^3(m_{\mathcal{P}}+m_{\mathcal{V}})^2}\frac{4}{3}e_f^2|V(0)|^2$$





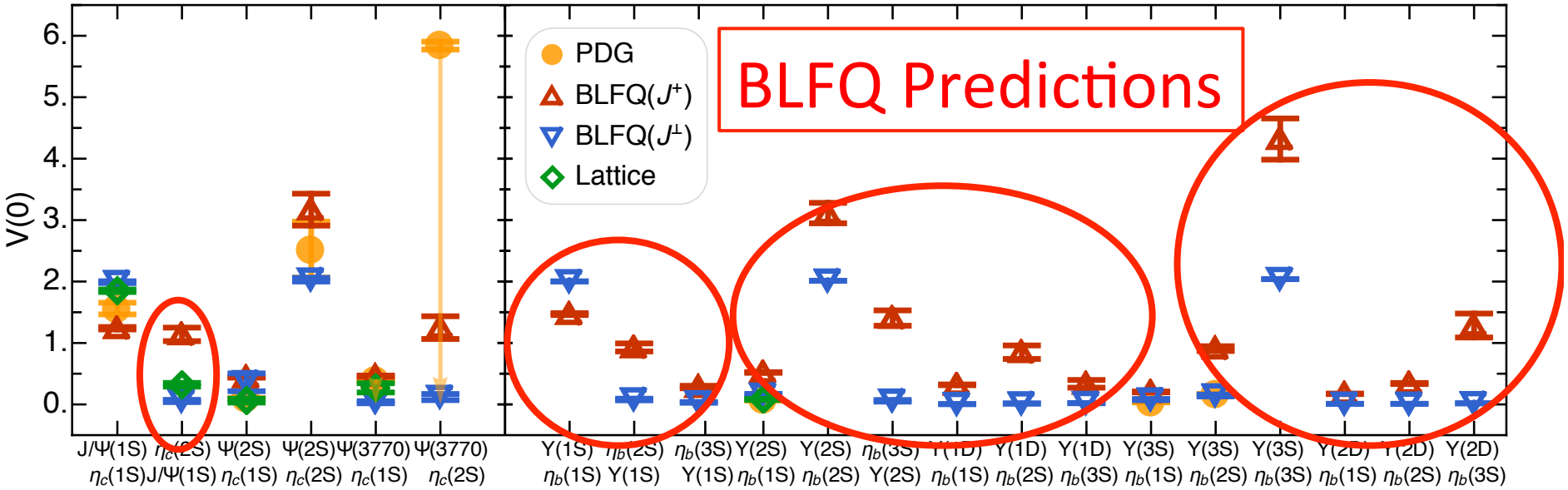


Light-front wavefunctions: Yang Li, Pieter Maris, James P. Vary. Phys. Rev. D 96, 016022 (2017).

Radiative transitions of heavy quarkonia

$$V^{+}(0) = -\frac{\sqrt{2}(m_P + m_V)}{4\pi} \sum_{\sigma_1,\sigma_2} \int \mathrm{d}r_\perp \int_0^1 \mathrm{d}x \int \mathrm{d}\theta\, \tilde{\psi}^{\sigma_1,\sigma_2}_{\mathcal{V},m_j=1}(r_\perp,\theta,x)\tilde{\psi}^{\sigma_1,\sigma_2 *}_{\mathcal{P}}(r_\perp,\theta,x)(1-x)r_\perp^2 \cos\theta$$

$$V^{\perp}(0) = \frac{m_P + m_V}{4\pi m_V} \sum_{\sigma_2} \int \mathrm{d}r_\perp \int_0^1 \mathrm{d}x \int \mathrm{d}\theta\, \tilde{\psi}^{\uparrow,\sigma_2}_{\mathcal{V},m_j=0}(r_\perp,\theta,x)\tilde{\psi}^{\uparrow,\sigma_2 *}_{\mathcal{P}}(r_\perp,\theta,x)\frac{r_\perp}{x}$$



[PDG] Chin. Phys. C40, 100001 (2016).
[Lattice] J. J. Dudek, et al., Phys. Rev. D 73, 074507 (2006), Phys. Rev. D 79, 094504 (2009); D. Bečirević, et al., Journal of High Energy Physics 2015, 14 (2015). C. Hughes et al., Phys. Rev. D 92, 094501 (2015).

B_c Meson System in BLFQ

Shuo Tang, Y. Li, P. Maris, J. P. Vary in preparation

Effective Hamiltonian

with holographic confinement and one gluon exchange interaction

$$H_{\rm eff} = \frac{\vec{k}_\perp^2 + m_q^2}{x} + \frac{\vec{k}_\perp^2 + m_{\bar{q}}^2}{1-x} + \kappa^4 \vec{\zeta}_\perp^2 - \frac{\kappa^4}{(m_q^2 + m_{\bar{q}}^2)^2} \partial_x \big(x(1-x)\partial_x\big) + V_g$$

Adopt running coupling up to the leading order

$$\alpha_s(Q^2) = \frac{1}{\beta_0 \ln(Q^2/\Lambda^2 + \tau)} \triangleq \frac{\alpha_s(M_Z^2)}{1 + \alpha_s(M_Z^2)\beta_0 \ln(\mu_{\rm IR}^2 + Q^2)/(\mu_{\rm IR}^2 + M_Z^2)}$$

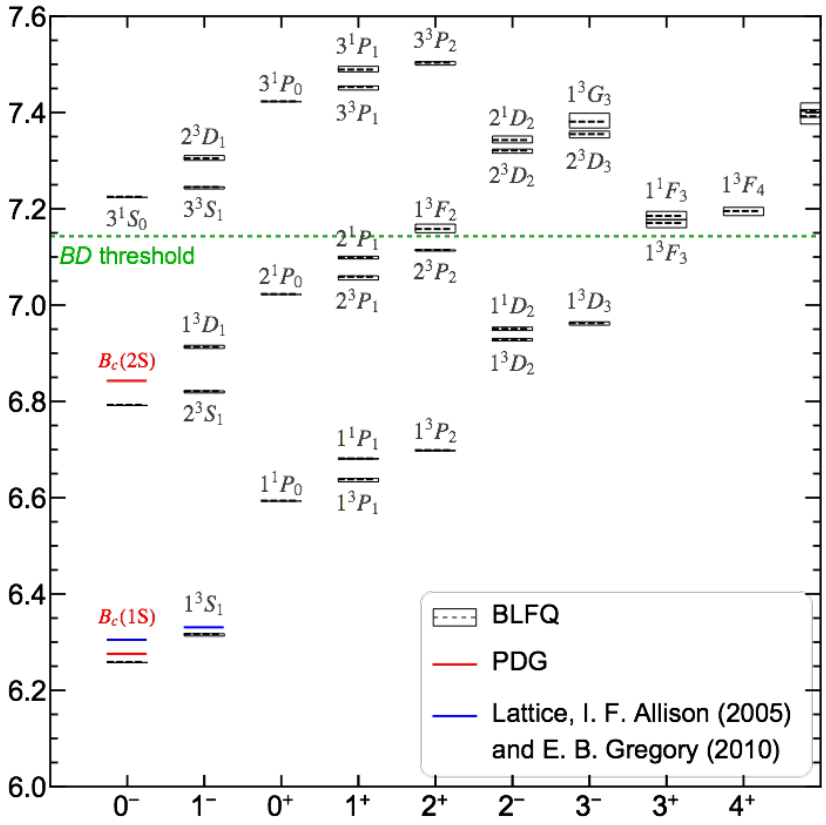
: asymptotic freedom at UV

Mass Spectrum

Without doing further parameter fitting for B_c, we make use of the parameters from charmonium and bottomonium.

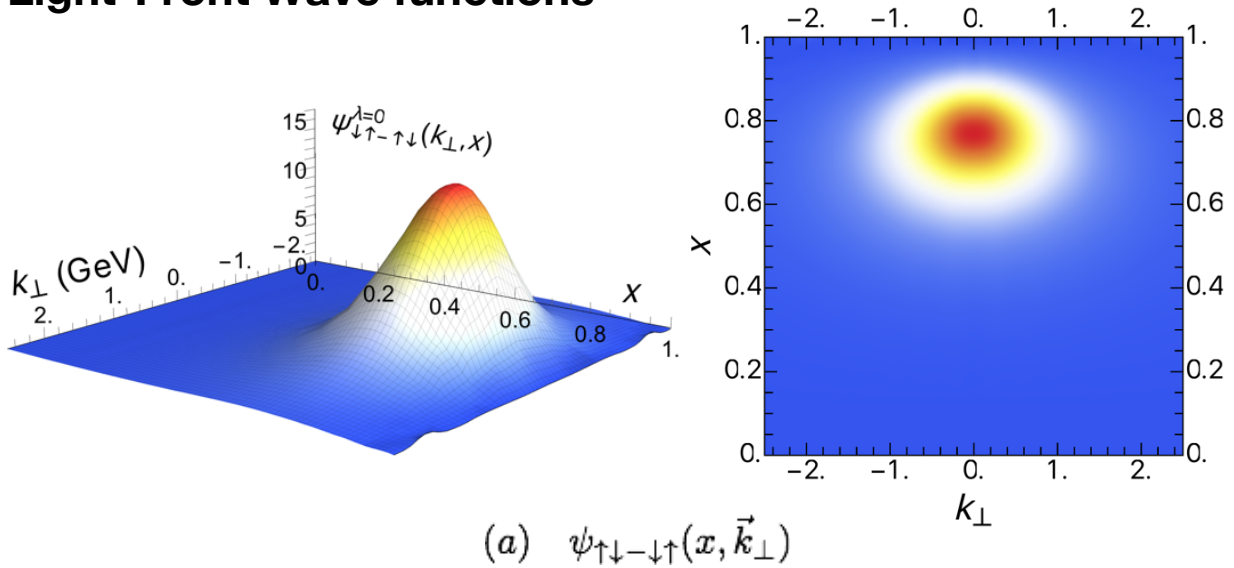
	N_f	κ (GeV)	m_c (GeV)	m_b (GeV)	rms (MeV)	$\delta_J M$ (MeV)
$c\bar{c}$	4	0.966	1.603	–	31	17
$b\bar{b}$	5	1.389	–	4.902	38	8
$b\bar{c}$	4	1.196	1.603	4.902	37	6

(HQET motivated: $\kappa_{b\bar{c}} = \sqrt{(\kappa_{b\bar{b}}^2 + \kappa_{c\bar{c}}^2)/2}$)

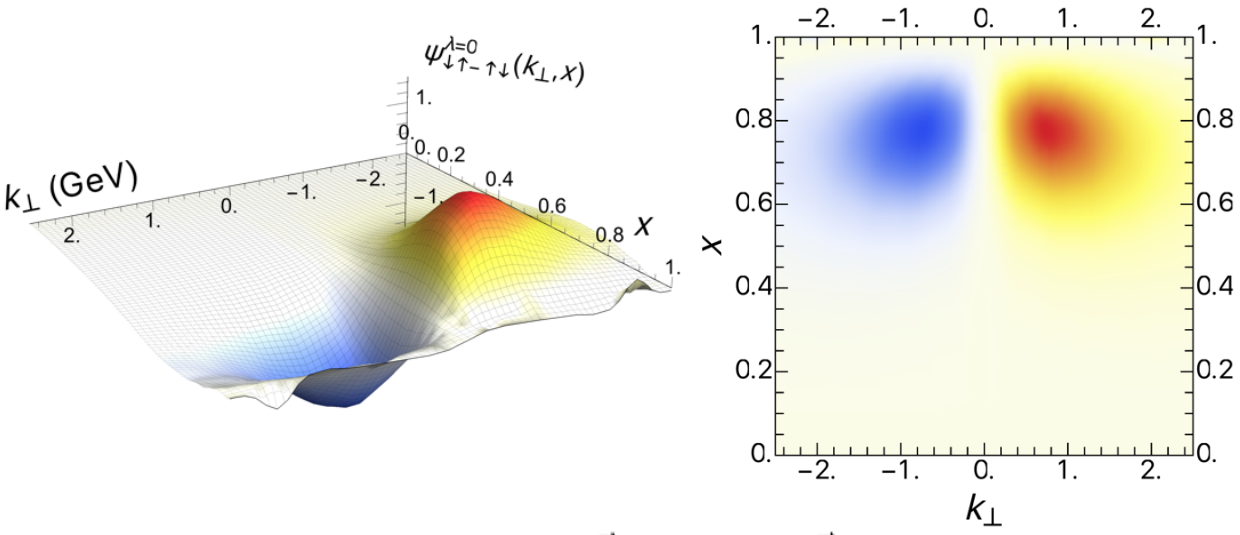


B_c Meson System in BLFQ

Light-Front Wave functions



$(a)\quad \psi_{\uparrow\downarrow-\downarrow\uparrow}(x,\vec{k}_\perp)$



$(b)\quad \psi_{\downarrow\downarrow}(x,\vec{k}_\perp)=\psi^{*}_{\uparrow\uparrow}(x,\vec{k}_\perp)$

LFWFs of the B_c ground-state pseudo scalar (1S), as functions of x and $k_\perp$.

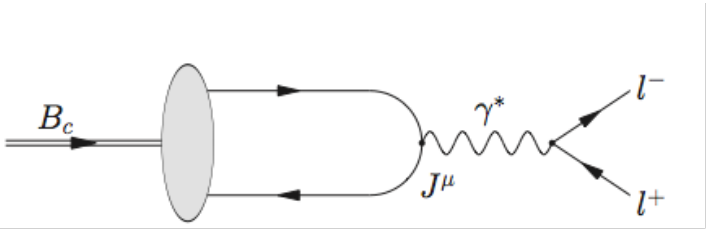
Two independent components are shown in upper and lower panels:
Component $\psi_{\uparrow\downarrow-\downarrow\uparrow}$ is related to the non-relativistic wave function, while $\psi_{\downarrow\downarrow}=\psi^{*}_{\uparrow\uparrow}$ is of pure relativistic origin and becomes negligible in non-relativistic limit.

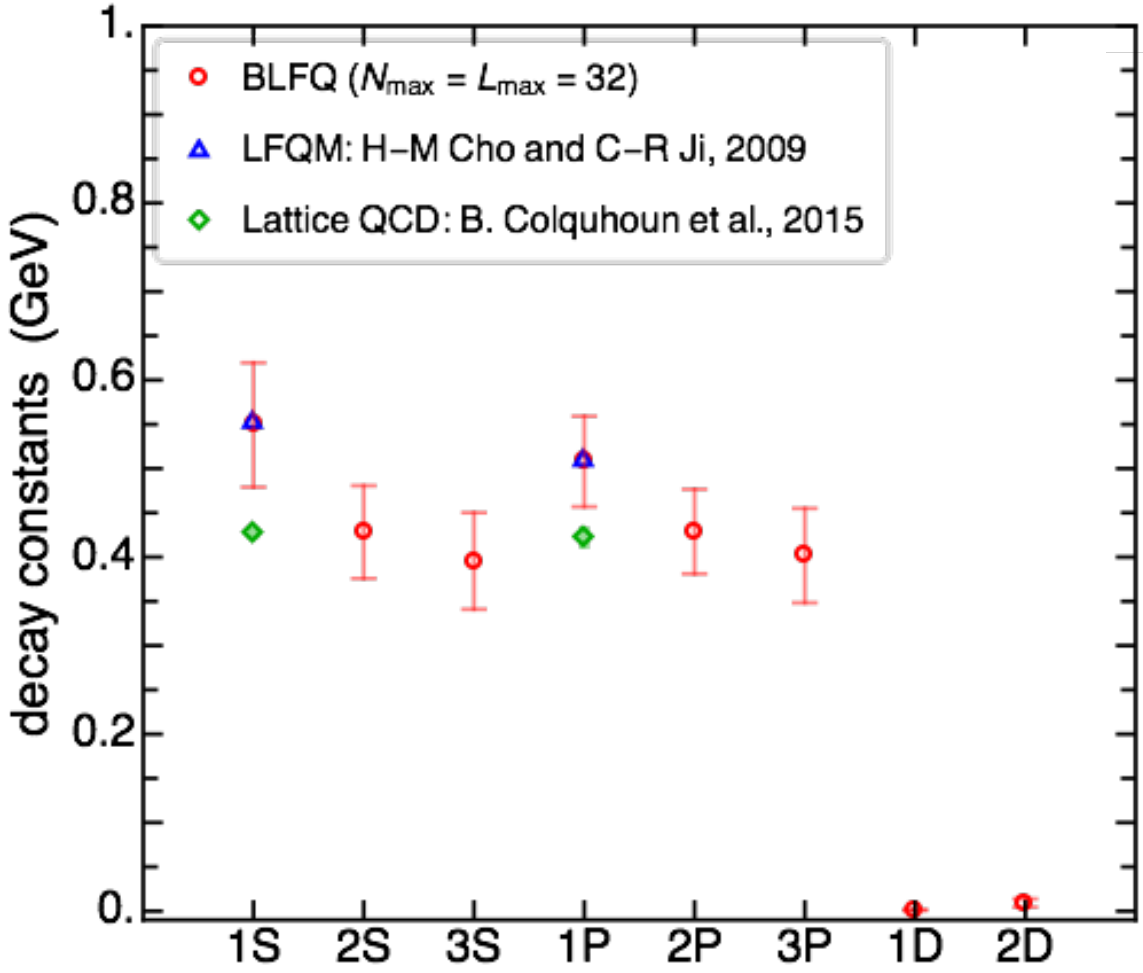
All of the LFWFs are centered at $x=m_b/(m_b+m_c)$.

B_c Meson System in BLFQ

Decay Constant

$$\frac{f_{P,V}}{2\sqrt{2N_c}} = \int_0^1 \frac{dx}{2\sqrt{x(1-x)}} \int \frac{d^2k_\perp}{(2\pi)^3} \psi^{(\lambda=0)}_{\uparrow\downarrow\mp\downarrow\uparrow}(x, \vec{k}_\perp)$$





Decay constants for vector and pseudo-scalar Bc mesons. The results are obtained with N_{max}=L_{max}=32, corresponding to UV cutoffs:

$$\Lambda_{\rm UV} \triangleq \kappa\sqrt{N_{\rm max}} \approx m_b + m_c$$

The length of error bars:

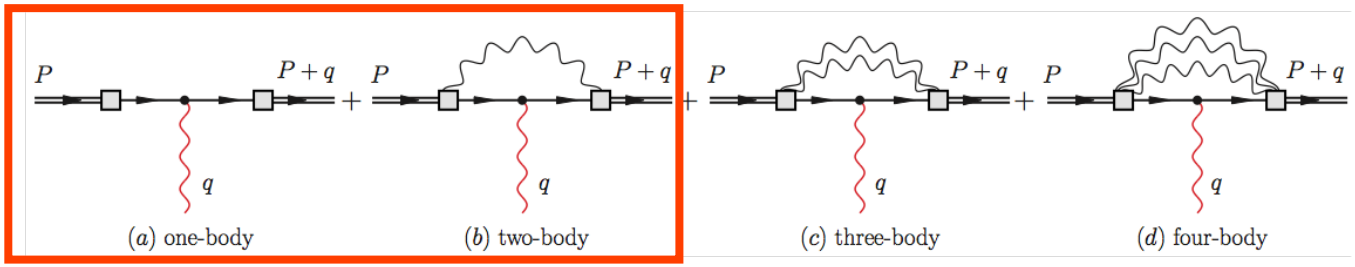
$$\Delta f = 2\left|f(N_{\rm max}=32) - f(N_{\rm max}=16)\right|$$

which is taken to indicate the sensitivity to basis truncation.

Form Factors of Electron

Shuo Tang, Y. Li, X. Zhao, J. P. Vary in preparation

Fock Sector Truncation



$$|e_{\rm phys}\rangle = c_1|e\rangle + c_2|e\gamma\rangle$$

Basis Truncation

Longitudinal: $P^+ = \sum_i p_i^+ \propto K_{\rm tot}$

Transverse: $\sum_i 2n_i + |m_i| + 1 \le N_{\rm max}$

Electro-magnetic form factors We took the "good current" ("+" component) for calculation

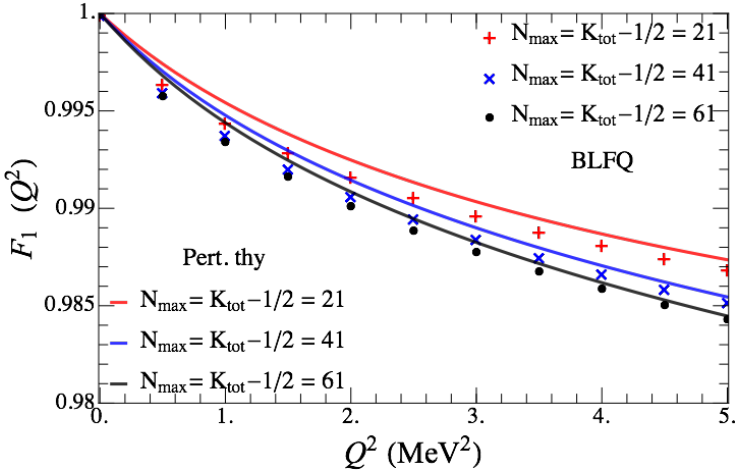
$$F_1(q^2) = \langle\Psi(P+q,\uparrow)|\frac{J^+(0)}{2P^+}|\Psi(P,\uparrow)\rangle; \qquad -(q^1-iq^2)\frac{F_2(q^2)}{2M} = \langle\Psi(P+q,\uparrow)|\frac{J^+(0)}{2P^+}|\Psi(P,\downarrow)\rangle;$$

Gravitational form factors

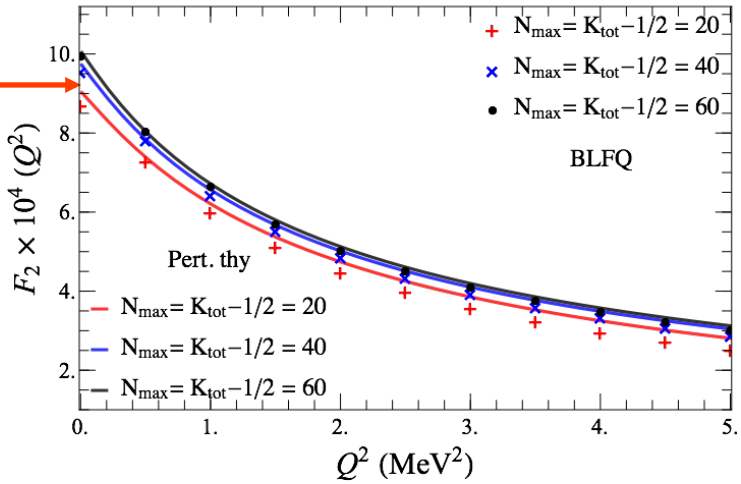
$$A(q^2) = \langle\Psi(P+q,\uparrow)|\frac{T^{++}(0)}{2(P^+)^2}|\Psi(P,\uparrow)\rangle; \qquad -(q^1-iq^2)\frac{B(q^2)}{2M} = \langle\Psi(P+q,\uparrow)|\frac{T^{++}(0)}{2(P^+)^2}|\Psi(P,\downarrow)\rangle;$$

Form Factors of Electron

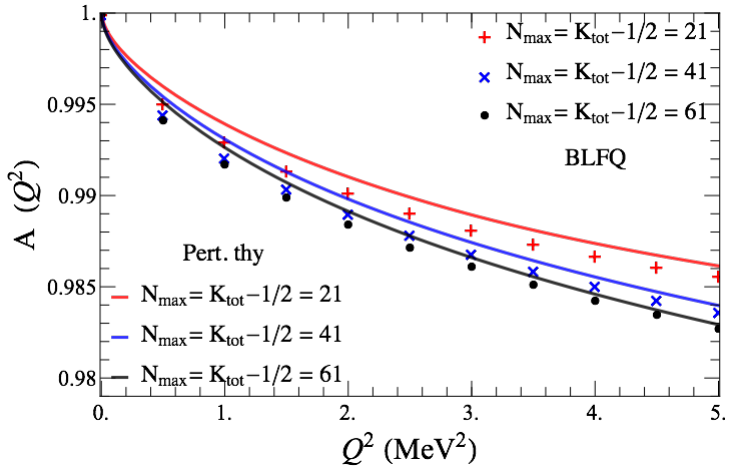
Electro-magnetic form factor



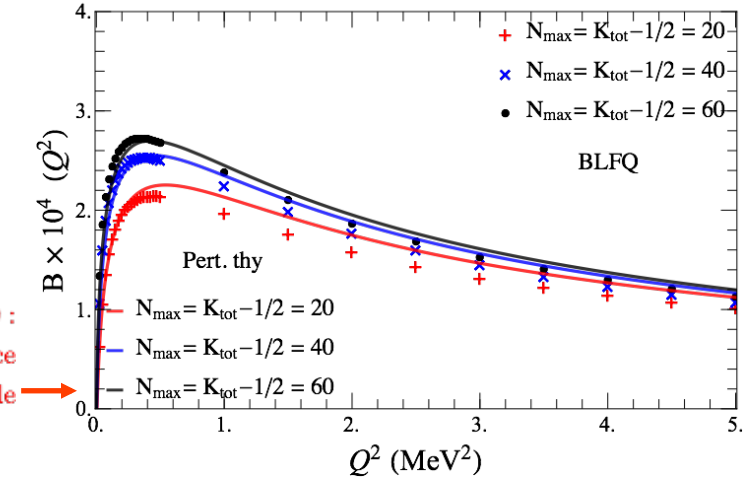
$F_2(q^2=0) \equiv$ Anomalous Magnetic Moment



Gravitational form factor



$B(q^2=0)=0$: Equivalence principle



* Form factors as functions of Q^2 obtained from BLFQ and Light Cone perturbation theory (Stanley J. Brodsky, et al.)

* In the perturbative calculation, UV and IR cutoffs are imposed on the transverse momenta to match the BLFQ basis truncation.
UV cutoff $\Lambda_k \sim b\sqrt{N_{max}}$ IR cutoff $\lambda_k \sim b/\sqrt{N_{max}}$

* We average odd and even N_{max} to reduce artifacts for F_1 and A.

Hadron Tomography

[Adhikari, et al., Phys. Rev. C 93, 055202 (2016); & in preparation]

- Generalized parton distributions (GPDs) [Ji '97 & '98]

$$H(x,\zeta,t) = \tfrac{1}{2}\int \frac{\mathrm{d}z^-}{2\pi} e^{ixP^+z^-} \langle P'|\overline{\psi}(-\tfrac{1}{2}z)\gamma^+\psi(+\tfrac{1}{2}z)|P\rangle\Big|_{z^+=z^\perp=0}$$

$q = P' - P$, $\zeta = q^+/P^+$, $t = q^2$.

  - DVCS, SIDIS, ..., spin physics

- Impact parameter dependent GPDs: [Burkardt '01]

$$q(x,\vec{b}_\perp) = \int \frac{\mathrm{d}^2\Delta_\perp}{(2\pi)^2} e^{i\vec{\Delta}_\perp\cdot\vec{b}_\perp} H(x,\zeta=0,t=-\Delta_\perp^2).$$

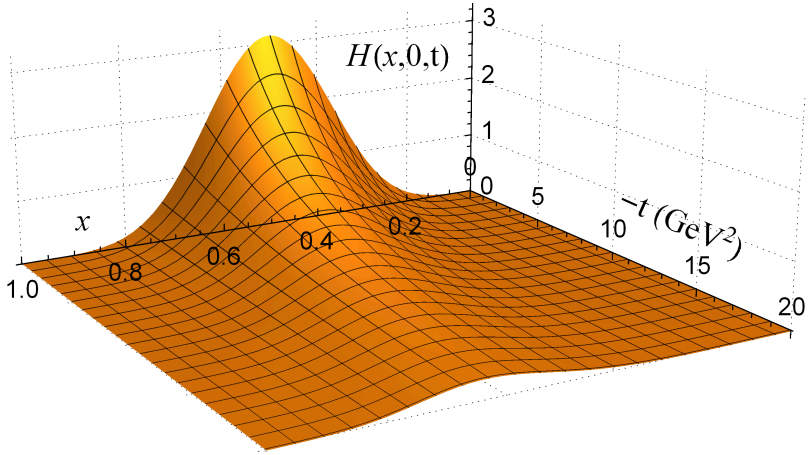
  - partonic interpretation: $\int d^2b_\perp \int_0^1 dx\, q\left(x,\vec{b}_\perp\right) = 1.$

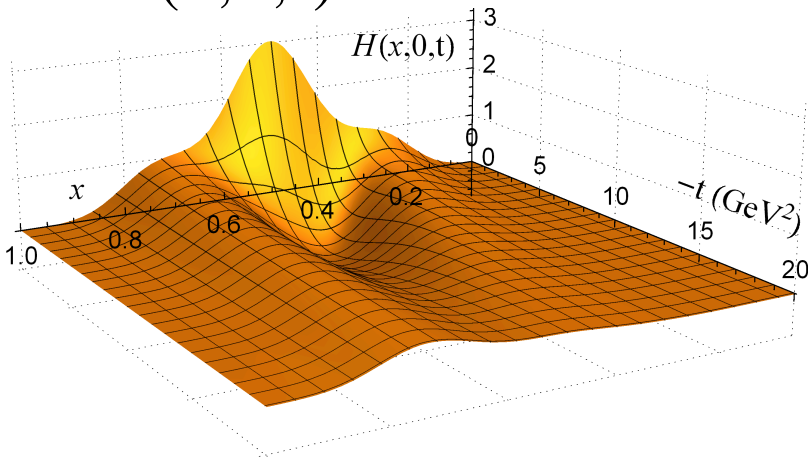
Spin non-flip GPDs $H(x,0,t)$

$$H(x,\xi=0,t=-\vec{\Delta}_\perp^2) = \sum_{\lambda_q,\lambda_{\bar{q}}} \int d^2\vec{k}_\perp\, \psi^*(\vec{k}'_\perp,x,\lambda_q,\lambda_{\bar{q}})\psi(\vec{k}_\perp,x,\lambda_q,\lambda_{\bar{q}}).$$

Charmonium

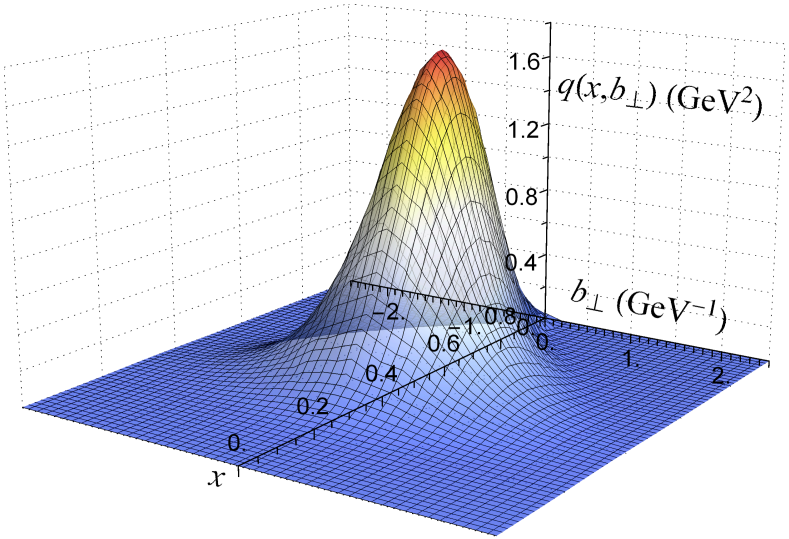
Spin non-flip GPDs $H(x,0,t)$

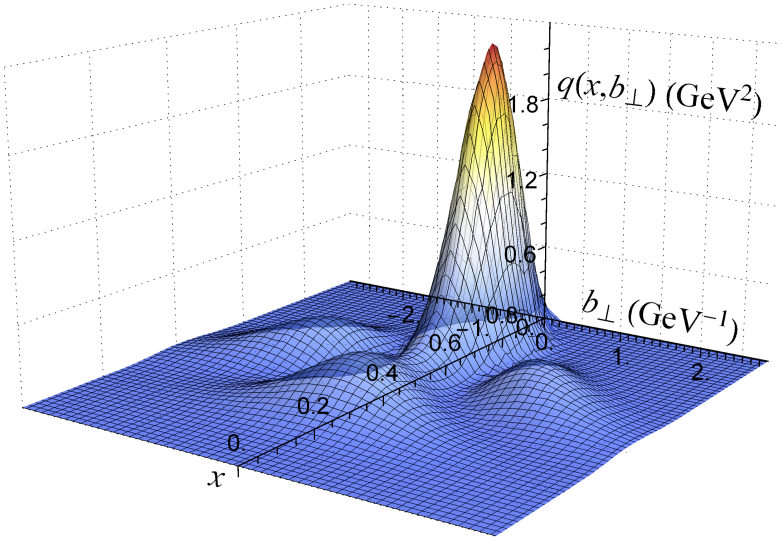




$J/\psi \;\; 1^3S_1\left(1^{--}\right)$

$\psi(2S) \;\; 2^3S_1\left(1^{--}\right)$





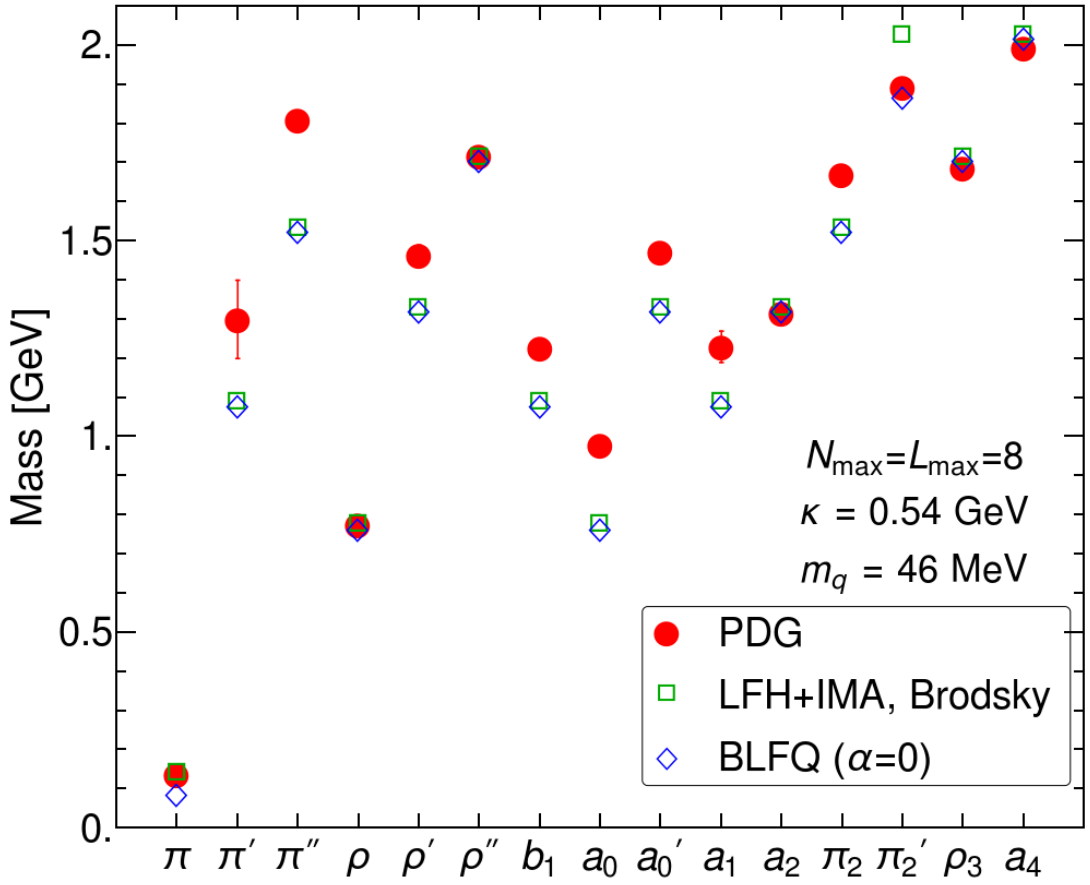
Adhikari, et al, in preparation

Light Mesons

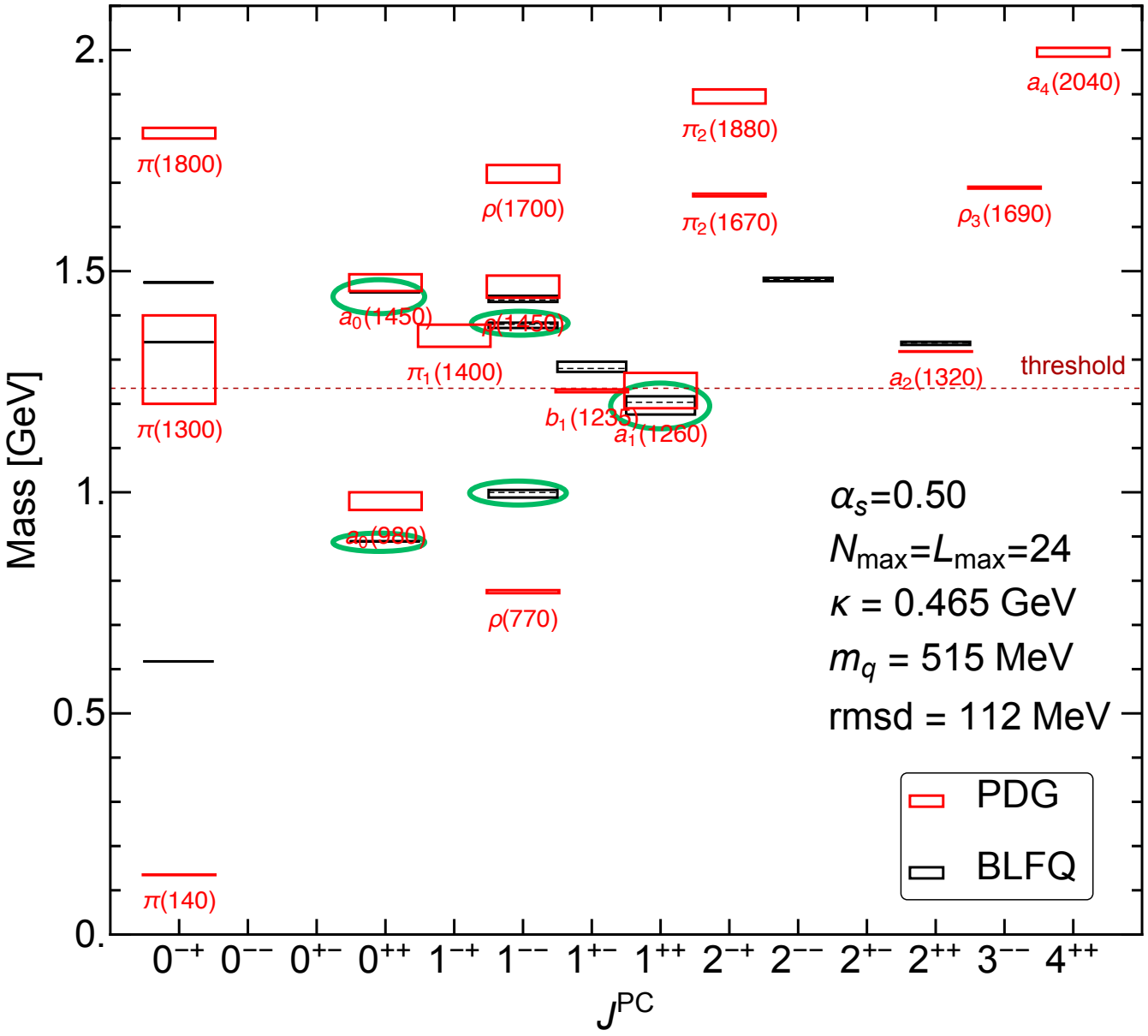
Wenyang Qian, et al., In preparation

Our focus on the light unflavored sector, with isospin = 1.

BLFQ spectrum at α=0 limit, basically Light-front Holography (LFH) + Longitudinal confinement ansatz (LCA) with no gluon exchange, is comparable to the results from LFH with invariant mass term (IMA).

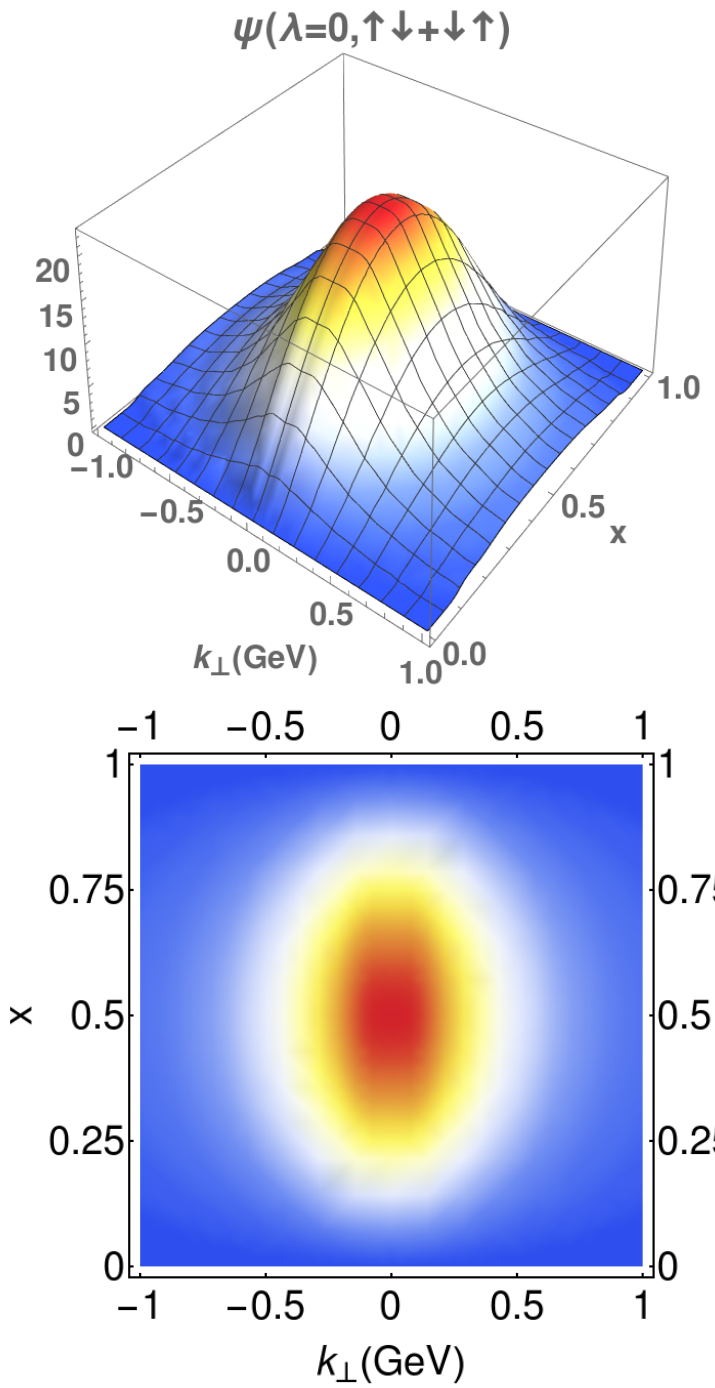


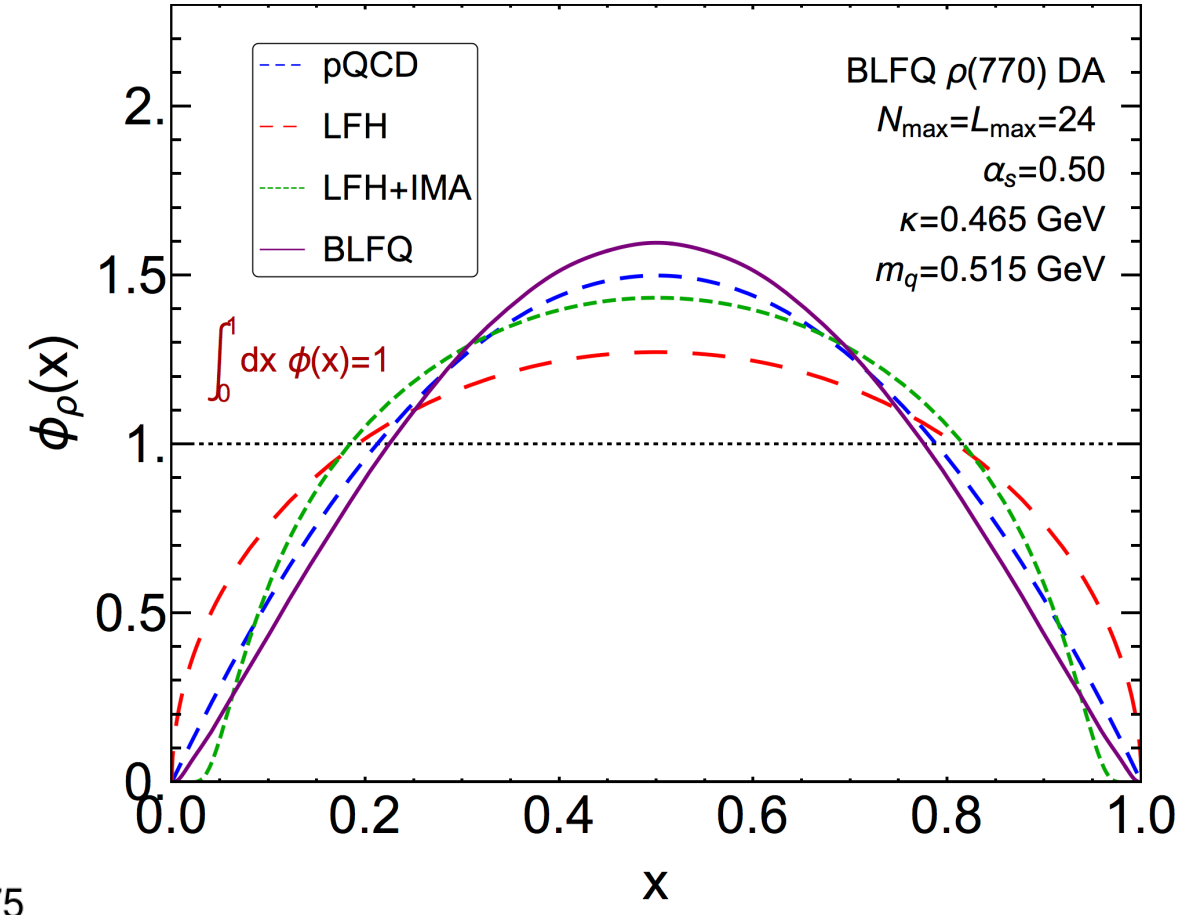
BLFQ results at α=0 limit. Differences between BLFQ and LFH+IMA results are less than one percent for the all the states, except for $\pi(130)$ and $\pi_2(1880)$.



BLFQ spectrum at realistic α_s=0.50. The values for confining strength κ and quark mass m_f are obtained by fitting five PDG experimental masses: ρ, ρ′, a_0, a_0′, a_2, indicated on the plot in green circles.

Red boxes represent the mass spread of the states identified in PDG; black boxes are the BLFQ calculations.

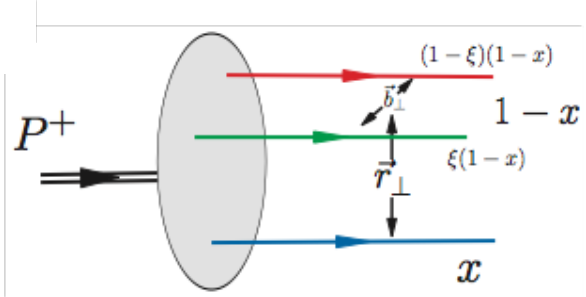




Light-front wave function (LFWF) and distribution amplitude of the ρ(770) meson. The left panels visualize the dominant spin-triplet LFWF at polarization λ=0, in momentum space. On the top right, we calculate its distribution amplitude, and compared with pQCD and AdS/QCD results.

Baryons

Anji Yu, et al., in preparation



$$\vec{k}_\perp \equiv \vec{k}_{1\perp} = \frac{x_1\vec{p}_{2\perp} - x_2\vec{p}_{1\perp}}{x_1+x_2} \qquad \vec{b}_\perp \equiv \vec{b}_{1\perp} = \vec{r}_{2\perp} - \vec{r}_{1\perp}$$

$$\vec{p}_\perp \equiv \vec{k}_{2\perp} = \vec{p}_{3\perp} - x_3\vec{P}_\perp \qquad \vec{r}_\perp \equiv \vec{b}_{2\perp} = \frac{\vec{r}_{3\perp} - \vec{R}_\perp}{1-x_3}$$

$$\vec{P}_\perp = \sum_{a=1}^{3}\vec{p}_{a\perp} \qquad \vec{R}_\perp = \sum_{a=1}^{3} x_a\vec{r}_{a\perp}$$

Effective Hamiltonian

Kinetic energy — Harmonic oscillator confining potential

$$H_{\rm eff} = \frac{\vec{p}_\perp^{\,2}+m_3^2}{x} + \frac{1}{1-x}\left[\vec{p}_\perp^{\,2} + \frac{\vec{k}_\perp^{\,2}+m_2^2}{\xi} + \frac{\vec{k}_\perp^{\,2}+m_1^2}{1-\xi}\right] + \kappa^4 x(1-x)\vec{r}_\perp^{\,2} + \kappa^4(1-x)\xi(1-\xi)\vec{b}_\perp^{\,2}$$
$$- \frac{\kappa^4}{(m_1+m_2+m_3)^2}\left[\partial_x\big(x(1-x)\partial_x\big) + \frac{1}{1-x}\partial_\xi\big(\xi(1-\xi)\partial_\xi\big)\right] + \text{const.}$$

Longitudinal confining potential

Mass eigenvalue

$$\left[M_{\rm L}^2 = (m_1+m_2)^2 + \frac{m_1+m_2}{m_1+m_2+m_3}\kappa^2(2{\rm L}+1) + \frac{\kappa^4}{(m_1+m_2+m_3)^2}{\rm L}({\rm L}+1)\right]$$

$$E_{{\rm n}_1,{\rm m}_1,{\rm n}_2,{\rm m}_2,{\rm L},{\rm l}} = (m_3+M_{\rm L})^2 + 2\kappa^2(2{\rm n}_1+|{\rm m}_1|+2{\rm n}_2+|{\rm m}_2|+2)$$
$$+ \frac{M_{\rm L}+m_3}{m_1+m_2+m_3}\kappa^2(2{\rm l}+1) + \frac{\kappa^4}{(m_1+m_2+m_3)^2}{\rm l}({\rm l}+1) + \text{const.}$$

Eigenfunction

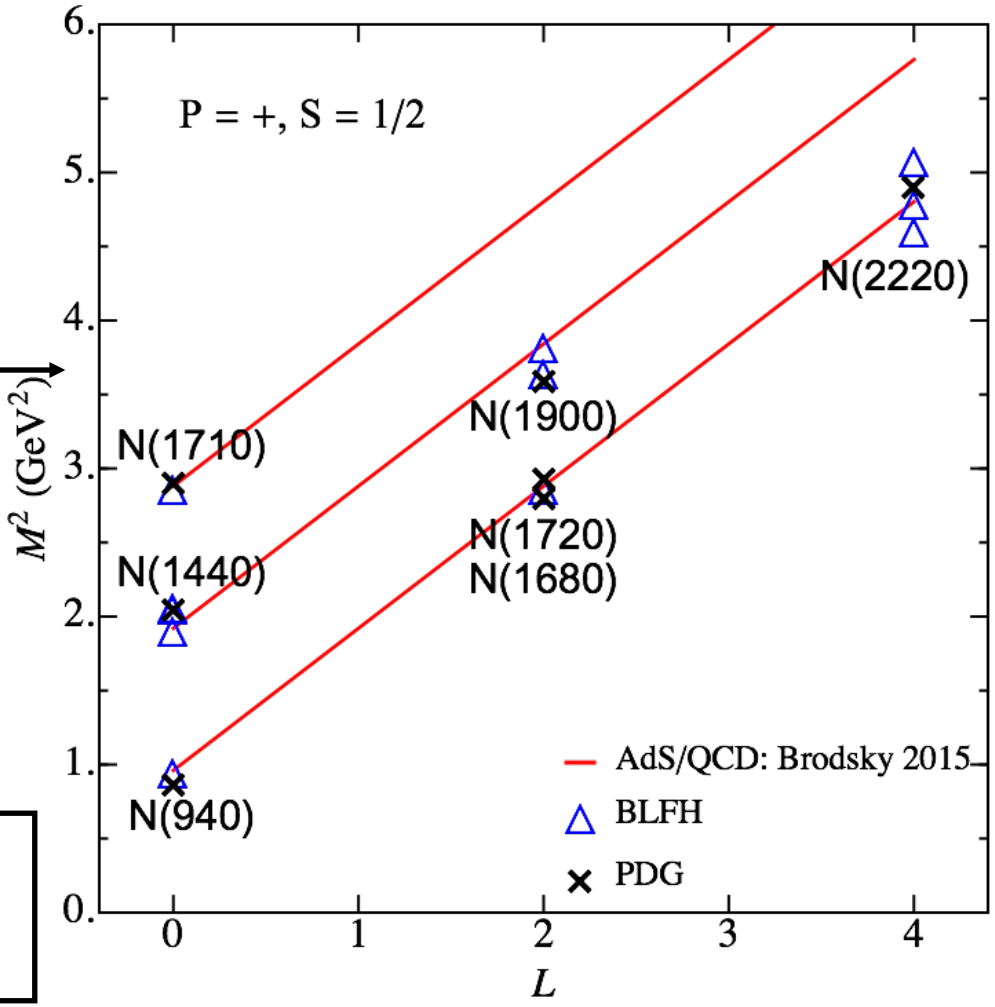
$$\Phi_{{\rm n}_1{\rm m}_1{\rm n}_2{\rm m}_2{\rm L}{\rm l}}(\vec{p}_\perp, x, \vec{k}_\perp, \xi) = \phi_{{\rm n}_1{\rm m}_1}(\vec{q}_{1\perp})\chi_{\rm L}^{(A,B)}(\xi)\phi_{{\rm n}_2{\rm m}_2}(\vec{q}_{2\perp})\chi_{\rm l}^{(\alpha_{\rm L},\beta)}(x).$$

$$A = 2m_1(m_1+m_2+m_3)/\kappa^2, B = 2m_2(m_1+m_2+m_3)/\kappa^2,$$
$$\alpha_{\rm L} = 2M_L(m_1+m_2+m_3)/\kappa^2, \beta = 2m_3(m_1+m_2+m_3)/\kappa^2$$

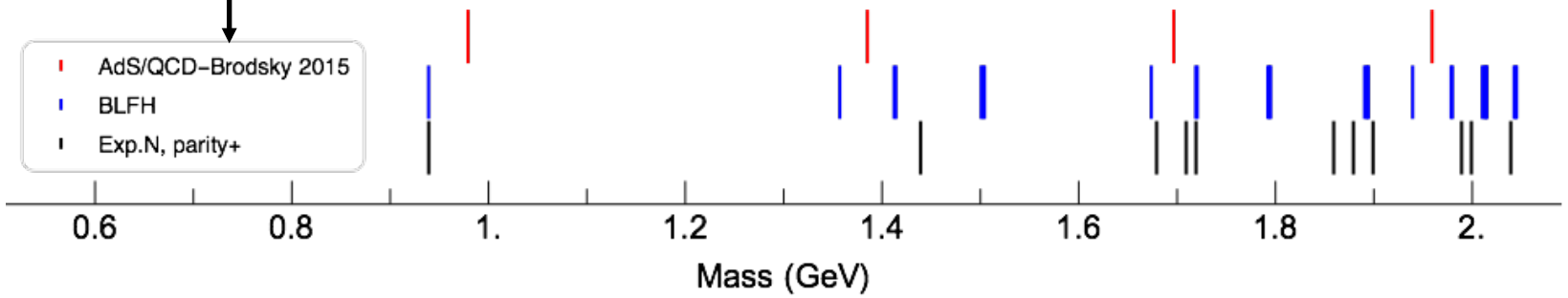
$$\vec{q}_{1\perp} \equiv \frac{\vec{k}_\perp}{\sqrt{(1-x)\xi(1-\xi)}} \qquad \vec{q}_{2\perp} \equiv \frac{\vec{p}_\perp}{\sqrt{x(1-x)}}$$

Baryons

1. All mass eigenvalues in AdS/QCD can be covered by BLFH under appropriate mass shift
2. Spin has not been assigned yet, we choose the nearest mass states to present. Great chances that we are able to find desired spin configuration since exited states are highly degenerate.



Baryon excitation spectrum of positive parity nucleons: PDG vs BLFH with even $(L+l+m_1+m_2)$, $\kappa = 0.49$ GeV, $m = 0.35$ GeV, matching the ground state value to mass of proton by mass shift.



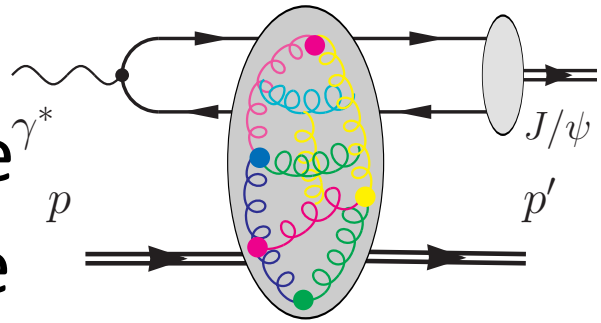
Brodsky, S. J., de Téramond, G. F., Dosch, H. G., & Erlich, J., "Light-front holographic QCD and emerging confinement", *Physics Reports*, *584*, 1 (2015)

Measuring VM LFWF

- Diffractive VM production

$$\sigma_{\rm tot} \sim \Psi_{photon} \circledast \sigma_{\rm dipole} \circledast \Psi_{VM}$$

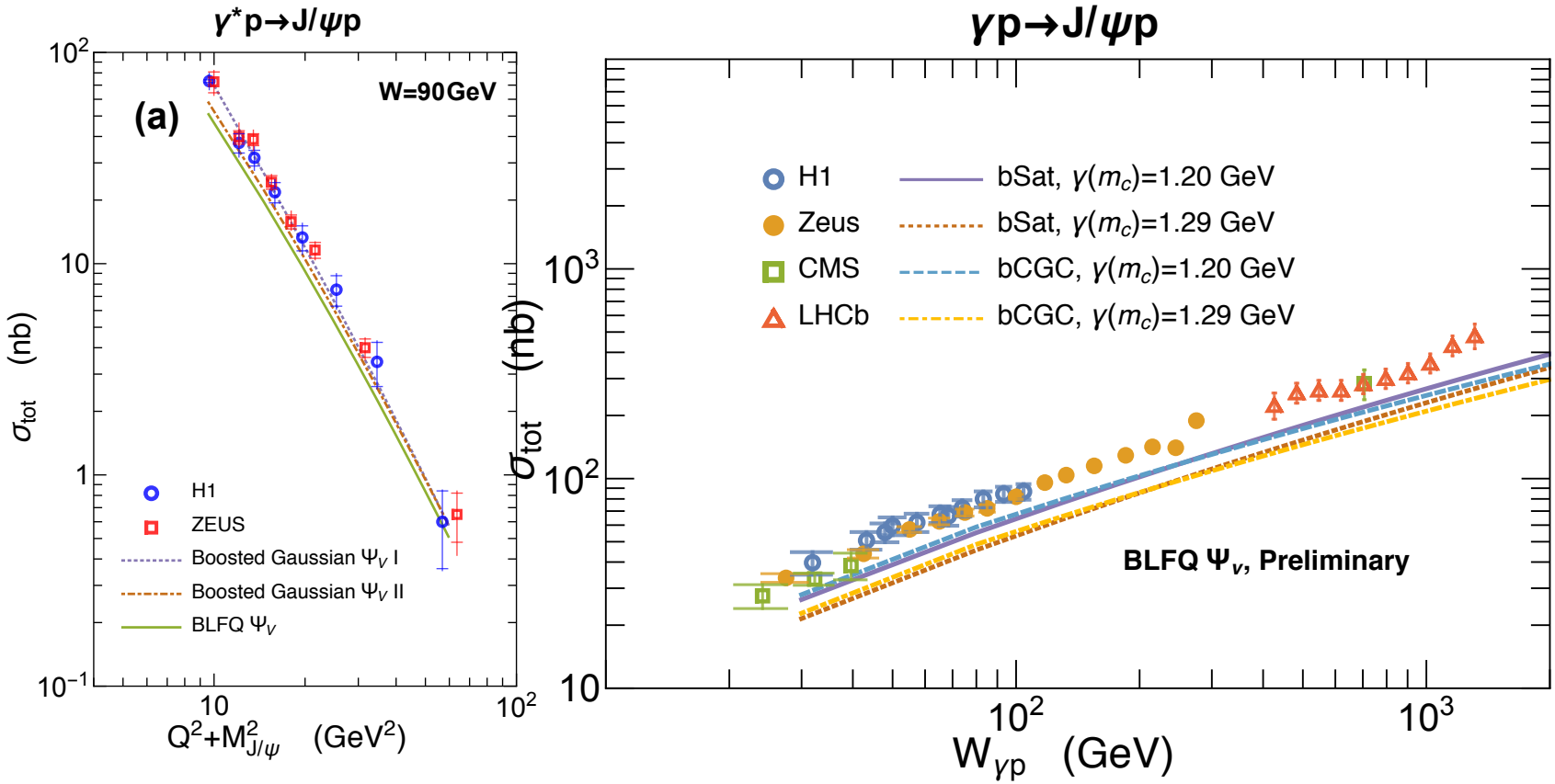
- Photon LFWF can be calculated from first principles.
- Dipole cross section can be obtained by measuring the inclusive DIS cross section.
- Provide measurements of VM LFWF and gluon distribution.



A. Mueller, '90
N. Nikolaev, '91
K. Golec-Biernat et al., '99

Confront existing data

- In agreement with HERA, RHIC and LHC data.

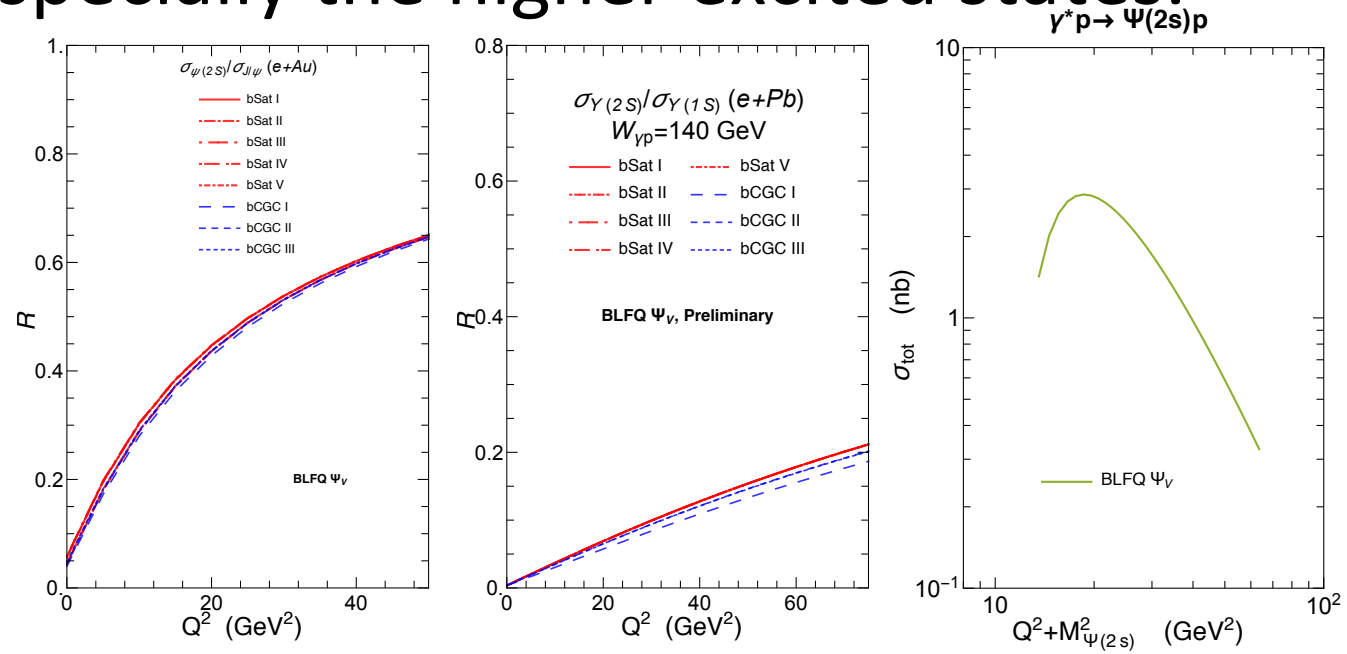


Chen, Li, Maris, Tuchin and Vary, PLB 769, 477, 2017

Prediction for future experiment

Guangyao Chen, et al., in preparation

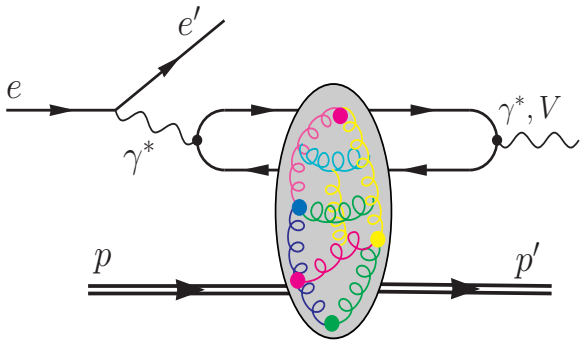
- Electron Ion Collider--high luminosity, wide kinematic range.
- Enable precision measurement of VM LFWF, especially the higher excited states.

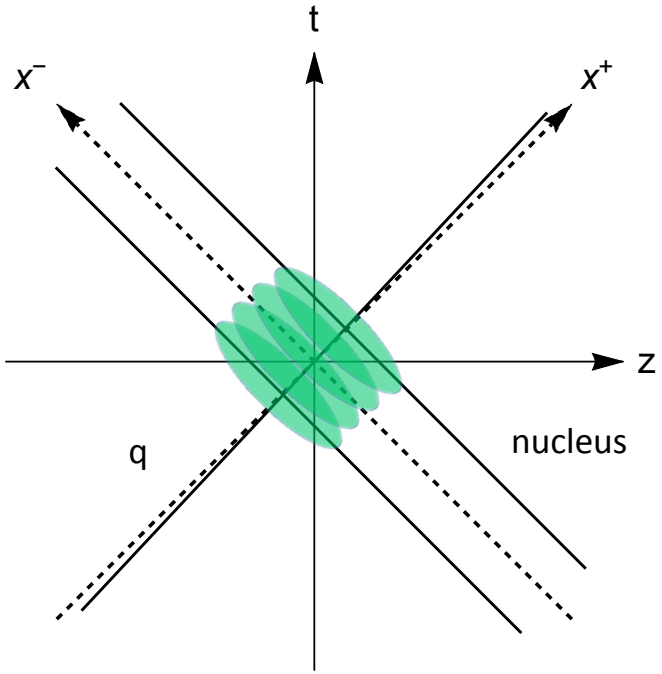


Chen, Li, Maris, Tuchin and Vary, PLB 769, 477, 2017

Quark jet scattering on Color Glass Condensate

Meijian Li, et al., in preparation





Space-time picture of qA scattering

qA scattering, an investigation inside deep inelastic scattering.

- A Hamiltonian formalism (tBLFQ) to investigate QCD dynamics
- Event-by-event study with Color Glass Condensate

$$\langle \rho_a(\vec{x}_\perp)\rho_b(\vec{y}_\perp)\rangle = \delta_{ab}\delta^2(\vec{x}_\perp - \vec{y}_\perp)g^4\mu^2$$

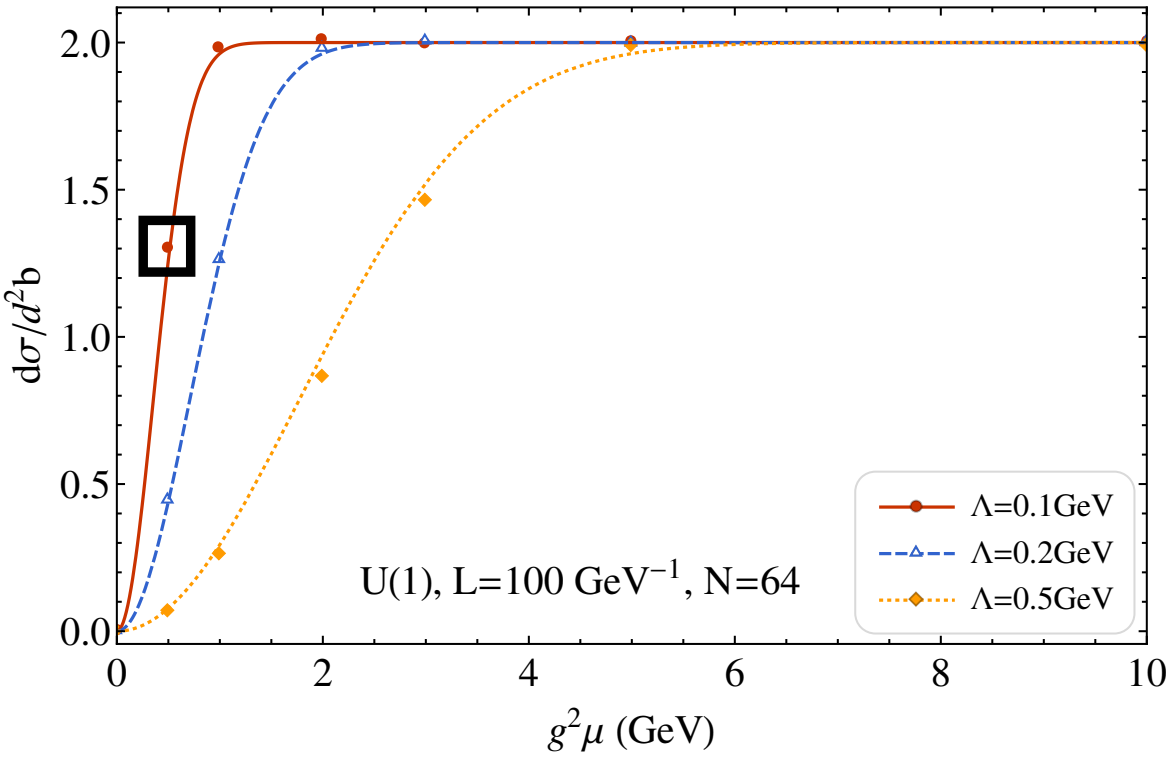
- Explicit time evolution of quark

$$|\psi, x^+\rangle_I = T_+ \exp(-\frac{i}{2}\int_0^{x^+} \mathrm{d}z^+ V_I)\,|\psi, 0\rangle_I$$

- Wavefunction provides full information of the system

$$O(x^+) =_I \left\langle \psi; x^+ \middle| \hat{O} \middle| \psi; x^+ \right\rangle_I$$

Quark jet scattering on Color Glass Condensate



Total cross section of quark jet in the U(1) case. (i) Dots are calculation from tBLFQ formalism, each averaged over 50 events. L is the transverse grid length. N is the transverse grid number. is the infrared cutoff. (ii) Curves are analytical calculation by LSZ formalism*.

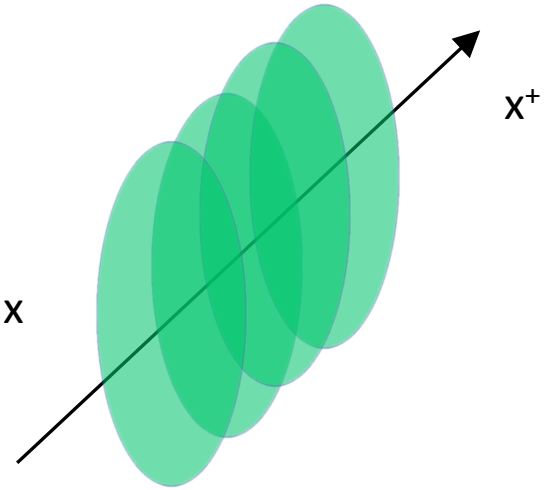
** A. Dumitru and J. Jalilian-Marian. Phys. Rev. L 89.2 (2002): 022301.*

Quark jet scattering on Color Glass Condensate

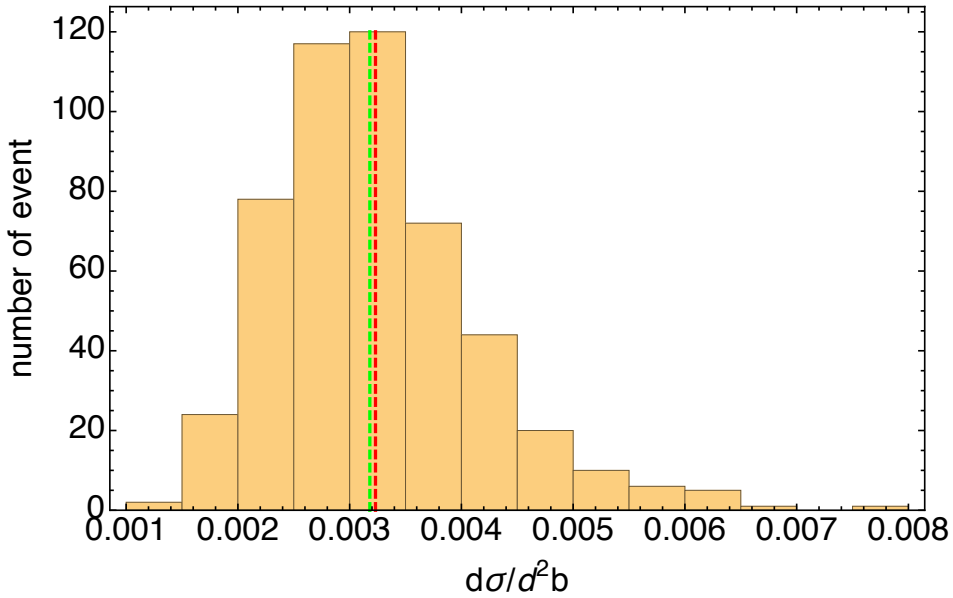
- **Time evolution by small time steps**

$$\mathcal{T}_+ \exp(-\frac{i}{2}\int_0^{x^+} \mathcal{M}) = (1-\frac{i}{2}\mathcal{M}\delta x^+)\cdots(1-\frac{i}{2}\mathcal{M}\delta x^+)$$

$\mathcal{M}_{\beta\beta'} = \langle\beta'|\, V_I \,|\beta\rangle$ is the interaction Hamiltonian matrix



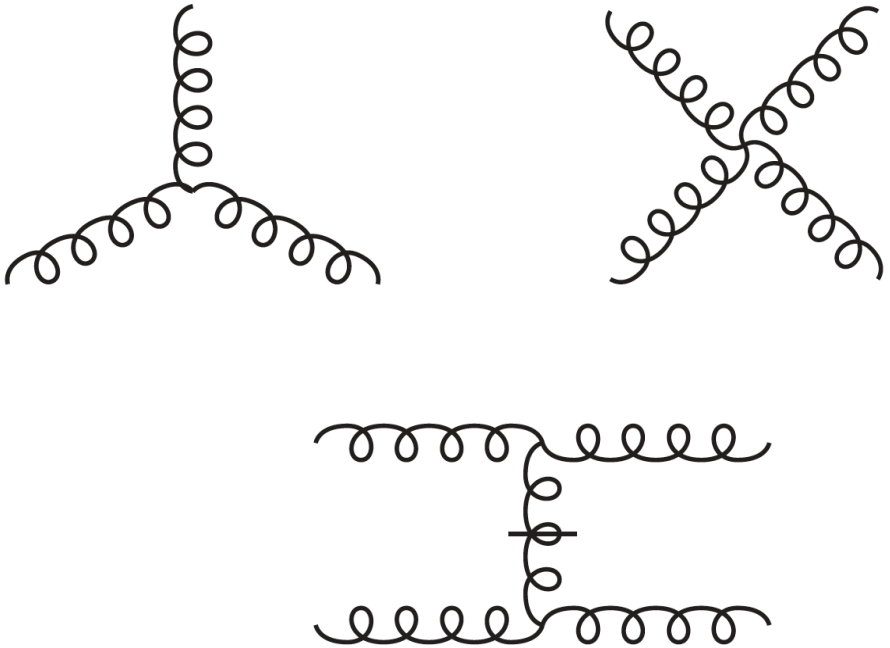
- **Event-by-event distribution**

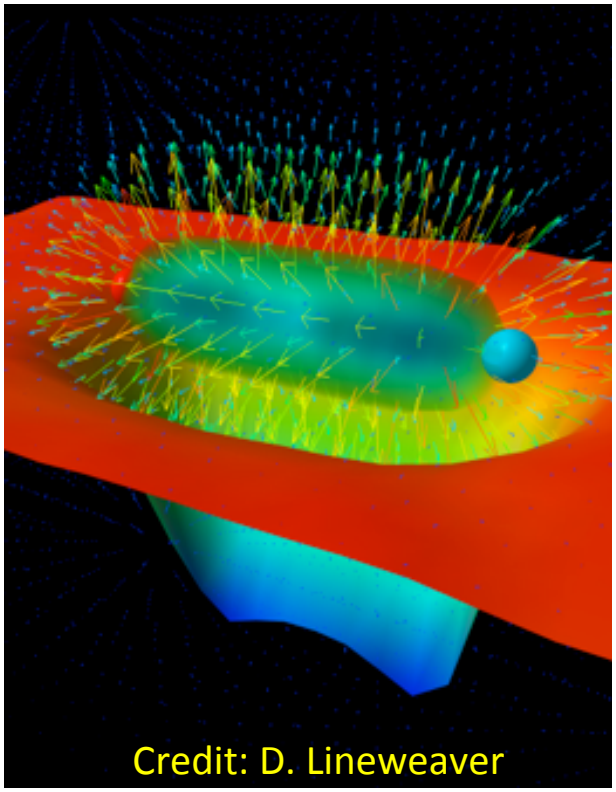


A distribution of total cross section in the U(1) case with one layer at x^+. Red line is the mean value of all 500 events.
Green line is from analytical calculation.
GeV, =0.1 GeV.

Full Basis Light-Front Quantization (FBLFQ)
UV and IR regulators but no additional Fock space truncation

Pure glue sector QCD – Glueballs?

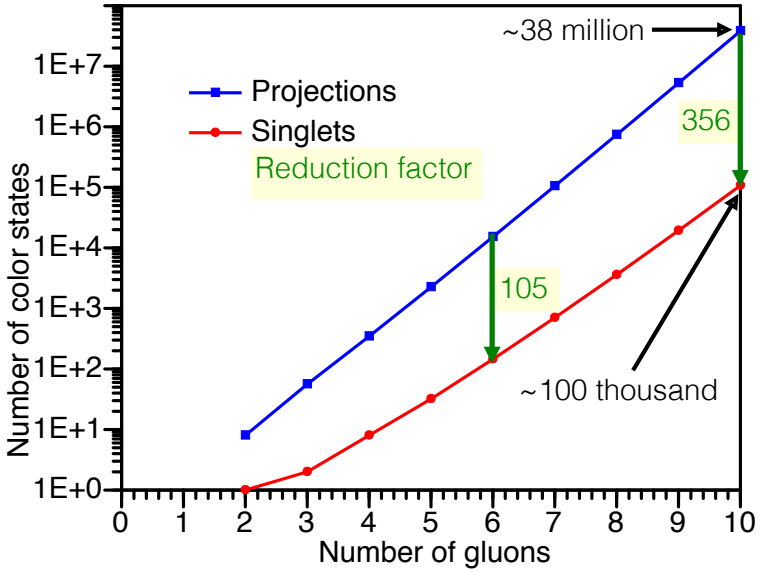




Retain only the triple gluon vertex in initial test application
PRELIMINARY RESULTS

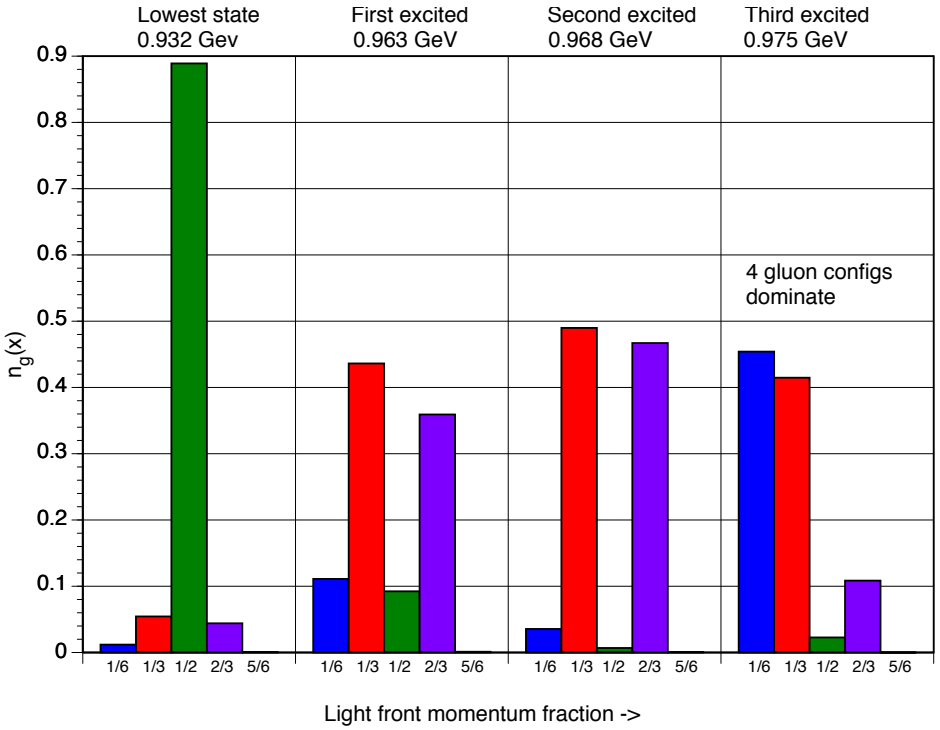
Color basis space dimensions of each multi-gluon space-spin configuration

Figure extended from Vary, et al., 2010



Distribution functions for 4 lowest mass eigenstates

$N_{max} = K = 6$
b=0.5 GeV
g_S=0.5
m_g = 0.25 GeV



Hamiltonian matrix dimension ~ 2000;
Calculation runs in 3 mins on laptop.

Next step: add 2 other vertices

Summary and Outlook

Hamiltonian Light-Front QCD is producing descriptions and predictions currently using models of confinement

- Bound states and transitions of the hadrons are described
- Time-dependent reaction applications are underway
- Future – expand the Fock spaces (e.g. F-BLFQ)
- Future – develop counterterms and remove regulators
- Future – additional methods for emergent phenomena
- Efficient utilization of supercomputing resources

LC2017 provides us with opportunities to preview/compare LF methods for solving LF QCD